A Dynamic North-South Model of Demand-Induced Product Cycles*

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Abstract

This paper presents a dynamic North-South general-equilibrium model with non-homothetic preferences. Innovation takes place in the rich North while firms in the poor South at random imitate products manufactured in the North. The model is able to generate endogenous product cycles as described by Vernon (1966), where the different stages of the product cycle are not only determined by supply side factors but also by the distribution of income between North and South. We simulate comparative statics results of changes in Southern labor productivity, and changes in inequality across regions. Furthermore, we provide suggestive evidence for the product cycle stages.

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1 Introduction

In his seminal article Vernon (1966) explained international trade patterns with product cycles. He hypothesized that new goods would be introduced in countries with high per capita incomes (catering to the needs of such a market), after a while demand for these goods emerges abroad (as incomes grow) and they are exported. Later on, goods are imitated by less advanced countries, which have a relative cost advantage, such that the production moves there. Hence, goods that were once exported by rich countries are eventually imported by them. In a follow-up paper, Vernon (1979) explicitly emphasized the role of the demand side in shaping the typical product cycle:

> In the early part of the post-war period, the US economy was the repository of a storehouse of innovations not yet exploited abroad, innovations that responded to the labour-scarce high-income conditions of the US market. As the years went on, other countries eventually achieved the income levels and acquired the relative labour costs that had prevailed earlier in the United States. As these countries tracked the terrain already traversed by the US economy, they developed an increasing demand for the products that had previously been generated in response to US needs. That circumstance provided the consequences characteristically associated with the product cycle sequence ... (Vernon 1979, p. 260).

It has been several decades since Vernon stressed the importance of the demand side for product cycles.[1] However, this is the first contribution to the literature, we are aware of, that provides a demand-based dynamic model which is able to generate the three stages of the product cycle as Vernon (1966) described: (1) a product is exclusively produced and consumed in the North, (2) a product is produced in North and exported to South and (3) a product is imitated and exported from South to North. This chapter describes a dynamic general-equilibrium model of two regions, a wealthy North, and a poor South. Households have non-homothetic preferences over differentiated products such that consumption patterns differ across regions. In particular, households in North can afford to consume more and newer products than households in South. Monopolistic firms in North innovate new products (horizontal innovations) whereas competitive firms in South randomly target Northern products for imitation. Trading products across regions is costless.[2] In the steady state, products go on average through the following cycle. A new product is developed and introduced in North. Only after a certain time have households in South become rich enough to afford a "new" product that is produced in North. This demand lag is endogenously determined and depends, ceteris paribus, positively on the degree of inequality across regions and negatively on the innovation rate.[3] In other words, if Southern households, relative to

[1]Two early studies of the product cycle that deserve mention can be found in Wells (1966) and Hirsch (1967), both doctoral students of Vernon at Harvard University.

[2]Obviously, this is a simplifying assumption. Its implications are discussed in Section 4.4.

[3]Our use of the term "demand lag" differs from Posner (1961). He thinks of the demand lag as the delay in the acceptance of foreign goods in the domestic market, i.e. foreign goods might not be considered perfect

Northern households, are poor the demand lag is long. Similarly, if incomes grow at a low rate the demand lag is long too. At this stage of the product cycle North is exporting the product. As time elapses further South eventually masters the technology to manufacture the product itself. Southern firms chose at random Northern products to imitate that have not yet been copied. They must invest resources in order to reverse engineer the production process of the chosen product. Once they have invested the necessary resources, they enter into price competition with the innovating firm in North. Because they have a cost advantage due to lower wages, they can underbid the Northern innovator and capture the whole market. Hence, South becomes an exporter of that product. The average time span a product is being manufactured in North is determined endogenously. In sum, we get on average a product cycle as described by Vernon (1966): At the stage of a new product, products are manufactured and consumed in North, at the mature stage they are exported to South, and eventually, at the stage of the standardized product they are manufactured in South and exported to North.

The remainder of the paper is structured as follows. In Section 2, we distinguish our contribution from the existing literature. Section 3 presents suggestive evidence for the product cycle described in the introductory Section 1 by studying six major consumer durables. In Section 4, we introduce the model and solve for the steady state, and transitional dynamics. Comparative statics results of changes in Southern productivity, and changes in inequality across regions are discussed in Section 5. Section 6 extends the model towards hierarchic preferences, and learning-by-doing. Eventually, Section 7 concludes.

2 Related Literature

The theory of Vernon (1966) grew out of dissatisfaction with classical trade theories, which explain trade between countries with differences in relative factor endowments (Heckscher-Ohlin) or differences in relative productivities (Ricardo). On the one hand, these theories missed characteristics like countries per capita incomes (Burenstam-Linder 1961) that are thought to be important determinants of international trade (Markusen 1986; Bernasconi 2013), and on the other hand, struggled to explain observed trade flows (Leontief 1953, 1956). Since Vernon put forward his verbal theory of the product cycle, a number of economists have both formalized the product cycle theory in theoretical models as well as put it to empirical tests.

One of the first to study product cycles in a theoretical model was Krugman (1979). In his model, an advanced North introduces new products at a constant exogenous rate, i.e. the product space expands, and a less advanced South copies these goods also at a constant exogenous rate. Higher per capita income in North depends on quasi rents from the Northern monopoly in new goods, i.e. North must continually innovate to maintain its relative and absolute position. Later, Grossman and Helpman (1991) extended Krugman's (1979) model,

substitutes for home-produced goods until some time elapses. We define the demand lag as the time it takes in the poor South for incomes to grow sufficiently such that households there can afford to buy goods produced in North, abstracting from differences in tastes.

and endogenized innovation and imitation rates. In their model, long-run growth is faster the larger the resource base of the South and the more productive its resources in learning the production process. The reason is that profits during the monopoly phase are higher when a smaller number of Northern firms compete for resources in the manufacturing sector, which outweighs the effect of a higher risk-adjusted interest rate since profits accrue on average for a shorter period of time. Both models focus on the supply-side aspect of the product cycle theory, i.e. how the diffusion of technology and the determination of relative wages depend on technology and preference parameters of the model (for a more recent example see Acemoglu, Gancia and Zilibotti 2012). However, in both models, demand patterns in North and South are identical because agents have homothetic constant-elasticity-of-substitution (CES) preferences. In other words, the consumption basket demanded in North is simply an inflated clone of the one in South. "This is clearly at odds with the fact, stressed by Vernon, that new goods are not typically consumed in the South until later in the cycle" (Stokey 1991, p. 63). Hence, Stokey (1991) focuses on the demand side. In her static model with vertically differentiated goods North manufactures high-quality products whereas South manufactures low-quality products. Since agents have non-homothetic preferences all of the products manufactured in the North are consumed domestically but only the lower-quality products are exported. Stokey (1991) is interested in the effect of population size and productivity changes on relative wages, production and trade patterns, respectively, and the terms of trade. Kugler and Zweimüller (2005) build a dynamic North-South model where households have non-homothetic preferences. Their model is close to our setup. However, the model in Kugler and Zweimüller (2005) is not a full-fledged general-equilibrium model since interest rates are exogenously determined. Furthermore, the focus of their analysis is on the cross-sectional composition of aggregate demand rather than on product cycles.

