Investment and Market Dominance

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First Draft: October, 1999.
This Draft: August, 2000.

Abstract: This paper analyzes a model of oligopolistic competition with ongoing investment. It incorporates the following models as special cases: incremental investment, patent races, learning-by-doing, and network externalities. We investigate circumstances under which a …rm with low costs or high quality will extend its initial lead through further cost-reducing or quality-improving investments. In many commonly-studied oligopoly games, such investments are strategic substitutes. We derive a new comparative statics result that applies to games with strategic substitutes, and we use the result to derive conditions under which leading …rms invest more than lagging …rms. Finally, we highlight plausible countervailing e¤ects that arise when investments of leading …rms are less e¤ective than lagging …rms, or in dynamic games when …rms are su¢ciently patient.

JEL Classi…cation Numbers: L11, L13, L41, O30.

Keywords: Oligopoly games, strategic substitutes, innovation, investment, increasing dominance, market concentration.

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We are grateful to the co-editor, Joe Harrington, two anonymous referees, as well as Rabah Amir, Zava Aydemir, Kyle Bagwell, Abhijit Banerjee, Men-Andri Benz, Stefan Bühler, Luis Cabral, Thomas Gehrig, Bob Gibbons, Ken Judd, Aija Leiponen, Paul Milgrom, Wally Mullin, Peter Neary, Andreas Polk, Mike Riordan, and seminar audiences at the 8th World Congress of the Econometric Society (Seattle), CIRANO, Freiburg, MIT, Munich, St. Gallen, Torino (EARIE) and Zürich for helpful discussions. Thomas Borek provided excellent research assistance. Athey thanks the NSF (SBR-9631760 and SES-9983820) for …nancial support.

1 Introduction

This paper studies the incentives of …rms to make cost-reducing and demand-enhancing investments, such as investments in product and process innovation. We compare the investment incentives of leading …rms with those of lagging …rms, and we use our results to make predictions about the dynamics of market structure. We examine conditions under which weak increasing dominance emerges, whereby leading …rms invest more into improving their state. We also identify conditions that imply strong increasing dominance, whereby leading …rms increase their market share. To accomplish this goal, we introduce a new comparative statics result, based on the theory of supermodular games, that applies to multi-player games with strategic substitutes.

The empirical evidence about the dynamics of concentration illustrates that a variety of phenomena are possible. Increasing dominance of the market leader is common in markets characterized by network e¤ects or learning-by-doing,[1] industries where …rms engage in process or product innovation,[2] and advertising-intensive industries.[3] On the other hand, even in such settings, lagging …rms sometimes undertake large investments and capture the lead.[4]

To provide insight about these patterns, the paper identi…es scenarios where increasing dominance is likely to emerge, while highlighting the potential for competing forces. A variety of speci…c models have been used in the literature to analyze increasing dominance;[5]

[1] Network e¤ects have been widely cited as the force that led VHS to take over the market for videocassette recorders in the 1980s; more recently, policymakers have focused on the role of network e¤ects in computer operating systems and applications. Seminal models of network e¤ects include Katz and Shapiro (1986) and Farrell and Saloner (1986). Learning-by-doing has been analyzed by Cabral and Riordan (1994), who provide a number of examples.

[2] For example, the literature on the product life cycle (e.g. Klepper, 1996) emphasizes concentration increases in mature markets because some …rms manage to reduce production costs faster than others. Bagwell and Ramey (1994) argue that in the retail sector, leading …rms (e.g. Wal-Mart) made a variety of cost-reducing investments as they gained in market share.

[3] See Sutton (1991) or Bagwell and Ramey (1994).

[4] Gruber's (1994) account of the semiconductor industry highlights a range of dynamic patterns within and across generations of memory chips. IBM lost its early lead in the mainframe computer market to market entrants in the 1960's and 1970's (see Sutton 1998, ch.15), in spite of strong network e¤ects. The aircraft industry is another example for a technology-intensive industry characterized by technological leapfrogging between a small number of …rms, resulting in leadership reversals and signi…cant shifts in market shares.

[5] For example, Flaherty (1980) …nds increasing dominance in an incremental investment model where …rms make cost-reducing investments prior to Cournot competition. Klepper (1996) …nds increasing dominance in mature phases of the product life cycle. In Bagwell et al. (1997), …rms use low prices today to signal

in this paper, we seek develop a general theoretical model that incorporates many of these as special cases.[6;7] In our model, two or more oligopolists compete over time, and …rms may invest in each period. In some cases, investment precedes product market competition; for example, …rms may invest in product or process innovation, which may either result in small incremental improvements relative to the earlier state, or else in major breakthroughs (as in patent races). We also allow for each …rm to make multiple, complementary investments, for example, in both product and process innovation. In other cases, investment is interpreted as a product market choice that a¤ects the future state of the …rm; for example, in a learning-by-doing or network externality setting, …rms "invest" by choosing a level of output that exceeds the level that maximizes pro…ts in a single period. Firms might also lower prices in order to acquire loyal customers.

Consider …rst the case where investment precedes product market competition. In a simple example, …rms play a two-stage game in each period, …rst investing in cost reduction and then competing in the product market. The state of the …rm is the cumulative extent of cost reduction from some reference level. We show that in many models, a …rm's equilibrium demand is decreasing in the opponent's state of cost reduction, and increasing in the …rm's own state of cost reduction. The equilibrium markup behaves similarly. If these forces dominate (as in a model where the equilibrium demand and the markup are linear), then the following conditions will hold: (i) investments are strategic substitutes; (ii) the greater the opponent's state of cost reduction, the lower are the incentives of a given …rm to invest; and (iii) the greater is a …rm's own state of cost reduction, the greater are the incentives to invest. This logic holds irrespective of whether product market choices are strategic substitutes or complements. Thus, a wide variety of commonly-used models satisfy conditions (i)-(iii), including Bertrand and Cournot competition with di¤erentiated goods and linear demand,

that they have low costs and will also have low prices in the future. Cabral and Riordan (1994) study learning-by-doing in a duopoly model.

[6]Budd, Harris, and Vickers (1993) also seek to uncover general properties of …rm dynamics. They consider a continuous-time game between two …rms, where each …rm's pro…ts depends on di¤erence between the two …rms' state variables. They link the slope of the pro…t functions to investments. However, they do not identify primitive conditions on oligopoly models that lead to increasing dominance, and the results do not immediately extend to more than two …rms. See also Ericson and Pakes (1995), who propose a dynamic model of strategic investment, entry, and exit.

[7]Cabral (1999) proposes an alternative theory, based on the insight that leaders have an incentive to undertake R&D investment with returns that are correlated with the laggard's, while laggards desire an opportunity to leapfrog, which can be accomplished using independent investments.

as well as many horizontal and vertical quality differentiation models.

Although it may seem intuitive that forces (i)-(iii) would favor weak increasing dominance, such a result is not immediate when there are more than two firms. In contrast to games with strategic complementarities (Topkis (1979), Milgrom and Roberts (1990, 1994), Vives (1990a)), where all choices are mutually reinforcing, multi-player games with strategic substitutes incorporate effects that can complicate comparative statics analysis.[8] Much of the existing literature restricts attention to two-player games for just this reason.[9] In this paper, we identify an alternative assumption: the firms' profit functions must be "exchangeable." That is, they must satisfy a certain kind of symmetry, whereby all differences among firms are summarized by the state variables. Our result represents an extension of the comparative statics literature, and it may also be applied to other games with strategic substitutes (such as tournaments or games of strategic trade policy).

Using this new result, we show that when firms are myopic, or when they commit to investment plans in advance, forces (i)-(iii) lead to weak increasing dominance, so long as (iv) investment costs are not too much higher for leading firms. Further, in the special case where each firm's equilibrium demand and markup are linear in the state variables, weak increasing dominance implies strong increasing dominance (leading firms increase their market shares) if market demand does not grow when each firm's state variable increases by the same amount.

We then enrich the study of dynamics to allow for Markov-perfect equilibria, where each firm's investment strategy is conditioned on the current-period's state variables. In a benchmark case, with only two firms and quadratic payoff functions, forces (i)-(iv) lead to weak increasing dominance. More generally, we identify additional effects that may work for or against increasing dominance, so that increasing dominance may be overturned if firms are sufficiently forward-looking.

We argued above that when investments precede product market competition, it is common to find that investments are strategic substitutes. However, in cases where a firm's

[8] Observe that with strategic substitutes, an increase in player 1's action has direct negative effects on all opponents; but the resulting decrease in player 2's action might lead to an even larger increase in player 3's action, or vice versa.

[9] Novshek (1985) takes this approach when analyzing Cournot oligopoly. Amir (1996) and Davis (1999) use the tools of supermodular games to analyze Cournot oligopoly with more than two firms; but, unlike us, they maintain the assumption that each firm cares only about the sum of opponent output.

investment is a product market choice that affects the future (i.e. learning-by-doing), investments may be strategic complements, as when firms compete in prices; then, a different approach is required. If forces (ii)-(iv) remain, existing comparative statics results for games with strategic complementarities do not immediately imply weak increasing dominance.[10] Nevertheless, if payoffs are exchangeable and each firm's payoff is quasi-concave in its own investment, weak increasing dominance holds for myopic firms if (i') investments are strategic complements, and conditions (ii)-(iv) hold.

Finally, we apply our framework to a variety of examples. We analyze incremental investment, patent races, learning-by-doing, and switching costs, identifying forces that support and oppose increasing dominance. In some models, competing effects arise when the investments of leading firms are less effective. For example, in models of learning-by-doing or incremental innovation, leading firms may eventually exhaust the opportunities for learning or improvement. In models of radical innovation, such as patent races, lagging firms may see a larger improvement from adopting an innovation.

We proceed as follows. Section 2 introduces the model. Section 3 contains the main results. Section 4 studies applications. Section 5 concludes.

2 The Model

This section first specifies a model of dynamic oligopoly with investment.[11] We then introduce two maintained assumptions, and finally analyze properties of product market profits that will be relevant for our analysis of increasing dominance.

[10] To see why, notice that forces (ii)-(iv) imply that an increase in firm 1's state variable has the direct effect of increasing the investment of firm 1 and decreasing those of all opposing firms. Yet, strategic complementarity implies an indirect effect, where the increase in firm 1's investment increases the returns to investment for all other firms.

[11] In many ways, our model is similar to the framework for empirical work proposed by Ericson and Pakes (1995). They also specify a general dynamic model where firms make investments, product market competition is represented in reduced form, and differences among firms are summarized by state variables. A difference is that they focus on entry and exit; although entry and exit can be incorporated in our model, we do not consider it explicitly.

2.1 Setup

There are T periods, $t = 1, ..., T$ $(T \leq \infty)$, and I firms, $i = 1, ..., I$. In each period t, firm i is characterized by a state variable $Y_i^t \in \mathcal{Y}_i$, where $\mathcal{Y}_i$ is a partially ordered subset of $\mathbb{R}^n$, typically $\mathbb{R}$.[12] We assume that $Y_i^t \geq Y_j^t$ implies that firm i has greater market share than firm j in period t. As such, the state variable might represent the cumulative cost reduction achieved by a firm, product quality or stock of loyal customers, the number of product variants offered by a firm, or, a combination of demand and cost parameters.

