Assignment Reversals:

Trade, Skill Allocation and Wage Inequality*

Thomas Sampson†

London School of Economics

November 2013

Abstract

The allocation of skilled labor across industries shapes inter-industry wage differences and wage inequality. This paper shows that the ranking of industries by workforce skill differs between developed and developing economies and builds a multi-sector assignment model to explain the causes and consequences of this fact. Heterogeneous agents leverage their ability through their span of control over an homogeneous non-labor input. In equilibrium, higher skill agents obtain larger spans of control by sorting into sectors where the cost per efficiency unit of input is lower. Consequently, skill allocation is endogenous to country-sector specific variation in input productivity and costs. This generates novel implications for how technical change and trade affect wages. Since scale is complementary to skill, wage inequality rises following shocks that cause agents to increase their spans of control, such as input augmenting productivity growth and input trade liberalization. Moreover, when the sector skill ranking differs across countries each country has a comparative advantage in its high skill sector and output trade integration causes the relative wage of high skill workers, and wage inequality within the high skill sector, to increase in both countries.

*I am grateful to Pol Antràs, Arnaud Costinot, Elhanan Helpman, Marc Melitz, Nathan Nunn and seminar participants at Cambridge, CREI, Harvard, LSE, Stockholm IIES, Toronto Rotman, Warwick, the World Bank and the European Economic Association meetings in Oslo for helpful comments and suggestions.

†Centre for Economic Performance, Department of Economics, London School of Economics. E-mail: t.a.sampson@lse.ac.uk

1 Introduction

What determines the allocation of skill across sectors? Answering this question is a necessary step towards explaining the structure of employment and wages. For example, inter-industry wage differences are mostly due to variation in workforce composition (Krueger and Summers 1986; Abowd, Kramarz and Margolis 1999; Gibbons et al. 2005), while the effects of technical change and globalization on the wage distribution depend on how skill levels vary across sectors. In particular, the classic Stolper-Samuelson theorem linking trade integration to the skill premium relies on the assumption that the ranking of sectors by skill intensity is constant across countries.

Contradicting this assumption, industry level data implies the ranking of sectors by workforce skill varies systematically across countries. Define the "wage rank correlation" to be the rank correlation of a country's industry wages with industry wages in the US. Figure 1 shows wage rank correlations plotted against income.[1] Although the correlation is always positive, it is strongly increasing in income. While industrialized countries have similar industry wage structures to the US, the industry wage ranking varies substantially between low and high income countries. Section 2.1 shows that the correlation observed in Figure 1 is a robust feature of industry wage data sets.

Under the assumption that inter-industry wage differences primarily reflect workforce skill,[2] Figure 1 implies the existence of an important phenomenon: assignment reversals. I define an assignment reversal to occur whenever the ranking of sectors by workforce skill differs across countries.[3] The goal of this paper is to explain the cause of assignment reversals and understand the implications of assignment reversals for skill allocation and the wage structure.

Existing theories that incorporate skill allocation in multi-sector economies do not admit the possibility of assignment reversals because they neither endogenize the determinants of the allocation of skill across sectors nor allow for variation in the ranking of sectors by workforce skill.[4] Instead multi-sector models customarily impose restrictions on the production technology that ensure the existence of an exogenous and invariant ranking of sectors by workforce skill. In comparative advantage assignment models (Sattinger

[1]The wage data is from the UNIDO Industrial Statistics database and covers 42 countries and 127 ISIC 4 digit manufacturing industries in 2000. Income is from the Penn World Tables 6.3. See Section 2.1 and Appendix C for a complete description of the data.

[2]Cross-country data on workforce skill is not available at the level of disaggregation used in Figure 1. However, Section 2.1 shows there exists cross-country variation in the ranking of industries by workforce education at a more aggregate level.

[3]Kurokawa (2011) documents the existence of an assignment reversal between the US and Mexico.

[4]An exception is Murphy, Shleifer and Vishny (1991) who discuss the possibility of cross-country assignment reversals in the allocation of talent between rent seeking and entrepreneurial activities.

1975; Ohnsorge and Trefler 2007; Costinot and Vogel 2009; Acemoglu and Autor 2011) the critical assumption is that sectors can be ordered such that the production technology is log supermodular in labor skill and a sector index. The ordering of sectors then uniquely determines the sector skill ranking. In the Heckscher-Ohlin model the relevant restriction is the no factor intensity reversals assumption. A possible explanation for assignment reversals is the existence of Heckscher-Ohlin skill intensity reversals. However, skill intensity reversals result from cross-country skill premium variation and in Section 2.2 I find no evidence that the assignment reversals evident in Figure 1 are driven by differences in skill premia.

To explain assignment reversals this paper starts from the idea that the returns to skill depend on the extent to which skilled workers leverage their ability through their span of control over other production inputs. This idea has motivated work on the allocation and pay of managerial talent (Rosen 1982; Garicano and Rossi-Hansberg 2006; Gabaix and Landier 2008; Terviö 2008), but has not been used to study the allocation of skill across sectors. Since a larger span of control raises the returns to skill, skilled workers sort into sectors where the inputs that complement skill are cheaper or more productive. Assignment reversals then occur when the ranking of sectors by input costs or productivity varies across countries. To formalize this argument the paper introduces two new features to the assignment literature. First, it marries Roy (1951) to Becker (1973) by allowing for both multiple sectors and matching between two factors of production with non-zero opportunity costs: heterogeneous labor and an homogeneous intermediate input.[5] Second, it explains the equilibrium assignment in terms of an observable sector level characteristic: intermediate input productivity.

Consider a competitive economy with a continuum of agents who differ along a single dimension of heterogeneity called skill and sort across a finite number of sectors. In comparative advantage assignment models the production technology is assumed to take the form:

$$y(\theta, k) = g(\theta)F(\theta, k),$$

where y is the output of a skill θ agent working in sector k. Provided F is log-supermodular there is positive assortative matching of high skill agents to high k sectors. I extend this framework by allowing for production to use both labor and an intermediate input that can be interpreted as materials, land or capital. A

[5] Sattinger (1979) considers the problem of matching heterogeneous workers to machines of different quality when all worker-machine pairs produce the same output good and machines are in fixed supply. However, in existing models with multiple sectors either production uses a single input as in the comparative advantage assignment models discussed below or production combines different types of labor in fixed quantities (Grossman and Maggi 2000; Grossman 2004).

production team comprising one agent working with a quantity x of intermediate input in sector k produces output:

$$y(\theta, k) = g(\theta)F(\theta, Q_k x),$$

where Q_k represents intermediate input productivity in sector k, g is strictly increasing in θ and F exhibits constant returns to scale. Variation in Q induces changes in the cost per efficiency unit of intermediate input and higher Q is equivalent to a reduction in the price of the intermediate input. The restriction on g implies the existence of increasing returns to skill. Importantly, the quantity of intermediate input used by each agent is endogenous and is chosen to maximize profits.

In equilibrium, a log-submodular production function implies positive assortative matching between skill and sectoral intermediate input productivity. Interestingly, this result reverses the condition on F required for positive assortative matching in comparative advantage assignment models. The switch is a consequence of making intermediate input quantity endogenous. The choice of x determines an agent's span of control. Since there are increasing returns to skill, higher skill agents leverage their abilities by having larger spans of control. F is strictly log-submodular if and only if the elasticity of substitution between inputs exceeds unity and this substitutability implies that agents with greater spans of control produce relatively more in sectors where the cost per efficiency unit of intermediate input is low. Consequently, positive assortative matching is the efficient allocation. This is an example of the scale of operations effect discussed in Sattinger (1993). By contrast, if each agent must work with the same quantity of intermediate input, substitutability mandates that high skill agents work with low productivity intermediate inputs and log-submodularity of F implies negative assortative matching.[6]

The assignment solution unpacks the determinants of labor assignment and gives a testable prediction linking the allocation of skill to observable intermediate input properties. In particular, assignment reversals occur whenever the ranking of sectors by the cost per efficiency unit of intermediate input varies across countries. What could cause such variation? First, Ricardian intermediate input augmenting technology differences at the country-sector level. For example, in France land is more productive for producing wine than rice, while in Bangladesh the reverse is true. Similarly, countries with better contracting institutions are likely to have higher relative productivity in sectors that use contract intensive intermediate inputs. Second,

[6]Similarly, if the production function is strictly log-supermodular the equilibrium assignment exhibits positive assortative matching if the intermediate input quantity is fixed and negative assortative matching if it is endogenous.

intermediate input price variation. Trade costs imply that the price of imported intermediates increases with distance from the exporting country, which raises relative input costs in sectors that are intensive in imported intermediates. Interpreting Figure 1 through the lens of the assignment model implies that intermediate input productivity levels and costs are similar across high income countries, but differ substantially between high and low income economies.[7]

The equilibrium assignment also has important implications for the distribution of wages:

1. Labor's share of output is decreasing in worker skill and, therefore, in wages – a correlation that is observed empirically.

2. At any given skill level, the returns to skill (the elasticity of wages with respect to skill) are higher in sectors with greater intermediate input productivity. Consistent with this prediction Gibbons et al. (2005) estimate that the returns to skill are greater in occupations that employ higher skill workers.

Moreover, I show that the span of control is a sufficient statistic for both wage inequality and labor's share of output. Whenever agents' spans of control increase wage inequality rises and labor's output share falls. Thus, in the assignment model changes in the distribution of wages across agents are associated with changes in the distribution of income across factors. Also, it is straightforward to use the model to study the causes of variation in wage inequality.

To understand the broader implications of the assignment reversals model, I embed the assignment problem in general equilibrium and analyze the effects of technical change and trade integration. Technical change that reduces the cost per efficiency unit of intermediate input is complementary to skill in two ways. First, a positive technology shock increases the skill level of agents assigned to a sector whenever it changes the ranking of sectors by the cost per efficiency unit of intermediate input. Consequently, any sector that experiences a sufficiently large positive technology shock will become a high skill sector. Second, when the cost per efficiency unit of intermediate input falls agents increase their spans of control and this disproportionately benefits more skilled agents who are better able to leverage their abilities. In general equilibrium technical change in either sector raises the returns to skill and wage inequality within both sectors. In addition, the increase in leverage caused by technical change reduces labor's output share.

Trade integration in the assignment model can take two forms: intermediate input trade and output trade. Trade in intermediates enables importers to purchase the intermediate input at lower cost. Thus, it

[7] Hsieh and Klenow (2007) argue that low income countries have low relative productivity in investment good production. Their evidence does not address within-country, cross-sector input productivity differences, but their conclusions are consistent with the idea that income convergence goes hand-in-hand with input productivity and cost convergence.

is equivalent to input cost reducing technical change and causes an increase in within sector returns to skill and wage inequality in the importing country. Suppose we interpret the intermediate input as capital. Since imports account for the majority of equipment investment in most lower and middle income countries (Eaton and Kortum 2001; Caselli and Wilson 2004), the model predicts that the liberalization of capital trade has led to higher inequality in these countries. This prediction is consistent with the findings of Csillag and Koren (2009) and Parro (2010) who show that capital imports increase the relative wage of high skill labor.

The effects of output trade liberalization are particularly interesting when there is an assignment reversal across countries. In a two country, two sector model I show that when the ranking of sectors by the cost per efficiency unit of intermediate input differs across countries: (i) both countries have a comparative advantage in their high skill, high wage sector regardless of their relative factor endowments; (ii) trade liberalization causes the high skill sector to expand in both countries and both countries export the output of their high skill sector, and; (iii) in both countries trade liberalization causes wage levels and wage inequality to increase in the high skill sector and decrease in the low skill sector. Thus, assignment reversals overturn the Stolper-Samuelson predictions. Since industry data implies assignment reversals occur most frequently between North and South, the assignment reversals model provides a new explanation for why the effects of trade liberalization on wage inequality in many unskilled labor abundant developing countries have been inconsistent with the Stopler-Samuelson theorem.[8] Regardless of whether there are assignment reversals, the assignment model also predicts that output trade integration raises wage inequality within the export sector of both countries. Brambilla et al. (2012) find that in Latin American countries the skill premium is higher in industries that export a greater share of output.

These results demonstrate that although this paper is related to the existing open economy multi-sector assignment literature (Grossman and Maggi 2000; Ohnsorge and Trefler 2007; Costinot 2009; Costinot and Vogel 2009), incorporating non-labor factors of production and assignment reversals not only explains the allocation of skill, but also leads to novel implications concerning the effects of technical change and globalization on labor markets. Technical change is skill-biased whenever it reduces the cost per efficiency unit of input; input trade liberalization is equivalent to skill-biased technical change, and; assignment reversals overturn the Stolper-Samuelson theorem. The common thread linking these results is that when the skill

[8]Goldberg and Pavcnik (2007) summarize empirical work linking trade liberalization episodes to higher wage inequality in developing economies. For alternative mechanisms explaining how trade can increase wage inequality see, for example, Davis (1996); Feenstra and Hanson (1996); Manasse and Turrini (2001); Yeaple (2005); Matsuyama (2007); Verhoogen (2008); Helpman, Itskhoki and Redding (2010); Monte (2011), and; Sampson (2013). None of these papers endogenize the sectoral skill allocation.

allocation is driven by leverage, scale is complementary to skill, and shocks that lead to increases in scale also raise wage inequality.

The remainder of the paper is organized as follows. Section 2 motivates the theory by using industry data to demonstrate the existence of cross-country assignment reversals. Section 3 develops and solves the assignment problem in partial equilibrium. Section 4 embeds a two sector version of the assignment problem in general equilibrium and analyzes the impacts of technical change and variation in the skill supply, while Section 5 extends the model to a two country open economy setting and studies the effects of output and intermediate input trade. Finally, Section 6 concludes.

2 Assignment reversals evidence

The allocation of skill across sectors can be inferred from data on industry wages and workforce characteristics. Since, for most countries, industry wage data is available at a more disaggregated level than measures of workforce skill, this section starts by treating an industry's mean wage per employee as a measure of the average skill of the industry's workforce. Under this assumption, I demonstrate that industry wage data implies the existence of cross-country assignment reversals that are systematically related to countries' income levels. I then show that using available data on observable measures of workforce skill leads to the same conclusion. Finally, I consider the possibility that the assignment reversals found in the data are due to Heckscher-Ohlin skill intensity reversals. I find no support for this hypothesis.

2.1 International wage structure comparisons

The assumption that inter-industry wage differences primarily reflect differences in workforce skill, rather than variation in industry specific rents is supported by the empirical literature on inter-industry wage differences. Krueger and Summers (1986) find that observable worker characteristics alone account for around half of inter-industry wage differences in the US. Moreover, once panel data is used to also control for unobservable worker characteristics, the explanatory power of workforce composition rises further (Abowd, Kramarz and Margolis 1999; Abowd, Creecy and Kramarz 2002).

Studies of inter-industry wage differences have generally concluded that the pattern of industry wages is highly correlated across countries. For example, Krueger and Summers (1986) find that in eight of the thirteen countries they consider the correlation of log wages with the US exceeds 0.8,[9] leading them to conclude that the "wage structure is amazingly parallel in looking at data for different countries" (p.1). However, the consensus found in the literature has emerged primarily from comparisons between industrialized economies. Noting that four of the five countries with correlations below 0.8 are non-industrialized economies Krueger and Summers (1986) caution that the wage structure in mature capitalist economies is "different from that of Communist or less developed economies" (p.2).

Figure 1, discussed previously in the introduction, shows that this claim continues to hold when looking at industry wage data for a broader sample of countries than considered by Krueger and Summers (1986). Remember that Figure 1 shows wage rank correlations (the correlation between the ranking of industries by wage levels in a given country and the ranking in the US) plotted against income levels (expressed as log differences from US income). Regressing the wage rank correlation on the income difference gives a significant slope coefficient of 0.13 (Table 1, column a). The positive association is robust to weighting observations by industry employment shares when calculating the wage rank correlations (column b) and to computing correlations using wages instead of wage ranks (columns c and d). An alternative approach to quantifying the similarity between a country's inter-industry wage structure and that of the US is to compute the proportion of industry pairs in which the ranking of industries by wage levels is the same as in the US. Figure 2 shows that for a country such as France the proportion exceeds 80%, but for the poorest country in the sample, Bangladesh, it is only 61%. The relationship between income and the proportion of pairwise rank matches is positive and significant (column e). Industry wage data also implies that income convergence with the US is associated with convergence towards the US inter-industry wage structure. Regressing the change in the wage rank correlation on the change in income relative to the US for 70 countries between 1965 and 1995 gives a significant slope of 0.12 (column f).[10]

If poorer countries report less reliable data, these findings could be caused by measurement error. To allay this concern Figure 3 shows wage rank correlations plotted against income using industry wage data for 1995 taken from the EU KLEMS database. The EU KLEMS database is designed to provide accurate industry level data for use in growth accounting exercises. The database covers 29 countries (the EU-25 plus Australia, Japan, South Korea and the US) and, at its most disaggregated level, 29 manufacturing industries. Again, the wage rank correlation is strongly increasing in income, but the slope of the relationship is larger

[9] The correlations are calculated using wage data for around 20 manufacturing industries in 1981 or 1982.