Our model differs from the existing literature in the following ways. In contrast to e.g. Stokey (1991) or Flam and Helpman (1987) we focus on horizontal instead of vertical innovations. In addition, we differ from Stokey (1991), Matsuyama (2000) or Falkinger (1990) who build static Ricardian trade models where agents have (hierarchic) non-homothetic preferences by developing a dynamic general-equilibrium model with monopolistic competition. We model the demand side as Foellmi and Zweimüller (2006), whereas the supply-side is borrowed from Grossman and Helpman (1991). Incorporating non-homothetic preferences into these types of models enables us to formalize the product cycle hypothesis and analyze the effects of the demand side on the product cycle.

3 Motivation

3.1 Empirical and anecdotal evidence

There have been many attempts to test the product cycle hypothesis empirically. Among the first who found evidence for the product cycle theory were Wells (1969) for consumer durables and Hirsch (1967) for electronic products. Hirsch (1975) and Mullor-Sebastian (1983) find that industrial product groups behave according to the product cycle theory. More recently, Feenstra and Rose (2000) find evidence for product cycles by ranking goods (and countries) according to the year they are first exported (export) to the United States. In particular, they show that less sophisticated goods like furniture are imported early by the US, and that advanced countries like Canada, UK, German, Japan and France start to export early to the US. Furthermore, there is a negative correlation between real GDP per capita and the country ranking, i.e. high-income countries tend to have a low ranking, which means that they start exporting early to the US (i.e. are more advanced).

Besides empirical evidence there is anecdotal evidence for the product cycle hypothesis, e.g. products like color T.V's, computer games, or electric can openers seem to follow or have followed Vernon's product cycle. A typical example that is currently in an early stage of the life-cycle is the Tesla Roadster (the first all electrical sports car) which is produced by Tesla Motors Inc. in California, USA. It was exclusively available in the US for around two years before it became available in some Western European countries, Hong Kong, Japan and Australia. The product cycle theory predicts that demand abroad further increases as incomes grow, and eventually the car is being imitated and production is moving abroad where manufacturing costs are comparatively lower. In the following section, we are going to illustrate the product life-cycle with 6 major consumer durables of the 20th century such as the microwave oven, dishwasher, freezer, washing machine, dryer and VCR.

3.2 The product cycle of 6 major consumer durables

Instead of attempting to empirically test the product cycle hypothesis, we take a different route and provide suggestive evidence by looking at three distinct features characterizing the product cycle described by Vernon (1966) that should be observed in the data.[4] First, new products are not introduced in all countries simultaneously, with the lag of introduction varying negatively with GDP per capita. Second, as the production of goods migrates from North to South, North should start out as a net exporter of a given product, and over time become a net importer of that good. Third, production of a given good should be high in developed relative to developing countries early in the product cycle, and low later on. We interpret the patterns found in the data for these 6 consumer durables as suggestive evidence for the type of product cycles Vernon (1966) had in mind. In particular, we believe they are representative for the type of final consumer goods that the product cycle hypothesis is relevant. It is obvious that the theory does not apply to trade in e.g. agricultural commodities or natural resources.

We study 6 major consumer durables introduced in the 20th century, i.e. dishwasher, dryer, freezer, microwave oven, VCR, and washing machine. For each consumer durable we know the year of introduction in 16 European countries, i.e. Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, and United Kingdom. The dates of introduction for these consumer durables

[4]A conclusive analysis would require time series data on production and consumption at the product level across a large sample of countries - the gathering and analysis of such data is beyond the scope of this chapter.

were kindly provided by Tellis, Stremersch and Yin (2003).[5] Trade data are U.S. import and export data at the 5 digit SITC level from 1972-2006, which are provided by the Center for International Data at UC Davis (Feenstra 1996, 1997). GDP data are taken from Penn World Tables (PWT) 7.0, see Heston, Summers and Aten (2012). Production data are obtained from the Industrial Commodity Production Statistics Database (United Nations Statistics Division 2012). Internet research reveals that all of these products were first introduced in the United States: the electric dishwasher was introduced in 1950 (Hobart Corp.), the automatic electric clothes dryer in 1949 (Hamilton Manufacturing Corp., General Electric), the domestic deep freezer in 1949 (General Electric), the countertop microwave oven in 1967 (Amana Corp.), the VCR in 1965 (Sony, Ampex, RCA), and the automatic electric washing machine in 1947 (Bendix, General Electric).

3.2.1 Demand lags

Table 1 shows the year of introduction, defined by Tellis, Stremersch and Yin (2003) as the first year commercial sales for the corresponding product were registered, of the 6 consumer durables, and GDP per capita (PWT 7.0, PPP, 2005 USD) in the year the product was introduced in the United States. For example, the countertop microwave oven was first introduced in the US in 1967, and last introduced in Greece and Portugal in 1982. GDP per capita in 1967 was USD 19,522 in the US, whereas in 1967 Greece had a GDP per capita of USD 9,742 and Portugal one of USD 5,937.[6] We observe that the year of introduction varies across countries. It appears that on average, products were first introduced in countries with a high GDP per capita like the US and UK, and last introduced in countries with a low GDP per capita like Greece, Portugal and Spain. This is also suggested by Spearman's rank correlation coefficient between the year of introduction and GDP per capita in that year, shown at the bottom of Table 1.

Let us look closer at the microwave oven, which we consider a typical household appliance. In 1946, Percy Spencer, an American engineer, while working on radar technology for the U.S. defense company Raytheon Corporation accidentally discovered that microwaves are capable of heating food extremely quickly. The story goes that a candy bar in Spencer's pocket melted during an experiment. Spencer realized the commercial potential, especially for a high-income market like the US, of his discovery, and Raytheon filed for patents. In 1947, Raytheon produced the first commercial microwave oven named "Radarange", which was sold to restaurants etc. Twenty years later, in 1967, Amana, a division of Raytheon, introduced the first domestic countertop microwave oven, marking the beginning of the use of microwave ovens in American kitchens (see e.g. Osepchuk 1984).

Figure 1 below shows the relationship between the demand lag in years of the countertop microwave oven relative to the US across the 16 European countries, and their GDP per capita relative to the United States in 1967 (PPP, 2005), the year the microwave oven was introduced

[5] Unfortunately, we don't have data on the diffusion of the 6 consumer durables.

[6] Note that PWT 7.0 provides data from 1950-2009. Hence, we use data on GDP per capita in 1950 to approximate GDP per capita in the years 1947 and 1949.