Let $Y^t = (Y_1^t, ..., Y_I^t) \in \mathcal{Y} = \times_i \mathcal{Y}_i$. The initial state of the market, $Y^0 = (Y_1^0, ..., Y_I^0)$, is exogenously given. Given Y^{t-1}, in period t each firm chooses an action or "investment" variable $a_i^t \in A_i$. Let $a^t = (a_1^t, ..., a_I^t) \in A = \times_i A_i$. The state vector of firm i develops according to $Y_i^t = Y_i^{t-1} + y_i^t$, and we let $y^t = (y_1^t, ..., y_I^t)$. In the simplest case, $a_i^t = y_i^t$. However, in a model of learning-by-doing or network externalities, y_i^t might be a nonlinear function $h(a_i^t, Y_i^{t-1})$.[13] The payoff to firm i in period t, given (a^t, Y^{t-1}), is

$$\Pi^i(a^t, Y^{t-1}) = \pi^i(a^t, Y^{t-1}) - k(a_i^t, Y_i^{t-1}), \tag{1}$$

where k is an investment cost function.

The interpretation of the function $\pi^i(a^t, Y^{t-1})$ depends on the application. First, consider the Investment Simultaneous with Competition case, or ISC. In this case, π^i is the profit of firm i in the product market, and a_i^t represents firm i's choice of price or quantity. For example, in a learning-by-doing model, a_i^t might represent firm i's choice of output, and Y_i^{t-1} is the sum of output in previous periods. Higher actions today lead to lower costs tomorrow; this can be captured in the properties of h.

A second interpretation arises from the case where Investment Precedes Competition, or IPC. In this case, firms play a two-stage game in each period. In the first stage, each firm i chooses a_i^t, and bears cost $k(a_i^t, Y_i^{t-1})$. These choices determine y^t. In the second stage, product market competition takes place. We do not explicitly model this stage, but instead assume that an equilibrium to the product market competition game exists, and the firms always select an equilibrium that yields profit to firm i of $\tilde{\pi}^i(Y^t)$. Thus,

[12] Throughout the paper, we will order vectors using the standard, componentwise order: for $x, x' \in \mathbb{R}^n$, $x \geq x'$ if $x_i \geq x_i'$ for $i = 1, ..., n$.

[13] As we show in the working paper, the model and some of the main results can be generalized to the case where y^t is the realization of a random variable $\tilde{y}^t$, whose distribution depends on a^t and Y^{t-1}.

$\pi^i(a^t; Y^{t-1}) = \bar{\pi}^i(Y^{t-1} + h(a^t; Y^{t-1})).$ The IPC case simplifies further when $y_i^t = a_i^t$; so that $\pi^i(a^t; Y^{t-1}) = \bar{\pi}^i(Y^{t-1} + a^t).$ We refer to this model as the incremental investment model. The incremental investment model will receive special attention throughout the paper.

It will often be convenient to represent product market profits, $\bar{\pi}^i$; as the product of equilibrium demand for firm i, denoted $D^i : \mathcal{Y} \to \mathbb{R}$; and the price-cost difference (the "markup"), denoted $M^i : \mathcal{Y} \to \mathbb{R}$, so that $\bar{\pi}^i(Y) = D^i(Y) \cdot M^i(Y).$

2.2 Maintained Assumptions

To keep the exposition concise, we assume that all of the relevant functions are differentiable, though our main results do not rely on that assumption. Throughout the paper, we will use subscripts to note partial derivatives. Further, we maintain two critical assumptions. The first requires a definition.

Definition 1 The set of equilibria of the game satisfies **conditional uniqueness** if in each period t, for each $i; j$ and each Y^t; whenever $Y_i^t \neq Y_j^t$ and there exist two vectors of equilibrium actions, $a^*(Y^t)$ and $\tilde{a}^*(Y^t)$; such that $a^*_{-ij}(Y^t) = \tilde{a}^*_{-ij}(Y^t)$; then $a_i^*(Y^t) = \tilde{a}_i^*(Y^t)$ and $a_j^*(Y^t) = \tilde{a}_j^*(Y^t)$.

In words, conditional on firms $k \neq i; j$ playing equilibrium actions $a^*_{-ij}(Y^t)$; if firms i and j have different state variables, there is a unique equilibrium. We can then state:

The set of equilibria is nonempty for each Y^t; and conditional uniqueness holds. (UNQ)

Observe that (UNQ) is considerably weaker than an assumption that there is a unique equilibrium of the I-player game.[14] In particular, (UNQ) allows that the set $\{(a; Y) : a \in a^*(Y)\}$ has dimension $2(I - 2)$: For example, if $I = 3$; (UNQ) allows for a set of equilibria satisfying $a_1/a_2 = Y_1/Y_2$ and $a_2/a_3 = Y_2/Y_3$.[15]

[14] To place (UNQ) in the context of familiar "dominant-diagonal" conditions (see, for example, Tirole (1988), p. 226), if each firm's objective function is globally concave, a sufficient condition for conditional uniqueness is that $\left|\pi^i_{a_i;a_i}\right| > \left|\pi^i_{a_i;a_j}\right|$ for $i; j = 1; :::; I;\ i \neq j$. In contrast, the sufficient condition for uniqueness in the overall game is that for all i; $\left|\pi^i_{a_i;a_i}\right| > \sum_{i \neq j}\left|\pi^i_{a_i;a_j}\right|$: Of course, neither of these conditions are necessary, but the comparison is suggestive.

[15] See also the literature on existence and uniqueness in Cournot quantity games (e.g. Novshek (1985), Amir (1996), Davis (1999)). Each of these papers considers models where firms care only about the sum of opponent outputs. Vives (2000) provides conditions for uniqueness that are weaker than dominant-diagonal conditions (p. 47-48), and summarizes results on existence (p. 43).

Next, define a map $T_{jk} : R^n \to R^n$ that transposes two elements of a vector: Formally, if $\tilde{x} = T_{jk}(x)$; then $\tilde{x}_j = x_k$; $\tilde{x}_k = x_j$; and for all $i \neq j, k$; $\tilde{x}_i = x_i$. Then we have:

Definition 2 Consider a set of I functions, $f^i : \times_i X_i \to R$ for i=1,..,I. The functions are *exchangeable* if for all $i, j, k \in \{1, \ldots, I\}$ such that $i \neq j \neq k \neq i$, the following three conditions hold: $X_i = X_j$; $f^i(x) = f^j(T_{ij}(x))$; $f^i(x) = f^i(T_{jk}(x))$:

We maintain the following assumption:

$$\text{The firms' profit functions are exchangeable as functions of } (a^t, Y^{t-1}). \qquad \text{(EXCH)}$$

Exchangeability requires a kind of symmetry in the identities of firms: each firm i cares only about the actions and state variables of its opponents, but not about the match between an opponent's identity and actions/state variables. It implies that firm i's profits are the same as firm j's profits would be if firm j was in firm i's situation. Further, firm i's profits are unchanged if the actions and state variables of two opponents are exchanged. Thus, all differences among firms are summarized in the state variables.[16]

In general, the exchangeability assumption is consistent with models of Cournot oligopoly, vertical product differentiation, and differentiated product models where the cross-price effects are identical for all firms. For a simple example where (EXCH) fails, consider horizontally differentiated firms on a Hotelling line, with negative marginal costs as the state variable. The effect of an increase in firm j's cost on firm i's profit will depend on whether firm j is a near neighbor or a distant firm. Hence, for $I > 2$ firms, exchangeability will not hold, and for $I = 2$ firms, exchangeability holds only if firms are restricted to locate symmetrically around the midpoint of the interval.[17]

2.3 Properties of Profit Functions in the IPC Model

In this section, we argue that the following condition emerges in a range of oligopoly models:

$$\pi^i_{Y_i;Y_i} \geq 0 \text{ and } \pi^i_{Y_i;Y_j} \leq 0: \qquad \text{(WID-II)}$$

[16] Exchangeability is closely related to the concept of anonymity, as used in cooperative game theory and social choice theory (see, for example, Moulin (1988)).

[17] However, exchangeability does hold if state variables are assumed to be two-dimensional, described by costs and location.

Below, we show that (WID-II) creates a force in favor of weak increasing dominance. For the moment, we focus on the IPC case, where the analysis is particularly clear, and return to the ISC case in Section 4. Recall our decomposition $\pi^i(Y) = D^i(Y) \cdot M^i(Y)$, and observe that

$$\pi^i_{Y_i,Y_i} = 2D^i_{Y_i}M^i_{Y_i} + M^i D^i_{Y_iY_i} + D^i M^i_{Y_iY_i}. \tag{2}$$

Given that by assumption, a firm's market share is increasing in its state variable, we also expect that a firm's equilibrium demand is increasing in its state variable ($D^i_{Y_i} \geq 0$) in many applications. Similarly, a firm's equilibrium markup is increasing in its state variable ($M^i_{Y_i} \geq 0$) in many applications, for example when a higher Y_i corresponds to lower costs or higher quality. If so, then $D^i_{Y_i}M^i_{Y_i} \geq 0$. Intuitively, the positive effect on the markup that stems from, say, reduced marginal costs is enhanced by the positive effect of cost-reduction on demand. So long as neither markup nor demand is extremely concave in the own state variable, this effect dominates. Similarly, observe that:

$$\pi^i_{Y_i,Y_j} = D^i_{Y_i}M^i_{Y_j} + D^i_{Y_j}M^i_{Y_i} + M^i D^i_{Y_iY_j} + D^i M^i_{Y_iY_j} \tag{3}$$

In many applications, improvements in the opponent's state variable are bad for market share and the markup: $D^i_{Y_j} \leq 0$ and $M^i_{Y_j} \leq 0$. Then, the negative effect of a competitor's improvement on a firm's own markup has a greater impact on profits the higher one's own state, and hence the higher the firm's own demand ($D^i_{Y_i}M^i_{Y_j} \leq 0$). Similarly, the positive effect of a firm's state on its own markup has a greater impact on profits the lower the competitor's state and hence the higher one's demand ($D^i_{Y_j}M^i_{Y_i} \leq 0$). Unless the last two terms of (3) are large and positive, the first two effects dominate. This discussion implies:

Lemma 1 Suppose that D^i and M^i are linear functions of Y, and that $D^i_{Y_i} \geq 0$, $M^i_{Y_i} \geq 0$, $D^i_{Y_j} \leq 0$, and $M^i_{Y_j} \leq 0$. Then (WID-II) holds.

Clearly, the linearity assumption is much more special than required for (WID-II). Nevertheless, several familiar oligopoly models satisfy the requirements of Lemma 1.[18]

Lemma 2 In each the following models, (WID-II) holds:

(a) Bertrand or Cournot competition where each firm's marginal cost is constant, goods are

[18] Bagwell and Staiger (1994) establish that several of these models satisfy strategic substitutability, but they do not check convexity.

differentiated, firm i's demand is more sensitive to firm i's price than those of other firms, and demand is linear (Dixit, 1979), where Y_i represents either the negative of marginal cost or firm i's quality level.