[10] The wage data covers 28 ISIC 3 digit manufacturing industries. See Appendix C for further details.

than in the UNIDO data. Regressing the wage rank correlation on the income difference gives a slope of 0.26.

The results in Table 1 support Krueger and Summers' (1986) hypothesis that while developed countries have strikingly similar industry wage structures, this similarity does not extend to developing economies. Under the maintained assumption that inter-industry wage differences stem from variation in workforce skill, the cross-country variation in wage rank correlations implies that assignment reversals exist and occur more frequently between countries at different stages of development than between countries with similar income levels.

Unfortunately, cross-country data on industry workforce skill is not available at the same level of disaggregation as industry wage data. However, the IPUMS-International database of individual-level censuses does report both respondents' industry (at approximately the 1 digit level) and their educational attainment. From this data I calculated the share of workers in each country-industry pair who had completed secondary school and used this measure of industry skill intensity to compute the "skill rank correlation" of each country with the US. There is a positive association between skill rank correlations and income, although the slope of 0.018 is smaller than for the wage rank correlations considered above (Table 2, column a). A stronger positive association is found if tertiary education completion shares are used (column b). These results are consistent with the industry wage data, but more disaggregated workforce skill data is needed to directly observe assignment reversals.

2.2 Skill intensity reversals?

Could the cross-country variation in wage rank correlations be caused by Heckscher-Ohlin skill intensity reversals?[11] Consider a multi-sector Heckscher-Ohlin economy in which production uses two types of labor: skilled and unskilled. In each industry the skill intensity of production will depend on the skill premium and the elasticity of substitution between skilled and unskilled labor. If skill intensity reversals occur, then industries in which the elasticity of substitution is relatively high will be skilled labor intensive in countries with low skill premia and unskilled labor intensive in countries with high skill premia. In particular, if all industries use constant elasticity of substitution production technologies it is simple to show that the number of skill intensity reversals between any two countries is an increasing function of the difference between

[11] See Minhas (1962) and Leontief (1964) for analysis of the conditions under which factor intensity reversals may occur and a debate over their existence. The extensive literature on factor intensity reversals tends to conclude that capital intensity reversals are of limited empirical relevance, but has largely overlooked skill intensity reversals.

their skill premia.[12] Therefore, if variation in wage rank correlations is caused by Heckscher-Ohlin skill intensity reversals, it will be strongly correlated with variation in skill premia.

Internationally comparable measures of the skill premium are not available for the majority of the countries in the UNIDO sample used in Figure 1. However, differences in skill premia across countries are well explained by variation in human capital levels.[13] Therefore, to crudely examine whether cross-country differences in the inter-industry wage structure are due to Heckscher-Ohlin skill intensity reversals I regress the wage rank correlations shown in Figure 1 on countries' stocks of physical and human capital per capita.[14] There is a strong positive association between the capital stock and the wage rank correlation, but the human capital variable is insignificant (Table 3, column a). Similar results are obtained when the secondary school enrollment rate, which is available for a larger sample of countries, is used as a proxy for the skill premium (column b). These findings do not support the conjecture that Heckscher-Ohlin skill intensity reversals are driving cross-country variation in wage rank correlations. Therefore, I will now proceed to develop an assignment model of skill allocation that explains the causes and consequences of assignment reversals.

3 Assignment model

At the heart of this paper is an assignment problem. How do heterogeneous workers sort across sectors? To understand the mechanism that drives sorting this section introduces the assignment problem and characterizes the equilibrium assignment.

3.1 Assignment problem

Consider an economy facing the following assignment problem. There exists an heterogeneous factor that differs along a single dimension of heterogeneity indexed by θ. To be concrete, suppose the factor is labor and there are a continuum of agents with differing skill levels θ. Let $M(\theta)$ be the mass of agents with skill less than or equal to θ and suppose M has support on $\left(0, \bar{\theta}\right]$. Bounded support is the only restriction on the skill distribution M required to obtain the main results of the paper.

The economy comprises K productive activities in which labor can be employed. The interpretation of these productive activities depends on how the assignment problem is embedded in general equilibrium.

[12] See Reshef (2007) for a theoretical analysis of the causes and consequences of skill intensity reversals in such a model.

[13] See, for example, Fernàndez, Guner and Knowles (2005) and Brambilla et al. (2012).

[14] The physical and human capital variables are computed using the methodology of Caselli (2005) and are expressed as the log difference from US physical and human capital per capita, respectively. See Appendix C for details.

For consistency with the general equilibrium model in Section 4 I will refer to the productive activities as sectors, but they could also be tasks or occupations. Each sector produces a different good and the production technology varies across sectors. The assignment problem is to characterize the mapping of agents to sectors.

Suppose that in all sectors output is produced by production teams, each of which consists of one agent working with an intermediate input.[15] In particular, let the output of a skill θ agent working with x units of intermediate input in sector k, $y_k(\theta, x)$, be given by:

$$y_k(\theta, x) = g(\theta)F(\theta, Q_k x), \tag{1}$$

where g is non-negative, differentiable and strictly increasing and F is a twice differentiable, constant returns to scale function that is strictly increasing in both its arguments, strictly concave and satisfies $\lim_{\theta\to 0} \frac{\partial F}{\partial \theta} = \lim_{x\to 0} \frac{\partial F}{\partial x} = \infty$. Within a sector all production teams produce the same output good.

Four features of the production function are particularly noteworthy. First, the labor input to production is indivisible. If, instead, agents with different skill levels were perfect substitutes within production teams, θ would simply measure an agent's efficiency units of labor and there would be no assignment problem. Second, skill enters production symmetrically in every sector. Holding $Q_k x$ fixed, the marginal effect of skill on output is constant across sectors. Third, g captures the existence of increasing returns to ability. Fourth, Q_k is an intermediate input augmenting productivity term that represents the level of technology in sector k. This is the only source of cross-sector heterogeneity.[16] Note that $Q_k x$ can be interpreted as the quantity of intermediate input used measured in efficiency units. I assume that sectors are ordered by ascending Q_k such that sector one is the least technologically advanced and sector K the most.

Assume there is perfect competition in all markets, that all sectors must produce positive aggregate output and that the intermediate input is in perfectly elastic supply at cost p. Provided x is a choice variable there is no loss of generality in assuming all sectors use the same intermediate input since allowing for variation in input cost across sectors is equivalent to varying Q_k. We can now solve the assignment problem in partial equilibrium taking the intermediate input cost and the existence of positive aggregate demand for each sector's output as given. The general equilibrium model analyzed in Section 4 shows that that

[15]The model does not speak to where the boundaries of the firm may lie, so I will refer to the basic unit of activity as a production team.

[16]Appendix B shows how the production function can be generalized to incorporate multiple sources of cross-sector heterogeneity and demonstrates that the effect of intermediate input productivity on the equilibrium assignment is robust to this extension.

partial equilibrium assignment patterns are robust to endogenizing intermediate input supply and explicitly specifying output demand.

Formally, the production function in (1) is similar to that used by Rosen (1982) in a single sector model of firm hierarchies. In theory, the intermediate input could represent materials, land, machines or an homogeneous labor input. Cross-sector technology heterogeneity captures intermediate input augmenting productivity differences that cause variation in the productivity, or equivalently the cost per efficiency unit, of the intermediate input. Such variation could result from sector specific intermediate input augmenting technology investments or from sector specific differences in the price of intermediates. Following Rosen (1982) the form of the production function can be motivated by assuming that a skill θ agent supplies θ units of labor input and produces output of quality $g(\theta)$. $Q_k x$ denotes the quantity of intermediate input used, measured in efficiency units, and diminishing returns to intermediates result from spreading a fixed labor input over an increasing quantity of intermediate input. However, the fact that higher skill agents produce higher quality output means there are increasing returns to skill.

3.2 Equilibrium assignment

The equilibrium assignment depends crucially on whether the quantity of intermediate input used x is endogenous. Let us suppose that each agent can choose how much input to work with. Since there is perfect competition each agent's wage will equal the profit of her production team $\pi_k y_k(\theta, x) - px$, where π_k is the price of sector k output. Remembering that F has constant returns to scale we can define $f\left[s_k(\theta)\right] = \frac{1}{\theta} F(\theta, Q_k x)$, where $s_k(\theta) \equiv \frac{Q_k x}{\theta}$ is the span of control of a skill θ agent working in sector k. $s_k(\theta)$ measures the efficiency units of intermediate input used per unit of skill and captures the extent to which an agent leverages her ability by working with large amounts of the intermediate input. Note that f is strictly increasing and strictly concave. With this change of variables:

$$y_k(\theta) = \theta g(\theta) f\left[s_k(\theta)\right], \tag{2}$$

and profit maximization implies:

$$f'\left[s_k(\theta)\right] = \frac{p}{\pi_k Q_k g(\theta)}. \tag{3}$$

Invoking the concavity of f we have that span of control is strictly decreasing in the input cost p, but strictly increasing in the output price π_k, level of technology Q_k and skill θ. Span of control is increasing in θ only because $g' > 0$, that is because there exist increasing returns to ability.

$$\frac{ds_k}{d\left(\frac{\pi_k Q_k}{p}\right)} > 0, \qquad \frac{ds_k}{d\theta} > 0.$$

Henceforth, I will suppress the dependence of s_k on θ unless its inclusion is necessary to avoid confusion.

Choosing the optimal span of control solves the income maximization problem of an agent conditional on her sector, but how do agents sort across sectors? Using (3) we obtain that the wage $w_k(\theta)$ of an agent in sector k is:

$$w_k(\theta) = \pi_k \theta g(\theta) \left[f(s_k) - s_k f'(s_k) \right]. \tag{4}$$

In equilibrium each agent will choose to work in the sector where her wage is greatest and output prices will adjust to ensure a positive mass of agents is assigned to every sector.[17] Consider an agent choosing between two sectors k and l with $Q_k > Q_l$:

$$\frac{w_k(\theta)}{w_l(\theta)} = \frac{\pi_k}{\pi_l} \frac{f(s_k) - s_k f'(s_k)}{f(s_l) - s_l f'(s_l)}. \tag{5}$$

The requirement that neither sector offers a strictly higher wage at all skill levels is sufficient to generate useful restrictions on equilibrium output prices. Suppose $\pi_k \geq \pi_l$. Then, since $Q_k > Q_l$, we must also have $\pi_k Q_k > \pi_l Q_l$. However, noting that $f(s_k) - s_k f'(s_k)$ is strictly increasing in s_k and, therefore, in $\pi_k Q_k$, it follows that if both $\pi_k \geq \pi_l$ and $\pi_k Q_k > \pi_l Q_l$ sector k strictly dominates sector l. To avoid this possibility we must have $\pi_k < \pi_l$. The intuition is straightforward – if all sectors have positive employment it is not possible that one sector both uses the intermediate input more productively and has a higher output price than another. Similarly, to guarantee sector l does not dominate we must have $\pi_k Q_k > \pi_l Q_l$, which implies $s_k(\theta) > s_l(\theta)$. An agent's span of control is greater in the more technologically advanced sector.

To obtain the equilibrium assignment we can now differentiate (5) obtaining:

$$\frac{d}{d\theta} \left[\frac{w_k(\theta)}{w_l(\theta)} \right] \propto \epsilon^g(\theta) \left[\epsilon^f(s_k) - \epsilon^f(s_l) \right], \tag{6}$$

where $\epsilon^g(\theta)$ is the elasticity of g with respect to skill and $\epsilon^f(s)$ is the elasticity of f, and consequently of

[17] This requirement follows from the demand assumption that all sectors produce positive aggregate output.

output, with respect to the span of control. I will refer to $\epsilon^f(s)$ as the output elasticity. Equation (6) has two important implications. First, if $g' = 0$ (implying constant returns to ability) then all agents are indifferent between sectors and there is no sorting. Second, given $g' > 0$, the sign of the right hand side of (6) depends on whether the output elasticity is increasing or decreasing in the span of control. The properties of the output elasticity can be determined using the following lemma. The proofs of all lemmas and propositions are in Appendix A.

Lemma 1. *The following are equivalent: (i) F is strictly log-submodular; (ii) F has elasticity of substitution greater than one; (iii) $\epsilon^f(s)$ is strictly increasing in the span of control s.*

Similarly, strict log-supermodularity of F is equivalent to F having elasticity of substitution σ less than one and to $\epsilon^f(s)$ being strictly decreasing in s, while if $\sigma = 1$ then $\epsilon^f(s)$ is independent of s.[18] Following Acemoglu (2002) I will refer to labor and intermediates as gross complements if $\sigma < 1$ and gross substitutes if $\sigma > 1$. Note that σ need not be constant, but any restrictions on σ are assumed to hold globally.

We showed above that since $Q_k > Q_l$ span of control is higher in sector k than in sector l. Consequently, if the output elasticity is strictly increasing in the span of control then $\epsilon^f(s_k) > \epsilon^f(s_l)$ and (6) implies that the relative wage in sector k is strictly increasing in ability. Moreover, to ensure neither sector dominates the other there must exist a threshold $\tilde{\theta} \in \left(0, \bar{\theta}\right]$ such that agents with skill below $\tilde{\theta}$ strictly prefer sector l and agents with skill above the threshold strictly prefer sector k. Therefore, appealing to Lemma 1 it follows that when F is log-submodular there is positive assortative matching and high skill agents prefer the more technologically advanced sector. However, if F is log-supermodular the sorting pattern is reversed and high skill agents select into the low technology sector. If F has unit elasticity of substitution there is no sorting because all agents are indifferent between sectors.

What explains these assignment patterns? The higher an agent's skill, the larger her span of control in any given sector. Consequently, high skill agents select into the sector with a higher output elasticity. Moreover, holding skill constant, span of control is increasing in intermediate input productivity and when the factors of production are gross substitutes the output elasticity is increasing in span of control. Therefore, high skill agents sort into the high productivity sector. Intuitively, the lower cost per efficiency unit of intermediate input allows high skill agents to exploit the substitutability between factors and leverage their

[18] See Costinot (2009) for a definition and discussion of log-supermodularity and log-submodularity. In particular, I use the fact that F is strictly log-submodular if and only if $\frac{\partial^2 \log F}{\partial\theta\partial x} < 0$. Though implicit in Sattinger (1975) and Kugler and Verhoogen (2008), I am not aware of previous work that demonstrates explicitly the link between log-supermodularity, log-submodularity and the elasticity of substitution of a constant returns to scale production function.

ability by working with large quantities of intermediates. This is an example of a scale of operations effect (Sattinger 1993). However, if the factors of production are gross complements having a greater span of control reduces the output elasticity because the complementarity between factors diminishes the value of working with large quantities of intermediates when the labor input is fixed. Therefore, high skill agents reduce their spans of control by choosing the low technology sector.

The preceding discussion considers only two sectors. However, by comparing all pairs of sectors it is straightforward to extend the results to encompass K sectors. The ranking of sectors by productivity Q_k fully determines the ranking of output prices π_k and of $\pi_k Q_k$. With $Q_K > Q_{K-1} > \ldots > Q_1$, then in any equilibrium such that all sectors produce positive aggregate output:

(i) $\pi_1 > \pi_2 > \ldots > \pi_K$;

(ii) $\pi_1 Q_1 < \pi_2 Q_2 < \ldots < \pi_K Q_K$.

These orderings hold regardless of the value of σ. The role of the elasticity of substitution comes in determining how agents sort across sectors. As shown in the proof of Proposition 1, if $\sigma > 1$, meaning that F is strictly log-submodular, there is positive assortative matching:

(iii) $\exists\, 0 = \theta_0 \leq \theta_1 \leq \ldots \leq \theta_{K-1} \leq \theta_K = \bar{\theta}$ such that only agents with skill $\theta \in [\theta_{k-1}, \theta_k]$ are employed in sector k.[19]

This means that in equilibrium agents are partitioned by ability such that higher ability groups of agents select into higher technology sectors. If $\sigma < 1$ the sorting pattern is reversed and there is negative assortative matching. Proposition 1 characterizes the equilibrium assignment of agents to sectors when the quantity of intermediate input used is endogenous.