Country	Dishwasher		Dryer		Freezer		Microwave Oven		VCR		Washing Machine	
	Year	GDP pc	Year	GDP pc	Year	GDP pc	Year	GDP pc	Year	GDP pc	Year	GDP pc
Austria	1962	6296	1965	6296	1953	6296	1974	13575	1977	12651	1962	6296
Belgium	1960	7992	1966	7992	1956	7992	1974	13602	1975	12820	1955	7992
Denmark	1960	9366	1965	9366	1954	9366	1974	15980	1977	15145	1958	9366
Finland	1964	6192	1973	6192	1961	6192	1975	11590	1978	11203	1960	6192
France	1959	7107	1963	7107	1960	7107	1975	13578	1974	12522	1954	7107
Germany	1960	6251	1966	6251	1956	6251	1969	14348	1974	14377	1952	6251
Greece	1966	2576	1990	2576	1972	2576	1982	9742	na	na	1964	2576
Ireland	1965	5880	1963	5880	1958	5880	1976	8789	na	na	1966	5880
Italy	1961	5361	1968	5361	1965	5361	1975	12305	1976	11015	1957	5361
Netherlands	1960	9961	1968	9961	1960	9961	1971	16356	na	na	1960	9961
Norway	1961	9434	1970	9434	1957	9434	1976	16366	1977	14966	1960	9434
Portugal	1966	2614	1973	2614	1956	2614	1982	5937	na	na	1956	2614
Spain	na	na	1973	3796	1972	3796	1973	10215	1977	9321	1964	3796
Sweden	1959	10301	1969	10301	1953	10301	1973	17043	1977	16380	1958	10301
Switzerland	na	na	1966	13712	na	na	1973	22880	1978	22056	na	na
United Kingdom	1958	10447	1960	10447	1954	10447	1971	14886	1974	14365	1954	10447
United States	1950	13119	1949	13119	1949	13119	1967	19522	1965	18364	1947	13119
Spearman's corr coeff	-0.858		-0.545		-0.675		-0.651		-0.011		-0.463	

Source: Tellis et al. (2003), PWT 7.0

Table 1: Introduction of 6 major consumer durables across European countries

in the US. We observe that on average the lower a countries' GDP per capita relative to the US in 1967, the longer the time span until households in a country start purchasing the microwave oven. For example, the Netherlands had a GDP per capita in 1967 that was about 20 percent lower than the US and households started consuming microwave ovens 4 years later than the US. By comparison, Portugal's GDP per capita in 1967 was about 30 percent of that in the US and households began purchasing microwave ovens 15 years later. Note that countries below the line of fit have a higher average growth rate between 1950-1990 than countries above the line of fit (calculations based on PWT 7.0). Switzerland is an extreme outlier in the sense that even though its GDP per capita in 1967 was higher than the US GDP per capita, households first purchased the microwave oven 6 years later than households in the US.

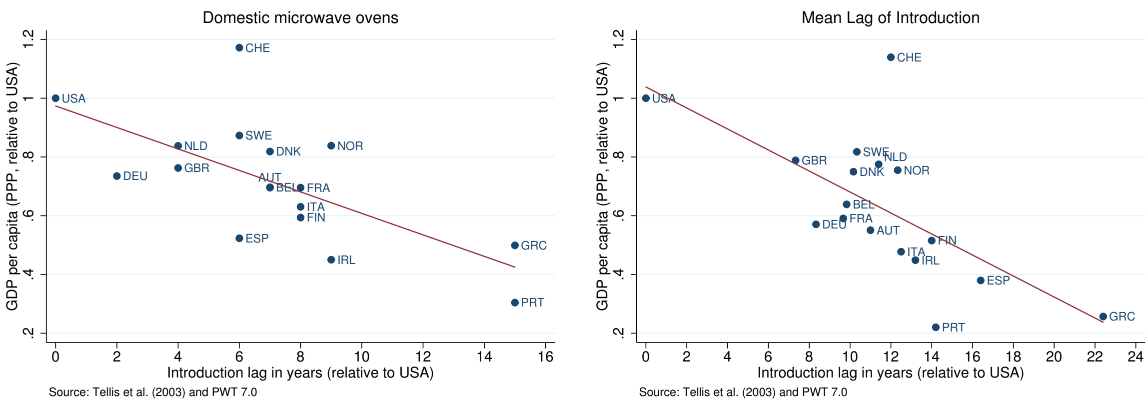

Figure 1: Relationship between GDP per capita and demand lag for the microwave oven (left-hand side) and across all 6 consumer durables (right-hand side)

The graphs for the other consumer durables look similar. For each country Figure 1 also plots the average of GDP per capita relative to the US in the year of introduction across all 6 consumer durables against the average lag in years in the introduction of these consumer durables. We conclude from Figure 1 that there is a negative correlation, suggesting that on average, in countries where households enjoy a high income, these consumer durables are purchased sooner.[7] Again, we notice that Switzerland is an extreme outlier that might be explained by its lacking integration into the European market at that time, its relatively highly regulated domestic market or its small population size. Hence, we also take into account (relative) population size, and for each product category j as well as across all product categories, estimate the following model by OLS:

$$\log\left(\Delta_{ij}\right) = \beta_0 + \beta_1 \log\left(\frac{GDPpc_{ij}}{GDPpc_{US}}\right) + \beta_2 \log\left(\frac{Pop_{ij}}{Pop_{US}}\right) + \varepsilon_{ij}.$$

In other words, we regress the introduction lag Δ (in logs) in each of the 6 product categories j on the (log of) GDP per capita in country i relative to the GDP per capita of the US and the (log of) population size of country i relative to the population size of the US, both in the year the US introduced the product category. The coefficient β_1 shows the importance of (relative) GDP per capita, holding relative population sizes constant. We expect β_1 to be negative. Table 2 below illustrates that (relative) GDP per capita has a negative effect on the introduction lag of all 6 products, controlling for (relative) population sizes.

Table 2: Correlation between (log) relative GDP per capita and (log) introduction lag Δ

	$\log(\Delta mean)$	$\log(\Delta dish)$	$\log(\Delta dryer)$	$\log(\Delta freeze)$	$\log(\Delta micro)$	$\log(\Delta vcr)$	$\log(\Delta wash)$
log(rel GDPpc)	-0.428	-0.399	-0.427	-0.702	-0.848	-0.124	-0.249
	(-3.95)	(-9.75)	(-3.61)	(-2.49)	(-2.88)	(-0.88)	(-1.45)
log(rel pop)	-0.109	-0.107	-0.098	0.094	-0.221	-0.108	-0.235
	(-2.41)	(-6.03)	(-1.77)	(0.75)	(-2.48)	(-3.86)	(-3.09)
adj. R^2	0.546	0.911	0.460	0.262	0.460	0.547	0.399
#obs	16	14	16	15	16	12	15

Notes: t-values in parentheses

3.2.2 Export performance

We now turn to the export performance of the United States in these product categories. In particular, we look at the value of US exports (X_{jt}) in product category j at time t relative to the value of US exports plus imports (I_{jt}) in category j at time t, i.e. $\widetilde{x}_{jt} \equiv X_{jt}/(X_{jt} + I_{jt})$. We note that if $\widetilde{x}_{jt} \in (0, 0.5)$ the US is a net importer in product category j at time t, and if $\widetilde{x}_{jt} \in (0.5, 1)$ the US is a net exporter. The product cycle hypothesis offers an explanation for

[7] We expected GDP per capita to be more important for product take-off (i.e. when a certain threshold of sales has been reached) than for the time of introduction. However, it seems that firms base their decisions to launch new products (and form their expectations about future sales performance) as much on the general level of development in a country (e.g. high average income level) as on the existence of a small group of rich people.

a falling export ratio $\widetilde{x}_{jt}$ in the data. The US should start out as a net exporter and become a net importer over time in a given product category. Again, in Figure 2, we take a look at the export performance of the United States in the product category of microwave ovens, as well as across all 6 consumer durables, both across the 16 European countries.