(b) Horizontal competition on the line (d'Aspremont et al., 1979) or on the circle (Salop, 1979) with quadratic transportation costs, where Y_i is as in (a), and $I = 2$.

(c) The Shaked/Sutton (1982) model of vertical quality differentiation with potentially different marginal costs, where Y_i represents firm i's marginal cost, and $I = 2$.[19]

(d) The Shaked/Sutton (1982) model of vertical quality differentiation where the market is covered, the firms have identical marginal costs, Y_i represents firm i's quality level, and $I = 2$.

A sketch of the proof can be found in the Appendix.[20] Parts (a)-(c) follow from Lemma 1. Part (d) is not quite as straightforward, because increasing quality does not necessary increase demand and the markup, in particular for a low-quality firm that reduces vertical differentiation by moving closer to its rival.[21] More generally, the literature has identified other examples where quality investments are strategic complements.[22]

It remains to check that the models described in Lemma 2 satisfy our maintained assumptions, (EXCH) and (UNQ). Note that these conditions must apply to the overall profit function Π^i, not just $\hat{\pi}^i$. In the special case of the incremental investment game, where $\Pi^i(a^t; Y^{t-1}) = \hat{\pi}^i(Y^{t-1} + a^t) - k(a_i^t; Y_i^{t-1})$, we can verify these conditions directly. Exchangeability always holds in models (a), (c), and (d), but as discussed above, it only holds in model (b) if there are two firms located symmetrically about the midpoint of the line. A sufficient condition for the conditional uniqueness requirement is $\Pi^i_{a_i,a_i}\Pi^j_{a_j,a_j} - \Pi^i_{a_i,a_j}\Pi^j_{a_j,a_i} \neq 0$ for $Y_i \neq Y_j$, $i \neq j$. In terms of primitives, this holds if k is sufficiently convex in a_i.

Models that satisfy the conditions of Lemma 1 have a number of special features, in addition to (WID-II). For example, linearity and (EXCH) implies that $\hat{\pi}^i$ depends on Y_{-i} only through $\sum_{j \neq i} Y_j$. Thus, if we establish that weak increasing dominance holds in such a model, it may not be clear which special features of the model are critical for the conclusion.

[19] In this case, condition WID-II does not necessarily hold when Y_i^t represents firm i's quality level.

[20] The analysis of (b)-(d) can be extended for $I > 2$, but a few parameter restrictions are necessary.

[21] Indeed, Ronnen (1991) considers the Shaked/Sutton model when the market is not covered, and finds conditions whereby vertical investments are strategic complements; similarly, the conditions do not necessarily hold if the firms have different marginal cost parameters.

[22] See, e.g., Vives (1990b) or Ellickson (1999). The former shows that investments can be strategic complements when they affect the slope of the demand curve in a differentiated Bertrand model. Leary and Neary (2000) show that strategic complements may also arise in models with cost reduction and spillovers.

Below, we show that for myopic firms, or when firms commit to long-term investment plans, the linearity assumptions can be relaxed without affecting conclusions about weak increasing dominance, so long as (WID-II) holds. However, when we consider Markov-perfect equilibria, simplifying assumptions like the ones contained in Lemma 1 may play a more important role.

3 Dominance results

In this section, we study conditions under which firms with higher state variables make higher investments in equilibrium. In such cases, we speak of weak increasing dominance. We begin by introducing an abstract theorem for all games with strategic substitutes. We then apply the theorem to our model under the assumption that firms are myopic, and develop an additional result for the case of strategic complements. Next, we consider the case where firms are far-sighted. Finally, we consider conditions under which weak increasing dominance implies strong increasing dominance, whereby higher investments by leading firms lead to higher market shares.

3.1 Games with Strategic Substitutes

Section 2.3 established that in a wide range of oligopoly models, $\hat{\pi}^i_{Y_i,Y_j} \leq 0$. For incremental investment games, this in turn implies that $\Pi^i_{a_i,a_j} \leq 0$: investments are strategic substitutes. The same forces are often present in more general investment games. To analyze such games, we prove a new comparative statics result. We introduce a slightly more abstract notation, allowing us to apply the same general results to both static and dynamic games.

Consider a game between I players. Denote player i's strategy space by X_i, with typical element x_i. Let $X_i = X_j$ for all i, j. Assume that X_i is a product set in $\mathbb{R}^N$, $N \leq 1$. For each player, there is an exogenous "state variable," $\theta_i \in \Theta_i$, where Θ_i is a product set in $\mathbb{R}^m$, $\theta = (\theta_1, \ldots, \theta_I)$; and $\Theta = \times_i \Theta_i$. Let the players' utility functions be given by $u^i : X \times \Theta \to \mathbb{R}$. To analyze this game, some terminology will be useful.

Definition 3 (Topkis, 1978) Let X, Y be partially ordered sets. A function $f : X \times Y \to \mathbb{R}$ satisfies increasing differences in (x, y) if for all $x^H > x^L$, $y^H > y^L$, $f(x^H, y^H) - f(x^L, y^H) \geq f(x^H, y^L) - f(x^L, y^L)$. If $Y = \times_i Y_i$ is a product set, $f: Y \to \mathbb{R}$ is supermodular in y if it satisfies increasing differences in (y_i, y_j) for all $i \neq j$.

If $f : \mathbb{R}^2 \to \mathbb{R}$ is smooth, it has increasing differences if and only if $f_{xy} \geq 0$.

Condition (UNQ) can be applied directly to this game, and we say that (EXCH$'$) holds if utility functions are exchangeable as functions of $(x; \mu)$. Consider now a third condition:

$$\text{For all } \mu \text{ and all } x_{-i};\ u^i(x_i; x_{-i}; \mu) \text{ is supermodular in } x_i. \qquad \text{(OSPM)}$$

This condition holds trivially unless players have multi-dimensional choices, as when they invest in both product and process innovation or make multi-period investment plans.

Games with strategic substitutes can be usefully contrasted against games with strategic complementarities.[23] Games with strategic complementarities are defined by the requirement that each u^i satisfies increasing differences in $(x_i; x_j)$ for all $j \neq i$. The following result is due to Topkis (1979).

Lemma 3 Suppose that: (i) (OSPM) holds; (ii) the players' actions are strategic complements, and (iii) u^i has increasing differences in $(x_i; \mu_j)$ for all j. For each μ, let $x^*(\mu)$ be the highest equilibrium. Then, $\mu^H > \mu^L$ implies $x^*(\mu^H) \geq x^*(\mu^L)$.

To see the intuition, suppose that increasing μ_i directly affects only firm i by increasing her incremental returns to investing. If opponents' actions were held fixed, firm i would then want to increase her action. However, such a change would lead all opponents to desire increases in their actions. Since such increases are mutually reinforcing in games of strategic complementarity, the equilibrium action vector must go up.

Now consider a game with strategic substitutes, defined by the requirement that each u^i satisfies increasing differences in $(x_i; -x_j)$ for $j \neq i$; that is, an increase in any opponent's action decreases the incremental return to a player's own action. Suppose that u^i has increasing differences in $(x_i; \mu_i)$, as before, but now suppose that increases in any opponent's state variable decrease the incremental return to acting, that is, u^i has increasing differences in $(x_i; -\mu_j)$ for all $j \neq i$. Although the comparative statics results of Lemma 3 do not generalize to games with strategic substitutes, we can still provide sufficient conditions for weak increasing dominance. The critical assumption for our result is exchangeability.

[23] For a discussion of this distinction for oligopoly games, see Bulow, Geanakoplos, and Klemperer (1985) and Fudenberg and Tirole (1984).

Theorem 4 *Suppose that (UNQ) and (EXCH′) hold. Suppose further that: (i) (OSPM) holds; (ii) the players' actions are strategic substitutes, and (iii) u^i has increasing differences in $(x_i;\theta_i)$ and $(x_i;-\theta_j)$ for $j \neq i$. Then $\theta_i > \theta_j$ implies that $x_i^*(\theta) \geq x_j^*(\theta)$.*

Proof. First suppose there are only two players. Define $\tilde{x} = (\tilde{x}_1;\tilde{x}_2) = (x_1;-x_2)$, $\tilde{\theta} = (\tilde{\theta}_1;\tilde{\theta}_2) = (\theta_1;-\theta_2)$; and let $\tilde{u}^i(\tilde{x};\tilde{\theta}) = u^i(\tilde{x}_1;-\tilde{x}_2;\tilde{\theta}_1;-\tilde{\theta}_2)$ for $i = 1;2$. Then the modified game satisfies the conditions of Lemma 3. For a given, $\tilde{\theta}$; let $\tilde{x}^*(\tilde{\theta})$ be the equilibrium of this game. Now, we compare two alternative parameter vectors, $\theta^1 = (\theta_H;\theta_L)$ and $\theta^2 = (\theta_L;\theta_H)$. Hence, $\tilde{\theta}^1 = (\theta_H;-\theta_L)$ and $\tilde{\theta}^2 = (\theta_L;-\theta_H)$. As $(\theta_H;-\theta_L) \geq (\theta_L;-\theta_H)$, and since (UNQ) implies that the equilibrium for each parameter vector is unique, Lemma 3 implies that $\tilde{x}_1^*(\theta_H;-\theta_L) \geq \tilde{x}_1^*(\theta_L;-\theta_H)$ and hence $x_1^*(\theta_H;\theta_L) \geq x_1^*(\theta_L;\theta_H)$. By exchangeability $x_2^*(\theta_H;\theta_L) = x_1^*(\theta_L;\theta_H)$, and the result follows.

Now suppose there are $I > 2$ players. Without loss of generality, consider players 1 and 2, and consider state variables θ' where $\theta'_1 > \theta'_2$. Let $\hat{\theta} = T_{12}(\theta')$. By (EXCH), there exists an equilibrium $x^*(\hat{\theta})$ such that $x^*_{-12}(\theta') = x^*_{-12}(\hat{\theta})$. Fix $x_{-12} = x^*_{-12}(\theta')$ and consider the game between players 1 and 2. Let $x^{**}(\theta')$ and $x^{**}(\hat{\theta})$ be equilibria of the two-player games where $\theta = \theta'$ and $\theta = \hat{\theta}$; respectively, and observe that (UNQ) implies that each of these equilibria is unique. By our exchangeability assumption, players $3;...;I$ are not affected by the reversal of the state variables of firms 1 and 2. Thus, the equilibrium of the two-player game is also an equilibrium when players $3;...;I$ are not constrained: $x^{**}(\theta') = (x_1^*(\theta');x_2^*(\theta'))$ and likewise for $\hat{\theta}$. But the argument for the two-player case implies that $x_1^{**}(\theta') \geq x_2^{**}(\theta')$. Q.E.D.