Proposition 1. *(Endogenous intermediate input quantity.) If the production function is strictly log-submodular then the equilibrium assignment of agents to sectors exhibits positive assortative matching. High skill agents are assigned to sectors with high levels of technology. If the production function is strictly log-supermodular then the equilibrium displays negative assortative matching.*

Proposition 1 gives a concrete, testable explanation for what determines the allocation of skilled labor across sectors – variation in intermediate input augmenting productivity. This motivation for sectoral sorting is not found in the existing assignment literature which only allows for a single factor of production. I explore below the implications of this result, but one consequence is immediately apparent. Assignment

[19] The inequalities in (iii) will be strict if there are no mass points in the distribution of θ.

reversals occur when the ranking of sectors by intermediate input productivity differs across countries. Section 5.1 analyzes the implications of assignment reversals for international trade theory.

It is useful to compare Proposition 1 with the predictions of the comparative advantage assignment literature (Sattinger 1975; Ohnsorge and Trefler 2007; Costinot 2009; Costinot and Vogel 2009; Acemoglu and Autor 2011). In this literature there is a single heterogeneous factor of production, the production function is Ricardian and log-supermodularity of the production function leads to positive assortative matching between the heterogeneous factor and sectors. For example, if the factor is labor then log-supermodularity of labor productivity in skill and some variable that indexes sectors implies that in equilibrium more skilled labor is assigned to sectors where the marginal effect of skill on labor productivity is greater. By contrast, Proposition 1 implies that log-submodularity of the production function implies positive assortative matching. To reconcile this apparent contradiction we must interpret the production function used in comparative advantage assignment models as a reduced form representation of the revenue function net of all non-labor input costs. This net revenue function is equivalent to the wage function $w_k(\theta)$ discussed above and differentiation of (4) shows that the wage is log-supermodular in θ and Q_k if and only if output is log-submodular and there are increasing returns to ability. An important contribution of this paper is to show how the properties of the net revenue function used in previous assignment models are related to the properties of the underlying production technology when there are two factors of production.[20]

The key assumption which ensures log-submodularity implies positive assortative matching is not that production uses an intermediate input, but that the input level x is endogenously chosen. Suppose instead that each agent must work with a fixed quantity $\tilde{x}$ of intermediate input. In this case each agent is assigned to the sector where she generates the greatest revenue, exactly as happens in the comparative advantage assignment literature. Wages are given by:

$$w_k(\theta) = \pi_k g(\theta) F(\theta, Q_k \tilde{x}) - p\tilde{x},$$

and comparing sectors k and l with $Q_k > Q_l$ we have that when $w_k(\theta) = w_l(\theta)$:

[20]Note that when production uses intermediate inputs a distinction must be made between the primitive production function given in (1) and the equilibrium output function $y_k(\theta, Q_k) = \theta g(\theta) f\left[s_k(\theta)\right]$ which gives output conditional on the optimal input choice. When F is log-submodular, the equilibrium output function can be either log-submodular or log-supermodular. However, the wage function will always be log-supermodular, which ensures positive assortative matching.

$$\frac{d}{d\theta}\left[\frac{w_k(\theta)}{w_l(\theta)}\right] \propto \epsilon^f(s_l) - \epsilon^f(s_k), \tag{7}$$

which is negative if F is strictly log-submodular and positive if F is strictly log supermodular by Lemma 1. Therefore, when the quantity of intermediate input used is exogenously fixed the equilibrium assignment is reversed and a log-submodular production function implies negative assortative matching.

Proposition 2. *(Exogenous intermediate input quantity.) If the production function is strictly log-submodular then the equilibrium assignment exhibits negative assortative matching between skill and intermediate input productivity. If the production function is strictly log-supermodular then the equilibrium displays positive assortative matching.*

To understand why fixing x reverses the sorting pattern observe that when F is log-submodular the inputs are gross substitutes and, if the input quantity is fixed, efficiency requires matching high skill agents with low technology sectors to take advantage of this substitutability. By contrast, if input choice is endogenous high skill agents leverage their ability by using more intermediate input. When there are increasing returns to skill and the inputs are gross substitutes, the leveraging effect is sufficiently strong that skill and technology are complements and this leads to positive assortative matching.[21]

The switch between positive and negative assortative matching triggered by allowing for input adjustability has potentially interesting implications for how institutional development affects the labor market. For example, consider an economy with a log-submodular production function. Suppose initially financial institutions are under-developed and borrowing constraints force all agents to work with a fixed quantity of the intermediate input. Under these circumstances high skill agents will work in low technology sectors. However, if credit markets develop to the point where agents can pledge some fraction of their income as collateral then more skilled agents will be able to work with greater quantities of input, sorting will reverse and financial development will precipitate dramatic changes in the labor market and the distribution of income.

For the remainder of this paper I will restrict attention to the most relevant case where intermediate input quantity is endogenous and there is positive assortative matching of high skill workers to high technology sectors. Therefore, I impose the following assumption.

[21] In the absence of increasing returns to skill the leveraging effect is weaker, span of control is independent of skill by equation (3) and, in equilibrium, agents are indifferent between sectors and there is no sorting.

Assumption 1. *(i) The production function is strictly log-submodular in labor skill θ and intermediate input quantity x. (ii) Intermediate input quantity x is a choice variable.*

3.3 Wage distribution

The equilibrium assignment has two important implications for the wage structure. First, from (2) and (4), labor's share of output is given by:

$$\frac{w_k(\theta)}{\pi_k y_k(\theta)} = 1 - \epsilon^f(s_k). \tag{8}$$

Since span of control is increasing in skill it follows from Lemma 1 that labor's output share is decreasing in skill, or equivalently wages, both within and across sectors. Moreover, intermediate input expenditure equals $\pi_k \theta g(\theta) f(s_k) \epsilon^f(s_k)$ which is increasing in θ, implying that intermediate input expenditure per worker is increasing in wages and decreasing in labor's share of output both within and across sectors. Proposition 3 summarizes these results.

Proposition 3. *Both across sectors and across production teams within sectors: (i) labor's share of output is strictly decreasing in wages, and; (ii) intermediate input expenditure per worker is strictly increasing in wages.*

Proposition 3 demonstrates that when there are multiple factors of production sorting causes cross-sectoral variation in income shares. The correlations predicted by Proposition 3 are observed at the industry level.[22] For example, using the NBER manufacturing database for 2000 and regressing labor's output share on the log wage gives a slope of -0.048 (robust t-statistic 3.7), while the elasticity of intermediate input expenditure to wages is 1.3 (robust t-statistic 11).[23] These correlations also provide support for maintaining Assumption 1 since if input quantity is fixed labor's output share is increasing in θ, while if input quantity is endogenous and the production function is log-supermodular labor's output share is increasing in skill within sectors, but has discontinuous downward jumps at the thresholds for sector assignment, meaning that the cross-sector correlation is in general ambiguous.

[22] See Slichter (1950) for some early evidence that there exists a negative correlation across industries between wages and labor's share of output.

[23] The regressions are estimated using 4 digit SIC 1987 industries. Log employment is included as a control in the intermediate input expenditure regression. Similar results are obtained using labor's share of value-added and capital investment instead of labor's output share and intermediate input expenditure, respectively.

The second important property of the equilibrium assignment comes from differentiating (4) which gives a very simple expression for the returns to skill:

$$\epsilon^{w_k}(\theta) = 1 + \frac{\epsilon^g(\theta)}{1 - \epsilon^f(s_k)}, \tag{9}$$

where $\epsilon^{w_k}(\theta) \equiv \frac{\theta w_k'(\theta)}{w_k(\theta)}$. Equation (9) implies that, holding θ constant, span of control is a sufficient statistic for the returns to skill. Moreover, Assumption 1 implies that the output elasticity is increasing in s_k, meaning that a higher span of control raises the returns to skill. Since an increase in $\pi_k Q_k$ leads to a higher span of control, it follows that the returns to skill are strictly increasing in $\pi_k Q_k$. Intuitively, when labor and intermediate inputs are gross substitutes, high skill agents are better able than low skill agents to take advantage of positive technology or output price shocks to increase production levels by working with greater quantities of intermediate input. Across sectors, $\pi_k Q_k > \pi_l Q_l$ if and only if $Q_k > Q_l$ implying that the returns to skill are higher in more technologically advanced sectors. Consistent with this prediction Gibbons et al. (2005) find that returns to skill are higher in more skilled occupations.

Let $w(\theta) \equiv \max_{1 \leq k \leq K} \{w_k(\theta)\}$ be an agent's equilibrium wage. The wage distribution depends on both the wage function $w(\theta)$ and the distribution of skill across agents. The model places no restrictions on the shape of the skill distribution, but equation (9), in combination with Lemma 2 below, allows us to characterize how shocks, such as technical change and trade liberalization, affect wage inequality when the skill distribution is held constant.

Lemma 2. *Let $w(\theta)$ and $\tilde{w}(\theta)$ be wage functions such that $\epsilon^w(\theta) > \epsilon^{\tilde{w}}(\theta)\, \forall\, \theta \in (\theta_a, \theta_b) \subseteq \left(0, \bar{\theta}\right]$. Then wage inequality among any subset of agents with skill levels in $[\theta_a, \theta_b]$ is higher under $w(\theta)$ than under $\tilde{w}(\theta)$ for any measure of inequality that respects scale independence and second-order stochastic dominance.*

Lemma 2 tells us that within-group wage inequality rises whenever both the returns to skill increase at all skill levels and membership of the group is unchanged. Adapting an approach used by Helpman, Itskhoki and Redding (2010) the proof of Lemma 2 relies on showing that, after a change in means, the wage distribution implied by $\tilde{w}(\theta)$ second-order stochastically dominates the distribution implied by $w(\theta)$. Combining Lemma 2 and equation (9) implies that the direction of the change in wage inequality within any group of agents is fully determined by variation in the span of control. This result will be used repeatedly below to characterize the effects of technical change and trade liberalization on wage inequality.

4 General equilibrium

To understand the determinants of wage inequality in the assignment model developed above this section embeds the assignment problem in general equilibrium and analyzes the effects of technical change and variation in the skill distribution. Section 5 then considers the consequences of trade integration by extending the model to an open economy setting.

To develop a general equilibrium version of the model I need to specify the intermediate input production technology and the source of demand for each sector's output. The assignment problem is sufficiently tractable to permit multiple alternative general equilibrium settings. For example, the productive activities agents undertake could be tasks, occupations or industries. Likewise, the intermediate input could represent land, materials, capital or homogeneous labor. For the remainder of this paper I assume that each productive activity constitutes a separate sector and there is an aggregate output good that can be used either for consumption or to produce the intermediate input. These assumptions are chosen primarily for their simplicity, allowing the paper to focus on identifying the new insights that arise from using the assignment model to determine the allocation of skill. However, in Appendix B I show that the main results continue to hold in a more complex model where agents are assigned to tasks and task outputs are used as factor inputs in a Heckscher-Ohlin model. This alternative set-up gives a version of the Heckscher-Ohlin model in which the ranking of industries by workforce skill is endogenous to the distribution of intermediate input productivity across tasks.

4.1 Assumptions

Suppose there are two sectors, $K = 2$, with $Q_2 > Q_1$ and assume the skill distribution has continuous support on $\left(0, \bar{\theta}\right]$ and no mass points.[24] Output from the two sectors is combined to produce a final good using a Cobb-Douglas technology:

$$Z = \left(\frac{Y_1}{\beta}\right)^{\beta} \left(\frac{Y_2}{1-\beta}\right)^{1-\beta}, \qquad \beta \in (0,1), \tag{10}$$

where Z is final good output and Y_k is aggregate output of sector k:

[24] This assumption is for ease of exposition. It is straightforward to solve the model when the skill distribution is discrete, but the notation is more cumbersome due to the necessity of keeping track of where agents work when they are indifferent between sectors.

$$Y_k = \int_{\theta_{k-1}}^{\theta_k} \theta g(\theta) f(s_k) dM(\theta). \tag{11}$$

This technology guarantees that all sectors must produce positive aggregate output. The final good can be used either for consumption or to produce the intermediate input. Each unit of final output can be transformed into γ units of intermediate input. This completes the specification of the economy. The use of a Cobb-Douglas final good production technology simplifies solving the model, but all the closed and open economy results obtained below continue to hold if the final good is produced using a general constant returns to scale technology. See Appendix B for details.

4.2 Equilibrium

Given Assumption 1 we know there is positive assortative matching between agents and sectors. Therefore, there exists a skill threshold θ_1 such that agents with skill below θ_1 work in sector one and agents with skill above θ_1 work in sector two.

To solve the model it is convenient to let the final good be the numeraire. This immediately implies that $p = \frac{1}{\gamma}$ and, from cost minimization using (10), that:

$$1 = \pi_1^{\beta} \pi_2^{1-\beta}. \tag{12}$$

Since $Q_2 > Q_1 \Rightarrow \pi_2 < \pi_1$ we must have that $\pi_2 < 1 < \pi_1$. In addition, (12) implies that $\frac{d\pi_1}{d\pi_2} < 0$. If the price of sector two output rises, then the price of sector one output falls. Cost minimization using (10) also gives the market clearing equations:

$$\beta Z = \pi_1 Y_1, \qquad (1-\beta) Z = \pi_2 Y_2. \tag{13}$$

Equations (3), (4), (11), (12) and (13) are sufficient to reduce the equilibrium to a system of two equations in the two unknowns, θ_1 and π_2. First, the wage equalization (WE) condition requires that an agent with ability θ_1 be indifferent between the two sectors. From (4) and (12) this implies:

$$f\left[s_1(\theta_1)\right] - s_1(\theta_1) f'\left[s_1(\theta_1)\right] = \pi_2^{\frac{1}{\beta}} \left(f\left[s_2(\theta_1)\right] - s_2(\theta_1) f'\left[s_2(\theta_1)\right] \right). \tag{WE}$$

Second, the output markets must clear. Using (11), (12) and (13) gives the market clearing (MC) condition:

$$\int_0^{\theta_1} \theta g(\theta) f(s_1) dM(\theta) = \frac{\beta}{1-\beta} \pi_2^{\frac{1}{\beta}} \int_{\theta_1}^{\bar{\theta}} \theta g(\theta) f(s_2) dM(\theta). \tag{MC}$$

In both equilibrium conditions s_1 and s_2 are defined by (3) and depend implicitly on π_2.

Figure 4 shows the (WE) and (MC) conditions in θ_1-π_2 space. The (WE) curve is downward sloping because an increase in π_2 makes sector two more profitable and, since $\frac{w_2(\theta)}{w_1(\theta)}$ is increasing in θ, this decreases the skill level at which agents are indifferent between sectors. The (MC) curve is upward sloping because a higher π_2 reduces the relative demand for sector two output, which necessitates the reallocation of labor to sector one. Together the two conditions define a unique equilibrium – see the proof of Proposition 4 for details.

Proposition 4. *There exists a unique closed economy equilibrium with a threshold skill θ_1 such that agents with skill above θ_1 work in the high technology sector and agents with skill below θ_1 work in the low technology sector.*

4.3 Technical change

How is the equilibrium affected by intermediate input augmenting productivity growth? The most dramatic effect occurs when technical change switches the ranking of sectors by input productivity. For example, an increase in intermediate input productivity in sector one from $Q_1 < Q_2$ to $Q_1' > Q_2$ precipitates an assignment reversal making sector one the high skill, high wage sector.

However, regardless of whether technological progress changes the sector technology ranking the equilibrium conditions can be used to show:[25]

$$\frac{d\left[\pi_k Q_k\right]}{dQ_j} > 0, \qquad \frac{d\pi_j}{dQ_j} < 0, \qquad \frac{d\pi_l}{dQ_j} > 0, \qquad j,k,l = 1,2, \quad l \neq j. \tag{14}$$

Unsurprisingly, productivity growth in a sector leads to a price decline in that sector and a price rise in the other sector. More importantly, productivity growth in either sector always increases $\pi_k Q_k$ in both sectors. Remembering equations (3) and (9) this implies the span of control $s_k(\theta)$ and returns to skill $\epsilon^{w_k}(\theta)$ rise in both sectors. Appealing to Lemma 2, the higher returns to skill increase within-group wage inequality among any group of agents who all work in the same sector and who do not switch sectors following the

[25] See the proof of Proposition 5 for details.

technology shock. Intuitively, intermediate input augmenting productivity growth raises the returns to skill in both sectors because it causes all agents to increase their spans of control, which disproportionately benefits high skill agents for whom the elasticity of output with respect to the span of control is greater.

