# Skills, Tasks and the Scarcity of Talent in a Global Economy

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April 2, 2014

**Abstract**

The scarcity of talent is a tremendous challenge for firms in the globalized world. This paper investigates the role of labor market imperfection in open economies for the usage of talent in the production process of firms. For this purpose, I set up a heterogeneous firms model, where production consists of a continuum of tasks that differ in complexity. Firms hire low-skilled and high-skilled workers to perform these tasks. How firms assign workers to tasks depends on factor prices for the two skill types and the productivity advantage of high-skilled workers in the performance of complex tasks. I study the firms' assignment problem under two labor market regimes, which capture the polar cases of fully flexible wages and a binding minimum wage for low-skilled workers. Since the minimum wage lowers the skill premium, it reduces the range of tasks performed by high-skilled workers, which increases firm-level productivity and reduces the mass of active firms. Whereas trade does not affect the firm-internal assignment of workers to tasks in a setting with fully flexible wages, it renders high-skilled workers a scarce resource and reduces the range of tasks performed by this skill type with negative consequences for firm-level productivity, if low-skilled wages are fixed by a minimum wage. In this case, trade leads to higher per-capita income for both skill types and thus to higher welfare in the open than in the closed economy, whereas – somewhat counter-intuitive – inequality between the two skill types decreases, as more low-skilled workers find employment in the production process.

**JEL codes:** F12, F16
**Keywords:** Scarcity of Talent, Firm-Internal Labor Allocation, Trade, Minimum Wages, Heterogeneous Firms

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# 1 Introduction

The organization of production within firms is a key determinant of firm performance. The ability to allocate scarce resources within the boundaries of firms in an efficient way is essential for firms to compete in modern economic life.[1] In recent years, the assignment of skills to tasks within the boundaries of firms seems to deteriorate, because of a growing mismatch between the skill needed in the production process and the talent available in the labor market.[2] Due to the scarcity of talent (and thus skill), firms must hire candidates that are under-qualified for the job, "just to fill a position quickly" (The Economist, 2006, p.139). This has negative consequences for a firm's productivity and competitiveness.[3] While there might be several reasons explaining the prevalence of skill scarcity, one of the major driving forces is globalization, because economic integration has increased the demand for high-skilled workers significantly (see Beechler and Woodward, 2009). With just a rudimentary presentation of the production process, existing trade models are not well equipped to analyze the scarcity of skill with its negative impact on firm productivity. Therefore, this paper takes a new approach and introduces the idea of a task-based firm-level production process into an otherwise standard model of the new trade theory. Modeling the assignment of workers to tasks and a discussion on how this assignment changes in an open economy is in the center of this paper's interest.

I model the task-based production process along the lines of Acemoglu and Autor (2011) and assume that firm output is assembled from a continuum of tasks that differ in complexity. For the performance of tasks, firms can hire low-skilled or high-skilled workers. High-skilled workers are more productive than low-skilled workers in the performance of all tasks and their relative productivity increases with the complexity of tasks. How firms assign skills to tasks depends on the productivity advantage of high-skilled workers and their skill premium. By altering the range of tasks performed by high-skilled workers, firms do not only affect their labor costs but also their productivity. To determine the optimal skill range, to manage the firm, and to organize the complex production process, firms need a fixed input of high-skilled workers. I embed this framework of task-based production into a trade model along the lines of Melitz (2003) that features monopolistic competition between heterogeneous firms.[4] Heterogeneity arises due to exogenous differences in total factor productivity, and I analyze to what extent this heterogeneity affects the firms' decision regarding the assignment of workers to tasks.

[1] In a recent paper by Giroud and Mueller (2012) it is shown that firm-level productivity increases, due to efficient resource reallocation within firms.

[2] According to a survey quoted by the Economist (2006), 62% of interviewed firms are worried about the scarcity of talent in the company. This empirical fact is also documented in a yearly conducted survey by the Manpowergroup. In the most recent survey they highlight that "[t]alent shortages affect more than one in three businesses globally" (Manpowergroup, 2013, p.4).

[3] Being asked how the scarcity of skill affects the overall organization, "39% [of managers] say that talent shortages reduce competitiveness and productivity in general" (Manpowergroup, 2013, p.10).

[4] The mechanisms in this model would also be effective in a Krugman (1980)-type model with homogeneous producers. However, in line with the recent literature in international economics and to contrast my results with the existing literature by highlighting the firm-internal adjustment mechanism, I conduct the analysis in a setting with heterogeneous firms along the lines of Melitz (2003).

I start with a characterization of the closed economy and consider perfectly flexible wages as a benchmark of my analysis. I then investigate the consequences of labor market imperfection for the firm-internal assignment of workers to tasks. I capture labor market imperfection in the simplest possible way and consider minimum wage. The minimum wage is only binding for low-skilled workers and leads to involuntary unemployment of this skill group. It lowers the skill premium and firms respond to this by assigning high-skilled workers to a broader range of tasks, including less complex ones. This increases firm-level productivity and leaves less resources for overhead services, thereby enforcing firm exit. Welfare declines and both skill groups and up being worse off in term of their per-capita income after the introduction of the minimum wage. The relative income of high-skilled workers increases, so that the minimum wage widens the income inequality in this model.

Under both labor market regimes the heterogeneity between firms does not affect the assignment decision, since all firms are price takers in the labor market and pay the same wages. This implies that the revenue ratio of any two firms is fully characterized by the (exogenous) differential in total factor productivity, a feature that is well known from other trade models with heterogeneous firms. Thus, my model is not equipped to shed new light on self-selection into exporting, and I abstract from any trade impediments in the open economy situation to focus on those aspects of the model that are new to the literature.[5] Being interested in the firm-internal adjustment to the globalization shock and its consequences for key macroeconomic variables, I assume that all firms are affected symmetrically. To be more specific, I abstract from any (fixed or variable) shipment costs and assume similar to Krugman (1980) that all firms end up being exporters in the open economy. Firms expand their demand for both skill types in order to serve foreign in addition to domestic consumers. In a situation with fully flexible wages this does not affect the firm-internal assignment of workers to tasks, while things are different when considering minimum wage. Whereas low-skilled labor supply is perfectly elastic at the given minimum wage, high-skilled labor supply is inelastic. The scarcity of high-skilled workers implies an increase in the skill premium, and firms respond to the relative factor price change by assigning low-skilled workers to a larger range of tasks, thereby lowering productivity. This points to a so far unexplored channel through which trade affects domestic production practices. Since firms replace high-skilled by low-skilled workers in the production of goods, more high-skilled labor is available for overhead services, and additional firms enter and produce in the open economy. Finally, trade increases real per-capita income of both skill types (high-skilled wages increase and low-skilled unemployment falls), raises welfare and lowers income inequality.

Since the endogenous assignment of workers to tasks provides a so far unexplored channel through which firms can absorb macroeconomic shocks, I investigate the channel in more detail by considering a comparative static exercise in an extension of the analysis. The comparative static exercise of interest is an increase of the high-skilled labor endowment. This addresses in a simple way the scope of policy interventions to overcome the scarcity of talent in the open economy by relaxing rules of immigration.[6] I assume that the endowment of high-skilled workers increases in

[5] For a discussion on the interaction between skill intensity and firm heterogeneity see Harrigan and Reshef (2011).
[6] Labor immigration is a prominent way to address scarcity of skill and is often proposed to policymakers. See, for

just one of the two economies. Interestingly, in a country of immigration the skill-intensity in the production process declines. This is the consequence of a magnification effect. An increase in the supply of high-skilled workers renders firm entry more attractive, and in the presence of external scale economies, the entry is so strong that the additional high-skilled labor demand for overhead services dominates the supply increase, so that less high-skilled labor is left for the production of goods. Hence, somewhat counter-intuitive, immigration of high-skilled workers aggravates the scarcity of this skill type in the production of goods. This reduces firm-level productivity. High-skilled workers see their income rising in absolute terms and relative to low-skilled workers. Furthermore, with more firms being active and low-skilled workers performing a broader range of tasks, unemployment decreases. With both skill groups being better off, immigration of high-skilled workers unambiguously increases welfare. However, due to trade linkages, immigration also exerts spillover effects on the partner country, with the respective consequences matching those of the country of immigration, except for the inequality between high-skilled and low-skilled workers, which falls in the partner country.

By shedding light on how trade affects the firm-internal labor allocation, this paper is related to a growing literature that analyzes how globalization shapes the organization of production, with the key finding of my analysis being that changes in the assignment of workers with different skills to tasks with differing complexity affects a firm's productivity level. This is a novel mechanism that differentiates the model presented here from other trade models with a task-based production function, including in particular studies on offshoring along the lines of Grossman and Rossi-Hansberg (2008).[7] In the Grossman and Rossi-Hansberg (2008) framework, production also consists of a continuum of low-skilled and high-skilled tasks. However, their model provides a perfect mapping between skills and tasks, as the set of tasks that can be performed by a skill type is exogenous: low-skilled workers are restricted to work in low-skill tasks and high-skilled workers are only assigned to high-skill tasks. In my framework, things are more sophisticated, because each skill type can in principle perform the whole range of tasks within a firm and the assignment decision depends on the relative performance of low-skilled and high-skilled workers in task production and on the respective factor costs.

By allowing for changes in the firm-internal assignment of workers to tasks, the paper contributes to a vivid discussion on how trade affects productivity. The seminal paper by Melitz (2003) proposes an increase in aggregate productivity due to a change in the composition of active producers, while leaving firm-level productivity unaffected. Bustos (2011) extends the Melitz-framework by allowing firms to invest into their technology. Since exporters gain market size in the open economy, they find it more attractive to invest into their technology, and hence end up having a higher productivity. In Helpman, Itskhoki, and Redding (2010), exporters extend their screening investments and thus have a better workforce composition and therefore higher productivity than in the closed economy. In Egger and Koch (2013) the expansion of screening leaves the workforce composition unaffected

instance, the report for several industrialized countries on "labour shortages and migration policy" conducted by the International Organization for Migration (IOM, 2012).