For the two-player case, the proof proceeds by observing that, by re-ordering the action set and state variable for one player, it is possible to convert the two-player game with strategic substitutes to a game with strategic complementarities.[24] Then, Lemma 3 can be used to compare the equilibrium choices under two scenarios: one where the first player's state variable is higher than the second player's, and one where the roles of the players are reversed. By (EXCH′) and (UNQ), the equilibrium in the second case is merely the equilibrium of the first case, with the roles of the players reversed. But when exchanging the state variables between the two players, Lemma 3 implies that decreasing the first player's state variable and increasing the second firm's state variable decreases the equilibrium choice for firm one, and

[24] Vives (1990a) and Amir (1994) use a similar approach to analyze Cournot oligopoly using the tools of supermodular games; see also Vives (2000).

increases the equilibrium choice for firm two. Thus, the player with the higher state variable must choose a higher action. (UNQ) plays an important role in drawing an unambiguous conclusion; with multiple equilibria, we can conclude only that some equilibrium satisfies weak increasing dominance.[25]

With more than two players, however, the problem is much more complex. Even if the conditions of Theorem 4 hold, an increase in μ_i does not necessarily lead to an increase in x_i and a decrease in x_j for $j \neq i$. Clearly the direct effect of μ_i supports the specified changes in the choice vectors, and one set of indirect effects does as well: when player i increases her action, all opponents have a lower incremental return to their actions. However, a second set of indirect effects could potentially dominate: when player j decreases her action, player k has an incentive to increase his. Whether player k is more sensitive to a change in player i's action or player j's action depends on the functional form and the value of μ.

We address these complexities as follows. We wish to compare the equilibrium choices for two vectors of state variables: the original vector, and a vector with the first two elements transposed. However, players three and higher are not affected when we reverse the roles of the first two players. Further, transposing the state variables of the first two players should merely transpose their equilibrium choices. Thus, we can hold fixed the actions of players three and higher at the equilibrium values for the original vector of state variables, and analyze the game between the first two players. Then, the logic of the two-player case applies: decreasing player 1's state variable and increasing player 2's state variable decreases the equilibrium choice of player 1 and increases the equilibrium choice of player 2.

Thus, exchangeability provides just enough structure to hold fixed the behavior of players three and higher, and focus on the two-player game. Without this assumption, we could find a counter-example, which might exploit asymmetries in the extent to which one player cares about the choices of the others. Although exchangeability is a strong assumption, it is much weaker than several alternatives that have been used in the existing literature. For example, it is common to consider two-firm models, or to assume that a firm's profit depends only on the sum of opponent actions (as in a Cournot model with perfect substitutes), so

[25] To see why, let $A_i = \{0,1\}$. If (UNQ) fails, both (0,1) and (1,0) can be equilibria for a given set of parameters. In fact, both might be equilibria for both parameter vectors (μ_H, μ_L) and (μ_L, μ_H); this is fully consistent with our assumption that actions are strategic substitutes. On the other hand, even when (UNQ) fails, if (0,1) is an equilibrium with (μ_H, μ_L), (EXCH) and Lemma 3 imply that (1,0) is an equilibrium as well. Thus, there exists an equilibrium satisfying weak increasing dominance.

that the game effectively becomes a two-player game.[26]

Beyond the applications considered in this paper, Theorem 4 may be of broader interest as a comparative statics result for games with strategic substitutes; for example, in the conclusion we discuss potential applications to strategic trade and tournaments.

Before proceeding, it is instructive to compare the approach pursued here with more standard approaches that might be used to reach the conclusion $\theta_i > \theta_j \Rightarrow x_i^*(\theta) \geq x_j^*(\theta)$. Consider an alternative set of sufficient conditions: (a) $\theta_i = \theta_j \Rightarrow x_i^*(\theta) = x_j^*(\theta)$; and (b) $x_i^*(\theta)$ is weakly increasing in θ_i and $x_j^*(\theta)$ is weakly decreasing in θ_i for $j \neq i$. If a particular game satisfies the requisite regularity conditions, condition (b) could be verified using the implicit function theorem. This approach would differ from ours in two respects. First, it would require additional (dominant-diagonal) conditions on second derivatives. Second, unlike the conditions of Theorem 4, (b) rules out situations where an increase in θ_i leads to an increase of some $x_j^*(\theta)$, $j \neq i$, even though such situations are quite plausible in games with strategic substitutes. Indeed, when there are three or more players, it is possible to construct examples where the conditions of Theorem 4 hold but condition (b) fails.

3.2 Weak Increasing Dominance for Myopic Firms

Using the results of the last subsection, we now give conditions for weak increasing dominance when firms are myopic. The case where firms are myopic serves as a useful polar case. We show that in a range of models, weak increasing dominance arises even absent the strategic incentive to invest in future market share. Such a finding may be relevant for antitrust policy, for example in evaluating claims of predatory behavior. Further, the results for myopic firms will also characterize market outcomes in cases where firms are forward-looking but fairly impatient.

Proposition 5 Suppose firms are myopic. Suppose that either (i) in the incremental investment game, (WID-II) holds and $\pi^i_{a_i,Y_i} \geq k_{a_i,Y_i}$; or more generally, (ii) for all $i \neq j$ and all (a, Y);

$$\pi^i_{a_i,a_j} \leq 0; \; \pi^i_{a_i,Y_j} \leq 0; \text{ and } \pi^i_{a_i,Y_i} \geq k_{a_i,Y_i}. \qquad \text{(WID)}$$

Then, weak increasing dominance holds: for all $i \neq j$, $Y_i > Y_j$ implies $a_i^*(Y) \geq a_j^*(Y)$.

[26] These properties have been exploited in the existence proof for Cournot oligopoly (Selten, 1970), and in the comparative statics analysis of Dixit (1986).

The Proposition is a direct application of Theorem 4. The conditions can be understood as follows. In the absence of adjustment costs, they require that in terms of expected product market profits, investments are strategic substitutes; higher levels of a firm's own state variable increase the marginal returns to investment; and higher levels of the opponent's state variable decrease the marginal returns to investment. When adjustment costs are considered, (WID) requires that the incremental adjustment cost must not increase too rapidly as the own state variable increases. As we saw in Section 2.3, if $k_{a_i;Y_i}$ is not too large, these conditions hold in many oligopoly models, and indeed, they are implied by (WID-II) in the incremental investment game.

Two extreme examples can be used to highlight scenarios under which the conditions on adjustment costs are likely to be satisfied. At one extreme, adjustment costs are entirely independent of the state variable. An example might be a pure incremental investment model (although even in the case of incremental investment, it is plausible that eventually, efficient firms find it difficult to further improve the production process). At the other extreme, the adjustment costs depend only on the target level of the state variable, not on the initial state. This type of adjustment cost is likely to arise if the firm invests in a radically different technology or product design, so that earlier expertise is of little use. Formally, suppose there exists a strictly increasing and convex function $\hat{k}$ such that $k(a_i^t; Y_i^{t-1}) = \hat{k}(Y_i^{t-1} + a_i^t) = \hat{k}(Y_i^t)$. In this case, the lower is a firm's state variable, the cheaper it is to attain a given increase in the state variable, a_i^t, potentially violating (WID).

Following Proposition 5, (WID) requires both $\pi^i_{a_i;a_j} \le 0$ and $\pi^i_{a_i;Y_j} \le 0$: In the incremental investment model, both of these two properties hold whenever $\tilde{\pi}_{Y_i;Y_j} \le 0$, and in most IPC models the two properties are closely related. In contrast, in the ISC case, firms might compete in prices, which are typically strategic complements, but yet $\pi^i_{a_i;Y_j} \le 0$ might hold (where a_i is the negative of the price). We now consider a result for that case.

If actions are strategic complements, but $\pi^i_{a_i;Y_j} \le 0$ and $\pi^i_{a_i;Y_i} \ge k_{a_i;Y_i}$, weak increasing dominance does not follow directly from Lemma 3 (or more generally, from the approaches proposed in the literature on supermodular games). To see why, observe that increasing Y_i has competing effects on the equilibrium. One effect is that the returns to firm i's own investment, a_i; go up; from there, an increase in a_i leads to self-reinforcing increases in the equilibrium values of a_j for $j \ne i$: However, these effects compete with the effect arising because Y_i decreases the returns on opponents' investments. Our next result uses exchangeability (as well as an additional quasi-concavity assumption) to resolve these issues.

Proposition 6 Suppose firms are myopic. Suppose that for each i, π^i is strictly quasi-concave in a_i, and for all $j \neq i$ and all (a, Y),

$$\pi^i_{a_i,a_j} \geq 0, \ \pi^i_{a_i,Y_j} \leq 0, \text{ and } \pi^i_{a_i,Y_i} \geq k_{a_i,Y_i}. \qquad \text{(WID-SC)}$$

Then, weak increasing dominance holds: for all $i \neq j$, $Y_i > Y_j$ implies $a^*_i(Y) \geq a^*_j(Y)$.

Proof. Suppose that weak increasing dominance is violated. Without loss of generality, suppose that the violation concerns firms 1 and 2. Then, there must exist $Y_H > Y_L$, $a_H > a_L$, and an equilibrium $a^*(Y)$ such that $(a_1, a_2, Y_1, Y_2) = (a_L, a_H, Y_H, Y_L)$. Hold fixed Y_{-12} and $a_{-12} = a^*_{-12}(Y)$, and suppress these in the notation. If $(a_1, a_2, Y_1, Y_2) = (a_L, a_H, Y_H, Y_L)$ is an equilibrium, then firm 2's first-order condition must be satisfied at that point. Then, by strict quasi-concavity, $\Pi^2_{a_2}(a_L, a_L, Y_H, Y_L) > 0$. Further, $\Pi^2_{a_2}(a_H, a_L, Y_L, Y_H) = 0$ by exchangeability, since $\Pi^1_{a_1}(a_L, a_H, Y_H, Y_L) = 0$ in equilibrium. Hence

$$\Pi^2_{a_2}(a_L, a_L, Y_H, Y_L) - \Pi^2_{a_2}(a_H, a_L, Y_L, Y_H) > 0. \qquad (4)$$

Since the choices are strategic complements, increasing a_1 increases $\Pi^2_{a_2}$, which by (4) implies that $\Pi^2_{a_2}(a_H, a_L, Y_H, Y_L) - \Pi^2_{a_2}(a_H, a_L, Y_L, Y_H) > 0$. However, (by the fundamental theorem of calculus), this contradicts (WID-SC), which requires $\Pi^2_{a_2Y_2} \geq 0$ and $\Pi^2_{a_2Y_1} \leq 0$. Q.E.D.

Proposition 6 can be applied in models of learning-by-doing where firms compete in prices, which are typically strategic complements; see Section 4.2. Further, the logic of the proposition can be applied even when (WID-SC) fails. The conclusions of Proposition 6 hold so long as (4) fails for any feasible equilibrium of the form $(a_1, a_2, Y_1, Y_2) = (a_L, a_H, Y_H, Y_L)$, where $Y_H > Y_L$ and $a_H > a_L$. Thus, (WID-SC) can be weakened, for example to allow for the case where $\pi^i_{a_i,a_j}$ is not too negative relative to the other effects.

Propositions 5 and 6 give conditions for weak increasing dominance. Reversing the ordering of the state variables gives conditions for "weak decreasing dominance," meaning that leaders invest less than laggards, as an immediate corollary.