Technical change can also take the form of an increase in the productivity γ of intermediate input production. From (3) we see that span of control depends on the cost per efficiency unit of intermediate input $\frac{1}{Q_k \gamma}$, meaning that an increase in γ is equivalent to equiproportional increases in both Q_1 and Q_2. Therefore, we have:

$$\frac{d\left[\pi_k \gamma\right]}{d\gamma} > 0, \qquad k = 1, 2,$$

implying that when γ increases the returns to skill rise in both sectors. Proposition 5 summarizes the effects of technical change.

Proposition 5. *Technical change that reduces the cost per efficiency unit of intermediate input in either sector raises the returns to skill in both sectors. Consequently, wage inequality increases within any group of agents who all work in the same sector and who do not switch sectors.*

Proposition 5 shows that in the assignment model technical change is skill-biased when it is either intermediate input augmenting or reduces the cost of the intermediate input. However, it's worth noting that such technical change is complementary to skill in two distinct senses. First, it increases the within-sector returns to skill in both sectors. Second, any sector which experiences a sufficiently large positive technology shock becomes the high skill sector, regardless of the skill level of its workers prior to the shock. In addition, observe from (8) that by increasing agents' spans of control technical change reduces labor's output share. Thus, the assignment model predicts that skill-biased technical change lowers labor's share of output.

Without placing restrictions on the shape of the skill distribution, or the functional form of the production technology, the effect of technical change on the skill threshold θ_1 and on inequality within groups that include agents who are induced to switch sectors by the technological shock is, in general, ambiguous. In particular, at skill levels such that agents switch from the high skill to the low skill sector following a technology shock the returns to skill can decrease. However, I show in the proof of Proposition 5 that whenever technical change causes the high skill sector to expand on the extensive margin ($d\theta_1 < 0$) wage inequality increases within all subgroups of the population.

4.4 Skill distribution

In contrast to economies with homogeneous factors of production, factor prices (the wage offered to agents with a given skill) differ across sectors in the assignment equilibrium. Consequently, the assignment model bears some resemblance to a specific factors model. However, because the equilibrium assignment is endogenous there are also important differences. These similarities and differences are well illustrated by the effects of shifts in the skill distribution. Let $M_k(\theta_1)$ denote the mass of agents assigned to sector k for a given equilibrium skill threshold θ_1. Consider a shock to the skill supply that increases $M_k(\theta_1)$. I will refer to such a shock as an increase in the supply of sector k agents, but it is important to note that the effect of a given shock on $M_k(\theta_1)$ depends not only on how the skill distribution changes, but also on the initial level of θ_1.

From the (MC) condition we see that an increase in $M_k(\theta_1)$ raises the supply of sector k output leading to a fall in π_k and a reallocation of agents out of sector k. Consequently, real wages, the returns to skill and within-group wage inequality fall in sector k and rise in sector l. For example, an increase in the supply of high skill agents reduces the real wage of all agents in sector two and reduces wage inequality within sector two, while having opposite effects on sector one.

Proposition 6. *An increase in the supply of agents with skills above the initial skill threshold θ_1 leads to: (i) an increase in θ_1; (ii) a decrease in real wages and wage inequality among agents who continue to work in the high skill sector, and; (iii) an increase in real wages and wage inequality among agents who continue to work in the low skill sector. An increase in the supply of agents with skills below θ_1 has opposite effects.*

As in a specific factors model, increasing the endowment of a sector specific factor reduces the price of that factor. However, unlike in the specific factors model there is also a reallocation of labor away from the sector that experiences a supply increase. Moreover, specific factors models do not speak to the determinants of within-sector wage inequality. Interestingly, the assignment model predicts that variation in the skill supply causes wage levels and wage inequality to move in the same direction within sectors and opposite directions across sectors.

5 Open economy

Let us now extend the model to include two countries: home and foreign. I will use an asterisk to denote foreign variables. Suppose the two countries are identical along all dimensions except: (i) cost per efficiency unit of intermediate input; (ii) skill distribution, and; (iii) population size. By Assumption 1 the production function F is log-submodular in both countries. The aim of this section is to study the effects of globalization when the cost per efficiency unit of intermediate input differs across both sectors and countries.

Differences in the cost per efficiency unit of intermediate input can result from both variation across countries and sectors in intermediate input augmenting productivity Q_k and variation across countries in the productivity of intermediate input production γ. Such variation captures in reduced form differences in country-sector specific knowledge stocks and capabilities and cross-country differences in the price at which intermediate inputs are sold.

In the open economy both the sectoral outputs and the intermediate input may be traded. To separate the effects of output and input trade I will start by analyzing output trade assuming the intermediate input is non-tradable and then proceed to allow trade in the intermediate input. Assuming each sector's output is freely traded implies that both the sectoral output prices and the final good price are equalized across countries. As above, I let the final good be the numeraire. I also assume that each country's skill distribution has continuous bounded support, but I allow the functional form and upper bound of the skill distribution to differ across countries. When comparing the closed and open economy equilibria I will use a tilde to denote autarky outcomes.

5.1 Assignment reversals

Let us start by considering the case where there is an assignment reversal across countries and the intermediate input is non-traded. In particular, suppose that home has higher productivity in sector two, $Q_1 < Q_2$, but foreign has higher productivity in sector one, $Q_1^* > Q_2^*$. This means that in autarky sector two is the high skill, high wage sector at home, while sector one is the high skill, high wage sector abroad. In addition, diversified production requires $\tilde{\pi}_2 < 1 < \tilde{\pi}_1$ and $\tilde{\pi}_1^* < 1 < \tilde{\pi}_2^*$ meaning:

$$\frac{\tilde{\pi}_2}{\tilde{\pi}_1} < 1 < \frac{\tilde{\pi}_2^*}{\tilde{\pi}_1^*},$$

which implies home has a comparative advantage in sector two and foreign has a comparative advantage in sector one. Therefore, when the ranking of sectors by intermediate input productivity differs across countries, each country has a comparative advantage in its high productivity sector, which is also its high skill, high wage sector.

We know from Section 3 that if $\pi_2 \geq \pi_1$ in the open economy equilibrium then in the home country sector two offers a strictly higher wage than sector one at all skill levels. Similarly, if $\pi_2 \leq \pi_1$ then sector one is strictly preferred to sector two by all foreign agents. Since free trade equalizes output prices across countries it follows that in the open economy at least one of the countries must specialize in its high skill sector. Without loss of generality, let us suppose that $\pi_2 \leq \pi_1$.[26] Then foreign specializes in sector one and equation (12) implies $\pi_2 \leq 1 \leq \pi_1$.

In the open economy output prices must satisfy (12) and equilibrium spans of control and wages are given by (3), (4) and their foreign equivalents. As in the closed economy, the open economy equilibrium can be reduced to a system of two equations in two unknowns, θ_1 and π_2. The wage equalization (WE) condition, which determines the skill threshold above which home agents select into sector two, is unchanged from the closed economy case. The difference is that output markets clear at the global, not the national, level. From (13) and its foreign equivalent global output market clearing requires:

$$Y_1 + Y_1^* = \frac{\beta}{1-\beta}\pi_2^{\frac{1}{\beta}}(Y_2 + Y_2^*),$$

and using (11), (12) and that foreign is specialized in sector one we obtain the open economy market clearing (MC′) condition:

$$\int_0^{\theta_1} \theta g(\theta) f(s_1) dM(\theta) + \int_0^{\bar{\theta}^*} \theta g(\theta) f(s_1^*) dM^*(\theta) = \frac{\beta}{1-\beta}\pi_2^{\frac{1}{\beta}} \int_{\theta_1}^{\bar{\theta}} \theta g(\theta) f(s_2) dM(\theta). \tag{MC′}$$

The only difference from the closed economy market clearing condition is the second term on the left hand side of (MC′), which represents foreign's sector one output. As in the closed economy, the (WE) curve is downward sloping and the (MC′) curve is upward sloping in θ_1–π_2 space and together they define a unique equilibrium. However, foreign production shifts the (MC′) curve upwards relative to the (MC) curve in the closed economy (see Figure 5). Therefore, globalization reduces the skill threshold above which home agents work in sector two, $\theta_1 < \tilde{\theta}_1$ and increases the home price of sector two output, $\pi_2 > \tilde{\pi}_2$.

[26] Equation (15) below gives a necessary and sufficient condition for this to be the equilibrium outcome.

For $\pi_2 \leq 1 \leq \pi_1$ to be the equilibrium outcome we must have that when $\pi_1 = \pi_2 = 1$, which implies both countries are specialized in their high productivity sector, there is not an excess supply of good one. Consequently, a necessary and sufficient condition for output prices to satisfy $\pi_2 \leq 1 \leq \pi_1$ in equilibrium is:

$$\int_0^{\bar{\theta}^*} \theta g(\theta) f(s_1^*) dM^*(\theta) \leq \frac{\beta}{1-\beta} \int_0^{\bar{\theta}} \theta g(\theta) f(s_2) dM(\theta), \tag{15}$$

where the spans of control are defined by (3) with $\pi_1 = \pi_2 = 1$. This condition tells us that if foreign is economically "small" relative to home then in the open economy equilibrium foreign specializes in its high productivity sector. In this context, an economy's size depends on how much output it can produce in its high productivity sector and smallness can result from having a relatively low population, relatively unskilled agents or relatively low intermediate input productivity in the high technology sector. Proposition 7 summarizes the structure of production in the open economy equilibrium.

Proposition 7. *When there is an assignment reversal across countries there exists a unique open economy equilibrium such that: (i) each country exports the output of its high skill sector; (ii) the smaller economy specializes in its high skill sector, and; (iii) compared to autarky the skill threshold above which agents select into the high skill sector is lower in both countries.*

Since each country has a comparative advantage in its high technology sector and high skill agents are matched to the high technology sector, the model predicts that the export sector is the high skill sector in both countries. This prediction is absent from models that do not include assignment reversals.[27] In addition, trade integration causes the high skill sector to expand on the extensive margin in both countries.

Comparing the open economy equilibrium to autarky we have $\tilde{\pi}_2 < \pi_2 < \tilde{\pi}_2^*$ and $\tilde{\pi}_1^* < \pi_1 < \tilde{\pi}_1$. Therefore, following trade integration each country experiences an increase in the price of its high skill sector and a decrease in the price of its low skill sector. From (3) and (4), these price changes increase the wages of agents in the high skill sector and decrease the wages of agents in the low skill sector. Whether agents who switch into the high skill sector following globalization obtain a higher wage than in autarky is ambiguous, but in each country there exists a skill threshold such that, following trade liberalization, the wage of all agents with skill below the threshold falls and the wage of all agents with skill above the

[27] Matsuyama (2007) presents a model in which export sectors are always more skill intensive than import sectors because, by assumption, export production uses a more skill intensive technology than production for domestic consumption.

threshold rises.[28] Thus, in contrast to the implications of the Stolper-Samuelson theorem, trade liberalization benefits high skill labor in both countries.

From equation (9) and Lemma 2 the price changes triggered by globalization increase the returns to skill in the high skill sector and decrease the returns to skill in the low skill sector. Consequently, in both countries, moving from autarky to free trade increases wage inequality within any group of agents employed in the high skill sector following trade liberalization and decreases wage inequality within any group of agents employed in the low skill sector following trade liberalization.[29] Since the smaller economy specializes in its high skill sector it experiences a pervasive rise in wage inequality – wage inequality increases within any subset of the population containing at least two agents with different skill levels. In addition, if equation (15) holds with equality, meaning the two economies are the same size, then both countries are fully specialized in the open economy equilibrium and trade integration causes a pervasive increase in wage inequality in both countries. Proposition 8 summarizes the effects of output trade on wages.

Proposition 8. *When there is an assignment reversal across countries moving from autarky to free trade causes each country to experience an increase in the price of its high skill good and a decrease in the price of its low skill good. Consequently, in both countries, wage levels and wage inequality increase in the high skill sector and decrease in the low skill sector.*

The observation that trade liberalization has coincided with increases in wage inequality in many unskilled labor abundant developing countries has prompted an extensive theoretical literature on alternatives to the Stolper-Samuelson theorem. Explanations of how trade integration can raise inequality in both developed and developing countries have invoked intra-industry offshoring (Feenstra and Hanson 1996), the existence of multiple cones of diversification in a Heckscher-Ohlin model (Davis 1996), trade induced intra-industry input quality upgrading (Verhoogen 2008; Kugler and Verhoogen 2008), higher skill intensity of export production (Matsuyama 2007) and intra-industry selection of high skill or high wage firms into exporting (Manasse and Turrini 2001; Yeaple 2005; Helpman, Itskhoki and Redding 2010; Monte 2011; Sampson 2013). This paper suggests a new mechanism – assignment reversals. In contrast to papers that focus solely on intra-industry effects, the assignment model can explain both increased wage inequality across sectors and increasing returns to skill within export sectors. In addition, the model predicts that trade

[28] See the proof of Proposition 8 for details.

[29] This prediction is consistent with evidence that labor market polarization has resulted in a compression of the bottom half of the US wage distribution together with increased dispersion in the top half (Autor, Katz and Kearney 2006).

integration between two economies will benefit high skill labor in both countries only when the countries are sufficiently dissimilar that the ranking of sectors by the cost per efficiency unit of intermediate input differs across countries. This may explain why the effects of trade integration on inequality have varied across developing countries.

5.2 No assignment reversals

Now suppose the ranking of sectors by the cost per efficiency unit of intermediate input is the same in both countries, meaning there are no assignment reversals. Assume that in both countries intermediate input productivity is higher in sector two than in sector one. In this case, comparative advantage will depend on the costs per efficiency unit of intermediate input and the skill distributions in both countries according to the autarky equilibrium conditions (WE) and (MC) and their foreign equivalents. The autarky price of sector two output is lower, ceteris paribus, in the country with: (i) higher relative productivity in sector two; (ii) lower absolute costs per efficiency unit of intermediate input, or; (iii) a greater proportion of high skill agents. Without loss of generality, assume $\tilde{\pi}_2 < \tilde{\pi}_2^*$, meaning home has a comparative advantage in sector two, while foreign has a comparative advantage in sector one. Therefore, in the open economy equilibrium home exports output from its high skill sector, while foreign exports output from its low skill sector.

Since the ordering of sectors by workforce skill is invariant across countries, trade-induced price changes cannot increase the price of the high skill good in both countries. Only the country with a comparative advantage in the high skill sector experiences an increase in the price of its high skill output. Open economy market clearing requires $\tilde{\pi}_2 < \pi_2 < \tilde{\pi}_2^*$ and from (12) this also implies $\tilde{\pi}_1^* < \pi_1 < \tilde{\pi}_1$. At home trade liberalization has similar effects to those experienced by both countries when there is an assignment reversal: the high skill sector expands on the extensive margin and the price changes benefit high skill labor. However, in foreign the price of high skill output declines, the price of low skill output increases and the low skill sector expands on the extensive margin. Consequently, trade liberalization benefits low skill labor – there exists a threshold such that the wage of all foreign agents with skill below the threshold is higher in the open economy than in autarky and the wage of all foreign agents with skill above the threshold is lower. Moreover, the returns to skill increase in the low skill sector and decrease in the high skill sector, meaning that trade liberalization increases wage inequality within any group of foreign agents employed in the low skill sector in autarky and decreases wage inequality within any group of foreign agents employed in the high skill sector following integration.

Proposition 9. *When the ranking of sectors by intermediate input productivity is the same in both countries there exists a unique open economy equilibrium such that: (i) one country exports its high skill, high productivity good and the other country exports its low skill, low productivity good, and; (ii) in both countries moving from autarky to free trade increases wage levels and wage inequality in the export sector and decreases wage levels and wage inequality in the import sector.*

Proposition 9 shows that in the absence of assignment reversals the effects of output trade integration on wage inequality between high and low skill agents are the same as those predicted by the Heckscher-Ohlin model. Inequality increases in the country which has a comparative advantage in the high skill good and decreases in the other country. However, in contrast to the Heckscher-Ohlin model, in the assignment model trade also affects within-group wage inequality. In particular, Propositions 8 and 9 show that in the assignment model trade liberalization increases returns to skill in the export sector and decreases returns to skill in the import sector of both countries, regardless of patterns of input productivity or comparative advantage.[30]

5.3 Intermediate input trade

Now let us consider the effects of trade in the intermediate input. In the absence of trade the price of the intermediate input is $\frac{1}{\gamma}$ at home and $\frac{1}{\gamma^*}$ in the foreign country. Without loss of generality, assume that foreign has higher productivity in intermediate input production than home implying $\gamma^* > \gamma$. Then home will import the intermediate input from foreign and moving from an equilibrium in which the intermediate is non-tradable to free trade in the intermediate is equivalent to increasing home's productivity in the intermediate input production sector from γ to γ^*.