[7]See, for instance, Kohler and Wrona (2011), Benz (2012), Grossman and Rossi-Hansberg (2012) or Wright (2014).

but improves the assignment of workers to tasks with positive productivity effects.[8] Despite several good theoretical arguments for a productivity-enhancing effect of exporting at the firm level, there is no conclusive empirical evidence for this link. For instance, reviewing 45 microeconometric studies, Wagner (2007) concludes that "exporting does not necessarily improve productivity" (p.1). My model provides a reasoning for the lack of evidence. Provided that wages are not fully flexible, trade raises demand for high-skilled workers, a fixed input in the provision of overhead services, and hence leaves less high-skilled resources for the production process. This worsens the skill composition with negative consequences for firm-level productivity.[9] [10]

The remainder of the paper is organized as follows. In Section 2, I introduce the model and characterize the equilibrium outcome in the closed economy. I start with a benchmark scenario, in which wages are fully flexible and then consider a model variant with a binding minimum wage. In Section 3, I provide insights into the impact of trade on the firm-internal assignment of skills to tasks when low-skilled wages are set by the government. In Section 4, I discuss how labor markets are linked in open economies and analyze how changes in endowments spill over to the partner country. Section 5 concludes with a brief summary of the most important results.

# 2 The closed economy

## 2.1 Model structure and firm-level analysis

Consider an economy that is populated by an exogenous mass of $L$ low-skilled and $H$ high-skilled workers and hosts two sectors of production: a final goods industry that assembles intermediates, and an intermediates goods industry, which employs labor for performing different tasks. The final good $Y$ is homogeneous and produced under perfect competition, according to a constant-elasticity-of-substitution (CES) production function (see Matusz, 1996):

$$Y = \left[ \int_{\omega \in \Omega} x(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where $x(\omega)$ denotes the quantity of intermediate good $\omega$ used in the production of $Y$, $\Omega$ represents the set of available intermediate goods with Lebesgue measure $M$, and $\sigma > 1$ denotes the (constant)

[8]An alternative mechanism, that relates firm productivity to the organization of workers in the production process is discussed by Caliendo and Rossi-Hansberg (2012). In their model it is the hierarchy structure within firms, i.e. the number of layers of management and the knowledge and span of control of each agent that is instrumental for firm performance.

[9]This mechanism is in line with the observation by Harrigan and Reshef (2011) that "empirical studies have failed to find large effects of trade liberalization on firm-level or plant-level skill upgrading" (p.3).

[10]Finally, by introducing a minimum wage for low-skilled workers, this paper is related to a sizable literature that accounts for different forms of labor market imperfections in the Melitz (2003)-framework (e.g. Davidson, Matusz, and Shevchenko (2008); Davis and Harrigan (2011); Egger and Kreickemeier (2009, 2012); Egger, Egger, and Markusen (2012); Felbermayr, Prat, and Schmerer (2011); Helpman and Itskhoki (2010); Helpman, Itskhoki, and Redding (2010)). In my model, productivity effects arise because labor market institutions affect low-skilled and high-skilled workers differently and therefore alters the firm-internal assignment of skills to tasks with consequences for firm-level productivity. Hence, the labor market imperfection leads to a reallocation of workers within firms, while in existing studies on heterogeneous firms, labor is reallocated between firms.

elasticity of substitution between the different variants of the intermediate.

Choosing the final good as numéraire, profits in the final goods industry are $Y-\int_{\omega\in\Omega} p(\omega)x(\omega)d\omega$, where $p(\omega)$ denotes the price of variety $\omega$. Maximizing these profits with respect to $x(\omega)$ gives intermediate goods demand[11]

$$x(\omega) = Y p(\omega)^{-\sigma}. \tag{2}$$

Intermediate goods producers compete with rival firms in a monopolistically competitive environment. Each firm produces a unique variety, by combining a continuum of tasks represented by the unit interval. I follow Acemoglu and Autor (2011) and use a simple Cobb-Douglas function to formalize the assembly of tasks in the production of intermediates:

$$x(\omega) = \phi(\omega) \exp\left[\int_0^1 \ln x(\omega, i) di\right], \tag{3}$$

where $\phi(\omega)$ is a firm's baseline productivity that measures the efficiency to coordinate and bundle tasks and $x(\omega, i)$ is the production level of task $i$ in firm $\omega$. Tasks are performed by low-skilled and high-skilled workers, $l(\omega, i)$ and $h(\omega, i)$ respectively, who are employed in a linear-homogeneous production function of the form[12]

$$x(\omega, i) = \alpha_l(i)l(\omega, i) + \alpha_h(i)h(\omega, i), \tag{4}$$

where $\alpha_l(i)$ and $\alpha_h(i)$ are the labor productivities of the two skill types, when performing task $i$. The task level production function in (4) implies that low-skilled and high-skilled workers are substitutes in the performance of tasks. However, the productivity of workers in performing a specific task differs, because workers differ in their abilities, while tasks differ in their skill requirements. To capture performance (i.e. productivity) differences across tasks between the two skill groups in a simple way, I impose the following assumption on absolute and comparative advantages in the performance of tasks:

**Assumption 1** *Denoting the labor productivity ratio between high- and low-skilled workers in task i by $\alpha(i) \equiv \alpha_h(i)/\alpha_l(i)$, it is assumed that $\alpha(i)$ is a twice differentiable, strictly increasing and convex function of i, i.e. $\alpha'(i) > 0$, $\alpha''(i) \geq 0$, with $\alpha(0) = 1$. To implement these properties in a tractable way, I consider $\alpha_l(i) = 1$ and $\alpha_h(i) = \alpha(i) = \exp[i]$ for all $i \in [0, 1]$.*

This assumption captures the idea that tasks can be ordered according to their complexity, with a higher index referring to higher complexity. A high-skilled worker that is assigned to the least complex task, is as productive as her low-skilled coworker, since her specific skills are not required

[11] Due to the choice of the numéraire, the CES price index corresponding to $Y$, $P = [\int_{\omega\in\Omega} p(\omega)^{1-\sigma} d\omega]^{1/(1-\sigma)}$, is equal to one.

[12] Acemoglu and Autor (2011) additionally account for medium-skilled workers in their model, since their main motivation is to analyze the observed increase of employment in high-skilled and low-skilled occupations relative to middle skilled occupations, which they call "job polarization". To keep the model tractable, I abstract from this third skill type, here.

for performing the respective task. Things are different in the case of a more complex task, where the higher skill level causes an absolute productivity advantage over low-skilled coworkers. Changes in the assignment of workers of different skill levels to the different tasks affect a firm's productivity level. This is a novel mechanism that plays a crucial role in the subsequent analysis and differentiates this model from other trade models with a task-based production function.

Intermediate goods producers maximize their profits according to a two-stage optimization problem. In a first step, firms assign skills to tasks and thereby determine the range of tasks performed by low-skilled and high-skilled workers, respectively. In a second step, they choose task-level output, which is equivalent to determining the task-level employment for a given skill assignment. In the subsequent analysis, I solve this two-stage problem through backward induction.

For a given assignment of workers to tasks, intermediate goods producers set task-level output $x(\omega, i)$, to maximize their profits

$$\pi(\omega) = p(\omega)x(\omega) - \int_0^1 x(\omega, i)c_k(\omega, i)di - fw_h, \tag{5}$$

subject to (2) and (3), where $c_k(\omega, i)$ denotes the unit costs of a firm $\omega$ performing task $i$ with the preassigned skill type $k = l, h$ and $f$ measures the fixed input of high-skilled labor that is required to manage the firm and organize the production process.[13] With a Cobb-Douglas production function, this gives the standard result of a constant cost share for each task. Furthermore, in the special case of each task entering the production function symmetrically, cost shares for all tasks are the same. To be more specific, substitution of (2) into the first-order condition $\partial\pi(\omega)/\partial x(\omega, i) = 0$ gives

$$\frac{\sigma - 1}{\sigma} p(\omega)x(\omega) = x(\omega, i)c_k(\omega, i). \tag{6}$$

A direct implication of the identical cost share is that the amount of workers of a specific skill type employed for performing tasks is the same for all tasks performed by workers of this skill type.[14]

With these insights at hand, I am now equipped to determine the optimal range of tasks performed by a specific skill type. For this purpose, I focus on the case of interior solutions and assume that both skill groups are used for the production of intermediates.[15] Since tasks are ordered according to their complexity, I can define a unique threshold task $z(\omega) \in (0, 1)$, for which the firm is indifferent between hiring low-skilled or high-skilled workers, at prevailing relative wages $s \equiv w_h/w_l$. To put it formally, the unit costs $c_k(\omega, z(\omega))$ of a firm $\omega$ performing task $z(\omega)$ are the same irrespective of the assigned skill type $k = l, h$. This implies $c_l(\omega, z(\omega)) = c_h(\omega, z(\omega))$ or,

[13] The assumption that high-skilled workers are needed to manage the firm and organize the production process is in line with the literature focusing on the internal organization of firms in economies with heterogeneous workers (see, for instance, Marin and Verdier, 2008, 2012).

[14] This can be seen from substitution of (4) and $c_k(\omega, i) = w_k/\alpha_k(i)$, with $k = l$ if $i < z$ and $k = h$ if $i \geq z$, into (6), which gives $w_l l(\omega, i) = w_l l(\omega)$ for all $i < z$ and $w_h h(\omega, i) = w_h h(\omega)$ for all $i \geq z$.

[15] Below, I will discuss a parameter constraint that needs to be fulfilled in order for such an interior solution to materialize.

equivalently

$$w_l = \frac{w_h}{\alpha_h(z(\omega))} \tag{7}$$

and establishes $s \equiv w_h/w_l = \alpha(z)$. Due to the absolute advantage of high-skilled workers in the performance of all tasks, the existence of an interior solution, $z(\omega) \in (0, 1)$, requires a skill premium, i.e. $s > 1$. Furthermore, due to the relative advantage of high-skilled workers in performing more complex tasks, it follows that low-skilled workers will be assigned to all tasks $i < z(\omega)$, while high-skilled workers will be assigned to all tasks $i \geq z(\omega)$. Notably, since all firms are price takers in the labor market and pay the same $w_h$, $w_l$, the threshold task $z(\omega)$ is the same for all intermediate goods producers, and hence I can write $z(\omega) \equiv z$ for all $\omega$. With the threshold task at hand, I can combine Eqs. (3) and (4) to rewrite firm output as

$$x(\omega) = \phi(\omega)\varphi(z) \exp\left[\int_0^z \ln l(\omega, i)di + \int_z^1 \ln h(\omega, i)di\right] \tag{8}$$

where $\varphi(z) \equiv \exp\left[\int_0^z \ln \alpha_l(i)di + \int_z^1 \ln \alpha_h(i)di\right] = \exp[(1-z^2)/2]$. According to (8), firm productivity consists of two parts: an exogenous baseline productivity $\phi(\omega)$ and the endogenous productivity term $\varphi(z)$, which varies with the assignment of skills to tasks, and thus is a function of threshold task $z$. From $\varphi'(z) = -\varphi(z) \ln \alpha(z) = -z\varphi(z)$ it follows that firms can raise their productivity when performing a larger range of tasks with high-skilled workers.[16] However, if $s > 1$, this comes at the cost of higher wages and is therefore not necessarily beneficial.