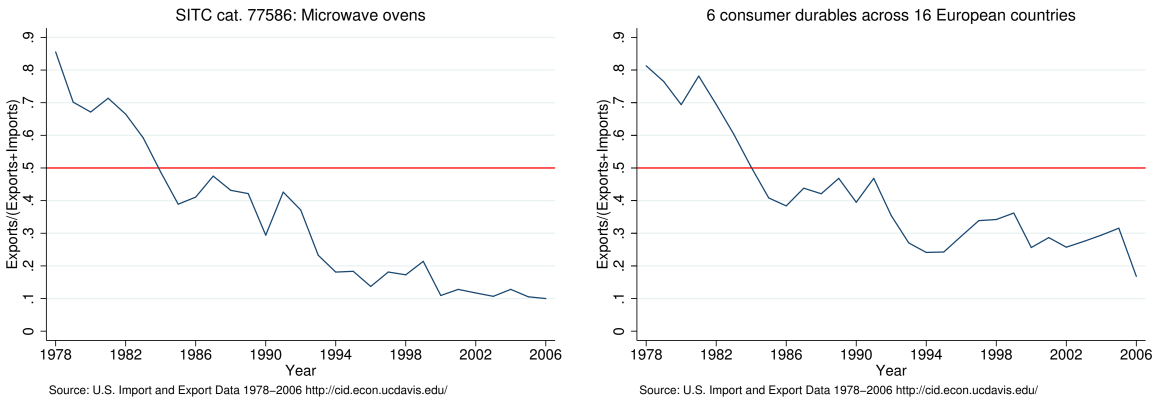

Figure 2: US export ratio in microwave ovens across 16 European countries (left-hand side) and across all 6 consumer durables (right-hand side)

We observe that the US starts out as a net exporter of microwave ovens at the beginning of the sample period in 1978 and ends up as a net importer at the end of 2006, switching around 1983/84. A possible interpretation for the decline in the export ratio is that firms in the 16 European countries mastered the technology to produce microwave ovens, and due to lower production costs were able to compete with US firms in their home markets as well as in the US market. In other words, US firms became less competitive in their export markets and/or European firms became more competitive in the US market, such that US exports relative to U.S. imports decreased. The export performance in the other products is similar, except for the domestic deep freezer where the export ratio $\widetilde{x}_{jt}$ follows an inverse U-shape over 1978-2006. The right-hand side panel in Figure 2 shows the export performance of the U.S. in the 16 European countries aggregated over the 6 product categories. We see that the value of US exports relative to its imports also declines across all 6 consumer durables combined.

3.2.3 Production patterns

Eventually, for the microwave oven and the washing machine we study production data for the United Kingdom, the United States, Japan, South Korea, Brazil, Russia, India, China, and Argentina during the time period of 1982-2008. The United Nations Commodity Statistics Yearbook (United Nations Statistics Division 2012) collects data on production of industrial commodities by country.[8] Unfortunately, we don't have historical data on the production of the microwave oven in China and India. Figure 3 plots the number of units (in millions)

[8]The data is collected through annual questionnaires sent to national statistical authorities. The data reported by the United Nations Commodity Statistics Yearbook reflect volume (and value) of production sold during the survey period, which is defined as the production carried out at some time, which has been sold (invoiced) during the reference period.

relative to US production for the microwave oven (left-hand side) and the washing machine (right-hand side) over the time period of 1982 to 2008. We make the following observations. First, U.S. production of both consumer durables is declining from the 1980s until 2008 relative to emerging countries like e.g. Brazil, Russia, India and China. Second, while emerging countries are catching up, the production of developed countries like the UK and Japan only moderately increases or even decreases. The relative increase in production of the washing machine is especially strong for China. However, also Indian production of washing machines is catching up slowly. Again, this is consistent with the product cycle hypothesis, which suggests that the production of microwave ovens and the washing machine should move from developed countries to developing countries as firms in these countries acquire the technology to produce microwave ovens and have lower production costs. Data limitations prevent us from investigating the production patterns for the rest of the consumer durables discussed above.

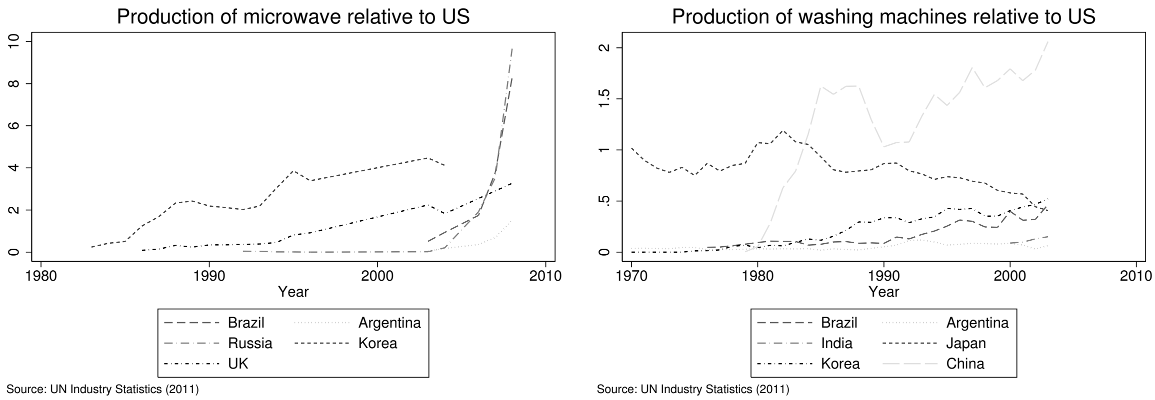

Figure 3: Production of microwave oven and washing machine relative to US

4 Model

4.1 Distribution and Endowments

The economy consists of two regions $i \in \{N, S\}$, an industrialized North (N) and a less developed South (S). The population size of the economy is L, a fraction β lives in the South and a fraction $(1-\beta)$ in the North. We assume that each household regardless of its residence inelastically supplies one unit of labor on the local labor market. This implies that aggregate labor supply in the South is given by βL, and by $(1-\beta)L$ in the North. Furthermore, suppose that each household holds domestic and foreign assets. Hence, income inequality is endogenously determined and originates from differences in labor and capital incomes across countries.

In order to study ceteris paribus effects of income inequality across countries we introduce a transfer system (e.g. foreign aid) between North and South so that each household in the North pays/receives a lump-sum tax/benefit $T_N(t)$, respectively $T_S(t)$. The transfer system must run a balanced budget in each period so that $(1-\beta)LT_N(t) = \beta LT_S(t)$, and transfers grow at the same rate as incomes. We will take $T_S(t)$ as the exogenous variable so that through the balanced budget condition $T_N(t)$ is endogenously determined.