An increase in γ reduces the cost per efficiency unit of intermediate input in both sectors by raising $\gamma\pi_k, k = 1, 2$. Consequently, spans of control increase in both sectors and, applying equation (9) and Lemma 2, this raises the returns to skill and wage inequality within each sector for agents whose sector of employment does not change. Thus, liberalization of intermediate input trade increases within sector wage inequality in the country that imports the intermediate by allowing workers to better leverage their skills, which disproportionately benefits more skilled agents. This result holds regardless of whether there are

[30]Brambilla et al. (2012) argue using Latin American data that the skill premium is higher in industries with a greater exports to output ratio.

assignment reversals across countries and regardless of whether sectoral outputs are traded or non-traded.[31] By contrast, the effect of intermediate input trade on wages in the intermediate exporter depends on the change in π_2 which is ambiguous at this level of generality.

Proposition 10. *Intermediate input trade raises the returns to skill in both sectors in the country that imports the intermediate input. Consequently, wage inequality increases within any group of agents who all work in the same sector and who do not switch sectors.*

Comparing Proposition 10 to Proposition 5 above shows that, for an importer, intermediate input trade has the same effects on wage inequality as an increase in the productivity of intermediate input production. This comparison highlights the fact that in the assignment model intermediates embody cross-country technology differences and intermediate input trade is equivalent to a technology transfer that raises the productivity of the importing country.

If the intermediate input is interpreted as capital the model predicts that reductions in the cost of capital imports increase within-sector returns to skill. This prediction receives support from two recent papers that estimate the impact of capital imports on wages. Csillag and Koren (2009) undertake structural estimation of a single sector model of worker assignment, similar to Sattinger (1979), using a rich matched employer-employee-imports data set from Hungary. They find that on average imported machines are more productive than domestic machines and are matched with higher skill workers. In addition, the returns to skill on the median productivity imported machine are 26% higher than on the median productivity domestic machine. Parro (2010) estimates the impact of capital imports on the skill premium using a calibrated version of the Eaton and Kortum (2002) model in which production uses skilled labor, unskilled labor and capital and there is capital-skill complementarity. The paper finds that from 1990-2007 reductions in capital trade costs and productivity growth in capital production each increased the skill premium by around 2 percentage points on average across countries.

From this perspective the assignment model can be viewed as a theory of capital-skill complementarity. However, it differs from existing models of capital-skill complementarity such as that estimated by Krusell et al. (2000) in four important ways. First, it provides a microfoundation for capital-skill complementarity in terms of agents leveraging their abilities by expanding their spans of control. Second, it is tractable when there are multiple sectors making it well-suited to studying the sectoral skill allocation. Third, it predicts

[31] When sectoral outputs are non-traded I assume the final good is freely tradable to ensure the trade balance condition holds.

labor's share of value-added is lower in high skill, high wage sectors – a correlation that holds empirically as documented in Section 3.3, but is not found in other work on capital-skill complementarity. Fourth, since it allows for a continuum of skill levels, the model can be used to study within-group wage inequality in addition to the wage gap between skilled and unskilled workers.

For ease of exposition I have assumed throughout the paper there is a unique intermediate input, implying the price of intermediates is constant across sectors within a country. However, it is simple to generalize the assignment model to allow for sector specific intermediate inputs. Suppose one unit of the final good can be transformed into γ_k units of the intermediate used in sector k implying $p_k = \frac{1}{\gamma_k}$ is the intermediate price in sector k. Then the cost per efficiency unit of intermediate input in sector k is $\frac{Q_k}{\gamma_k}$. It is straightforward to check that, provided the intermediate is non-tradable, variation in γ_k across sectors is equivalent to variation in Q_K. For example, the equilibrium assignment is as in Proposition 1, except that sectors are ranked by $\frac{Q_k}{\gamma_k}$ instead of Q_k.

However, this equivalence breaks down when the intermediate input is tradable because while intermediate input augmenting productivity Q_k is disembodied, variation in γ_k is embodied in the intermediate input and can be transferred across countries. Each country-sector pair will source its intermediate input from the lowest cost supplier. In the extreme case that there is no within-country variation in input augmenting productivity (i.e. $Q_k = Q_l \forall k, l$), free trade in intermediate inputs implies that the ranking of sectors by the cost per efficiency unit of intermediate input is the same in all countries. Consequently, there are no assignment reversals. Effectively, when all cross-sector technology differences are embodied in intermediate inputs, trade and technology transfer are perfect substitutes and intermediate input trade leads to global convergence in the ranking of sectors by workforce skill.[32]

6 Conclusions

The ranking of industries by workforce skill is generally assumed to be determined by invariant industry characteristics. However, industry wage and educational attainment data imply that the sectoral skill allocation differs dramatically between developed and developing economies. To explain the allocation of skill across sectors this paper develops a multi-sector assignment model that includes both labor and an homo-

[32] An earlier version of this paper (Sampson 2011) builds on this observation to provide evidence in support of the assignment model by showing that countries which, by virtue of their geographic location, have low cost access to imports of specific varieties of capital have relatively higher wages in sectors that use those capital varieties more intensively. This result is consistent with the prediction that sectors with lower costs per efficiency unit of intermediate input employ higher skill workers.

geneous non-labor input. The model formalizes the idea that high skill agents leverage their abilities via their spans of control over other production inputs. In equilibrium higher skill workers obtain larger spans of control by sorting into sectors where the cost per efficiency unit of the non-labor input is lower. Through endogenizing the ranking of sectors by workforce skill the model explains the existence of assignment reversals, which are driven by cross-country, cross-sector differences in input productivity levels and costs. The assignment model also provides a new framework for understanding how technical change and trade affect employment and wages. In contrast to the existing multi-sector assignment literature, the paper highlights the key role played by non-labor inputs in shaping the wage distribution. The returns to skill depend on the cost per efficiency unit of inputs and when inputs become more productive or cheaper wage inequality rises. More broadly, scale and skill are complements and shocks which enable workers to raise their spans of control lead to increased inequality.

The theoretical tools developed in this paper can be applied to a wide range of questions that feature multiple productive activities and matching between two factors of production with non-zero opportunity costs of forming a match. For example, when the intermediate input is interpreted as homogeneous unskilled labor, the model can be reinterpreted as a model of firm hierarchies. Consequently, the assignment reversals framework could be used to extend the single sector literature on globalization and firm hierarchies (Antràs, Garicano and Rossi-Hansberg 2006; Burstein and Monge-Naranjo 2009) to a multi-sector world. It would also be interesting to allow for endogenous technical change in intermediate input productivity and analyze the conditions under which profit maximizing R&D will lead to assignment reversals. Finally, the increasing availability of matched employer-employee data opens new opportunities for testing assignment models and estimating the role of non-labor inputs in determining labor sorting. Such estimates would also discipline attempts to quantify the relevance of alternative sorting mechanisms.

References

Abowd, John M., Robert H. Creecy, and Francis Kramarz. (2002) “Computing Person and Firm Effects Using Linked Longitudinal Employer-Employee Data.” Working Paper.

Abowd, John M., Francis Kramarz, and David N. Margolis. (1999) “High Wage Workers and High Wage Firms.” *Econometrica* 67 (2): 251-333.

Acemoglu, Daron and David Autor. (2011) “Skills, Tasks and Technologies: Implications for Employment and Earnings.” *Handbook of Labor Economics* 4: 1043-1171.

Acemoglu, Daron. (2002) “Directed Technical Change.” *Review of Economic Studies* 69 (4): 781-809.

Antràs, Pol, Luis Garicano, and Esteban Rossi-Hansberg. (2006) “Offshoring in a Knowledge Economy.” *Quarterly Journal of Economics* 121 (1): 31-77.

Autor, David H., Lawrence F. Katz, and Melissa S. Kearney. (2006) “The Polarization of the U.S. Labor Market.” *American Economic Review* 96 (2): 189-94.

Barro, Robert J., and Jong-Wha Lee. (2001) “International Data on Educational Attainment: Updates and Implications.” *Oxford Economic Papers* 53 (3): 541-63.

Becker, Gary. (1973) “A Theory of Marriage: Part 1.” *Journal of Political Economy* 81 (4): 813-46.

Brambilla, Irene, Rafael Dix-Carneiro, Daniel Lederman, and Guido Porto. (2012) “Skills, Exports, and the Wages of Seven Million Latin American Workers.” *World Bank Economic Review* 26 (1): 34-60.

Burstein, Ariel T. and Alexander Monge-Naranjo. (2009) “Foreign Know-how, Firm Control, and the Income of Developing Countries.” *Quarterly Journal of Economics* 124 (1): 149-195.

Caselli, Francesco. (2005) “Accounting for Cross-country Income Differences.” *Handbook of Economic Growth* 1: 679-741.

Caselli, Francesco and Daniel J. Wilson. (2004) “Importing Technology.” *Journal of Monetary Economics* 51 (1): 1-32.

Costinot, Arnaud, and Jonathan Vogel. (2010) “Matching and Inequality in the World Economy.” *Journal of Political Economy* 118 (4): 747-86.

Costinot, Arnaud. (2009) "An Elementary Theory of Comparative Advantage." *Econometrica* 77 (4): 1165-1192.

Csillag, Márton and Miklós Koren. (2009) "Machines and Machinists: The Effect of Imported Capital on Wage Inequality." Mimeo, Central European University.

Davis, Donald R. (1996) "Trade Liberalization and Income Distribution." NBER Working Paper.

Eaton, Jonathan and Samuel Kortum. (2001) "Trade in Capital Goods." *European Economic Review* 45 (7): 1195-1235.

Eaton, Jonathan, and Samuel Kortum. (2002) "Technology, Geography and Trade." *Econometrica* 70 (5): 1741-79.

Feenstra, Robert C. and Gordon H. Hanson. (1996) "Foreign Investment, Outsourcing and Relative Wages." In The Political Economy of Trade Policy: Papers in Honor of Jagdish Bhagwati, 89-127. Cambridge and London: MIT Press.

Fernandez, Raquel, Nezih Guner, and John Knowles. (2005) "Love and Money: A Theoretical and Empirical Analysis of Household Sorting and Inequality." *Quarterly Journal of Economics* 120 (1): 273-344.

Gabaix, Xavier, and Augustin Landier. (2008) "Why has CEO Pay Increased so Much?" *Quarterly Journal of Economics* 123 (1): 49-100.

Garicano, Luis, and Esteban Rossi-Hansberg. (2006) "Organization and Inequality in a Knowledge Economy." *Quarterly Journal of Economics* 121 (4): 1383-1435.

Gibbons, Robert, Lawrence F. Katz, Thomas Lemieux, and Daniel Parent. (2005) "Comparative Advantage, Learning, and Sectoral Wage Determination." *Journal of Labor Economics* 23 (4): 681-723.

Goldberg, Pinelopi Koujianou and Nina Pavcnik. (2007) "Distributional Effects of Globalization in Developing Countries." *Journal of Economic Literature* 45 (1): 39-82.

Grossman, Gene M. and Giovanni Maggi. (2000) "Diversity and Trade." *American Economic Review* 90 (5): 1255-1275.

Grossman, Gene M. (2004) "The Distribution of Talent and the Pattern and Consequences of International Trade." *Journal of Political Economy* 112 (1): 209-239.

Helpman, Elhanan, Oleg Itskhoki, and Stephen Redding. (2010) “Inequality and Unemployment in a Global Economy.” *Econometrica* 78 (4): 1239-1283.

Hsieh, Chang-Tai and Peter J. Klenow. (2007) “Relative Prices and Relative Prosperity.” *American Economic Review* 97 (3): 562-585.

Krueger, Alan B. and Lawrence H. Summers. (1986) “Reflections on the Inter-Industry Wage Structure.” NBER Working Paper.

Krusell, Per, Lee E. Ohanian, José-Victor Ríos-Rull, and Giovanni L. Violante. (2000) “Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis.” *Econometrica* 68 (5): 10291053.

Kugler, Maurice and Eric Verhoogen. (2008) “The Quality-Complementarity Hypothesis: Theory and Evidence from Colombia.” NBER Working Paper.

Kurokawa, Yohsinori. (2011) “Is a Skill Intensity Reversal a Mere Theoretical Curiosum? Evidence from the US and Mexico.” *Economics Letters* 112: 151-154.

Leontief, Wassily. (1964) “International Factor Costs and Factor Use.” *American Economic Review* 54 (4): 335-45.

Manasse, Paolo and Alessandro Turrini. (2001) “Trade, Wages, and ’Superstars’.” *Journal of International Economics* 54 (1): 97-117.

Matsuyama, Kiminori. (2007) “Beyond Icebergs: Towards a Theory of Biased Globalization.” *Review of Economic Studies* 74 (1): 237-253.

Minhas, Bagicha. (1962) “The Homohypallagic Production Function, Factor Intensity Reversals and the Heckscher-Ohlin Theorem.” *Journal of Political Economy* 70 (2): 138-156.

Monte, Ferdinando. (2011) “Skill Bias, Trade, and Wage Dispersion.” *Journal of International Economics* 83 (2): 202-218.

Murphy, Kevin M., Andrei Shleifer, and Robert W. Vishny. (1991) “The Allocation of Talent: Implications for Growth.” *Quarterly Journal of Economics* 106 (2): 503-530.

Ohnsorge, Franziska and Daniel Trefler. (2007) “Sorting it Out: International Trade with Heterogeneous Workers.” *Journal of Political Economy* 115 (5): 868-892.

Parro, Fernando. (2010) "Capital-Skill Complementarity and the Skill Premium in a Quantitative Model of Trade." Mimeo, University of Chicago.

Reshef, Ariell. (2007) "Heckscher-Ohlin and the Global Increase of Skill Premia: Factor Intensity Reversals to the Rescue." Mimeo, New York University.

Rosen, Sherwin. (1982) "Authority, Control, and the Distribution of Earnings." *Bell Journal of Economics* 13 (2): 311-323.

Roy, Andrew. (1951) "Some Thoughts on the Distribution of Earnings." *Oxford Economic Papers* 3: 135-146.

Sampson, Thomas. (2011) "Assignment Reversals: Trade, Skill Allocation and Wage Inequality." Centre for Economic Performance Discussion Paper 1105.

Sampson, Thomas. (2013) "Selection into Trade and Wage Inequality." American Economic Journal: Microeconomics, forthcoming.

Sattinger, Michael. (1975) "Comparative Advantage and the Distributions of Earnings and Abilities." *Econometrica* 43 (3): 455-468.

Sattinger, Michael. (1979) "Differential Rents and the Distribution of Earnings." *Oxford Economic Papers* 31 (1): 60-71.

Sattinger, Michael. (1993) "Assignment Models of the Distribution of Earnings." *Journal of Economic Literature* 31 (2): 831-880.

Slichter, Sumner H. (1950) "Notes on the Structure of Wages." *Review of Economics and Statistics* 32 (1): 80-91.

Terviö, Marko. (2008) "The Difference that CEOs Make: An Assignment Model Approach." *American Economic Review* 98 (3): 642-668.

Verhoogen, Eric A. (2008) "Trade, Quality Upgrading, and Wage Inequality in the Mexican Manufacturing Sector." *Quarterly Journal of Economics* 123 (2): 489-530.

Yeaple, Stephen Ross. (2005) "A Simple Model of Firm Heterogeneity, International Trade, and Wages." *Journal of International Economics* 65 (1): 1-20.

Appendix A – Proofs

Proof of Lemma 1

Since F is twice differentiable it is strictly log-submodular if and only if $\frac{\partial^2 \log F}{\partial\theta\partial x} < 0$. Differentiating F gives:

$$
\begin{aligned}
\frac{\partial^2}{\partial\theta\partial x}\log F(\theta, Q_k x) &= \frac{Q_k}{F^2}(FF_{\theta x} - F_\theta F_x), \\
&= \frac{Q_k}{F^2}FF_{\theta x}(1-\sigma),
\end{aligned}
$$

where the second line uses the fact that the elasticity of substitution of a twice differentiable, constant returns to scale function F is given by $\sigma = \frac{F_\theta F_x}{FF_{\theta x}}$. Since F has constant returns to scale and is strictly concave we must have $F_{\theta x} > 0$. Therefore, F is strictly log-submodular if and only if $\sigma > 1$.