Having solved a firm's two-stage optimization problem, I am now able to determine the profit-maximizing price. As shown in the appendix, this gives

$$p(\omega) = \frac{\sigma}{\sigma - 1} \frac{w_l^z w_h^{1-z}}{\phi(\omega)\varphi(z)}. \tag{9}$$

Noting that revenues of firm $\omega$ are given by $r(\omega) = p(\omega)x(\omega)$ and taking into account that $w_l$, $w_h$ and $\varphi(z)$ are the same for all producers, it follows from (2) and (9) that the revenue ratio of two firms 1 and 2 with productivity levels $\phi(\omega_1)$, $\phi(\omega_2)$ is given by $r(\omega_1)/r(\omega_2) = [\phi(\omega_1)/\phi(\omega_2)]^{\sigma-1}$. Hence, relative firm performance is fully characterized by the baseline productivity ratio. I can thus skip firm index $\omega$ from now on, and instead refer to firms by their $\phi$-levels.

Regarding firm entry, I follow the literature on heterogeneous firms along the lines of Melitz (2003) – with the mere difference that I consider a static model variant as in Helpman and Itskhoki (2010) and Helpman, Itskhoki, and Redding (2010) – and assume that the baseline productivity

[16] The literature provides different explanations for the positive link between skill intensity and firm performance, e.g. quality of management (Bloom and Van Reenen, 2011), knowledge spillovers (Audretsch and Feldman, 2004), complementarity of skill and capital (Autor, Levy, and Murnane, 2003) or the adaption of new technologies by high-skilled workers (Abowd, Haltiwanger, Lane, McKinney, and Sandusky, 2007). In my model it is a direct implication of Assumption 1, which can be motivated by the empirically well documented positive correlation between private and social returns to skill (see, for instance, Card, 1999; Psacharopoulos and Patrinos, 2004).

is drawn by firms in a lottery from the common Pareto distribution, $G(\phi) = 1 - \phi^{-k}$.[17] The participation fee for the lottery is $f_e w_h$ and this fee gives a firm a single productivity draw. Having revealed their productivity, producers decide upon setting up a plant and starting production by making the additional investment of $f$ units of high-skilled labor (see above). With revenues (and thus profits) increasing in baseline productivity, I can identify a cutoff productivity level, $\phi^*$, which separates active firms with $\phi \geq \phi^*$ from inactive ones with $\phi < \phi^*$. The profits from production of a firm with cutoff productivity $\phi^*$ are equal to zero by definition and I can thus characterize the marginal firm with cutoff productivity level $\phi^*$ by means of a zero profit condition $\pi(\phi^*) = 0$. This zero profit condition is usually referred to by the term zero-cutoff profit condition. In view of a Pareto distribution of baseline productivity levels, there is a proportional link between revenues of the marginal producer and average revenues of all active producers. As outlined in the appendix, this link can be used to establish the modified zero-cutoff-profit condition

$$\bar{\pi} = \frac{f w_h(\sigma - 1)}{k - \sigma + 1}, \tag{10}$$

where $k > \sigma - 1$ is required for a positive, finite value of $\bar{\pi}$. In equilibrium the costs of entering the productivity lottery, $f_e w_h$, must be equal to the expected profit of doing so, $\bar{\pi}(1 - G(\phi^*))$. This establishes the free entry condition

$$\bar{\pi} = f_e w_h (\phi^*)^k. \tag{11}$$

Combining (10) and (11), I can explicitly solve for cutoff productivity level $\phi^*$:

$$\phi^* = \left( \frac{f}{f_e} \frac{\sigma - 1}{k - \sigma + 1} \right)^{1/k}. \tag{12}$$

Eqs. (10) and (12) are the key firm-level variables, which are also informative for economy-wide variables. In particular, with $\phi^*$ at hand, I can calculate the productivity average $\tilde{\phi} \equiv [k/(k - \sigma + 1)]^{1/(\sigma-1)}\phi^*$, which is useful because key aggregate variables in this model of heterogeneous firms are the same as they would be in an otherwise identical model of homogeneous firms with productivity $\tilde{\phi}$: $R = Mr(\tilde{\phi})$, $\Pi = M\pi(\tilde{\phi})$, and, $Y = M^{\sigma/(\sigma-1)}x(\tilde{\phi})$ and $P = M^{1/(1-\sigma)}p(\tilde{\phi})$. With these insights at hand, I can now turn to study the general equilibrium outcome in my model.

## 2.2 General equilibrium with perfect labor markets

To solve for the general equilibrium outcome in the closed economy, I have to specify how wages are determined. I start with a benchmark scenario, in which wages of low-skilled and high-skilled workers are flexible and determined in perfectly competitive markets. Using the adding up condition, which simply states that adding up employment of a given skill type over all producers must give

[17] Corcos, Del Gatto, Mion, and Ottaviano (2012) provide evidence for the Pareto distribution, using firm level data for European countries.

total employment of the respective skill group, market clearing for low-skilled workers establishes[18]:

$$L = M \int_{\phi^*}^{\infty} L(\phi) \frac{dG(\phi)}{1 - G(\phi^*)} = zMs \frac{fk(\sigma - 1)}{k - \sigma + 1}, \tag{13}$$

whereas for high-skilled workers, I obtain

$$H = M \int_{\phi^*}^{\infty} H(\phi) \frac{dG(\phi)}{1 - G(\phi^*)} + Mf + M_e f_e = M \frac{fk}{k - \sigma + 1} [(1 - z)(\sigma - 1) + 1]. \tag{14}$$

Furthermore, there exists a third condition, which I have to consider for characterizing the general equilibrium outcome in the closed economy: I have to make sure that profit-maximizing price-setting is in accordance with firm entry. Following Egger, Egger, and Markusen (2012) I call the respective condition profit maximization condition and combine the solution for the CES price index, $P = M^{1/(1-\sigma)} p(\tilde{\phi})$, with the choice of numéraire, $P = 1$, and the price markup condition in (9), applied for the firm with productivity $\tilde{\phi}$. Using (12) and the definition of $\tilde{\phi}$, I can solve for

$$M = \left[ \frac{w_l^z w_h^{1-z} \zeta}{\varphi(z)} \right]^{\sigma - 1}, \tag{15}$$

where $\zeta \equiv [\sigma/(\sigma - 1)] \left[(k - \sigma + 1)/k\right]^{1/(\sigma-1)} \left\{f_e(k - \sigma + 1)/[f(\sigma - 1)]\right\}^{1/k}$ is a constant.

Putting together, there are hence four equations, namely (7) and (13)–(15) which jointly determine the four endogenous variables: $z$, $w_l$, $w_h$ and $M$. In the interest of readability, I defer technical details of the analysis to the appendix and develop a graphical tool to determine the general equilibrium variables of interest. Therefore, I first combine the two labor market clearing conditions. Dividing (13) by (14) and solving for the skill premium, I can compute

$$s = \frac{L}{H} \frac{(1 - z)(\sigma - 1) + 1}{z(\sigma - 1)}, \tag{16}$$

with $\lim_{z \to 0} s = \infty$, $s = L/[(\sigma - 1)H]$ if $z = 1$, and $ds/dz = -(\sigma L)/[(\sigma - 1)H(z)^2] < 0$. Intuitively, an increase in $z$ reduces demand for high-skilled relative to low-skilled workers and thus reduces the skill premium. Noting further that Eq. (7) establishes a positive link between $s$ and $z$; $s = \exp[z]$, as workers are paid according to their marginal product of labor, combining (7) and (16) therefore gives a unique solution for the skill premium and the threshold task in the closed economy.[19]

The thus determined equilibrium level of $z$ can be used in (14) to compute the equilibrium mass of firms. Thereby, the labor market clearing condition for high-skilled workers determines for a

[18] In view of constant markup pricing, labor costs are a constant share $(\sigma - 1)/\sigma$ of a firm's revenues: $w_l L(\phi) + w_h H(\phi) = r(\phi)(\sigma - 1)/\sigma$. Using $L(\phi) = zl(\phi)$, $H(\phi) = (1 - z)h(\phi)$ and accounting for $w_l l(\phi) = w_h h(\phi)$, further implies $L(\phi) = z[(\sigma - 1)/\sigma]r(\phi)/w_l$ and $H(\phi) = (1 - z)[(\sigma - 1)/\sigma]r(\phi)/w_h$, respectively. Finally, combining $w_h = \alpha(z)w_l$ and $M \int_{\phi^*}^{\infty} r(\phi)dG(\phi)/[1 - G(\phi^*)] = M\sigma kfw_h/(k - \sigma + 1)$ from the appendix and $M_e = M(\phi^*)^k$, allows me to compute (13) and (14).

[19] Since firms need high-skilled workers to manage the firm and organize the production process, a country's relative endowment with high-skilled workers must be sufficient large to guarantee that some workers are left for the performance of tasks. The respective parameter domain (see Lemma 1 below) is derived in the appendix.