4.2 Preferences

There is a continuum of differentiated products in the economy indexed by $j \in [0, \infty)$, where only a subset $N(t)$ is available on the market at each point in time. We assume differentiated products to be indivisible, and model consumption as a binary decision. Hence, households consume either 1 unit of product j at time t, or they don't consume that product at all. Instantaneous utility is non-homothetic and takes the following form

$$u\left(\{c(j,t)\}_{j=0}^{N(t)}\right) = \int_0^{N(t)} c(j,t)\, dj \tag{1}$$

where $c(j,t)$ is an indicator function that takes the value one if product j is consumed at time t, and zero otherwise. The indicator function $c(j,t)$ will be specific to the income group, i.e. the region. The specification of the instantaneous utility function contrasts with the constant-elasticity-of-substitution (CES) form as follows. With zero-one preferences households can only choose consumption along the extensive margin, i.e. choose how many different products they want to purchase, whereas with CES preferences they can only choose consumption along the intensive margin, i.e. how many units of each product they want to buy. In that sense, our preferences are no less special or general than CES preferences.[9] Furthermore, note that preferences in (1) are symmetric, i.e. no product is intrinsically better or worse than any other product. In other words, there is no explicit consumption hierarchy. This allows us to order products in ascending order from old to new, such that product j is developed before product j', where $j' > j$.[10]

The household's intertemporal objective function is given by

$$U(0) = \int_0^{\infty} \exp(-\rho t) \log u\left(\{c(j,t)\}_{j=0}^{N(t)}\right) dt \tag{2}$$

where $\rho > 0$ denotes the time preference rate. Note that intertemporal preferences given by (2) are homothetic. Households maximize their lifetime utility (2) subject to non-negativity

[9] For the sake of illustration, suppose that the whole product set available to households consists of the six consumer durables in Section 3. With the preferences specified in (1) wealthy households in the North would consume one unit of all consumer durables available whereas poor households in the South could not afford to consume all goods available, and for example could only purchase one washing machine and one freezer (some of the "older" goods available). With CES preferences Northern and Southern households would both consume all six consumer durables available. However, Northern households would purchase e.g. five units of each good whereas Southern households could only buy one unit each.

[10] Note that the same ordering would emerge if we assumed instantaneous utility to take the following form $u(c(j,t)) = \int j^{-\eta} c(j,t)\, dj$. The power function $j^{-\eta}$ implies that (instantaneous) marginal utility is falling in the index j, i.e. higher indexed goods yield lower marginal utility than lower indexed goods. The parameter $\eta \in (0,1)$ determines the "steepness" of the hierarchy, i.e. how fast marginal utility falls in index j. With these preferences households start consuming low-indexed goods (as they yield higher marginal utility) and expand consumption towards high-indexed goods until their income is used up. To keep the model simple, we will assume that such a hierarchy in consumption latently exists rather than explicitly modeling it. For a detailed discussion see Section 6.1.

constraints $c(j,t) \geq 0$ for all j, t, and to their lifetime budget constraint

$$\int_0^\infty \int_0^{N(t)} p(j,t)c(j,t)dj \exp\left(-\int_0^t r(s)ds\right) dt \leq a(0) + \int_0^\infty (w(t)+T(t)) \exp\left(-\int_0^t r(s)ds\right) dt$$

where $r(t)$ denotes the risk-free interest rate, $a(0)$ initial wealth, and $w(t)$ the market clearing wage rate. The solution to the household problem has been relegated to Appendix A.1. From the maximum principle conditions we derive the individual Marshallian demand function for product j:

$$c\left(j,t\right) = \begin{cases} 1 & p\left(j,t\right) \leq z\left(j,t\right) \\ 0 & p\left(j,t\right) > z\left(j,t\right) \end{cases} \tag{3}$$

where $z\left(j,t\right) \equiv \left[u\left(\cdot\right)\lambda(t)\right]^{-1}$ denotes the willingness to pay. Figure 4 below shows the individual demand curve (3) for product j. The costate variable, which can be interpreted as marginal utility of wealth at time t, is denoted by $\lambda(t)$. Households purchase one unit of a product if the price of that product does not exceed their willingness to pay. Since preferences are symmetric over all products the willingness to pay is identical for all products j. However, the willingness to pay depends on $\lambda(t)$, i.e. on the shadow price of (lifetime) income. Hence, consumption patterns differ across regions since by our distributional assumptions (lifetime) incomes are different across regions. Wealthy households in the North, with a lower equilibrium value of $\lambda(t)$, consume a larger set of products than poor households in the South.

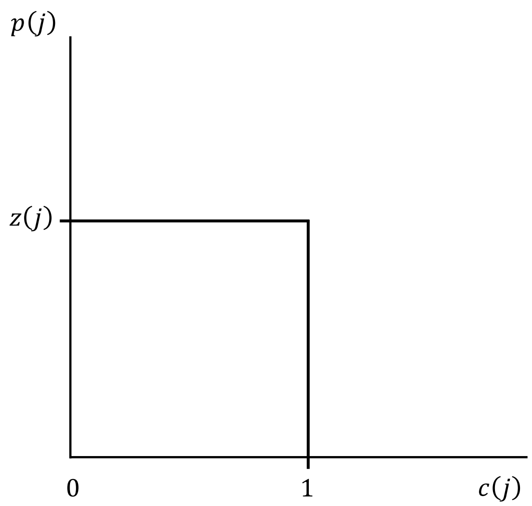

Figure 4: Individual demand

4.3 Technology and Trade Integration

4.3.1 Innovation technology in North

New products are designed and developed in high-income countries.[11] Each firm in the North is a single-product firm, which has access to the same innovation technology. The creation of a new product requires $F^N(t) = F^N/N(t)$ units of labor, once this set-up cost has been incurred, the firm has access to a linear technology that requires $b^N(t) = b^N/N(t)$ units of labor to produce one unit of output, with $F^N, b^N > 0$ being positive constants. Innovations obey an important spillover because they imply technical progress. We assume that the knowledge stock of this economy equals the number of known designs $N(t)$. The labor coefficients are inversely related to the stock of knowledge. New products are protected by infinite patents but face a positive probability of being copied by a Southern firm (patent infringement). We assume that firms in the North cannot license technology to Southern firms, or set up manufacturing plants in the South (i.e. engage in foreign direct investment).

4.3.2 Imitation technology in South and Transportation Costs

As in Grossman and Helpman (1991) we assume that each new product, which has been developed in the North at time t, faces the same positive probability of being imitated by a Southern firm at some time $\widetilde{T} > t$. At the time the product is developed date $\widetilde{T}$ is unknown. In other words, $\widetilde{T}$ is a random variable that represents the age of a product at the time of imitation. A Southern firm selects at random one of the existing products in the North, which has not yet been copied, for imitation. We assume that firms in the South benefit in reverse engineering and production from the total stock of knowledge (i.e. there are international knowledge spillovers). Imitation of a selected product requires $F^S(t) = F^S/N(t)$ units of labor, with $F^S > 0$. Investing $F^S(t)$ allows a Southern firm to learn the production process of the randomly chosen product with probability one. Hence, there is complete certainty for a Southern imitator that reverse engineering succeeds. Subsequent production of the copied good requires $b^S(t) = b^S/N(t)$ units of labor per unit. Finally, we assume that product markets are fully integrated and trade costs are zero.