Corollary 7 Suppose firms are myopic. Suppose that for all $i \neq j$ and all (a, Y),

$$\pi^i_{a_i,Y_j} \geq 0, \ \pi^i_{a_i,Y_i} \leq k_{a_i,Y_i},$$

and further, either (i) for all $i \neq j$, $\pi^i_{a_i,a_j} \leq 0$, or else (ii) for all $i \neq j$, $\pi^i_{a_i,a_j} \geq 0$ and π^i is strictly quasi-concave in a_i. Then for all $i \neq j$, $Y_i > Y_j$ implies $a^*_i(Y) \leq a^*_j(Y)$.

In words, for myopic …rms, if a higher state variable for …rm i decreases …rm i's investment incentives and increases …rm j's investment incentives, and …nally either actions are always strategic substitutes or else always strategic complements (and payo¤s are quasi-concave), then leading …rms will invest less than lagging …rms. An application of this result is presented in Section 4.3.

3.3 Weak Increasing Dominance for Far-Sighted Firms

In this section, we suppose that …rms are not myopic, but instead discount the future at rate $\pm > 0$. Further, we focus on models where (WID) holds, as in Proposition 5. Following an approach that is common in the literature,[27] we begin by analyzing the benchmark case of "open-loop" pure strategy Nash equilibria (OLE), where each …rm makes a deterministic investment plan at the beginning of the game, and this plan cannot be modi…ed later. Subsequently, we analyze closed-loop, or Markov-perfect, pure strategy Nash equilibria (MPE), where …rms can condition their actions on states in each period. We do not consider mixed strategy equilibria for either OLE or MPE.

Since our model is deterministic, every OLE is also a Nash equilibrium in the game where each player can condition his actions on the observed history of past play, although it may not be subgame-perfect. As the OLE omits the strategic e¤ects that might arise when …rms attempt to manipulate the future investments of opponents, it serves as a useful point of comparison. Further, if …rms are fairly impatient and must …x their investment plans several periods in advance, OLE may provide a good …rst approximation of behavior. Such advance planning might be required if research and development requires large capital expenditures or specialized technology, such as laboratories.[28]

Milgrom and Roberts (1990) showed that the theory of supermodular games can be applied to problems with a wide variety of choice sets, including problems where the agent chooses an in…nite sequence of actions. We use a similar approach, applying Theorem 4 to the in…nite-horizon problem.

[27] See, for example, Fudenberg and Tirole (1991), pp. 130-133.

[28] The open-loop approach might also be justi…ed if players can only observe each others' actions with a considerable time lag, so that the possibilities for responding to the behavior of competitors are limited, as may be plausible for R&D investments.

Proposition 8 Suppose that firms live for $T \leq 1$ periods and are far-sighted, and that (WID) holds. Suppose further that $Y_i^t = Y_i^{t-1} + a_i^t$, and that either (i) in the incremental investment game, (WID-II) holds; or more generally, (ii) for all $i \neq j$; and all (a, Y);

$$\pi^i_{Y_i,Y_i} \geq k_{Y_i,Y_i};\ \pi^i_{a_j,Y_i} \leq 0;\ \text{and}\ \pi^i_{Y_i,Y_j} \leq 0. \qquad \text{(WID-D1)}$$

If there is a conditionally unique OLE, denoted $\mathbf{a}^*(Y^0)$, $Y_i^0 > Y_j^0$ implies that for all t, $a_i^{t*}(Y^0) \geq a_j^{t*}(Y^0)$.

The proof is given in the Appendix. Proposition 8 imposes several conditions beyond those required in Proposition 5.[29] The functional restriction on the evolution of the state variable simplifies the problem, allowing us to consider directly the effect of today's action on all future periods. Condition (WID-D1) is required to guarantee that the following additional conditions hold: the actions of a given firm in two different periods are complementary in increasing the profit in all future periods (requiring $\pi^i_{Y_i,Y_i} \geq k_{Y_i,Y_i}$); and, across any pair of periods, the actions of the two firms are strategic substitutes (requiring $\pi^i_{a_j,Y_i} \leq 0$ and $\pi^i_{Y_i,Y_j} \leq 0$). In the incremental investment game, (WID-II) implies (WID-D1).

Observe that the approach of Proposition 8 relies on the fact that the interaction between a_i and a_j in today's profits (determined by $\pi^i_{a_i,a_j}$) reinforces the interaction in future profits, determined by $\pi^i_{a_j,Y_i}$ and $\pi^i_{Y_i,Y_j}$. The property that dynamic interactions unambiguously reinforce static ones also holds in models that satisfy part (ii) of Corollary 7, but not Proposition 6 or part (i) of Corollary 7.

While Proposition 8 illustrates that some aspects of dynamic competition reinforce our results about increasing dominance, by focusing on OLE the result ignores the incentives of firms to adjust their investment strategies over time in an attempt to manipulate the investment response of opposing firms. When we enlarge the strategy space of firms in the dynamic game to allow them to respond to current conditions, a variety of competing effects can emerge. The following result identifies a (strong) set of sufficient conditions for weak increasing dominance in a dynamic game between two firms and the horizon is finite.

[29] Note, as well, that the restriction (UNQ) may be more severe with infinite horizon, although it is satisfied in a number of specific models (e.g. Fershtman and Kamien (1987)). For continuous-time models where payoffs are quadratic, Engwerda (1998) identifies necessary and sufficient conditions for a unique OLE in finite-horizon games, and shows that the infinite-horizon problem may have multiple equilibria.

Proposition 9 Suppose: the assumptions of Proposition 8 hold; $T < \infty$; $I = 2$; each firm's investment is chosen from a compact subset of $\mathbb{R}$; and π^i is twice continuously differentiable. Restrict attention to MPE where strategies are exchangeable and continuously differentiable, assume that within this class UNQ holds, and let $a_i^t(Y^{t-1})$ denote the equilibrium strategy of player i in period t. Finally, assume that either (i) in the IPC case, the conditions of Lemma 1 hold and k is quadratic, or, more generally, (ii) for $i \neq j$ and for all (a, Y),

$$\pi^i_{Y_j, Y_j} \geq 0; \; \pi^i_{Y_j, a_j} \geq 0; \; \pi^i_{a_j, a_j} \geq 0; \; \pi^i_{a_j} \leq 0; \qquad \text{(WID-D2)}$$

and for all t, $a_i^t(Y^{t-1})$ is continuously differentiable, and

$$\frac{\partial}{\partial Y_i^{t-1}} a_j^t(Y^{t-1}) \text{ is nondecreasing in } (-Y_i^t, Y_j^t); \; \frac{\partial}{\partial Y_j^t} a_j^t(Y^{t-1}) \text{ is nonincreasing in } Y_j^{t-1}. \qquad \text{(WID-A)}$$

Then $Y_i^{t-1} > Y_j^{t-1}$ implies $a_i^t(Y^{t-1}) \geq a_j^t(Y^{t-1})$ (and moveover, $a_i^t(Y^{t-1})$ is nondecreasing in Y_i^{t-1} and nonincreasing in Y_j^{t-1}).

This result, proved in the Appendix, provides sufficient conditions for weak increasing dominance in a MPE, whereby each firm's investment in period t depends on the state variable in period t. Under the assumptions of Proposition 9, we can compare the MPE with the OLE, finding that the additional strategic effects reinforce the tendency towards weak increasing dominance. Although the result imposes strong restrictions, the conditions are similar to those used in much of the existing literature on MPE; for example, many existing studies restrict attention to two firms and quadratic payoffs.[30] However, when the assumptions are relaxed, the additional strategic effects may serve as mitigating factors. If firms are sufficiently patient, weak increasing dominance may be overturned.

Consider the role of each new assumption. First, as in most of the existing literature, we have restricted attention to exchangeable MPE in continuously differentiable strategies. In finite-horizon linear-quadratic models, this rules out equilibria where, despite the inherent symmetry in payoffs, firms adopt asymmetric strategies (for example, one firm is tougher than the other in all contingencies).

Second, consider the maintained assumption (UNQ). In a quadratic finite-horizon game, there is a unique MPE in linear strategies (Kydland (1975)). However, in general (UNQ)

[30] See, for example, Fershtman and Kamien (1987), Beggs and Klemperer (1992), and Jun and Vives (1999); Vives (2000) reviews the literature.

may be restrictive. Without it, we can no longer assert that every equilibrium satisfies weak increasing dominance. However, if there are multiple equilibria, we can still show that there exists an equilibrium where weak increasing dominance holds, as discussed in Footnote 25.

Third, we have imposed additional conditions on partial derivatives of the product market payoffs (WID-D2) and the policy functions (WID-A). They play a role because, to determine whether investments are strategic substitutes, we must verify that today's investments are substitutes in affecting tomorrow's profit. These restrictions are implied by the conditions of Lemmas 1 and 2. Condition (WID-A) is used to show that firm i's state variable increases the return's to firm i's future investments and decreases the returns to firm j's future investments, and further, today's investments by firm i and firm j are strategic substitutes in their effect on future profits. When per-period payoffs are quadratic, optimal policy functions are linear and so (WID-A) is satisfied trivially. In general, the third derivatives of the profit function can generate effects that compete with increasing dominance.

Fourth, consider the role of the assumption that there are only two firms. The proof exploits that fact that $I = 2$ implies that equilibrium policy functions are monotone: by Lemma 3, $\frac{\partial}{\partial Y_1^{t-1}}a_1^t(Y^{t-1}) \geq 0$ and $\frac{\partial}{\partial Y_1^{t-1}}a_2^t(Y^{t-1}) \leq 0$. In turn, this implies that today's incentives to manipulate future investments reinforce the incentives from today's profits. With more than 2 firms, policies are not necessarily monotone; but even if they are, competing effects can still arise. Consider the problem faced by a firm in the penultimate period; then, the interaction between a_i and a_j in firm i's objective function depends in part on the interaction between firm i's state variable and firm j's state variable in the final period. The cross-partial derivative $\frac{\partial^2}{\partial Y_i^{T-1}\partial Y_j^{T-1}}\Pi^i(a^T(Y^{T-1}); Y^{T-1})$ includes the following terms:

$$\Pi^i_{Y_i;a_k}(a^T(\cdot);\cdot)\frac{\partial}{\partial Y_j^{T-1}}a_k^T(\cdot) + \Pi^i_{a_j;a_k}(a^T(\cdot);\cdot)\frac{\partial}{\partial Y_i^{T-1}}a_j^T(\cdot)\frac{\partial}{\partial Y_j^{T-1}}a_k^T(\cdot).$$

If $\frac{\partial}{\partial Y_j^{T-1}}a_k^T(Y^{T-1}) \leq 0$, the first term is positive, creating a force opposing strategic substitutability between firm i's investment and firm j's investment. We have not specified the sign of $\Pi^i_{a_j;a_k}$. However, since all of these potentially competing effects are weighted by δ and balanced against effects arising in the present, for δ small enough, the competing effects will be outweighed and weak increasing dominance will hold even when $I > 2$.