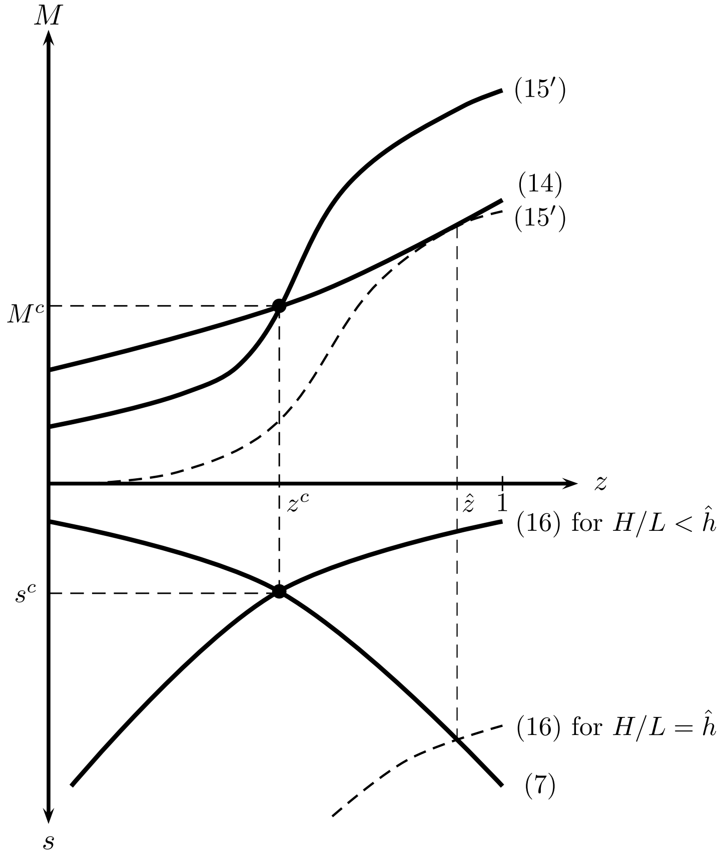

Figure 1: Equilibrium with fully flexible wages in the closed economy

given threshold task the mass of firms that can be active in equilibrium. Finally, accounting for (7), I can rewrite (15) as follows:

$$M = \left[\frac{w_l \zeta}{\beta(z)}\right]^{\sigma-1}, \tag{15'}$$

where $\beta(z) = \varphi(z)\alpha(z)^{-(1-z)} = \exp[(1-z)^2/2]$. Eq. (15′) determines for a given threshold task and a given mass of producers the low-skilled wage rate $w_l$ and thus the unit cost $w_l/[\phi\varphi(z)]$ that are consistent with the markup pricing condition in (9). These insights are summarized in the following Lemma:

**Lemma 1** *Provided that the relative supply of high-skilled workers is sufficiently high, with $H/L > \{(\sigma-1)\exp[1]\}^{-1}$, there exists a unique interior equilibrium, in which firms hire both skill types for the performance of tasks, i.e. $z \in (0,1)$.*

**Proof.** Analysis in the text and formal proof in the appendix. ■

Figure 1 provides a graphical illustration of how the four equations (7) and (13)-(15′) interact in determining the general equilibrium variables of interest.[20] To see how the equilibrium outcome is determined one has to start in the lower panel, where equilibrium values of $z$ and $s$ are represented by the intersection point of (7) and (16). I use index $c$ to refer to an equilibrium with competitive

[20]The appendix provides technical details on how Figure 1 is derived.

labor markets. Combining the equilibrium threshold level $z^c$ with (14) in the upper panel, then determines the equilibrium mass of firms $M^c$. Finally, given $z^c$ and $M^c$, the position of locus (15′) has to be adjusted in order to bring the low-skilled wage in accordance with constant markup-pricing and the price index corresponding to Eq. (1). Hereby, it is notable that a leftward shift of (15′) refers to an increase in $w_l$.

### 2.3 Equilibrium with a minimum wage for low-skilled workers

It is an empirically well documented fact for industrialized economies, that involuntary unemployment is especially persistent among low-skilled workers. Therefore, I introduce a (real) binding minimum wage $w$, that is set by the government for this skill type. This implies that the labor market clearing condition for low-skilled workers no longer holds, and the adding-up condition for low-skilled workers now determines unemployment. To be more specific, (13) is replaced by

$$(1-u)L = zMs\frac{fk(\sigma-1)}{k-\sigma+1} \tag{17}$$

with $u$ denoting the unemployment rate, which is positive if the minimum wage is binding. While (7), (14) and (15′) remain unaffected by this modification (except for $w_l$ being now determined exogenously by minimum wage $w$), the determination of the equilibrium values for $z$ and $M$ changes. In contrast to the previous section with fully flexible wages, (14) and (15′) now jointly determine the threshold task and the mass of active firms in the economy. Given $z$, (7) then determines the skill premium. Finally, substitution of (7) and (14) into (17), allows me to relate the unemployment rate to the computed skill premium:

$$u = 1 - \ln[s]s\frac{H(\sigma-1)}{L[(1-\ln[s])(\sigma-1)+1]}, \tag{18}$$

with $du/ds < 0$. To see how the general equilibrium variables are linked in the minimum wage economy, I can build on insights from Figure 1. Noting that the minimum wage is binding if and only if $w > w_l^c \equiv \underline{w}$, because otherwise firms would simply pay the competitive wage and unemployment would fall to zero, it is immediate that in the minimum wage economy locus (15′) is shifted leftwards relative to the benchmark scenario with competitive wages. Since low-skilled workers are more expensive in the minimum wage economy, firms assign them to a lower range of tasks and, hence $z < z^c$. This implies that more high-skilled labor is used as a variable input, leaving less resources for entering the lottery and to manage the firm and organize the production process which lowers the mass of competitors, i.e. $M < M^c$.[21]

With the solution for $z$ and $M$ at hand, skill premium $s$ in the minimum wage economy is determined in the lower right panel of Figure 2. Locus (18) in the lower left panel of Figure 2 finally

[21]The appendix provides a proof for these results and a discussion on the uniqueness and stability of an interior equilibrium with $z \in (0,1)$ in the minimum wage economy.

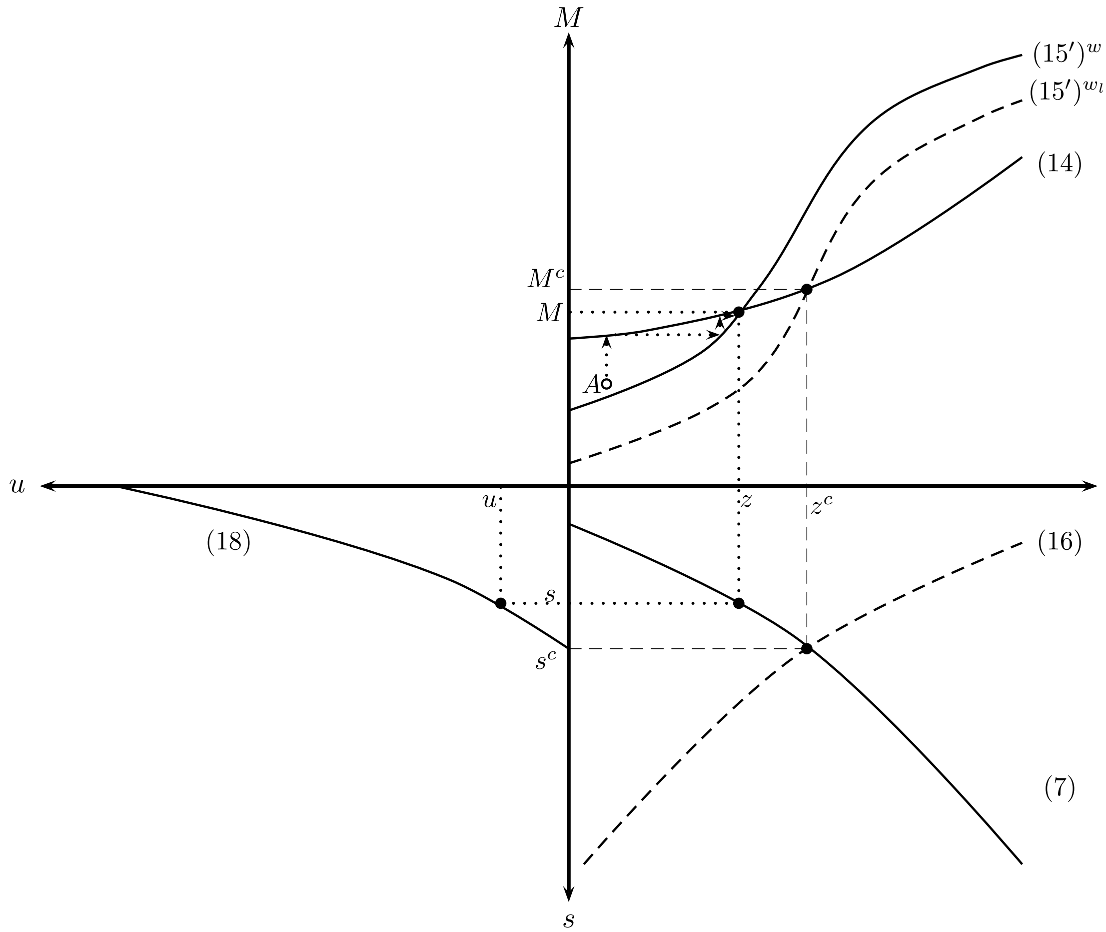


Figure 2: Equilibrium with a minimum wage for low-skilled workers in the closed economy

determines unemployment rate $u$ in the minimum wage economy.[22] With these insights, I am now equipped to discuss the group-specific effects of a binding minimum wage. Looking at the group of high-skilled workers, there are two counteracting effects on their income triggered by an increase in $w_l$. On the one hand, a higher wage for low-skilled workers, implies that high-skilled workers are employed for a larger range of tasks in all active firms. This labor demand stimulus is counteracted by a decline in the mass of firms entering the market, which lowers demand for both skill types ceteris paribus. To see which of the two effects dominates I can substitute $w_h = \alpha(z)w$ in (14) and $(15')$ to compute

$$w_h = \left[\frac{H(k-\sigma+1)}{fk}\right]^{\frac{1}{\sigma-1}} \zeta^{-1} \exp\left[\frac{1+z^2}{2}\right] \left[\frac{1}{(1-z)(\sigma-1)+1}\right]^{\frac{1}{\sigma-1}}. \tag{19}$$

Noting from above that introduction of a binding minimum wage lowers threshold task $z$, it follows from (19) that $dw_h/dw_l < 0$. Accordingly, high-skilled workers are worse-off in the minimum wage economy than in the benchmark model with competitive labor markets.

Regarding the group of low-skilled workers, there are winners and losers. Those, who keep their job in a minimum wage economy, see their income rising, whereas those who lose their job are

[22]Thereby, locus (16) is used to construct the intercept of locus (18) with the vertical axis at the skill premium $s = s^c$, which leads to $u = 0$.

worse off than in the competitive labor market scenario. To obtain a compulsory measure for the group-specific welfare of low-skilled workers, I can look at per-capita income $(1-u)w$. Substituting $s = w_h/w$ into (17), it is immediate that introduction of the minimum wage, by lowering $M$, $z$ and $w_h$, unambiguously lowers per-capita income (and thus welfare) of low-skilled workers.