4.4 Equilibrium

Depending on parameter values, two decentralized equilibria can emerge: (i) households in the South are too poor to afford any Northern products or (ii) they can afford at least some Northern products. In case (i) no trade equilibrium exists. Hence, we focus on the interesting case (ii), and assume in the following that households in the South can afford some Northern products. In proving the existence of the equilibrium, we will derive the necessary assumption

[11] In principle, one could think that both North and South have access to the innovation technology but that the South is sufficiently unproductive at developing new products compared to the North such that in equilibrium no innovation takes place in the South. Since it is difficult to measure research productivity, for illustration's sake, consider research input. World Bank (2012) data on research and development spending of low/middle and high income countries show that high income countries on average spent about 2.5 times as much on R&D in percent of their GDP than low and middle income countries during the period 2000-2007.

on parameters. Let us denote the set of all products available in the economy as $N(t) = N^N(t) + N^S(t)$, where $N^N(t)$ denotes the subset of products that have not yet been imitated by the South, and $N^S(t)$ the subset of products that have been copied by the South.

4.4.1 World demand

In the equilibrium we consider households in the North consume all products available in the market $N_N(t) = N(t)$, whereas households in the South consume only a subset of all products $N_S(t) \subset N(t)$, which includes all products manufactured in the South and some but not all Northern products. World demand for product j can be derived by horizontally aggregating individual demand (3) across regions. It is determined by:

$$C(j,t) = \begin{cases} 0, & p(j,t) > z_N(j,t) \\ (1-\beta)L, & z_S(j,t) < p(j,t) \leq z_N(j,t) \\ L, & p(j,t) \leq z_S(j,t) \end{cases} \tag{4}$$

where $z_i(j,t)$, with $i \in \{N, S\}$, denotes the willingness to pay of households in the North, and South, respectively. Since the willingness to pay is the same for all products j, aggregate demand is the same for all products. World demand (4) is depicted in Figure 5 below.

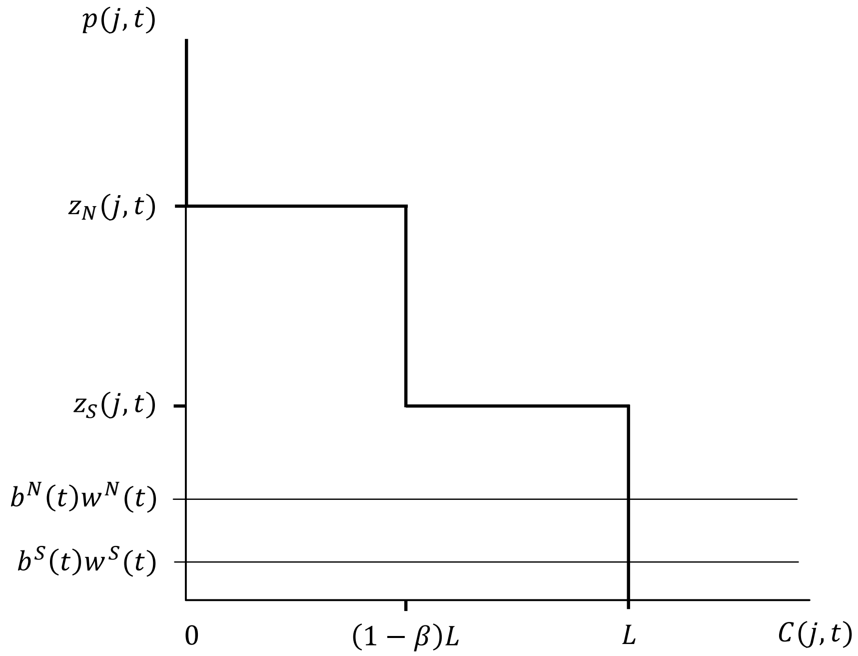

Figure 5: World demand

If the price of a product exceeds the willingness to pay of Northern households, there is no demand for that product. With a price between the willingness to pay of Southern and Northern households only the latter purchase the product. If the price falls short of the willingness to pay of households in the South everyone purchases it. Figure 5 is drawn under the assumption that the willingness to pay of Southern households exceeds marginal costs $b^N(t)w^N(t)$, which holds true in the equilibrium of interest.

4.4.2 Aggregate supply

Let us first consider the problem of a monopolistic firm j located in the North. Firm j maximizes operating profits

$$\pi^N(j,t) = \left[p(j,t) - w^N(t)b^N(t)\right] C(j,t) \tag{5}$$

subject to aggregate demand (4) by choosing a price $p(j,t)$ such that marginal revenue equals marginal cost. From Figure 5 and the discussion in the previous section it follows that there are two candidates for the price that maximizes profits (5). Firm j either sets a high price equal to the willingness to pay of Northern households $z_N(j,t)$ and sells exclusively to domestic households, or it sets a low price equal to the willingness to pay of Southern households $z_S(j,t)$ and serves both markets.

We assume that firms cannot price discriminate across regions. As there are no trade costs, arbitrageurs would take advantage of any price differential between North and South.[12] Thus, exporters set the same price in both regions. This implies that in equilibrium not all Northern firms export. To see this, suppose that at every point in time all Northern firms would set prices equal to the willingness to pay of Southern households and sell to everyone. In that case, households in the North would not exhaust their budgets, i.e. the shadow price of their (lifetime) income would become zero. That would imply an infinitely large willingness to pay for an additional product. Consequently, Northern firms had an incentive to deviate from selling to everyone and sell exclusively in the North. Hence, a situation where all Northern firms serve all households cannot be an equilibrium. Also, by the same argument, a situation where all Northern firms sell exclusively to Northern households cannot be an equilibrium as the willingness to pay of Southern households for a Northern product would become infinitely large.

In an equilibrium, where some Northern firms serve all households in both regions and others serve exclusively the domestic region, firms must be indifferent between selling only to Northern households and selling to all households at any point in time. Hence, the following arbitrage condition must hold

$$\left[z_N(j,t) - w^N(t)b^N(t)\right](1-\beta)L = \left[z_S(j,t) - w^N(t)b^N(t)\right]L. \tag{6}$$

In the aggregate, a measure n of firms sells in the North and South whereas $(1-n)$ firms sell only in the North. Due to symmetric preferences, however, the behavior of a single firm is indeterminate. Because we are free to order the different goods, we may think of the following

[12] The threat of arbitrage opportunities imposes a price setting restriction on firms. If there are no trade costs the price setting restriction is always binding. However, in the presence of iceberg trade costs the price setting restriction might not be binding. In particular, if the difference in per capita incomes between North and South were sufficiently low, all newly invented products would be exported to the South right away.

firm behavior at the micro level that generates the described outcome at the macro level: After developing a new product each firm starts marketing its product solely in the North and after a certain period of time has elapsed, i.e. the time it takes for incomes in the South to have grown sufficiently, begins exporting. In that case, there are at any point in time new products that are sold exclusively in the domestic market and older products that are exported as well. Section 6 discusses two possible extensions where the product cycle at the firm level is determinate. We argue that while the model would become substantially more complex the basic structure and intuition of the baseline model is preserved.