Finally, to prove that $\sigma > 1$ is equivalent to $\epsilon^f(s)$ being strictly increasing in s differentiate $\epsilon^f(s)$ to obtain:

$$
\begin{aligned}
\frac{\partial}{\partial s}\epsilon^f(s) &= \frac{1}{f^2}(ff' + sff'' - sf'^2), \\
&= \frac{1}{f^2}(F_\theta F_x - FF_{\theta x}), \\
&= \frac{FF_{\theta x}}{f^2}(\sigma - 1).
\end{aligned}
$$

Proof of Proposition 1

Consider the case where F is strictly log-submodular. For any $k \in \{1, \ldots, K\}$ the requirement that sector k produces positive aggregate output implies there exists $\theta \in (0, \bar{\theta}]$ such that agents with skill θ weakly prefer sector k to any other sector. Suppose the equilibrium assignment does not exhibit positive assortative matching. Then there exists $l' < l$ and $\theta_a, \theta_b \in (0, \bar{\theta}]$ with $\theta_b > \theta_a$ such that $w_l(\theta_a) \geq w_k(\theta_a)\,\forall k$ and $w_{l'}(\theta_b) \geq w_k(\theta_b)\,\forall k$.

However, $l > l' \Rightarrow Q_l > Q_{l'} \Rightarrow \pi_l Q_l > \pi_{l'} Q_{l'} \Rightarrow s_l > s_{l'}$. Since F is strictly log-submodular, $\epsilon^f(s)$ is strictly increasing in s and, therefore, it follows from equation (6) above that $s_l > s_{l'} \Rightarrow \frac{d}{d\theta}\left[\frac{w_l(\theta)}{w_{l'}(\theta)}\right] > 0\,\forall\theta$.

Consequently, $w_l(\theta_a) \geq w_{l'}(\theta_a) \Rightarrow w_l(\theta_b) > w_{l'}(\theta_b)$, which contradicts the assumption that there is not positive assortative matching.

An analogous argument can be used to prove that there is negative assortative matching when F is strictly log-supermodular.

Proof of Proposition 2

The proof follows the same reasoning used to prove Proposition 1 except $\frac{d}{d\theta}\left[\frac{w_k(\theta)}{w_l(\theta)}\right]$ is given by (7) instead of (6).

Proof of Proposition 3

Part (i) follows immediately from equation (8) and Lemma 1 after remembering that span of control is strictly increasing in θ.

To prove part (ii) let $x_k(\theta)$ denote intermediate input use by a skill θ agent working in sector k. From optimal input choice (3) we have:

$$px_k(\theta) = \pi_k \theta g(\theta) f(s_k) \epsilon^f(s_k), \tag{16}$$

which is strictly increasing in θ for given k. Now suppose $w_k(\theta) = w_l(\theta)$ and $Q_k > Q_l$. Then substituting (4) into (16) gives:

$$\begin{aligned} \frac{px_k(\theta)}{px_l(\theta)} &= \frac{\epsilon^f(s_k)}{\epsilon^f(s_l)} \frac{1-\epsilon^f(s_l)}{1-\epsilon^f(s_k)}, \\ &> 1, \end{aligned}$$

where the inequality follows from Lemma 1 and $s_k(\theta) > s_l(\theta)$. Thus, input expenditure per worker increases discontinuously at the skill thresholds that separate sectors. It follows that input expenditure per worker is strictly increasing in θ.

Proof of Lemma 2

Let Ω be an arbitrary subset of agents with skill levels in $[\theta_a, \theta_b]$. If the mass of agents in Ω is concentrated at a single point, then there is no inequality between members of Ω. Assume this is not the case and let $\theta_{\min} = \inf\{\theta \in \Omega\}$ and $\theta_{\max} = \sup\{\theta \in \Omega\}$. Clearly, $\theta_{\max} > \theta_{\min}$.

Let $\hat{w}(\theta) = C\tilde{w}(\theta)$ where C is chosen to ensure $\mathbb{E}_\Omega \hat{w}(\theta) = \mathbb{E}_\Omega w(\theta)$ and $\mathbb{E}_\Omega$ denotes an expectation taken over the subset Ω. Obviously, $\epsilon^{\hat{w}}(\theta) = \epsilon^{\tilde{w}}(\theta)\, \forall\theta$. Since $\epsilon^{w}(\theta) > \epsilon^{\hat{w}}(\theta)\, \forall\theta \in (\theta_{\min}, \theta_{\max})$ we have that if $w(\theta') = \hat{w}(\theta')$ with $\theta' \in \Omega$ then $w(\theta) > \hat{w}(\theta)\, \forall\theta > \theta', \theta \in \Omega$ and $w(\theta) < \hat{w}(\theta)\, \forall\theta < \theta', \theta \in \Omega$. Remembering that $\mathbb{E}_\Omega \hat{w}(\theta) = \mathbb{E}_\Omega w(\theta)$ it immediately follows that $w(\theta)$ and $\hat{w}(\theta)$ satisfy a single-crossing property on $[\theta_{\min}, \theta_{\max}]$ with $w(\theta_{\min}) < \hat{w}(\theta_{\min})$ and $w(\theta_{\max}) > \hat{w}(\theta_{\max})$.

Consequently, the wage distribution over Ω induced by $\hat{w}(\theta)$ second-order stochastically dominates the distribution induced by $w(\theta)$. Since $\hat{w}(\theta)$ and $\tilde{w}(\theta)$ are identical up to a change in scale it follows that for any measure of inequality that respects scale independence and second-order stochastic dominance wage inequality among members of Ω is higher when wages are given by $w(\theta)$ than when wages are given by $\tilde{w}(\theta)$.

Proof of Proposition 4

Since the skill distribution has no mass points the (MC) condition implies that $\pi_2 \to 0$ as $\theta_1 \to 0$ and $\pi_2 \to \infty$ as $\theta_1 \to \bar{\theta}$. The (WE) condition implies that $\pi_2 < 1\, \forall\theta_1 \in \left(0, \bar{\theta}\right]$ since if $\pi_2 \geq 1$ all agents obtain a strictly higher wage in sector two than in sector one. Differentiating the (WE) condition gives:

$$\left(\pi_2^{\frac{1}{\beta}} f\left[s_2(\theta_1)\right] - f\left[s_1(\theta_1)\right]\right) \epsilon^g(\theta_1)\hat{\theta}_1 - s_1(\theta_1) f'\left[s_1(\theta_1)\right]\left(\hat{Q}_1 + \hat{\gamma}\right) = \\ -\pi_2^{\frac{1}{\beta}} s_2(\theta_1) f'\left[s_2(\theta_1)\right]\left(\hat{\pi}_2 + \hat{Q}_2 + \hat{\gamma}\right) - \left(f\left[s_1(\theta_1)\right] - \beta s_1(\theta_1) f'\left[s_1(\theta_1)\right]\right)\frac{\hat{\pi}_2}{\beta}, \tag{17}$$

where $\hat{\theta}_1 \equiv \frac{d\theta_1}{\theta_1}$ and analogous definitions hold for other variables. Differentiating the (MC) condition gives:

$$C_1\hat{\theta}_1 = C_2\left(\frac{1-\beta}{\beta}\hat{\pi}_2 - \hat{Q}_1 - \hat{\gamma}\right) + C_3\left(\hat{\pi}_2 + \hat{Q}_2 + \hat{\gamma}\right) + C_4\hat{\pi}_2, \tag{18}$$

where:

$$
\begin{aligned}
C_1 &\equiv \left(f\left[s_1(\theta_1)\right] + \frac{\beta\pi_2^{\frac{1}{\beta}}}{1-\beta} f\left[s_2(\theta_1)\right] \right) \theta_1^2 g(\theta_1) dM(\theta_1) > 0, \\
C_2 &\equiv \int_0^{\theta_1} \theta g(\theta) \frac{f'\left[s_1(\theta)\right]^2}{-f''\left[s_1(\theta)\right]} dM(\theta) > 0, \\
C_3 &\equiv \frac{\beta\pi_2^{\frac{1}{\beta}}}{1-\beta} \int_{\theta_1}^{\bar{\theta}} \theta g(\theta) \frac{f'\left[s_2(\theta)\right]^2}{-f''\left[s_2(\theta)\right]} dM(\theta) \geq 0, \\
C_4 &\equiv \frac{\pi_2^{\frac{1}{\beta}}}{1-\beta} \int_{\theta_1}^{\bar{\theta}} \theta g(\theta) f\left[s_2(\theta)\right] dM(\theta) \geq 0.
\end{aligned}
\tag{19}
$$

The derivations of (17) and (18) use $\hat{\pi}_1 = -\frac{1-\beta}{\beta}\hat{\pi}_2$, which follows from differentiating (12). Note that (17) and (18) allow for variation in Q_1, Q_2 and γ. This is not necessary to prove Proposition 4, but will be needed for the proof of Proposition 5.

Let $\hat{Q}_1 = \hat{Q}_2 = \hat{\gamma} = 0$. Note that: (i) $\frac{\pi_2^{\frac{1}{\beta}} f[s_2(\theta_1)]}{f[s_1(\theta_1)]} = \frac{1-\epsilon^f[s_1(\theta_1)]}{1-\epsilon^f[s_2(\theta_1)]} > 1$ since the span of control is higher in sector two, and; (ii) $f\left[s_1(\theta_1)\right] > s_1(\theta_1) f'\left[s_1(\theta_1)\right]$. Therefore, it follows from (17) that the (WE) curve is strictly downwards sloping on $\left(0, \bar{\theta}\right]$. In addition, equation (18) implies that the (MC) curve is strictly upward sloping on $\left(0, \bar{\theta}\right]$. Combining these results with the boundary conditions above proves that the (WE) and (MC) curves have a unique intersection on $\left(0, \bar{\theta}\right)$.

Proof of Proposition 5

Let $\hat{\gamma} = 0$. Suppose $\hat{Q}_1 = 0$, but $\hat{Q}_2 > 0$. Then, if $\hat{\pi}_2 \geq 0$ equation (17) implies $\hat{\theta}_1 < 0$, but equation (18) implies $\hat{\theta}_1 > 0$ – a contradiction. Therefore, we must have $\hat{\pi}_2 < 0 \Rightarrow \hat{\pi}_1 > 0$. Now suppose $\hat{\pi}_2 < 0$ and $\hat{\pi}_2 + \hat{Q}_2 \leq 0$. Then equation (17) implies $\hat{\theta}_1 > 0$, but equation (18) implies $\hat{\theta}_1 < 0$ – a contradiction. Therefore, we must have $\hat{\pi}_2 + \hat{Q}_2 > 0$. Similar reasoning shows that if $\hat{Q}_1 > 0$ and $\hat{Q}_2 = 0$ then (17) and (18) together imply $\hat{\pi}_2 > 0, \hat{\pi}_1 < 0$ and $\hat{\pi}_1 + \hat{Q}_1 = -\frac{1-\beta}{\beta}\hat{\pi}_2 + \hat{Q}_1 > 0$. This proves the claims made in equation (14).

Given $\frac{d[\pi_k Q_k]}{dQ_j} > 0, j, k = 1, 2$ equations (3) and (9) together imply that $\frac{d\epsilon^{w_k}(\theta)}{dQ_j} > 0 \, \forall \theta, j, k = 1, 2$. Lemma 2 is then sufficient to conclude that technological progress increases within-group inequality among any group of agents who all work in the same sector and do not switch sectors following the technology shock.

If $\frac{d\theta_1}{dQ_j} < 0$, agents switch from sector one to sector two following an increase in Q_j. Since $s_2(\theta) >$

$s_1(\theta)\,\forall\theta$, equation (9) implies $\epsilon^{w_2}(\theta) > \epsilon^{w_1}(\theta)\,\forall\theta$. Remembering that $\frac{d\epsilon^{w_k}(\theta)}{dQ_j} > 0\,\forall\theta, j, k = 1, 2$ this means that an increase in Q_j unambiguously increases $\epsilon^{w}(\theta)$ at any value of θ such that agents switch from sector one to sector two following the shock. It immediately follows that $\frac{d\epsilon^{w}(\theta)}{dQ_j} > 0\,\forall\theta$. Therefore, Lemma 2 implies that income inequality increases among all subsets of agents.

Finally, observe from (17) and (18) that the case where $\hat{\gamma} = \chi \neq 0$ is equivalent to having $\hat{\gamma} = 0$ and $\hat{Q}_1 = \hat{Q}_2 = \chi$. It immediately follows that $\frac{d[\pi_k \gamma]}{d\gamma} > 0, k = 1, 2$ implying that an increase in γ has the same qualitative effects on the returns to skill and wage inequality as an increase in either Q_1 or Q_2.

Proof of Proposition 6

Without loss of generality suppose $M_2(\theta_1)$ increases. The (WE) condition is independent of the skill distribution. Holding θ_1 and π_2 constant, a rise in $M_2(\theta_1)$ does not affect the left hand side of the (MC) condition, but increases the right hand side. Consequently, the (MC) curve shifts outwards implying that θ_1 increases, π_2 falls and π_1 rises. Equations (3), (4) and (9) then imply that for workers who do not switch sectors the wage level and returns to skill fall in sector two and rise in sector one. Finally, appealing to Lemma 2 shows that wage inequality within workers who do not switch sectors falls in sector two and rises in sector one.

Proof of Proposition 7

The (WE) condition is the same as in the closed economy. It is a strictly downward sloping curve on $\left(0, \bar{\theta}\right]$. Let π_2^{WE} be the value of π_2 at which the (WE) curve intersects the $\theta_1 = 0$ axis. Obviously, $\pi_2^{WE} \leq 1$. The (MC′) condition implies $\pi_2 \to \infty$ as $\theta_1 \to \bar{\theta}$. Differentiating the (MC′) condition gives:

$$C_1\hat{\theta}_1 = C_2\left(\frac{1-\beta}{\beta}\hat{\pi}_2 - \hat{Q}_1 - \hat{\gamma}\right) + C_3\left(\hat{\pi}_2 + \hat{Q}_2 + \hat{\gamma}\right) + C_4\hat{\pi}_2 + C_5\left(\frac{1-\beta}{\beta}\hat{\pi}_2 - \hat{Q}_1^* - \hat{\gamma}^*\right), \quad (20)$$

where C_1, C_2, C_3 and C_4 are defined by (19) and:

$$C_5 \equiv \int_0^{\bar{\theta}^*} \theta g(\theta)\frac{f'\left[s_1^*(\theta)\right]^2}{-f''\left[s_1^*(\theta)\right]} dM^*(\theta) > 0.$$

Equation (20) implies that the (MC′) curve is strictly upward sloping on $\left(0, \bar{\theta}\right]$. Let $\pi_2^{MC'}$ be the value of π_2 at which the (MC′) curve intersects the $\theta_1 = 0$ axis. Equation (15) implies $\pi_2^{MC'} \leq 1$. If $\pi_2^{MC'} < \pi_2^{WE}$

then the (WE) condition and the (MC′) condition must have a unique intersection on $\left(0,\bar{\theta}\right)$ and this gives the open economy equilibrium. If $\pi_2^{MC'} \geq \pi_2^{WE}$ then equilibrium is given by $\theta_1 = 0$ and $\pi_2 = \pi_2^{MC'}$ and in equilibrium both countries specialize in their high productivity sector. This proves the existence of a unique open economy equilibrium.

The remainder of Proposition 7 follows immediately from the discussion in the main body of the paper.

Proof of Proposition 8

Consider the home country and assume home is not specialized in equilibrium. Since $\tilde{\pi}_2 < \pi_2$ and $\tilde{\pi}_1 > \pi_1$, equations (3) and (4) imply that $w_2(\theta) > \tilde{w}_2(\theta)$ and $w_1(\theta) < \tilde{w}_1(\theta)\,\forall\theta$. In addition, $0 < \theta_1 < \tilde{\theta}_1$ and the continuity of w and $\tilde{w}$ imply $w(\theta_1) < \tilde{w}(\theta_1)$ and $w(\tilde{\theta}_1) > \tilde{w}(\tilde{\theta}_1)$. Moreover, $\epsilon^w(\theta) > \epsilon^{\tilde{w}}(\theta)\,\forall\theta \in (\theta_1, \tilde{\theta}_1)$. Therefore, invoking continuity once more, w and $\tilde{w}$ must intersect exactly once on $(\theta_1, \tilde{\theta}_1)$. Trade liberalization reduces the wage of all agents with skill below the intersection and increases the wage of all agents with skill above the intersection.