While the skill premium $s = \alpha(z)$ is lower in the minimum wage economy than in the benchmark model with competitive labor markets, setting $w > w_l^c$ increases the return to high-skilled workers relative to the expected income of low-skilled workers. This can be seen from rewriting Eq. (17) as follows

$$\frac{w_h}{(1-u)w} = \frac{L}{zM}\frac{k-\sigma+1}{fk(\sigma-1)} \tag{20}$$

and noting from the discussion above that the introduction of a binding real minimum wage lowers both $z$ and $M$. Finally, since both skill types end up with a lower per-capita income in a minimum wage economy, compared to a situation with fully flexible wages, it immediately follows that welfare, measured by per-capita income $W \equiv I/(H+L)$, where $I = (1-u)Lw + Hw_h$ denotes aggregate labor income, is reduced. Proposition 1 summarizes the insights of introducing a binding real minimum wage for low-skilled workers and completes the discussion of the closed economy.

**Proposition 1** *For a binding minimum wage $w \in (\underline{w}, \overline{w})$, there exists a unique and stable interior equilibrium with $z < z^c$ and $M < M^c$ if $H/L > \hat{h}$. Introduction of the minimum wage lowers welfare relative to the benchmark of an economy with a competitive labor market and it generates involuntary unemployment of low-skilled workers. Looking at the group-specific effects, the introduction of a binding minimum wage $w \in (\underline{w}, \overline{w})$ lowers welfare of high-skilled and low-skilled workers and – although lowering the skill premium – increases the relative income of high-skilled workers.*

**Proof.** Analysis in the text and formal discussion in the appendix. ■

# 3 The open economy

## 3.1 Basic structure

It is the purpose of this section to shed light on the assignment of skills to tasks and a firm's production process if the country under consideration opens up to trade. I thereby discuss the consequences of trade between two countries indexed by $j = 1, 2$, whose economies are characterized as in the previous section, and focus on a situation when there is a binding minimum wage for low-skilled workers in both countries.[23] To keep the analysis tractable, I thereby abstract from any trade impediments and assume that all firms export. This simplification seems to be justified, because in my model the revenue ratio of any two firms and thus the export decision is fully characterized by baseline productivity levels, and hence my model is not equipped to shed new light on the exporting decision of firms (see, for instance, Melitz, 2003; Bernard, Redding, and Schott, 2007; Melitz and

[23]The supplement with a discussion on trade with perfect labor markets is available upon request.

Ottaviano, 2008). Therefore, I prefer the more parsimonious structure without self-selection of firms into exporting in order to focus on those aspects of the model that are new in the literature.

When the country opens up for trade, intermediate goods producers can raise their profits by selling to the foreign market. Abstracting from trade impediments, $Y$ and $P$ are identical to all firms irrespective of their home country. Furthermore, without selection into exporting, trade does not alter the firm entry mechanism, so that (10)-(12) still hold after a country's movement from autarky to trade. As discussed in the previous section, the cutoff productivity is independent of the labor market regime, hence $\phi_1^* = \phi_2^* \equiv \phi^*$ and $\tilde{\phi}_1 = \tilde{\phi}_2 \equiv \tilde{\phi}^*$ hold in the minimum wage economy. Constant markup pricing in both economies implies $\pi_j(\phi^*) = r_j(\phi^*)/\sigma - fw_{hj} = 0$, and therefore $r_1(\phi^*)/\sigma = fw_{h1}$ and $r_2(\phi^*)/\sigma = fw_{h2}$. Accounting for (2), (7) which is the same as in the closed economy, (9) and the definition of $\beta(z)$ I can compute

$$\frac{w_1}{w_2} = \left[\frac{\alpha(z_2)}{\alpha(z_1)}\right]^{\frac{1}{\sigma}} \left[\frac{\beta(z_1)}{\beta(z_2)}\right]^{\frac{\sigma-1}{\sigma}} = \exp\left\{\frac{z_2 - z_1}{2}\left[2 - \frac{\sigma-1}{\sigma}(z_1 + z_2)\right]\right\}, \tag{21}$$

which determines $z_1$ relative to $z_2$ in the open economy and implies $z_1 < z_2$ if $w_1 > w_2$. To compare prices of the marginal firms in the two countries, I first substitute (7) into (9), which entails $p_j(\phi^*) = \sigma w_j/[(\sigma-1)\phi^*\beta(z_j)]$. As the cutoff productivity is the same in both economies, I get $p_1(\phi^*)/p_2(\phi^*) = w_1\beta(z_2)/[w_2\beta(z_1)]$. Accounting for (21) and the definition of $\beta(z)$ then gives

$$\frac{p_1(\phi^*)}{p_2(\phi^*)} = \left[\frac{\alpha(z_2)\beta(z_2)}{\alpha(z_1)\beta(z_1)}\right]^{\frac{1}{\sigma}} = \exp\left[\frac{z_2^2 - z_1^2}{2\sigma}\right]. \tag{22}$$

To analyze the impact of intermediates trade on the general equilibrium variables of interest, note first that both adding up conditions for low-skilled and high-skilled workers are the same as in the closed economy. However, as the final good is now assembled with intermediate varieties from both countries, the corresponding price index and therefore (15′) need to be adjusted. In the open economy the mass of available intermediate varieties has changed to $M_t = M_1 + M_2$, implying that the price index in the open economy is given by $P = [M_1 p_1(\tilde{\phi})^{1-\sigma} + M_2 p_2(\tilde{\phi})^{1-\sigma}]^{1/(1-\sigma)}$. Accounting for (7) and (9) together with $P = 1$ this can be written as (see the appendix)

$$M_j = \left[\frac{w_j \zeta}{\beta(z_j)}\right]^{\sigma-1} \left[1 + \frac{M_{-j}}{M_j}\left(\frac{p_j(\phi^*)}{p_{-j}(\phi^*)}\right)^{\sigma-1}\right]^{-1}. \tag{23}$$

This equation still establishes a positive relationship between the mass of producers and the threshold task in the home country $j$, for given values of $z$ and $M$ in the foreign country $-j$. With these insights, I am now equipped to study the impact of trade on the variables of interest. I thereby start with a situation, in which both countries are fully symmetric and governments set the same binding minimum wage, $w_1 = w_2$ and postpone a discussion of country asymmetries to the extension in Section 4.

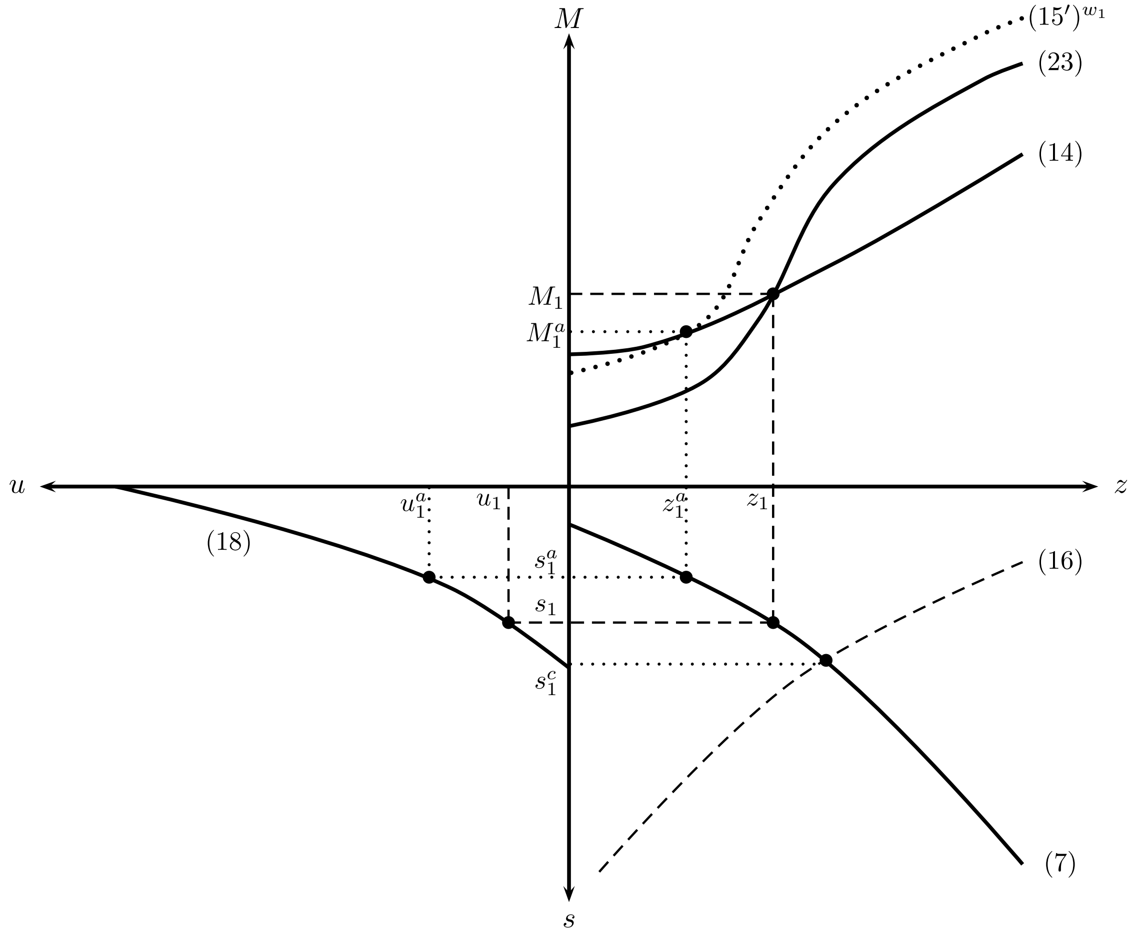

Figure 3: Trade between two fully symmetric minimum wage economies

## 3.2 Trade between two minimum wage economies

To determine the eight endogenous variables $s_j$, $z_j$, $M_j$ and $u_j$, for $j = 1, 2$, I can make use of (7), (14) and (17) – applied to both economies – (21) and (23).[24] As discussed in the closed economy, the labor market clearing condition for high-skilled workers and the profit maximization condition now jointly determine the mass of firms and the threshold task. With perfect symmetry between the two economies, (23) reads $M_j = [w_j\zeta/\beta(z_j)]^{\sigma-1}(1/2)$ and is shifted rightwards relative to its closed economy counterpart in Figure 3, implying that $M$ and $z$ are increased compared to the autarky scenario. Since all other things equal, final goods producers have access to more differentiated intermediate goods, final output increases due to a standard division of labor effect. This stimulates demand for intermediate goods, according to (2), and therefore aggregate labor demand for each skill type. While the factor price for low-skilled workers is fixed and remains unaffected, the wage rate for high-skilled workers will increase.[25] Thus, the relative factor costs have changed in favor of low-skilled workers and firms respond to the cost increase by raising the threshold task to $z > z^a$. A lower skill intensity implies that more high-skilled workers are left to entering the lottery and to manage the firm and organize the production process, so that the mass

[24] Note that (23) can only be used for one country. Applying it for the other country simply confirms that $P_1 = P_2 = 1$.