The Northern firm, which develops product j at time t, faces a positive probability that its product will be copied by a Southern firm. After a product has been imitated, the Southern firm maximizes operating profits

$$\pi^S(j,t) = \left[p(j,t) - w^S(t)b^S(t)\right] C(j,t)$$

where $C(j,t) = L$ is given by (4). After the firm in the South has copied the Northern product j it enters into a price competition with the Northern firm currently producing j (the innovating firm). This forces the Southern firm to set a limit price equal to the marginal costs of the competing firm in the North. Hence, optimal prices of Southern products are equal to $w^N(t)b^N(t)$.[13]

4.4.3 Labor markets

Labor is immobile across regions but regional labor markets are assumed to be perfect. In particular, in the North labor is completely mobile between production and R&D, and in the South between production and reverse engineering. Labor market clearing in the North demands that

$$(1-\beta)\,L = g(t)F^N + b^N L\,[n(t) - m(t)] + (1-\beta)\,b^N L\,[1-n(t)] \tag{7}$$

where we defined $g(t) \equiv \dot{N}(t)/N(t)$, and the share of goods consumed and produced in the South, respectively, as $n(t) \equiv N_S(t)/N(t)$ and $m(t) \equiv N^S(t)/N(t)$. The first term in (7) on the right-hand side denotes labor demand from the R&D sector, the second term labor demand from the production of older Northern products consumed by all households in both regions, and the third term labor demand from the production of newer Northern products exclusively consumed by Northern households. Similarly, labor market clearing in the South requires

$$\beta L = g^S(t)m(t)F^S + m(t)b^S L \tag{8}$$

[13]The wide-gap case discussed in Grossman and Helpman (1991) where Southern firms can set the monopoly price cannot occur here since $z_S(t) > w^N(t)b^N(t) > w^S(t)b^S(t)$ in equilibrium as otherwise, no firm in the North would export to the South. Our case is similar to their narrow-gap case where Southern firms charge prices marginally below the marginal cost of Northern firms.

where we defined $g^S(t) \equiv \dot{N}^S(t)/N^S(t)$. The right-hand side in (8) denotes labor demand from reverse engineering, and production of imitated products which are consumed by all households in both regions.

4.4.4 Capital markets

We assume that international capital markets are perfect, hence, interest rates equalize across regions. The expected present discounted value of profits of product j that was introduced at time t is determined by equation (9) below, given the instantaneous rate of imitation $\mu(t) \equiv \dot{N}^S(t)/N^N(t)$. We make the standard assumption of free entry into product development in the North. Hence, the expected value of product j must equal R&D costs $w^N(t)F(t)$,

$$v^N(j,t) = \int_t^{\infty} \exp\left(-\int_t^s (r(\tau)+\mu(\tau))\, d\tau\right) \pi^N(j,s)\, ds = w^N(t)F^N(t). \tag{9}$$

Note that profits are discounted using the risk-adjusted interest rate $r(\tau)+\mu(\tau)$, where $r(\tau)$ is the risk-free interest rate and $\mu(\tau)$ the risk premium. Since we assume capital markets to be perfect, households can diversify away the idiosyncratic risk of a Northern firm of being copied by holding a portfolio of shares in all Northern firms. Free entry also prevails in the reverse engineering sector in the South, which is not an uncertain activity, so that their present discounted value of profits $v^S(j,t)$ must equal the imitation cost $w^S(t)F^S(t)$,

$$v^S(j,t) = \int_t^{\infty} \exp\left(-\int_t^s r(\tau) d\tau\right) \pi^S(j,s)\, ds = w^S(t)F^S(t). \tag{10}$$

4.4.5 Asset holdings and balance of payments

The balance of payments in present value terms is determined by

$$\begin{aligned} 0 &= \int_0^{\infty} \left\{ \left[(1-\beta)LN^S(t)\omega^N b^N - \beta L \left(N_S(t) - N^S(t)\right) z_S(t) \right] \right. \\ &+ \left. \beta L T_S(t) \right\} \exp\left(-\int_0^t r(s)ds\right) dt \end{aligned} \tag{11}$$

where the first term in brackets on the right-hand side denotes the trade balance and the second term net transfer payments. We assume that net foreign assets (portfolio investments) are zero.[14] Note that if $T_S(t) > 0$ for all t, the South runs a (permanent) trade deficit, i.e. the value of its exports falls short of the value of its imports.

4.5 Steady state

The economy is in a steady state if Northern firms introduce new products at a constant rate g and Southern firms imitate at a constant rate μ. In steady state, shares of resources devoted

[14] Because of equal interest rates, consumption growth is identical across regions in steady state. Hence, net foreign assets will remain zero forever. If net foreign assets are non-zero, T_S is to be interpreted as sum of transfer and interest payments. For a formal derivation see Appendix A.4.

to R&D and production are constant, and the fraction of Northern products that have not yet been imitated is constant. Furthermore, prices of Northern and Southern products and therefore, profits of Northern firms are constant. Let us choose the marginal costs of production of Northern firms as the numeraire, and set $w^N(t)b^N(t) = 1$ for all t.

First, we turn to the first-order conditions of the household's maximization problem. It follows that the optimal evolution of consumption of Northern and Southern households, i.e. the Euler equation, in steady state is given by

$$g = r - \rho \tag{12}$$

which implies equal growth rates in North and South. Households budget constraints in steady state are given in Appendix A.3.

Now, consider the equilibrium in the labor markets. The resource constraint in the South (8) becomes

$$\beta L = gmF^S + mb^S L. \tag{13}$$

A higher fraction of products that have been imitated m implies that there is more imitation activity in the South so that on average Northern products are copied sooner, ceteris paribus. This tends to depress innovation activity in the North implying a lower g. The resource constraint of the North (7) can be written as follows in the steady state

$$(1 - \beta) L = gF^N + Lb^N (n - m) + (1 - \beta) Lb^N (1 - n) \tag{14}$$

where n denotes the "consumption gap" between South and North. Note that a higher share of the South in total production m releases resources from the production sector in the North that can be reallocated to the R&D sector, ceteris paribus. This allows the North to introduce new products at a higher rate g. Furthermore, a higher consumption share of the South n induces a reallocation from the R&D sector to the production sector in the North to satisfy the additional demand for existing Northern products by the South, thereby depressing innovation in the North, ceteris paribus.

Next, a fixed inter-sectoral allocation of labor implies that prices of Northern products must be constant in steady state. We denote the price of a new product that is sold exclusively to Northern households as z_N. Since all firms face the same demand curve and have the same cost structure, z_N is identical for all new products $j \in (N_S(t), N(t)]$. From the arbitrage condition (6) follows that prices for all old Northern goods $j \in (N^S(t), N_S(t)]$, which are sold to all households, are also constant and determined by $z_S = \beta + (1 - \beta) z_N$. Moreover, this implies that profits are constant over time. Prices of Southern products $w^N(t)b^N(t)$ are equal to 1 due to our choice of numeraire. This is consistent with the steady state, else demand for Southern labor would change over time.