From (3) we have that $\tilde{\pi}_1 > \pi_1 \Rightarrow \tilde{s}_1(\theta) > s_1(\theta)\,\forall\theta$. Equation (9) then implies $\epsilon^{\tilde{w}}(\theta) > \epsilon^w(\theta)\,\forall\theta < \theta_1$. Applying Lemma 2 this means that wage inequality within any subset of agents who work in sector one after trade liberalization is lower in the open economy than in the closed economy. By contrast, $\tilde{\pi}_1 Q_1 < \tilde{\pi}_2 Q_2 < \pi_2 Q_2 \Rightarrow \epsilon^{\tilde{w}}(\theta) < \epsilon^w(\theta)\,\forall\theta > \theta_1$. Consequently, trade liberalization increases wage inequality within any subset of agents who work in sector two in the open economy.

Similar reasoning can be used to prove the analogous results for the home country when $\theta_1 = 0$ and for the foreign country.

Proof of Proposition 9

Equilibrium is defined by the wage equalization (WE) condition and its foreign equivalent, which are the same as in autarky, and by the global output market clearing condition:

$$\int_0^{\theta_1} \theta g(\theta) f(s_1) dM(\theta) + \int_0^{\theta_1^*} \theta g(\theta) f(s_1^*) dM^*(\theta) =$$
$$\frac{\beta}{1-\beta}\pi_2^{\frac{1}{\beta}} \left[\int_{\theta_1}^{\bar{\theta}} \theta g(\theta) f(s_2) dM(\theta) + \int_{\theta_1^*}^{\bar{\theta}^*} \theta g(\theta) f(s_2^*) dM^*(\theta)\right].$$

From the foreign wage equalization condition, θ_1^* is strictly decreasing in π_2. Given this relationship it is easy to differentiate the market clearing condition, as was done in the proofs of Propositions 4 and 7, and show that it defines a strictly upward sloping relationship between θ_1 and π_2. The market clearing condition also implies that when $\theta_1 = \bar{\theta}, \pi_2 > \tilde{\pi}_2^* > \tilde{\pi}_2$ implying that in θ_1–π_2 space the market clearing curve sits above the home (WE) curve when $\theta_1 = \bar{\theta}$. Let π_2^{WE} be the value of π_2 at which the home (WE) curve intersects the $\theta_1 = 0$ axis. Let $\pi_2^{MC''}$ be the value of π_2 at which the market clearing curve intersects the $\theta_1 = 0$ axis. If $\pi_2^{MC''} < \pi_2^{WE}$ then the home (WE) condition and the market clearing condition must have a unique intersection on $\left(0, \bar{\theta}\right)$ and this gives the open economy equilibrium. If $\pi_2^{MC''} \geq \pi_2^{WE}$ then equilibrium is given by $\theta_1 = 0$ and $\pi_2 = \pi_2^{MC''}$. This proves the existence of a unique open economy equilibrium.

In addition, since the global market clearing condition is simply the sum of the home autarky market clearing condition (MC) and its foreign equivalent we cannot have $\pi_2 \leq \tilde{\pi}_2$ or $\pi_2 \geq \tilde{\pi}_2^*$. In the former case there is excess global supply of good one, and in the later there is excess global supply of good two. Therefore, $\tilde{\pi}_2 < \pi_2 < \tilde{\pi}_2^*$. The remainder of the proof follows from the discussion in the main body of the paper and from using reasoning analogous to that applied in the proof of Proposition 8 to characterize the effect of moving from autarky to free trade on wage levels and wage inequality.

Proof of Proposition 10

First, consider the case when sectoral outputs are non-traded. In this case equilibrium is given, as in autarky, by the (WE) and (MC) conditions at home and their foreign equivalents abroad, but with $p = \frac{1}{\gamma^*}$ in both countries. Therefore, for home intermediate input trade is equivalent to experiencing an increase in γ in autarky. The result then follows immediately from applying Proposition 5.

When the sectoral outputs are traded there are multiple cases to consider depending on whether there exists an assignment reversal and whether production in each country is diversified. However, since the same reasoning applies in each case I will only give the proof for the case considered in Section 5.1 where an assignment reversal exists and foreign is specialized in producing good one. In this case equilibrium is given by the (WE) and (MC′) conditions and differentiating these conditions gives (17) and (20). Intermediate input trade implies $\hat{\gamma} > 0$ while $\hat{Q}_1 = \hat{Q}_2 = \hat{\gamma}^* = 0$. Now, if $\hat{\pi}_2 \geq 0$ equation (17) implies $\hat{\theta}_1 < 0$ and equation (20) then implies $\hat{\pi}_1 + \hat{\gamma} > 0$. Alternatively, if $\hat{\pi}_2 < 0$ and $\hat{\pi}_2 + \hat{\gamma} \leq 0$ then equation (17) implies $\hat{\theta}_1 > 0$, but equation (20) implies $\hat{\theta}_1 < 0$ – a contradiction. It follows that if $\hat{\pi}_2 < 0$ then $\hat{\pi}_2 + \hat{\gamma} > 0$.

Therefore, in both cases we have $\hat{\pi}_k + \hat{\gamma} > 0, k = 1, 2$. Proposition 10 then follows immediately from applying equations (3) and (9) and Lemma 2.

Appendix B – Theoretical extensions

Cross-sector heterogeneity

It is straightforward to modify the production technology in (1) to allow for sources of cross-sector heterogeneity other than differences in intermediate input productivity. Suppose that production in sector k requires a team of N_k workers and that if each worker has skill θ output is given by:[33]

$$y_k(\theta, x) = g(\theta) A_k \left[\lambda_k \left(B_k \theta \right)^{\frac{\sigma-1}{\sigma}} + (1-\lambda_k) \left(Q_k x \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}. \tag{1'}$$

Given Assumption 1 we must have $\sigma > 1$. This formulation allows for cross-sector heterogeneity in team size N_k, Hicks-neutral productivity A_k, labor augmenting productivity B_k, intermediate input productivity Q_k and the labor intensity of production λ_k. I restrict the production function to be a constant elasticity of substitution (CES) technology in order to introduce the CES parameter λ_k. If λ_k is not included in the analysis then the results below hold without imposing functional form restrictions on F.

The same reasoning employed to derive Proposition 1 can be used to characterize the equilibrium assignment when output is given by ($1'$). The structure of equilibrium is unchanged, but agents sort across sectors based not on the ranking of sectors by Q_k, but on the ranking of sectors by V_k where:

$$V_k \equiv \left(\frac{1-\lambda_k}{\lambda_k} \right)^{\frac{\sigma}{\sigma-1}} \frac{N_k Q_k}{B_k}.$$

Higher ability agents are assigned to sectors with higher V_k. Consequently, skill levels and wages are higher, ceteris paribus, in sectors with: (i) higher intermediate input productivity; (ii) lower labor augmenting productivity; (iii) larger production teams, and; (iv) lower labor intensity.

Interestingly, different forms of technical change have contrasting implications for sorting across sectors. Whereas increases in intermediate input productivity tend to draw more skilled workers into a sector, labor augmenting technical change has the opposite effect. To understand this result, remember that when Assumption 1 holds and output is given by (1) higher ability agents sort into sectors with higher spans of control. If we redefine the span of control to equal the number of efficiency units of intermediate input used per efficiency unit of skill, $s_k(\theta) \equiv \frac{Q_k x}{B_k \theta}$, this insight remains true under the production technology ($1'$).

[33] This specification assumes that in equilibrium all members of a team have the same skill level. This will necessarily be the case if, for example, a team inherits the skill level of its least able member.

Labor augmenting technical change reduces an agent's optimal span of control and, therefore, has the opposite effect to increases in intermediate input productivity. Similarly, higher labor intensity is equivalent to a simultaneous rise in labor augmenting productivity and fall in intermediate input productivity and decreases the optimal span of control. Meanwhile, higher team size increases the output price by raising labor costs, thereby leading to a greater optimal span of control. Finally, the equilibrium sorting pattern does not depend on Hicks-neutral productivity A_k because A_k is multiplicatively separable from the production function.

Generalized final good technology

Suppose that instead of equation (10), the final good production function is given by:

$$Z = H(Y_1, Y_2),$$

where H is a constant returns to scale function that is strictly increasing in both its arguments, strictly concave and satisfies $\lim_{Y_k \to 0} \frac{\partial H}{\partial Y_k} = \infty$, $k = 1, 2$. Obviously, introducing this final good technology does not affect the existence of positive assortative matching between high skill agents and high technology sectors.

Let $\zeta \equiv \frac{Y_2}{Y_1}$. Then cost minimization in final good production and the choice of the final good as numeraire together imply $\frac{d\pi_2}{d\pi_1} = -\frac{1}{\zeta} < 0$ and:

$$\frac{h'(\zeta)}{h(\zeta) - \zeta h'(\zeta)} = \frac{\pi_2}{\pi_1}, \tag{21}$$

where $h(\zeta) \equiv H(1, \zeta)$. Since H is strictly concave, (21) implies that ζ is a strictly decreasing function of $\frac{\pi_2}{\pi_1}$.

As in the Cobb-Douglas case, equilibrium reduces to a wage equalization condition and a market clearing condition. The wage equalization condition is still given by equation (WE) above, while the market clearing condition is:

$$\int_0^{\theta_1} \theta g(\theta) f(s_1) dM(\theta) = \frac{1}{\zeta} \int_{\theta_1}^{\bar{\theta}} \theta g(\theta) f(s_2) dM(\theta).$$

By differentiating this expression and using $\zeta'\left(\frac{\pi_2}{\pi_1}\right) < 0$, it is straightforward to show that the market clearing condition defines an upward sloping curve in θ_1-π_2 space and that Propositions 4, 5 and **??** continue to hold.

To solve the open economy model note that the open economy market clearing condition is:

$$Y_1 + Y_1^* = \frac{1}{\zeta}(Y_2 + Y_2^*),$$

where ζ is given by (21). In addition, when there is an assignment reversal across countries foreign will specialize in good one if and only if:

$$\bar{\zeta}\int_0^{\bar{\theta}^*} \theta g(\theta) f(s_1^*) dM^*(\theta) \le \int_0^{\bar{\theta}} \theta g(\theta) f(s_2) dM(\theta)$$

where $\frac{h'(\bar{\zeta})}{h(\bar{\zeta})-\bar{\zeta}h'(\bar{\zeta})} = 1$ and $\pi_1 = \pi_2 = h'(\bar{\zeta})$.

Using these expressions we can solve for the open economy equilibrium following the same reasoning applied in the Cobb-Douglas case and Propositions 7, 8, 9 and 10 continue to hold.

Heckscher-Ohlin assignment model

Consider the following variant of the Heckscher-Ohlin model. There are two industries and two factors of production and each industry has a Cobb-Douglas technology:

$$Z_j = \left(\frac{Y_{1j}}{\mu_j}\right)^{\mu_j}\left(\frac{Y_{2j}}{1-\mu_j}\right)^{1-\mu_j}, \qquad \mu_j \in (0,1), \quad j = 1,2,$$

where Z_j is output of industry j and Y_{kj} is the quantity of factor k used in industry j. Assume $\mu_1 > \mu_2$ meaning that industry one is factor one intensive. Now, suppose that the factors of production do not represent the economy's endowments, but must be produced. Factor k is the output of task k and task production is governed by the assignment problem in Section 3. Finally, suppose that output from the two industries is combined to produce a final good, which can either be consumed or used as the intermediate input in task production. Output of the final good is given by:

$$Z = \left(\frac{Z_1}{\beta}\right)^{\beta}\left(\frac{Z_2}{1-\beta}\right)^{1-\beta}, \qquad \beta \in (0,1).$$

In this set-up factor supplies are endogenous to the equilibrium of the assignment problem. Suppose task two has higher intermediate input productivity than task one, $Q_2 > Q_1$. Then, given Assumption 1, high skill agents will be assigned to task two and low skill agents will perform task one.

Following the same logic used to solve for equilibrium in Section 4.2, it is easy to show that the closed economy equilibrium of this Heckscher-Ohlin assignment model can be characterized by the same (WE) and (MC) conditions derived in Section 4.2, except that the parameter β is replaced by $\mu_1\beta + \mu_2(1-\beta)$. Consequently, the model has a unique closed economy equilibrium featuring positive assortative matching between agents and tasks and the effects of technical change on the returns to skill and wage inequality are as described in Section 4.3.

In the baseline model all workers in the high productivity sector have higher skill than any worker in the low technology sector. However, in this Heckscher-Ohlin variant each industry employs both high skill workers to perform task two and low skill workers to perform task one. The equilibrium wage function ensures that employers are indifferent between all workers assigned to a particular task. Therefore, I will assume that the skill distribution of workers employed in each task is the same in both industries. Under this assumption the average wage w_j in industry j is:

$$w_j = \frac{\bar{w}_1 + \nu_j \bar{w}_2}{1 + \nu_j},$$

where $\bar{w}_k$ is the average wage of agents assigned to task k and:

$$\nu_j \equiv \frac{1-\mu_j}{\mu_j} \frac{\mu_1\beta + \mu_2(1-\beta)}{1-\mu_1\beta - \mu_2(1-\beta)} \frac{M(\bar{\theta}) - M(\theta_1)}{M(\theta_1)}.$$

Unsurprisingly, the mean industry wage is a weighted average of the mean task wages. Note that $\mu_1 > \mu_2 \Rightarrow \nu_1 < \nu_2$. Therefore, the mean industry wage is higher in the industry that is intensive in the high skill task. As in the baseline model, shocks to intermediate input productivity which switch the productivity ranking across tasks will reverse the ranking of industries by average wages and average employee skill. It can also be shown that labor's share of output is lower in the industry that is intensive in the high skill task.

Appendix C – Data

UNIDO's Industrial Statistics database contains employment and compensation data for 127 ISIC Revision 3 manufacturing industries at the 4 digit level. The database starts in 1990, but country coverage varies over time. The wage variable is defined as the ratio of Wages and salaries to Employment. The sample used in the paper is selected as follows: (i) for each country the data used is from the latest year between 1995 and 2000 for which wage data is reported; (ii) all industries reporting negative wages and salaries, or with fewer than 10 employees, were dropped; (iii) only countries with data on at least 60% of industries were included.[34] The final sample covers 43 countries including the US.[35] Wage data for the US is available from 1997-2000. The statistics shown in Figure 1 and Figure 2 are calculated using US data for the same year in which a country reported data, unless the data is from 1995 or 1996, in which case US data from 1997 is used.

UNIDO's Industrial Statistics database does not include long time series of industry data at the 4 digit level. Consequently, changes in wage rank correlations are computed using wage data for 3 digit ISIC Revision 2 manufacturing industries. The 3 digit data covers 28 industries and I drop country-year observations with wage data for fewer than 80% of industries. I use data from 1965-1995 and compute annualized changes between the first and the last year in which a country is included in the data set. Only countries for which the first and the last year are at least 10 years apart are included.

The EU KLEMS data is taken from the March 2008 release of the database. The industry wage rate is defined as the ratio of Compensation of employees to Total hours worked by employees. The data for 1995 covers 29 countries and, at the most disaggregated level available, 29 manufacturing industries. I use the NAICS-based data for the US and drop Luxembourg from the sample since it has a higher income than the US.

The IPUMS-International data includes all 34 countries for which a census taken between 1995 and 2005 is available. I use the internationally harmonized educational attainment and industry of employment variables and drop all respondents for whom either educational attainment or industry is unknown. I drop

[34] Informal data examination suggests there is substantial noise in the Industrial Statistics database. The 60% coverage cut-off is designed to select for countries that produce relatively comprehensive industrial statistics, since such countries are likely to report higher quality data. It also reduces the selection bias that may arise if there is non-randomness in which industries report data. The results in the paper do not depend on the exact value of the cut-off.

[35] The sample countries are: Austria, Azerbaijan, Bangladesh, Belgium, Canada, Colombia, Denmark, Ecuador, Egypt, Finland, France, Germany, Hungary, India, Indonesia, Iran, Italy, Japan, Kyrgyzstan, Latvia, Lebanon, Lithuania, Malaysia, Mexico, Morocco, Netherlands, New Zealand, Norway, Peru, Portugal, Singapore, Slovakia, Slovenia, South Korea, Spain, Sweden, Thailand, Turkey, Ukraine, United Kingdom, US, Vietnam and Zimbabwe.

the industry labeled “Other industry, n.e.c.”, leaving 15 industries covering the entire economy. Mali is not included in the regressions reported in Section 2.1 because its extremely low skill rank correlations make it a clear outlier.

Capital stock per capita is computed from the Penn World Tables 6.3 using the perpetual inventory method as implemented by Caselli (2005). Human capital per capita is computed from the Barro and Lee (2001) educational attainment data set. Average years of schooling for the population 25 and over is converted to human capital following the methodology in Caselli (2005).