To understand why difficulties arise with $I > 2$ firms in MPE, but not OLE, recall that in the proof of Theorem 4, we held fixed the actions of all players except two, and analyzed a two-player game. When considering Markov-perfect strategies, we can still hold fixed the strategies of all players except two. However, since strategies are plans contingent on state variables, firms take into account their ability to manipulate each opponent's future investment behavior by changing future state variables. Thus, competing effects arise due to the interaction between the investments of firm i and firm j in manipulating firm k's future investment.[31]

Finally, consider relaxing the assumption that $T < \infty$. If (UNQ) holds when $T = \infty$, and the equilibrium is continuously differentiable (e.g., linear), the approach of Proposition 9 can be extended. However, with an infinite horizon, there are typically multiple equilibria (although there may be a unique equilibrium within the class of continuous or differentiable strategies[32]). If there are multiple equilibria, one equilibrium of particular interest (if it exists) is an equilibrium attained by taking the limit of first-period strategies as the horizon T approaches infinity. Since the properties required for weak increasing dominance are preserved by limits, such an equilibrium would satisfy weak increasing dominance under the assumptions of Proposition 9. However, in general the limit of the first period strategies as $T \to \infty$ may not exist; and even if it does, the limit may not be an equilibrium.[33]

In summary, we find that when (WID) holds and firms are forward-looking, the basic forces from our static model remain present; and if the firms are forced to commit to strategic investment plans in advance, the incentives leading to weak increasing dominance are reinforced. Thus, our results about weak increasing dominance are perhaps most salient when firms are impatient, or when investment plans are inherently long-term. However, if firms are far-sighted and if they adjust their investments in response to the evolution of the state variable, then our results must be qualified. Competing effects may arise when firms are sufficiently patient, and either there are more than two firms, or else the policy functions are nonlinear. In specific models, it may be possible to make definitive predictions about

[31] Another approach to extending the results to $I > 2$ firms is to exploit functional form assumptions of profit functions. When payoff functions are quadratic (as in part (i)), optimal policy functions are linear. In the IPC model, this (together with (EXCH)) implies that each firm's payoffs depend only on the sum of opponent investments. Then we can analyze the model as a two-player game, where player i treats all opponents as a single player, and weak increasing dominance obtains under the conditions of part (i), so long as the sum of opponent investments is nondecreasing in the sum of opponent state variables (which is always true if $\rho_{Y_i;Y_j}$ is close enough to 0).

[32] See, for example, Beggs and Klemperer (1982); for continuous-time models, see Jun and Vives (1999), or Miravete (1999).

[33] For continuous-time models, see, e.g., the discussion in Lockwood (1996).

when these competing effects are outweighed.[34]

3.4 Strong Increasing Dominance

Weak increasing dominance does not necessarily imply strong increasing dominance, whereby leading firms increase their market share over time. When state variables do not depreciate over time, this holds if higher investments by leading firms lead to higher market shares. However, we caution that if state variables depreciate quickly enough, market shares might converge to $1/I$ over time even if leading firms invest more and have larger market shares in each period.

For simplicity, we focus on the incremental investment game where state variables do not decrease over time, and suppose that firm i's market share is decreasing in firm j's state variable. Let $\mathbf{1}$ denote $(1, \ldots, 1)$ and let D denote total demand. Then a leading firm i that invests more than its competitors always increases its market share if and only if $\frac{\partial}{\partial \alpha} \left. \frac{D^i(Y+\alpha \mathbf{1})}{D(Y+\alpha \mathbf{1})} \right|_{\alpha=0} \geq 0$ whenever $Y_i \geq \max_{i \neq j} Y_j$. This is equivalent to

$$\left. \frac{\frac{\partial}{\partial \alpha} D^i(Y+\alpha \mathbf{1})}{\frac{\partial}{\partial \alpha} D(Y+\alpha \mathbf{1})} \right|_{\alpha=0} \geq (\leq) \frac{D^i(Y)}{D(Y)} \qquad (5)$$

whenever $\frac{\partial}{\partial \alpha} D(Y+\alpha \mathbf{1}) \geq (\leq) 0$; that is, whenever the effect of a uniform increase (respectively, decrease) in all state variables is to increase (respectively, decrease) industry demand, so that investments are demand-creating (respectively, demand-stealing). Since by definition, $D^i/D > 1/I$ for a leading firm, this requires that when investments are demand-creating, a simultaneous increase in all state variables increases firm i's demand faster than that of the average firm. However, this condition may be difficult to satisfy. For example, consider a special case: equilibrium demand is linear in states and exchangeable. In this case, $\frac{\frac{\partial}{\partial \alpha} D^i(Y+\alpha \mathbf{1})}{\frac{\partial}{\partial \alpha} D(Y+\alpha \mathbf{1})} = 1/I$. Then (5) holds for a leading firm if and only if $\frac{\partial}{\partial \alpha} D(Y+\alpha \mathbf{1}) \big|_{\alpha=0} \leq 0$, that is, investments are demand-stealing. Other things being equal, therefore, it is more likely that weak increasing dominance implies strong increasing dominance in industries where investments are demand-stealing (e.g. advertising) rather than demand-creating (e.g. cost-

[34] Of course, increasing dominance may arise even when the sufficient conditions of our propositions are not satisfied. For instance, in a model of Cabral (1999), increasing dominance arises despite the fact that expected payoffs are concave in the state variable for leaders and convex for laggards.

reducing investments in products with fairly elastic demand).[35] However, these conditions may or may not favor weak increasing dominance, so that we cannot o¤er an unconditional prediction about whether strong increasing dominance is more likely when investments are demand-stealing.[36]

4 Extensions and Applications

This section considers extensions and applications of our results. We identify primitive conditions under which weak increasing dominance holds, and we further highlight sources of competing e¤ects.

4.1 Incremental Innovation and Patent Races

We have already analyzed the incremental investment model above in some depth. In this section, we sketch how the results can be extended naturally in several directions.

Consider …rst extending the incremental investment model to allow for incremental innovation with uncertain returns. The working paper (Athey and Schmutzler, 1999) establishes conditions under which Proposition 5 can be extended to the stochastic case. For example, suppose that each …rm's investment does not directly a¤ect other …rms, and leads to a …rst-order stochastic dominance increase in the distribution over its own investment returns. If investment returns are independent or a¢liated across …rms, then when …rms are myopic, leading …rms invest more and are more likely to extend their lead; however, a lagging …rm may, by chance, overtake a leader. If that occurs, the new leader will have higher investment incentives in the next stage of the game. If …rms are far-sighted, the game can be analyzed

[35]Our observations have a rough parallel in the patent race literature. In Gilbert and Newbery (1982), only one …rm can receive a patent, so that one …rm's gain is another …rm's loss in terms of innovation. In Reinganum (1983), more than one …rm may receive a patent, though one …rm's chances are diminished by others' investments. In the former but not the latter case, there is increasing dominance, where the former case is more analogous to "demand-stealing."

[36]A similar approach can be applied to determine whether leading …rms increase their share of revenue or per-period pro…ts. Consider the question of whether …rm i's share of product market pro…ts increases for leading …rms when they invest more. Following the logic above, we ask whether a simultaneous increase in all state variables increases …rm i's pro…ts more than those of the average …rm. On the one hand, if (WID-II) holds, the pro…t of a leading …rm increases more than average in response to an increase in its own state variable. On the other hand, increases in opposing …rms' state variables hurt a leading …rm more than average. In general, the results are ambiguous.

using the approach of Proposition 9; again, the main results carry through.

Now suppose that firms can innovate in more than one dimension, for instance in cost reduction and quality improvement. Recall that Theorem 4 applies to such multi-dimensional investments provided they are complementary; Propositions 5 and 8 can be similarly extended. It remains to consider conditions under which cost-reducing and quality-enhancing investments are complementary. To this end, Lemma 1 extends directly: if equilibrium markups and demand are both linear, and if they are both increasing in own-investments and decreasing in opponent investments, the conditions required by Theorem 4 will be fulfilled. In the working paper (Athey and Schmutzler, 1999), we show that these conditions hold in models (a) and (b) of Lemma 2. The results also extend if investment returns are stochastic, so long as the returns are independent or affiliated across investments and across firms. Thus, the finding that product and process innovation are complements for a monopolist (see, e.g. Athey and Schmutzler (1995)) extends to many oligopoly models, with the additional prediction that leading firms will be more innovative in both dimensions.[37]

Now consider patent races. The existing literature suggests that there are a variety of competing effects that may work for or against increasing dominance.[38] In the working paper (Athey and Schmutzler, 1999), we provide a formal analysis of a patent race model; here, we simply discuss the sources of competing effects through the lens of our model. Suppose that a_i^t is firm i's R&D investment directed at cost reduction, and suppose that $-Y_i^t$ is the firm's marginal cost. The firms invest in hopes of achieving a given level of cost $\hat{c} < -\max\left\{Y_1^{t-1}, \ldots, Y_I^{t-1}\right\}$: If the effect of investment on the probability of attaining the patent is the same for all firms, leading firms pay more for each unit of expected reduction in marginal cost. However, at least for the case of two firms,[39] the signs of $\pi^i_{a_i,a_j}$ and $\pi^i_{a_i,Y_j}$ are those required for (WID). When the opposing firm has a lower state variable, a given firm has higher investment incentives under (WID-II), because the gain to winning the patent is larger. Firm investments are strategic substitutes if the investments of firm i and firm j are substitutes in their effect on the probability that firm i receives the patent.

Because of the competing effects in patent race models, it may be more promising to

[37] Of course, any of the many sources of competing effects identified above can undermine this result, such as those arising from adjustment costs or dynamics.

[38] See, e.g. Reinganum (1985), Vickers (1986), and Beath et al (1987)). For example, in Vickers (1986), weak increasing dominance arises in Bertrand but not Cournot models.

[39] Additional competing effects may arise with more than two firms, as discussed in the working paper.

apply the approach of the proof of Proposition 6, verifying that condition (4) fails in the relevant range.

4.2 Learning-by-doing

As Cabral and Riordan (1994) and Sutton (1998) argue in some detail, numerous real-world industries are characterized by learning-by-doing effects, whereby costs are monotone decreasing functions of previous output levels. Cabral and Riordan (1994) consider a model of learning-by-doing with two firms competing in prices over an infinite horizon. There is a single unit of demand, only one firm produces in a given period, and the opportunities for learning are exhausted after a finite number of units have been produced. In their model, weak increasing dominance always arises even though learning may be more effective for lagging firms. In this section, we analyze learning-by-doing in a somewhat different model.[40] We allow for downward-sloping demand, more than two firms, and we consider cases where firms compete in quantities as well as prices; however, for simplicity, we focus on the cases where either firms are myopic, or else they live only two periods. We highlight the competing effects that may arise.

First, consider quantity competition. Denote the output level as a_i^t. The state variable is the sum of prior output experience, i.e. $Y_i^t = \sum_{\tau=1}^{t} a_i^{\tau}$: Learning-by-doing leads to cost reduction from a reference level $\bar{c}$, through a function $r(\cdot)$ which is increasing and concave in prior output. Finally, $k^i \equiv 0$, as all costs and benefits of increasing output are borne through the product market profit. Suppose that the inverse demand curve is given by $P^i(a)$ (decreasing in a_i and increasing in a_j for $j \neq i$), so that $\pi^i(a^t; Y^{t-1}) = \left[P^i(a^t) - \bar{c} + r(Y_i^{t-1})\right] a_i^t$.