[25] To see this, remember from (19), that $dw_h/dz > 0$.

of local intermediate goods producers increases in both countries relative to the closed economy. The higher $z$ furthermore implies an increase in the skill premium, as can be seen in the lower right panel of Figure 3. Finally, the adjustment in $z$ and $M$ contribute to an increase in low-skilled labor employment and a decline in unemployment rate $u$ as depicted in the lower left panel of Figure 3.[26] From inspection of Eq. (20) there are counteracting effects on the relative per-capita income of high-skilled workers. However, solving (14) for $M$ and substituting the respective expression into (20), I can compute

$$\frac{w_h}{(1-u)w} = \frac{Lfk[(1-z)(\sigma-1)+1]}{Hz(k-\sigma+1)} \tag{24}$$

which, according to $z > z^a$, implies that trade unambiguously lowers relative per-capita income of high-skilled workers. Finally, the increase in the wage rate for high-skilled workers and the reduction in the unemployment rate trigger an increase in welfare. These findings are summarized in the following proposition:

**Proposition 2** *With a binding minimum wage for low-skilled workers, a country's opening up to trade with a symmetric partner country reduces the unemployment rate for low-skilled workers and increases the real wage, the skill premium and the relative per-capita income of low-skilled workers. Welfare is unambiguously higher in the open economy than in the closed economy and all active firms produce a broader range of tasks with low-skilled workers, which reduces observed labor productivity.*

**Proof.** Analysis in the text. ■

The results in Proposition 2 demand further discussion. First, there is a crucial difference to the findings in the literature on heterogeneous firms. Usually, the claim in the literature is that trade liberalization has a positive impact on economy-wide labor productivity by relocating productive factors towards high-productive firms, which have access to export markets and thus benefit disproportionately from trade liberalization (see Melitz, 2003). Thereby, the firm-level productivity stays constant, but selection to more productive firms increases the economy-wide productivity. In my model, this channel is closed, as trade is costless and all firms participate in exporting. This leaves the relative performance of any two firms unaffected, and hence there is no relocation of productive factors towards high-productive firms. Here, productivity effects arise at the firm-level due to adjustments in the assignment of workers to tasks. Thereby, the existence of labor market imperfections are instrumental for the impact of trade on firm productivity. With differences in the wage setting institutions among low-skilled and high-skilled workers, a higher labor demand in the open economy changes the relative factor return and therefore leads to adjustments in the assignment of skills to tasks with consequences for a firm's labor productivity.

[26] The reduction in the unemployment rate is also present in the Egger, Egger, and Markusen (2012) framework, where production consists of a single task performed by one type of workers. Similar to this paper, the positive impact is a consequence of external scale economies in the production of the final good.

# 4 Extension

The previous section has shed light on how trade between two symmetric countries effects the assignment of workers to tasks and thereby changes the production process and productivity of intermediate producers, with a particular focus on the consequences of these firm-level adjustments for economy wide variables. The aim of this section is to analyze on how labor markets are linked in the open economy. I focus on trade between two minimum wage economies as discussed in the previous subsection. Starting from a scenario with fully symmetric countries, I discuss how migration of high-skilled workers into the foreign economy spill over to the domestic country.[27]

As shown in the previous section, the opening up to trade leads to a skill downgrading at the firm level with negative consequences on firm productivity. To reduce the scarcity of skill in the labor market, countries may try to relax their immigration rules to attract more high-skilled workers. If country 2 reduces its immigration rules, the increase in $H_2$ would allow for additional firm entry, so that $M_2$ goes up.[28] However, with low-skilled labor supply being not a binding constraint in the minimum wage economy, the additional demand for low-skilled labor at the extensive margin – triggered by the additional firm entry – does not increase the factor return of low-skilled workers implying that intermediate goods producers have no incentive to reduce the range of tasks performed by low-skilled workers. Moreover, there is a magnification effect in the sense of $dM_2/dH_2 > 0$, so that the range of tasks performed by low-skilled workers increases. Hence, with a binding real minimum wage an increase in $H_2$ reduces the range of tasks performed by high-skilled workers, and hence does not have the intended effect.[29] The increase in $z_2$ implies a fall in productivity for intermediate goods producers. The implications for the wage rate of high-skilled workers can be seen when rewriting Eq. (7) as $w_{h2} = w_2 \exp[z_2]$. Since $z_2$ rises in $H_2$, a higher supply of high-skilled workers increases $w_{h2}$. Hence, high-skilled workers gain in relative[30] and absolute terms. However, also low-skilled workers gain from the additional supply of $H_2$, due to an increase in the employment level, implying a higher per-capita income $(1 - u_2)w_2$ of this skill group.[31] This is intuitive as the demand for low-skilled workers is stimulated by an increase in $z_2$ and $M_2$. Finally, since both skill groups benefit, immigration of high-skilled workers leads to higher welfare in the foreign country.

The higher mass of intermediate goods producers in country 2, now exerts an impact on the domestic country 1, according to (23). The increase in $M_2$ shifts locus (23) of country 1 rightwards in Figure 3, whereas it leaves loci (7), (14) and (16) and thus also (18) unaffected. As a consequence, the range of tasks performed by low-skilled workers in country 1 must increase.[32] As less high-skilled workers are used as a variable input, which implies a fall in productivity of all active producers,

[27]From inspection of (13) any change in $L$ is fully absorbed in the unemployment rate but leaves $z$ and $M$ unaffected. Thus, I restrict the discussion to the interesting case where countries differ with respect to $H$ and therefore $z$, $s$ and $M$.

[28]To see this, note that an increase in $H_2$ would shift locus (14) upwards in Figure 3.

[29]In the appendix, I provide a formal prove of $dz_2/dH_2 > 0$.

[30]This can bee seen from (20). Accounting for $dz_2/dH_2 > 0$ and $dM_2/dH_2 > 0$, the relative per-capita income of high-skilled workers $w_{h2}/[(1 - u_2)w_2]$ clearly increases.

[31]This can be immediately inferred, by substituting (7), $\exp[z_2] = w_{h2}/w_2$, into (13).

[32]In the appendix, I provide a formal prove of $dz_1/dH_2 > 0$.

more high-skilled workers are left to provide the input for the lottery and to manage the firm and organize the production process, thus $M_1$ increases. The increase in labor demand for low-skilled workers, triggered by the firm-internal adjustment of skills to tasks and additional firm entry, implies that the unemployment rate falls in country 1. Looking at high-skilled workers, they experience an increase in the real wage. However, from inspection of (24), it follows that the increase in $z$ unambiguously reduces relative per-capita income $w_h/[(1-u)w]$. Furthermore, the increase in group-specific per-capita income levels $(1-u)w$ and $w_h$ provides a welfare stimulus in the domestic country. These findings are summarized in the following proposition.

**Proposition 3** *Starting from an open economy equilibrium with minimum wages in both countries, an increase in the supply of high-skilled workers in one country increases the mass of firms there and reduces the range of tasks performed by high-skilled workers in both countries. This lowers productivity of all active firms. Low-skilled workers face a lower unemployment rate, while high-skilled workers face an increase in the real wage which raises welfare in both countries. Relative per-capita income of high-skilled workers increases in the country with high-skilled migration but falls in the partner country.*

**Proof.** Analysis in the text and the formal proof in the appendix. ■

# 5 Conclusion

This paper sets up a heterogeneous firms model along the lines of Melitz (2003), in which production is modeled as a continuum of tasks, which differ in complexity. Firms hire low-skilled and high-skilled workers for the performance of tasks, who differ in their ability to perform these tasks, with high-skilled workers having an absolute advantage in the performance of all tasks, which increases in the complexity of tasks. How firms organize the firm-internal production process by assigning skills to tasks depends on the respective factor costs and the productivity advantage of high-skilled workers in performing more complex tasks. Accounting for a task-based production process, allows me to discuss a so far unexplored adjustment margin, through which firms respond to exogenous shocks.

I use this framework to analyze how imperfections in the labor market affect the firm-internal assignment of skills to tasks in the closed economy. After characterizing the autarky equilibrium outcome with fully flexible wages for both skill types, I introduce a (real) minimum wage, that is set by the government for low-skilled workers and causes involuntary unemployment of this skill type. As relative factor prices are changed and low-skilled task production becomes more costly, firms assign high-skilled workers to a broader range of tasks. This firm-internal skill upgrading improves a firm's labor productivity. However, as more high-skilled workers are employed for the performance of tasks, less of them are left to manage firms and the mass of firms therefore declines. Firm exit triggers a decline in aggregate output, income and welfare. I use the model to discuss how trade between two countries affects the firm-internal production process. Only when low-skilled wages are set by a binding minimum wage, trade exerts an impact on the firm-internal assignment process. The opening up to trade raises goods demand for each firm due to external scale economies in the production of final goods. When the factor price for low-skilled workers is fixed, the skill premium increases implying that high-skilled task production becomes relatively unattractive. Firms respond in broadening the range of task production with low-skilled workers, which reduces labor productivity of each firm. Aside from this negative productivity effect, trade increases the mass of producers in each country and reduces the unemployment rate of low-skilled workers. This causes an increase in per-capita income of both skill types, with high-skilled workers benefiting disproportionately. As a consequence aggregate output, income and welfare are stimulated. After discussing the movement from autarky to trade I show how changes in local endowments spill over to the partner country. Thereby, I show that an increase in the supply of high-skilled workers in one country increases the mass of firms there and reduces the range of tasks performed by high-skilled workers in both countries. This lowers productivity of all active firms. Low-skilled workers face a lower unemployment rate, while high-skilled workers face an increase in the real wage which raises welfare in both countries. Relative per-capita income of high-skilled workers increases in the country with high-skilled migration but falls in the partner country.

The discussion in the main text abstracts from the fact that firms do also organize their production process geographically, by offshoring part of tasks that can be produced at lower costs abroad. Without costs of shifting production abroad and transporting, task trade would lead to factor price equalization among the partner countries. In contrast to the findings above, trade in tasks therefore implies that only the highest minimum wage remains binding. As firms shift the production of tasks performed by low-skilled workers to the low wage economy, this increases labor demand for that skill type there, and the incentive to shift tasks is present until the market clearing wage abroad is equal to the minimum wage at home. The offshoring of low-skilled tasks would therefore increase the domestic unemployment rate. Moreover, to the extent that task trade also affects relative factor prices, firms will respond by adjusting the assignment of skills to tasks.