Table 1: International wage structure comparisons

Dependent variable:	Wage rank correlation		Wage correlation		Proportion pairwise rank matches	Δ Wage rank correlation	Wage rank correlation
	(a)	(b)	(c)	(d)	(e)	(f)	(g)
Income	0.13 ***	0.075 ***	0.14 ***	0.089 ***	0.055 ***		0.26 ***
	(0.016)	(0.024)	(0.028)	(0.031)	(0.0068)		(0.064)
Δ Income						0.12 **	
						(0.052)	
Constant	0.75 ***	0.79 ***	0.70 ***	0.76 ***	0.78 ***	0.0049 ***	0.69 ***
	(0.025)	(0.037)	(0.042)	(0.045)	(0.011)	(0.0012)	(0.036)
R^2	0.57	0.19	0.36	0.19	0.58	0.06	0.41
N	42	42	42	42	42	70	25

Robust standard errors in parentheses.
Income is expressed as the log difference from US income.
* indicates coefficient statistically significant at 10% level; ** at 5% level, and; *** at 1% level.
Columns (a)-(e) use UNIDO Industrial Statistics wage data covering 127 4 digit manufacturing industries in 2000.
Columns (b) and (d) use correlations calculated using industry employment shares as weights.
Column (f) uses UNIDO Industrial Statistics wage data covering 28 3 digit manufacturing industries and calculates changes between 1965 and 1995.
Column (g) uses EU Klems wage data covering 29 manufacturing industies in 1995.

Table 2: International industry skill comparisons

Dependent variable:	Skill rank correlation	
	Secondary	**Tertiary**
	(a)	(b)
Income	0.018 *	0.046 **
	(0.010)	(0.020)
Constant	0.89 ***	0.93 ***
	(0.015)	(0.024)
R^2	0.11	0.33
N	32	32

Robust standard errors in parentheses.
Income is expressed as the log difference from US income.
* indicates coefficient statistically significant at 10% level; ** at 5% level, and; *** at 1% level.
Education data is from IPUMS-International and covers 15 industries.

Table 3: Wage structure and factor endowments

Dependent variable:	Wage rank correlation	
	(a)	(b)
Human capital	-0.0029	
	(0.14)	
Secondary enrollment rate		-0.022
		(0.066)
Physical capital	0.085 ***	0.095 ***
	(0.028)	(0.015)
Income		
Constant	0.72 ***	0.71 ***
	(0.038)	(0.025)
R^2	0.53	0.54
N	32	41

Robust standard errors in parentheses.
Human capital, secondary enrollment rate and physical capital are expressed as the log difference from their respective US values.
* indicates coefficient statistically significant at 10% level; ** at 5% level, and; *** at 1% level.
UNIDO Industrial Statistics wage data covers 127 4 digit manufacturing industries in 2000.

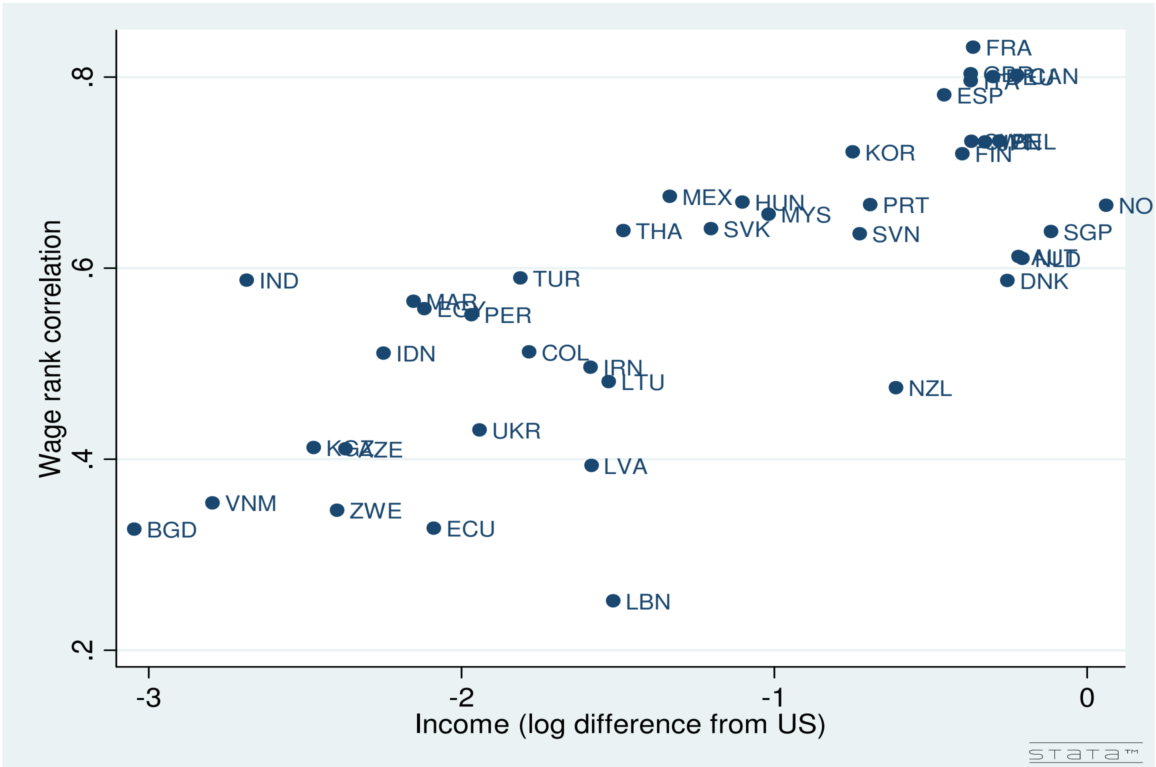

Figure 1: Wage rank correlations – UNIDO 2000

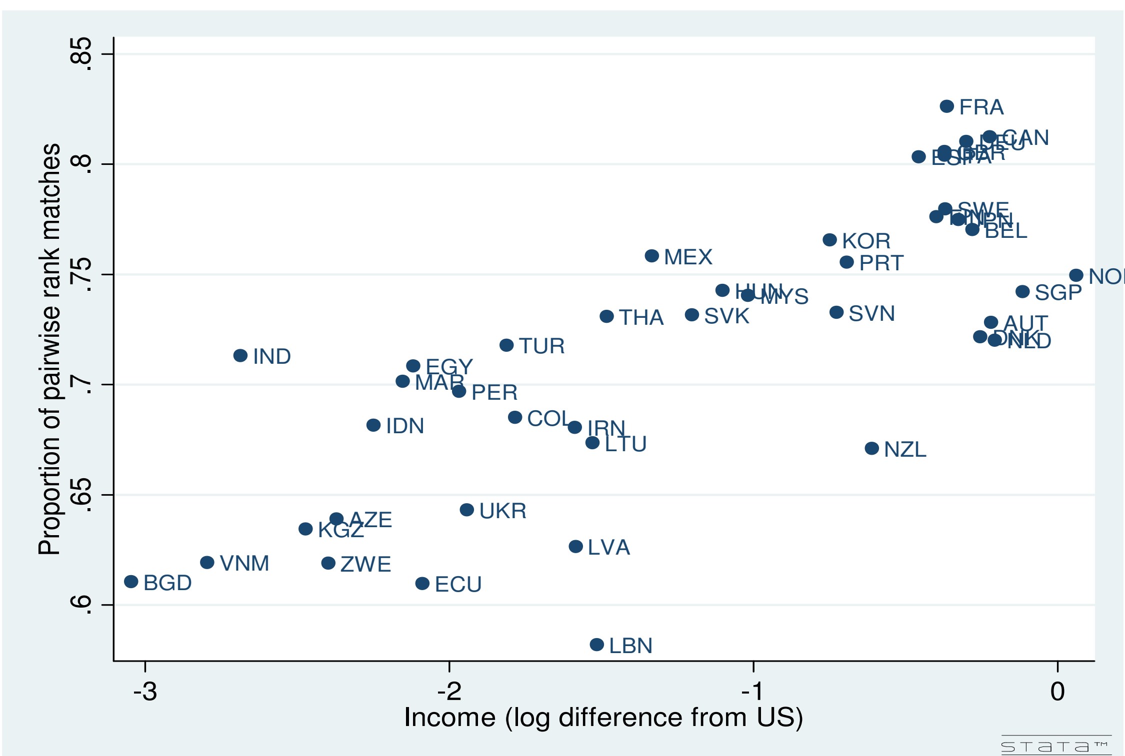

Figure 2: Pairwise industry wage rank matches – UNIDO 2000

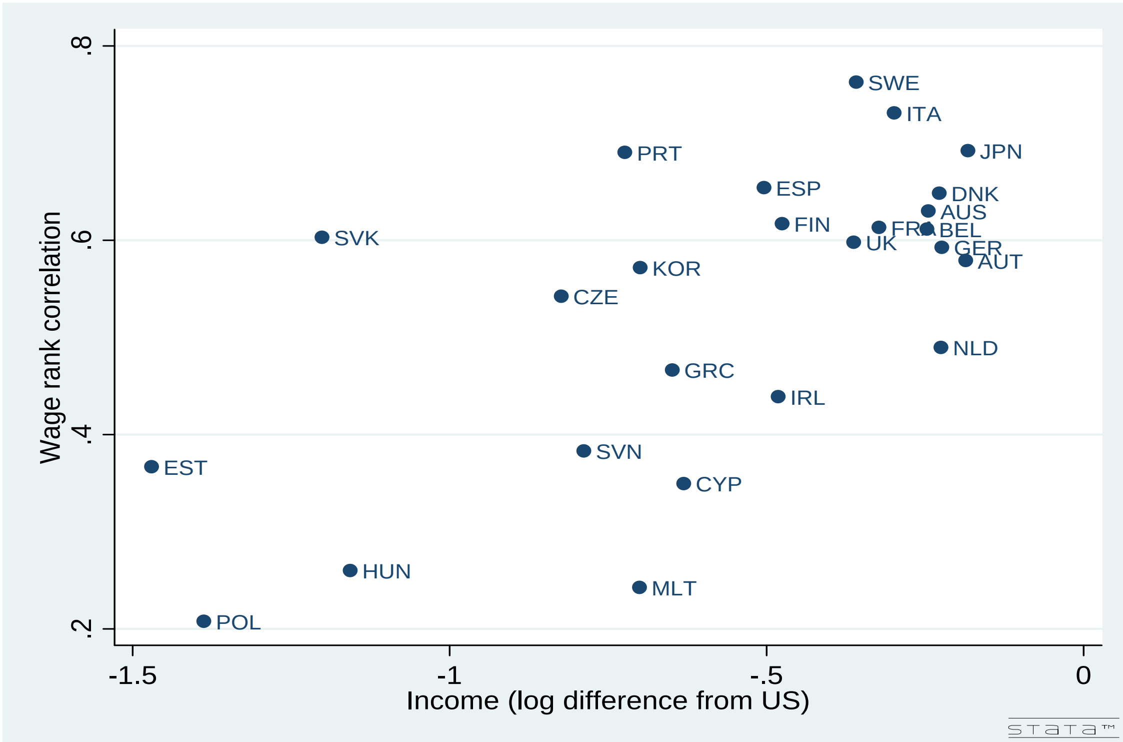

Figure 3: Wage rank correlations – EU KLEMS 1995

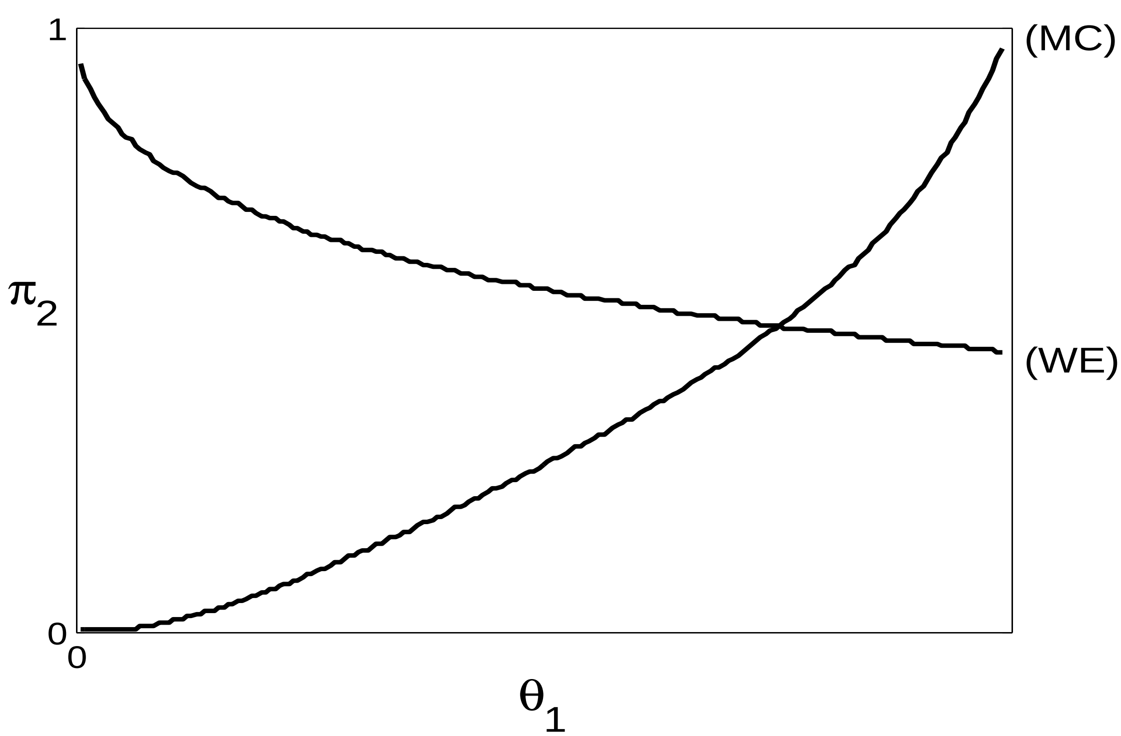

Figure 4: Closed economy equilibrium

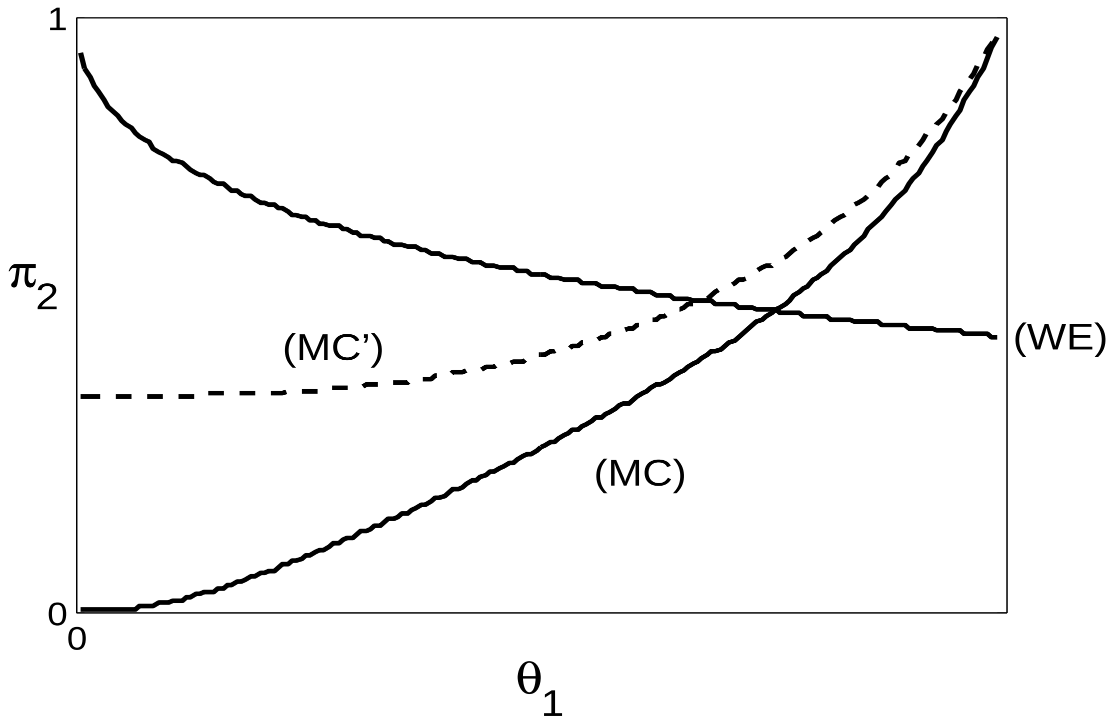

Figure 5: Open economy equilibrium