Consider conditions under which (WID-II) holds. If the goods are perfect substitutes, it is standard to assume that $\pi^i_{a_i;a_j} < 0$ in order to guarantee existence of equilibrium (see Novshek 1985). More generally, quantity choices are substitutes if P^i is linear, or if $P^i_{a_i;a_j} \le 0$ for $j \neq i$. Further, $\pi^i_{a_i;Y_i} = r' > 0$: an increase in output is more valuable for a low cost firm. Finally, $\pi^i_{a_i;Y_j} = 0$. Hence, by Proposition 5, weak increasing dominance holds when $\delta = 0$ without additional assumptions (beyond (UNQ) and (EXCH)).

[40] It is simple to reinterpret the model presented in this section as a model of network externalities, as in Farrell and Saloner (1986) and Katz and Shapiro (1986). In this case, the state variable (previous sales) affects consumer demand rather than cost.

For forward-looking firms, the long-run profit of firm i can be written as

$$LR^i\left(a^1;Y^0\right) = \pi^i\left(a^1;Y^0\right) + \delta\phi^i(\bar{c} - r(a_1^1 + Y_1^0);\ldots;\bar{c} - r(a_I^1 + Y_I^0));$$

where $\phi^i(c)$ is the profit for cost structure c when firms play a one-shot quantity game. Applying Theorem 4, weak increasing dominance will obtain if the following conditions hold for all $i \neq j$:

$$\frac{\partial^2 LR^i}{\partial a_i^1 \partial Y_i^0} = \pi_{a_i;Y_i}\left(a^1;Y^0\right) + \delta\phi^i_{c_i;c_i}(\cdot)\left(r'(a_i^1 + Y_i^0)\right)^2 + \delta\phi^i_{c_i}(\cdot)r''(a_i^1 + Y_i^0) \geq 0;$$

$$\frac{\partial^2 LR^i}{\partial a_i^1 \partial Y_j^0} = \delta\phi^i_{c_i;c_j}(\cdot)r'(a_i^1 + Y_i^0)r'(a_j^1 + Y_j^0) \leq 0;$$

$$\frac{\partial^2 LR^i}{\partial a_i^1 \partial a_j^1} = \pi_{a_i;a_j}\left(a^1;Y^0\right) + \delta\phi_{c_i;c_j}(\cdot)r'(a_i^1 + Y_i^0)r'(a_j^1 + Y_j^0) \leq 0:$$

We argued above that the properties of π work in favor of these inequalities. As long as (WID-II) holds, the same is true for the terms involving second derivatives of ϕ^i: the lower the opponent's state variable or investment today, the greater expected output tomorrow, and thus the greater the returns to "investing" in lower cost for tomorrow. However, in the first inequality, $\phi^i_{c_i}(\cdot)r''(a_i^1 + Y_i^0) \leq 0$, where $r'' \leq 0$ reflects the slow-down in learning for a better firm. For weak increasing dominance to arise with forward-looking firms, it is therefore important that the product market effects dominate. If the learning curve is approximately linear, as might be true in initial stages of learning, then increasing dominance holds; more generally, leading firms will produce more when δ is low enough.[41]

A natural question to ask is how these results change when firms compete in prices instead of quantities; it might seem that when "investments" are strategic complements, competing effects would arise. We can address this question by modifying the model above, so that firms compete in a differentiated Bertrand pricing game with linear demand: Then, prices are always strategic complements in the myopic case. Further, letting $a_i = -p_i$; it is straightforward to verify that $\pi^i_{a_i;Y_i} \geq 0$ and $\pi^i_{a_i;Y_j} = 0$: Then, Proposition 6 implies that weak increasing dominance holds when firms are myopic. In the two-period case, it can be shown that prices are still strategic complements if $r'(0)$ is small enough. Thus, weak

[41] Even when weak increasing dominance holds, strong increasing dominance is not guaranteed in the model described above, following the arguments of Section 3.4. Since demand is downward-sloping, cost reduction increases equilibrium demand. In contrast, Cabral and Riordan (1994) assume that total demand is independent of production costs, so that "investment" is purely business-stealing.

increasing dominance holds if r is not too concave in the relevant region, or if δ is small enough.[42]

4.3 Pricing Games with Adjustment Costs

Pricing games provide examples where investments are strategic complements. Beggs and Klemperer (1992) analyze a duopoly model with customer switching costs where decreasing dominance emerges. In their model, differentiated duopolists compete in prices $(-a_i)$ for new customers in each period, where prices are strategic complements ($\pi^i_{a_i,a_j} \geq 0$). Consumers continue to buy one unit of the good in future periods. Because of switching costs, consumers who choose one firm initially stick with this firm in the future. A firm can thus "invest" in acquiring loyal customers (Y_i) by setting low prices. Firms that already have many loyal customers find it relatively unattractive to reduce prices, as they suffer greater losses in revenues from these loyal customers ($\pi^i_{a_i,Y_i} \leq 0$). Adjustment costs are not an issue in this model ($k \equiv 0$). Finally, there is no direct interaction between the number of customers acquired by the competitor in the past and the own price in the present period, so that $\pi^i_{a_i,Y_j} = 0$. By Corollary 7, decreasing dominance arises when firms are myopic, even when there are more than two firms; further, Proposition 9 applies to the duopoly problem (reversing the sign of a_i), so that decreasing dominance holds for far-sighted firms when demand is close to linear. The approach of Proposition 9 can also be used to show that with more than two firms, decreasing dominance holds when δ is close to 0.

In a somewhat related paper, Jun and Vives (1999) analyze a duopoly model where differentiated firms compete in a Bertrand pricing game in each period (with linear demand and zero marginal cost), and each firm faces adjustment costs when changing its output from the previous period. In our framework, we let $-a_i$ be the price for firm i, let $Y_i^t = a_i^t$, and let $\Pi^i(a^t, Y^{t-1}) = q^i(-a^t)(-a_i^t) - \kappa(q^i(-Y^{t-1}) - q^i(-a^t))$, where κ is an increasing and convex adjustment cost function. Then, (WID-SC) holds in this model, and weak increasing dominance holds for myopic firms. Intuitively, a firm that had a large demand yesterday finds it more profitable to lower price today. However, for sufficiently far-sighted firms, prices may be complements within a period and substitutes across periods. In a continuous-time

[42] However, recalling our discussion following Proposition 9, the techniques developed in this paper may not be directly applicable in a more general dynamic model when (WID-SC) holds. In particular, prices may be complements within a period but substitutes across periods, potentially generating competing effects.

game, Jun and Vives (1999) …nd that the prices of the two …rms converge to the same level.[43]

5 Conclusions

This paper analyzes oligopolistic …rms that can engage in demand-enhancing or cost-reducing activities. It provides conditions under which a …rm is likely to increase an initial advantage over competitors. In many oligopoly models, leading …rms expect a larger market share, increasing the returns to further investment. We identify a natural set of conditions under which leading …rms tend to invest more when …rms are myopic or must commit to investment plans in advance, and when adjustment costs are not too much higher for leading …rms (as might be true for incremental investment).

On the other hand, a variety of competing e¤ects are possible. In some models (e.g. radical innovation or patent races), lagging …rms may receive a greater change in their state from the same level of investment. Furthermore, when …rms are su¢ ciently far-sighted and condition their investments on observed actions of competitors, competing e¤ects may arise when there are more than two …rms, and when the …rms attempt to manipulate the future investments of their opponents.

Our results have policy implications. In markets where increasing dominance is expected, apparently anti-competitive behavior, such as predatory pricing, mergers, and acquisitions might be of particular concern. However, in our model, …rms gain market share through investments that may bene…t consumers, such as cost reduction or quality improvements. Thus, the welfare e¤ects of dominance are ambiguous.[44] In the context of trade policy, where a home country might wish to improve its own market share, our results can be used to identify markets where increasing dominance is likely, so that subsidies have limited long-term impact unless the home country becomes a market leader.

More generally, Theorem 4 represents a contribution to the literature that uses lattice-theoretic tools to analyze strategic behavior. In the present context, we have argued that many commonly-studied oligopoly models have forces that favor strategic substitutability. In part to circumvent the complexities of multi-player games with strategic substitutes, existing

[43] Recalling our discussion in Section 3.4, this is not inconsistent with weak increasing dominance (or even a scenario where the leading …rm has greater market share in each period). In this model, the direct e¤ect of an investment on the state variable disappears after one period.

[44] See Cabral and Riordan (1994) and Bagwell et al. (1997) for a more detailed analysis of this point.

studies of games with strategic substitutes often impose a variety of simplifying assumptions, for example restricting attention to two-player games or considering only games where payoffs depend on the sum of opponent's choices. In this paper, we have shown that a more general assumption suffices, namely, exchangeability of the firm profit functions.

Finally, we expect that the techniques developed in this paper can be fruitfully applied in a variety of other problems in industrial organization outside of the oligopoly context. For example, the players could be workers in a firm engaged in repeated tournaments for promotions, where human capital investments are possible in each period. We might also analyze the dynamic evolution of strategic trade policies, such as export subsidies, even in the absence of innovation. In the working paper (Athey and Schmutzler, 1999), we consider the game between the governments of two countries, where the governments choose to subsidize their export industries. We provide conditions under which export subsidies are strategic substitutes. Further, we show that when the initial positions of countries are not too different, leading countries have a higher incentive to subsidize exports than lagging countries. This potentially undermines the standard infant industry protection argument.

6 Appendix

Proof of Lemma 2:(a) Suppose that inverse demand functions are given by $p_i = \alpha_i - \beta q_i - \gamma \sum_{j \neq i} q_j$, where q_i is firm i's output and $\beta > \gamma$. Define $K = \frac{1}{(2\beta - \gamma)(2\beta + (I-1)\gamma)}$, and let c_i represent firm i's marginal cost. For the case of quantity competition, tedious calculations show that

$$\frac{\partial M_i}{\partial c_i} = -K\left(2\beta^2 + \beta\gamma I - 2\beta\gamma\right); \frac{\partial M_i}{\partial c_j} = -\frac{\partial M_i}{\partial \alpha_j} = \beta\gamma K; \frac{\partial M_i}{\partial \alpha_i} = \left(2\beta^2 + (I-2)\beta\gamma\right)K;$$

$$\frac{\partial D_i}{\partial c_i} = -\frac{\partial D_i}{\partial \alpha_i} = -(2\beta + (I-2)\gamma)K; \frac{\partial D_i}{\partial c_j} = -\frac{\partial D_i}{\partial \alpha_i} = \gamma K$$

Hence markups and quantities are linear in $(c_1, \ldots, c_I)$ and $(\alpha_1, \ldots, \alpha_I)$. For $\beta > \gamma$, they are also increasing in c_i and α_j, and decreasing in c_j and α_i, so that Lemma 1 can be applied. For price competition, the calculations similar.

(b) Suppose that firm i is located at $w_i \in [0,1]$, where consumers value the good at v_i. Transportation costs for a consumer at w are given by $t(w - w_i)^2$. Assuming that in equilibrium the entire interval is covered, there exist functions d_i and m_i such that equilibrium markup and demand can be written as

$$D_i = \frac{(v_i - c_i - v_j + c_j + d_i(w_i, w_j, t))}{6t|w_j - w_i|}; \quad M_i = \frac{(v_i - c_i - v_j + c_j + m_i(w_i, w_j, t))}{3}.$$

Hence, when $c_i; v_i$ or $v_i - c_i$ is used the a state variable, the equilibrium markup and demand are both linear functions of these state variables. Similar reasoning shows that when firms are located on a circle, demand and markup are linear functions of marginal costs (see Eswaran and Gallini (1996) for details).