Clearly, to keep the analysis tractable, this framework relies on several simplifying assumptions, which help keeping the analysis tractable and allow me to concentrate on the firm-internal adjustment margin and how firms adjust their task-based production process, which is the focus of this paper. By shedding light on this so far unexplored channel, I hope that my findings encourage further research on the organization of labor within firms.

## Acknowledgements

I am grateful to Hartmut Egger for insightful guidance and continuous encouragement while writing this paper. I also thank Carsten Eckel and participants at the European Trade Study Group Meeting in Birmingham, the Göttingen Workshop on International Economics, the IO and Trade Seminar at the Department of Economics at the University of Munich and the Brown Bag seminar at the University of Bayreuth for helpful comments. Financial support by the Bavarian Graduate Program in Economics (BGPE) is gratefully acknowledged.

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# Appendix

## Derivation of Eq. (9)

First, integrating (6) over the unit interval, shows that prices are set as a constant markup $\sigma/(\sigma-1)$ over variable unit costs $C(\omega)/x(\omega)$: $p(\omega) = [\sigma C(\omega)]/[(\sigma-1)x(\omega)]$, where $C(\omega) \equiv \int_0^1 x(\omega,i)c_k(\omega,i)di$ are a firm's total variable labor costs. Second, it follows from (4), (6) and (7) that $w_l l(\omega) = w_h h(\omega)$. This implies

$$s = \frac{l(\omega)}{h(\omega)} = \frac{1-z}{z}\frac{L(\omega)}{H(\omega)}, \tag{25}$$

where $L(\omega) = \int_0^z l(\omega,i)di = zl(\omega)$ and $H(\omega) = \int_z^1 h(\omega,i)di = (1-z)h(\omega)$ are firm $\omega$'s total low-skilled and high-skilled variable labor input, respectively. Accordingly, a firm's skill intensity is given by $H(\omega)/L(\omega) = (1-z)/[z\alpha(z)]$ and thus decreasing in $z$. Putting together, I can thus write a firm's total variable labor costs as $C(\omega) = [wL(\omega)/H(\omega) + w_h]H(\omega)$, while this firm's output is given by $x(\omega) = \phi(\omega)\varphi(z)\{[(1-z)/z]L(\omega)/H(\omega)\}^z h(\omega)$. Substitution of (25), then gives me for the variable unit cost of this firm: $C(\omega)/x(\omega) = w_l^z w_h^{1-z}/[\phi(\omega)\varphi(z)]$, which is equal to the marginal cost of the respective producer. Constant markup pricing therefore gives (9).

## Derivation of Eq. (10)

Aggregate revenues of all intermediate producers equal

$$R = M\int_{\phi^*}^{\infty} r(\phi)\frac{dG(\phi)}{1-G(\phi^*)}. \tag{26}$$

Accounting for $r(\phi)/r(\phi^*) = (\phi/\phi^*)^{\sigma-1}$ and using the Pareto distribution for parameterizing $G(\phi)$, I can compute average revenues $\bar{r} = R/M$ as follows:

$$\bar{r} = r(\phi^*)\frac{k}{k-\sigma+1} = \frac{\sigma k f w_h}{k-\sigma+1}, \tag{27}$$

where the second equality follows from the fact that constant markup pricing implies $\pi(\phi) = r(\phi)/\sigma - fw_h$, while the marginal firm makes zero profits $\pi(\phi^*) = 0$. Therefore, average profits in the market, $\bar{\pi} = \bar{r}/\sigma - fw_h$, can be expressed as (10). *QED*

## Derivation details to Lemma 1

To guarantee an interior solution with $z \in (0,1)$, $\exp[1] > L/[(\sigma-1)H]$, and therefore

$$\frac{H}{L} > \frac{1}{(\sigma-1)\exp[1]} \tag{28}$$

must hold. In this case, the skill premium determined by (7) is larger than the skill premium determined by (16), when the two equations are evaluated at $z = 1$, and hence (7) and (16) have an intersection given in $(s, z)$-space, see above. This is the parameter domain, I am focusing on in my analysis.

### Derivation details to Figure 1

In Figure 1 it is taken into account that for a given $w_l$, both (14) and (15′) establish a positive link between the threshold task $z$ and the mass of producers $M$. Differentiating (14), I can compute

$$\left.\frac{dM}{dz}\right|_{Eq.(14)} = M\frac{\sigma-1}{(1-z)(\sigma-1)+1} > 0, \qquad \left.\frac{d^2M}{dz^2}\right|_{Eq.(14)} = 2M\frac{(\sigma-1)^2}{[(1-z)(\sigma-1)+1]^2} > 0, \quad (29)$$

which implies that locus (14) establishes a positive and convex relationship between $M$ and $z$, as depicted in the upper panel of Figure 1. Furthermore, differentiating (15′) gives

$$\left.\frac{dM}{dz}\right|_{Eq.(15')} = M(\sigma-1)(1-z) > 0, \qquad \left.\frac{d^2M}{dz^2}\right|_{Eq.(15')} = M(\sigma-1)\left[(\sigma-1)(1-z)^2-1\right], \quad (30)$$

with $d^2M/dz^2|_{Eq.(15')}$ being positive for small levels of $z$ if $\sigma > 2$ and negative for high ones. This establishes the S-shape of locus (15′) in the upper panel of Figure 1, while the relationship is concave for $\sigma < 2$.[33] The lower panel of Figure 1 captures (7) and (16) in the $(s, z)$-space.

In Figure 1, (15′) is plotted such that it intersects (14) at $(M^c, z^c)$ from below. As outlined in the subsection 2.3, this is a prerequisite for a stable equilibrium in a minimum wage economy. To shed further light on this issue, it is notable that

$$\left.\frac{dM}{dz}\right|_{Eq.(15')}^{z=z^c, w_l=w_l^c} \gtreqless \left.\frac{dM}{dz}\right|_{Eq.(14)}^{z=z^c} \quad (31)$$

is equivalent to $\hat{z} \gtreqless z^c$, with

$$\hat{z} \equiv \frac{2\sigma-1-\sqrt{4\sigma-3}}{2(\sigma-1)} \quad (32)$$

and $\hat{z} \in (0,1)\ \forall\ \sigma > 1$. It therefore follows that in the competitive equilibrium locus (15′) intersects locus (14) from below if $z^c < \hat{z}$, requiring that

$$\frac{H}{L} > \frac{(1-\hat{z})(\sigma-1)+1}{\hat{z}(\sigma-1)\exp[\hat{z}]} \equiv \hat{h}, \quad (33)$$

which provides a more restrictive parameter constraint than (28). This is illustrated in Figure 1 where the dashed curve in the upper panel indicates a scenario with $H/L = \hat{h}$ and $z = \hat{z}$. Starting from such an outcome, an increase in $H/L$ – due to a decline in $L$ for a given $H$ – shifts locus (16) inwards and locus (15′) to the left in Figure 1, thereby establishing an equilibrium in which (15′)

[33] Throughout the paper, $\sigma > 2$ is assumed for illustrative reasons, while in principle, $\sigma > 1$ is sufficient for establishing the results.

intersects (14) from below.[34]

## Derivation details for Section 2.3

Provided that the relative supply of high-skilled workers is sufficiently large, i.e. $H/L > \hat{h}$ as discussed in the previous subsection, locus (15′) intersects (14) from below, implying that a leftward shift of locus (15′) gives $z < z^c$ and $M < M^c$.[35]

For an interior solution with $0 < z < 1$, I have to restrict the possibility that the minimum wage is so high that employment of low-skilled workers becomes eventually unattractive even for the least complex task, resulting in $z = 0$. To rule out such a corner solution, I focus on a parameter domain for which (14) and (15′) intersect at some $z \in (0, 1)$. This is the case if $w < \overline{w} \equiv \exp[1/2]\zeta^{-1}[H(k - \sigma + 1)/(fk\sigma)]^{1/(\sigma-1)}$. If an interior equilibrium with $z \in (0, 1)$ exists in the minimum wage economy, it follows from the properties of (14) and (15′) that the equilibrium is unique. Furthermore, the equilibrium is stable, as can be inferred from considering a point like A in the upper right panel of Figure 2. In point A, the mass of firms is too low for a given $z$, and $M$ will increase until it is consistent with the labor market clearing condition for high-skilled workers. However, in view of (15′), the prevailing $z$ is now too small for a given $M$. Hence, with constant markup pricing $z$ must increase in order to restore $P = 1$. This mechanism continues until the intersection point of (14) and (15′) is reached.

## Derivation details for Eq. (23)

Starting from $P = [M_j p_j(\tilde{\phi})^{1-\sigma} + M_{-j} p_{-j}(\tilde{\phi})^{1-\sigma}]^{1/(1-\sigma)}$ I can account for $P = 1$ to obtain $1 = M_j p_j(\tilde{\phi})^{1-\sigma} + M_{-j} p_{-j}(\tilde{\phi})^{1-\sigma}$. Noting that $p_j(\tilde{\phi}) = [w_{lj}\zeta]/\beta(z_j)$, according to (7) and (9), this can be rewritten as in (23). To show that (23) still establishes a positive link between $z_j$ and $M_j$ for given foreign values of $z_{-j}$ and $M_{-j}$, rewrite (23) as

$$M_j = \left(\frac{w_{lj}\zeta}{\beta(z_j)}\right)^{\sigma-1} \left\{1 + \frac{M_{-j}}{M_j}\left[\frac{w_{lj}}{w_{l-j}}\frac{\beta(z_{-j})}{\beta(z_j)}\right]^{\sigma-1}\right\}^{-1} \tag{34}$$

and thus as

$$(w_{lj}\zeta)^{\sigma-1} = M_j \beta(z_j)^{\sigma-1} + M_{-j}\left[\frac{w_{lj}}{w_{l-j}}\beta(z_{-j})\right]^{\sigma-1}. \tag{35}$$

[34]Of course, the analysis above does not ensure that (14) and (15′) have a unique intersection point. Looking at the shapes of the two loci (14) and (15′), I cannot rule out that there exists a second intersection point to the right of $(z^c, M^c)$. However, in such an intersection point $z > z^c$ and $M > M^c$ must hold, and this is inconsistent with an equilibrium, as can be seen when substituting (7) into (13) to obtain $L = ze^z Mfk(\sigma - 1)/(k - \sigma + 1)$. Since the latter holds if $z = z^c$ and $M = M^c$, it must be violated if $z > z^c$ and $M > M^c$, rendering an intersection point to the right of $(z^c, M^c)$ inconsistent with market clearing for low-skilled workers.