(c),(d) In this model,[45] firms sell products of different qualities v_i. Customers differ in their valuation θ for quality. This taste parameter is distributed uniformly across the interval $[\underline{\theta}; \bar{\theta}]$, where $\bar{\theta} \geq 2\underline{\theta}$. It is straightforward to show that the conditions of Lemma 1 hold for $Y_i = \bar{c} - c_i$ where $\bar{c}$ is some reference cost level. Now consider $Y_i = v_i$, in the boundary case that costs are identical. Profits can be written as

$$\pi^i = \begin{cases} \frac{1}{9}(v_j - v_i)[\bar{\theta} - 2\underline{\theta}]^2 & \text{for } v_i < v_j \\ \frac{1}{9}(v_i - v_j)[2\bar{\theta} - \underline{\theta}]^2 & \text{for } v_i \geq v_j \end{cases}.$$

Thus $\pi^i_{Y_i;Y_j} = 0$, wherever this is defined. It suffices to show that for $Y_i \geq Y_j$; any incremental investment dy increases profits more for firm i than for firm j. This follows because the value of such an investment is $[2\bar{\theta} - \underline{\theta}]^2 dy$ for firm i, whereas for firm j it is

$$\max\left[(\bar{\theta} - 2\underline{\theta})^2 dy; (dy - Y_i + Y_j)(2\bar{\theta} - \underline{\theta})^2 - (Y_i - Y_j)(\bar{\theta} - 2\underline{\theta})^2\right].$$

Proof of Proposition 8: Let $\tilde{a}_i = \left(a_i^1; \dots; a_i^T\right)$; $\tilde{a}_j = \left(a_j^1; \dots; a_j^T\right)$ and $\tilde{a} = (\tilde{a}_i; \tilde{a}_j)$. We need to show that $Y_r^0 > Y_s^0$ implies $\tilde{a}_r \geq \tilde{a}_s$. To apply Theorem 4, note that the long-run profit function of firm i can be written as

$$LR^i\left(\tilde{a}; Y^0\right) = \Pi^i(a^1; Y^0) + \delta \Pi^i(a^2; Y^0 + a^1) + \cdots + \delta^{t-1}\Pi^i(a^t; Y^0 + a^1 + \cdots + a^{t-1}) + \cdots$$

(OSPM) requires:[46]

$$\frac{\partial^2 LR^i}{\partial a_i^t \partial a_i^s} = \delta^{t-1}\Pi^i_{a_i;Y_i} + \delta^t \Pi^i_{Y_i;Y_i} + \cdots + \delta^{T-1}\Pi_{Y_i;Y_i} \geq 0 \text{ for } s < t.$$

To guarantee strategic substitutes, we require:

$$\frac{\partial^2 LR^i}{\partial a_i^t \partial a_j^t} = \delta^{t-1}\Pi^i_{a_i;a_j} + \delta^t \Pi^i_{Y_i;Y_j} + \cdots + \delta^{T-1}\Pi^i_{Y_i;Y_j} \leq 0 \text{ for } t \in \{1; \dots; T\}.$$

$$\frac{\partial^2 LR^i}{\partial a_i^t \partial a_j^s} = \delta^{s-1}\Pi^i_{a_j;Y_i} + \delta^s \Pi^i_{Y_i;Y_j} + \cdots + \delta^{T-1}\Pi^i_{Y_i;Y_j} \leq 0 \text{ for } t \in \{1; \dots; T\} \text{ and } s > t.$$

$$\frac{\partial^2 LR^i}{\partial a_i^t \partial a_j^s} = \delta^{t-1}\Pi^i_{a_i;Y_j} + \delta^t \Pi^i_{Y_i;Y_j} + \cdots + \delta^{T-1}\Pi^i_{Y_i;Y_j} \leq 0 \text{ for } t \in \{1; \dots; T\} \text{ and } s < t.$$

[45] To be precise, we follow a slightly modified model by Tirole (1988, p. 296).

[46] Here and in the following, mixed partials differ according to where they are evaluated; as this does not affect our results, we drop the arguments.

The increasing differences conditions require the following:

$$\frac{\partial^2 LR^i}{\partial a_i^t \partial Y_i^0} = \delta^{t-1}\pi^i_{a_i,Y_i} + \delta^t \pi^i_{Y_i,Y_i} + \cdots + \delta^{T-1}\pi^i_{Y_i,Y_i} \geq 0 \text{ for } t \in \{1,\dots,T\}$$
$$\frac{\partial^2 LR^i}{\partial a_i^t \partial Y_j^0} = \delta^{t-1}\pi^i_{a_i,Y_j} + \delta^t \pi^i_{Y_i,Y_j} + \cdots + \delta^{T-1}\pi^i_{Y_i,Y_j} \leq 0 \text{ for } t \in \{1,\dots,T\}$$

All of these conditions are implied by (WID) and (WID-D1).

Proof of Proposition 9: Let $V^{i,t}(Y^{t-1})$ be the value of the firm in period t, and let $a^t(Y^{t-1})$ be the equilibrium policy vector in period t.
Step 1: Show that condition (i) in the Proposition implies (ii). Let

$$B^i(V) = \pi^i(a^*, Y) + \delta V^i(a^* + Y)$$

where a^* is the Nash equilibrium of the auxiliary static game where player i's payoffs are given by $\pi^i(a, Y) + \delta V^i(a + Y)$. Then if π^i is quadratic, $B^i(V)$ maps quadratic functions into quadratic functions, as the equilibrium strategies a^* will be linear. Since $V^{i,T}(Y^{T-1})$ is quadratic, the value function in each period will therefore be quadratic, and the policy functions linear and continuously differentiable. (WID-A) and (WID-D2) therefore hold.
Step 2: Establish conditions required for weak increasing dominance in the penultimate period, $T-1$.

Since the payoffs and policy functions are continuously differentiable in each period, the value function is continuously differentiable for each t. Assume for notational simplicity that the value function and policy function in each period are twice differentiable. Consider the properties required for increasing dominance. As π^i satisfies (WID), (WID-D1), and (WID-D2) by assumption, analogous conditions hold replacing π^i with $\pi^i + \delta V^{i,t}$, if:

$$V^{i,t}_{Y_i,Y_j} \leq 0,\ V^{i,t}_{Y_i,Y_i} \geq 0;\ \text{and } V^{i,t}_{Y_j,Y_j} \geq 0. \tag{WID-V}$$

Consider first whether $V^{i,T}$ satisfies (WID-V). Observe that $V^{i,T}(Y^{T-1}) = \pi^i(a^T(Y^{T-1}); Y^{T-1})$. Differentiating and using the envelope theorem as described above, for $i \neq j$, $\frac{\partial^2}{\partial Y_i^{T-1}\partial Y_j^{T-1}}\pi^i(a^T(Y^{T-1}); Y^{T-1})$ can be written as follows:[47]

$$\pi^i_{Y_i,Y_j} + \pi^i_{Y_i,a_i}\frac{\partial}{\partial Y_j^{T-1}}a_i^T(Y^{T-1}) + \pi^i_{Y_i,a_j}\frac{\partial}{\partial Y_j^{T-1}}a_j^T(Y^{T-1})$$
$$+\pi^i_{Y_j,a_j}\frac{\partial}{\partial Y_i^{T-1}}a_j^T(Y^{T-1}) + \pi^i_{a_j}\frac{\partial^2}{\partial Y_i^{T-1}\partial Y_j^{T-1}}a_j^T(Y^{T-1})$$
$$+\pi^i_{a_i,a_j}\frac{\partial}{\partial Y_i^{T-1}}a_j^T(Y^{T-1})\frac{\partial}{\partial Y_j^{T-1}}a_i^T(Y^{T-1}) + \pi^i_{a_j,a_j}\frac{\partial}{\partial Y_i^{T-1}}a_j^T(Y^{T-1})\frac{\partial}{\partial Y_j^{T-1}}a_j^T(Y^{T-1})$$

[47] Here and in the following, we drop the arguments $(a^T(Y^T); Y^T)$ to simplify the exposition.

Note that $\frac{\partial}{\partial Y_j^{T-1}} a_j^T(Y^{T-1}) \geq 0$ and $\frac{\partial}{\partial Y_j^{T-1}} a_i^T(Y^{T-1}) \leq 0$ by Lemma 3. Thus, the following are sufficient to guarantee that $\frac{\partial^2}{\partial Y_i^{T-1}\partial Y_j^{T-1}} \Pi^i(a^T(Y^{T-1}), Y^{T-1}) \leq 0$ for $i \neq j$: WID, WID-D1, WID-D2, and WID-A (observing that by exchangeability, the signs of the derivatives of a_j^T can be inferred).

Similarly, $\frac{\partial^2}{\partial (Y_i^{T-1})^2} \Pi^i(a^T(Y^{T-1}), Y^{T-1})$ is given by:

$$\Pi^i_{Y_i,Y_i} + \Pi^i_{Y_i,a_i}\frac{\partial}{\partial Y_i^{T-1}}a_i^T(Y^{T-1}) + 2\Pi^i_{Y_i,a_j}\frac{\partial}{\partial Y_i^{T-1}}a_j^T(Y^{T-1}) + \Pi^i_{a_j}\frac{\partial^2}{\partial (Y_i^{T-1})^2}a_j^T(Y^{T-1})$$
$$+ \Pi^i_{a_i,a_j}\frac{\partial}{\partial Y_i^T}a_j^T(Y^{T-1})\frac{\partial}{\partial Y_i^T}a_i^T(Y^{T-1}) + \Pi^i_{a_j,a_j}\left(\frac{\partial}{\partial Y_i^T}a_j^T(Y^{T-1})\right)^2$$

This is positive under the same set assumptions; finally, these assumptions are also sufficient to guarantee that $\frac{\partial^2}{\partial (Y_j^{T-1})^2} \Pi^i(a^T(Y^{T-1}), Y^{T-1})$ is positive, as can be verified in a similar way. Since (WID-V) holds for V^T, weak increasing dominance holds in period $T-1$.

Step 3: Use induction to show that WID-V holds for each t.

The assumptions of the Proposition guarantee that WID-A holds for all t, and that WID-V holds for $t = T$. Suppose that WID-V holds for arbitrary t. Then, following similar arguments to above, and since $V^{i,t}$ is continuously differentiable for each t, the derivatives of $\Pi^i(a^t(Y^{t-1}), Y^{t-1}) + \delta V^{i,t}(a^t(Y^{t-1}) + Y^{t-1})$ can be analyzed following the approach of Step 3. Thus, if WID-V holds in period t, it will hold for $t-1$. By induction, WID-V holds for all t.

Step 4: The assumptions of the Proposition together with WID-V imply that in each period t, Theorem 4 applies to guarantee weak increasing dominance.

7 References

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