[35]From the analysis in the previous subsection I know that an outcome with $z > z^c$ and $M > M^c$ is inconsistent with an equilibrium.

This allows me to define the implicit function

$$\Gamma(z_j, M_j) \equiv M_j \beta(z_j)^{\sigma-1} + M_{-j} \left[\frac{w_{lj}}{w_{l-j}} \beta(z_{-j})\right]^{\sigma-1} - (w_{lj}\zeta)^{\sigma-1}. \tag{36}$$

Applying the implicit function theorem to (36), gives me $dM_j/dz_j = -[\partial\Gamma(\cdot)/\partial z_j]/[\partial\Gamma(\cdot)/\partial M_j] = M_j(\sigma-1)(1-z_j) > 0$. *QED*

### Derivation details for Section 4

Consider $w_1 = w_2$. Then, (21) establishes the implicit function

$$\Gamma^1(z_1, z_2) \equiv 1 - \exp\left\{\frac{z_2 - z_1}{2}\left[2 - \frac{\sigma-1}{\sigma}(z_1 + z_2)\right]\right\} = 0. \tag{37}$$

Furthermore, allowing for $H_1 \neq H_2$, I can substitute $M_1$ and $M_2$ from (14) into (36) and account for the definition of $\beta(z)$ to formulate the implicit function

$$\Gamma^2(z_1, z_2, H_1, H_2) \equiv H_1 \frac{\exp\left[\frac{\sigma-1}{2}(1-z_1)^2\right]}{(1-z_1)(\sigma-1)+1} + H_2 \frac{\exp\left[\frac{\sigma-1}{2}(1-z_2)^2\right]}{(1-z_2)(\sigma-1)+1} - \frac{fkw_1^{\sigma-1}\zeta^{\sigma-1}}{k-\sigma+1} = 0. \tag{38}$$

Applying the implicit function theorem to (37), gives $dz_2 = -dz_1\Gamma^1_{z_1}/\Gamma^1_{z_2}$. Furthermore, applying the implicit function theorem to (38) and accounting for the previous result, allows me to calculate

$$\frac{dz_1}{dH_2} = -\frac{\Gamma^2_{H_2}}{\Gamma^2_{z_1} - \Gamma^2_{z_2}\Gamma^1_{z_1}/\Gamma^1_{z_2}}. \tag{39}$$

The respective partial derivatives are given by

$$\begin{aligned}
\Gamma^1_{z_1} &= 1 - \frac{\sigma-1}{\sigma} z_1 > 0, \quad \Gamma^1_{z_2} = -\left(1 - \frac{\sigma-1}{\sigma} z_2\right) < 0, \\
\Gamma^2_{z_1} &= H_1 \exp\left[\frac{\sigma-1}{2}(1-z_1)^2\right] \frac{\sigma-1}{[(1-z_1)(\sigma-1)+1]^2}[z_1 - (1-z_1)^2(\sigma-1)], \\
\Gamma^2_{z_2} &= H_2 \exp\left[\frac{\sigma-1}{2}(1-z_2)^2\right] \frac{\sigma-1}{[(1-z_2)(\sigma-1)+1]^2}[z_2 - (1-z_2)^2(\sigma-1)] \left(\frac{w_1}{w_2}\right)^{\sigma-1}, \\
\Gamma^2_{H_2} &= \frac{\exp\left[\frac{\sigma-1}{2}(1-z_2)^2\right]}{(1-z_2)(\sigma-1)+1} > 0.
\end{aligned}$$

The signs of $\Gamma^1_{z1}$, $\Gamma^1_{z2}$ and $\Gamma^2_{H_2}$ need no further discussion. To determine the sign of $\Gamma^2_{z_1}$ and $\Gamma^2_{z_2}$, note first that the requirement for a stable equilibrium is given by the same condition $z_j < \hat{z}$ – with $\hat{z}$ determined by (32) – in the closed as well as the open economy. Noting further that the sign of $\Gamma^2_{z_j}$ is determined by the sign of $g(z_j) \equiv z_j - (1-z_j)^2(\sigma-1)$, it follows from $g(0) = -(\sigma-1) < 0$, $g(\hat{z}) = 0$, and $g'(z_j) = 1 + 2(1-z_j)(\sigma-1) > 0$ that both $\Gamma^2_{z_1} < 0$ and $\Gamma^2_{z_2} < 0$ are negative in the relevant parameter domain.

Accounting for $\Gamma^2_{H_2} > 0$, $\Gamma^2_{z_1} < 0$, $\Gamma^2_{z_2} < 0$, $\Gamma^1_{z_1} > 0$ and $\Gamma^1_{z_2} < 0$, it follows immediately that

$dz_1/dH_2 > 0$. Moreover, with $dz_2 = -dz_1\Gamma^1_{z_1}/\Gamma^1_{z_2}$, $dz_2/dH_2 > 0$ holds. *QED*

## Supplement

(Not intended for publication)

### Trade with perfect labor markets

With fully flexible wages, the eight endogenous variables in the open economy, $w_{lj}$, $w_{hj}$, $z_j$ and $M_j$, for $j = 1, 2$ are determined by condition (7) and the labor market clearing conditions (13) and (14) – applied to the two economies – Eq. (21) and finally the profit maximization condition in the open economy, Eq. (23), applied for country $j$. To illustrate the equilibrium in the open economy, I can employ the graphical tool. If wages are set in perfectly competitive markets, the equilibrium threshold task and the skill premium are jointly determined by (7) and (16), which are plotted in the lower panel in Figure 4. As both loci remain unaffected by an opening up to trade, the skill premium and the threshold task are the same as in the closed economy, i.e. $s_a^c = s_j^c$ and $z_a^c = z_j^c$, where index $a$ refers to autarky variables. Moreover, since the labor market clearing condition for high-skilled workers and thus locus (14) remains unaffected as well, also the mass of firms in country $j$ stays constant, i.e. $M_a^c = M_j^c$. These findings indicate, that the intersection point between loci (14) and (23) in the upper panel of Figure 4 is the same as in the closed economy equilibrium. According to (21) and (22), prices for the cutoff firm in each market are identical when both countries are fully symmetric, implying that (23) reads $M_j = [w_{lj}\zeta/\beta(z_j)]^{\sigma-1}(1/2)$. Hence, compared to its closed economy counterpart in (15′), the profit maximization condition (23) is shifted rightwards for any given wage rate for low-skilled workers $w_{lj}^a$. Opening up to trade raises the mass of available intermediate varieties to $M_t = M_1 + M_2$. This increases country-specific output $Y$ and stimulates demand for each firm, according to (2). Therefore, aggregate labor demand for each skill type is stimulated. With fully flexible wages, $w_l$ must increase, to bring the economy back to $z_j^c = z_a^c$ and $M_j^c = M_a^c$. According to $w_h = w_l\alpha(z)$ the wage rate for high-skilled workers increases by the same extend, so that the skill premium remains at the autarky level. Similar to Krugman (1979), trade between two fully symmetric countries therefore leads to a positive income and thus welfare effect, while leaving all other variables of interest unaffected. These findings are summarized in the following proposition.

**Proposition 4** *If wages are fully flexible, a country's opening up to trade with a symmetric partner country has no impact on the skill premium, the firm internal assignment of skills to tasks and the mass of active firms. However, trade increases the real wage for both skill types and thus welfare.*

**Proof.** Analysis in the text. ■

The findings from Proposition 4 do not hinge on the assumption that both countries are symmetric in their relative endowments with high-skilled and low-skilled workers. This can be easily inferred from the discussion above. As any change in the supply of $L$ or $H$ in Foreign, leaves the position of loci (7), (14) and (16) in Home unaffected, it does not affect $z_j$, $s_j$ and $M_j$. Thus, when wages are fully flexible, trade between two countries that differ in their relative endowments still exerts only a positive income and welfare effect, but does not change the other variables of

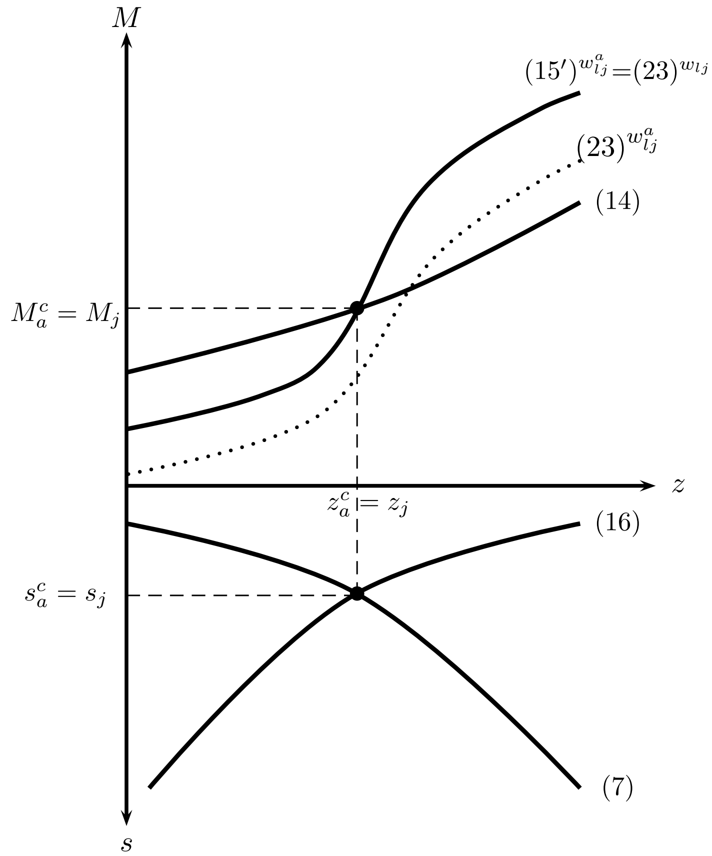

Figure 4: Trade with fully flexible wages

interest. Moreover, the strength of these effects depends on the stimulus in labor demand for the two skill types. The higher the mass of intermediate producers in the foreign country, the larger is the positive demand shock by opening up for trade from a domestic country's perspective. As $M_{-j}$ is increasing in $L_{-j}$ and $H_{-j}$, welfare effects at Home are therefore increasing in the size of the foreign factor markets.