Let us consider the *average* life cycle in steady state of some product j, which is introduced at time t. At the time of introduction product j is sold at price z_N exclusively to Northern households. At time $t + \Delta$, where $N(t) = N_S(t + \Delta) = N_S(t) \exp(g\Delta)$, the Northern firm producing good j lowers the price to $\beta + (1-\beta)\, z_N$ and exports it to the South. Therefore, the average demand lag equals $\Delta = -\log(n)/g > 0$, decreasing in the consumption share n and the innovation rate g. We consider the case where $\widetilde{T} > t + \Delta$ for all t. In other words, on average Northern products are exported to the South for some time before they are copied by a Southern firm.[15] Notice that in steady state $\widetilde{T}$ follows an exponential distribution. Thus, the average time span product j is being produced in the North is determined by $1/\mu$. Hence, our assumption above implies that the demand lag is shorter than the time span product j is being manufactured in the North, i.e. $1/\mu > \Delta$. Due to lower production costs in the South, Southern firms can set a price marginally below 1, the marginal costs of Northern firms. Hence, the Northern firm stops producing product j and the product is now exported to the North. Of course, this discussion is only relevant for the *average* product. By the random nature of imitation there will be some products that are imitated before households in the South become rich enough to afford them. Hence, those products will skip the export stage. The average life cycle of product j in terms of sales volume is depicted in Figure 6 below.

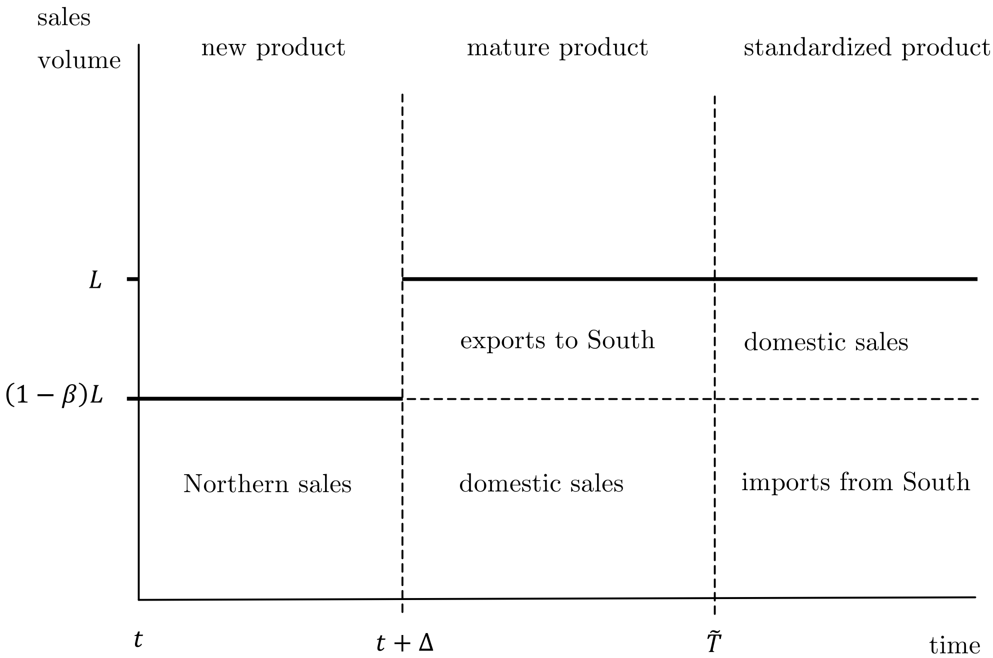

Figure 6: Average life cycle (in terms of sales volume)

From the definition of the imitation rate $\mu = \dot{N}^S(t)/N^N(t)$ we can express the production share of the South in the total number of differentiated products as

$$m = \frac{\mu}{g + \mu} \tag{15}$$

which must be constant in the steady state. Next, the zero-profit condition (9) together with

[15]Note that in the other case with $\widetilde{T} \le t + \Delta$, goods would on average skip the export stage. We consider the case in the text to be the interesting one.

the arbitrage condition (6) in the North implies that in the steady state the value of a firm is equal to the expected present discounted value of its future profits

$$\frac{[z_N - 1]\,(1-\beta)\,L}{r+\mu} = \frac{F^N}{b^N}. \tag{16}$$

Similarly, in the South the zero-profit condition (10) yields

$$\frac{\left[1-\omega^S b^S\right] L}{r} = \omega^S F^S \tag{17}$$

where $\omega^S(t) \equiv w^S(t)/N(t)$ is constant since wages in the South grow at rate g. Last, in steady state, the balance of payments (11) becomes

$$(n-m)\,[\beta + (1-\beta) z_N]\,\beta = m(1-\beta) + \beta T \tag{18}$$

where $T \equiv T_S(t)/N(t)$. Note that due to Walras' law the balance of payments is implied by the budget constraints, the zero-profit conditions and the resource constraints.

Equations (12) - (18) in the unknowns g, μ, n, m, r, z_N, and ω^S fully characterize the steady state. We can reduce this system to 2 equations in 2 unknowns m and g. The first equation, the *RS-curve*, describes a steady state relationship between g and m that is consistent with labor market clearing in the South:

$$m = \frac{\beta L}{g F^S + b^S L}. \tag{19}$$

The second equation, the *NA-curve*, describes a steady state relationship between g and m that is consistent with labor market clearing in the North, balance of payments, free entry in the North, and the no arbitrage condition

$$\left(1 + \rho\frac{F^N}{b^N L} + \frac{g}{1-m}\frac{F^N}{b^N L}\right)\left((1-\beta)\left(\frac{1}{b^N} - 1 + m\right) - g\frac{F^N}{b^N L}\right) = m(1-\beta) + \beta T. \tag{20}$$

To guarantee that the *NA-curve* defined by (20) has a positive x-axis intercept in the (m, g) space we make the following assumption.

Assumption 1. $\left(1+\rho\frac{F^N}{b^N L}\right)(1-\beta)\left(\frac{1}{b^N} - 1\right) \geq \beta T.$

Proposition 1. *Given Assumption 1 holds, a steady state equilibrium with positive growth rate g and a constant share of imitated products m exists.*

Proof. The *RS-curve* (19) is downward sloping in the (m, g)-space. To determine the shape of the *NA-curve* we rewrite (20) as $NA(m, g) = 0$. The left hand side of this equation is a quadratic function in g with inverted U-shape. If Assumption 1 holds, $NA(m, g)$ has a negative and a positive solution for g. Thus, $NA_g(m, g) < 0$ at the relevant solution. Further, differentiation shows that $NA_m(m,g) = \frac{g}{(1-m)^2}\frac{F^N}{b^N L}\left((1-\beta)\left(\frac{1}{b^N} - 1 + m\right) - g\frac{F^N}{b^N L}\right) + \left(\rho\frac{F^N}{b^N L} + \frac{g}{1-m}\frac{F^N}{b^N L}\right)(1-\beta) > 0$. Hence, the *NA-curve* has a positive slope and positive intercept with the x-axis and a negative intercept with the y-axis in the (m, g)-space. Figure 7 below depicts the graphical solution of the steady state.[16] □

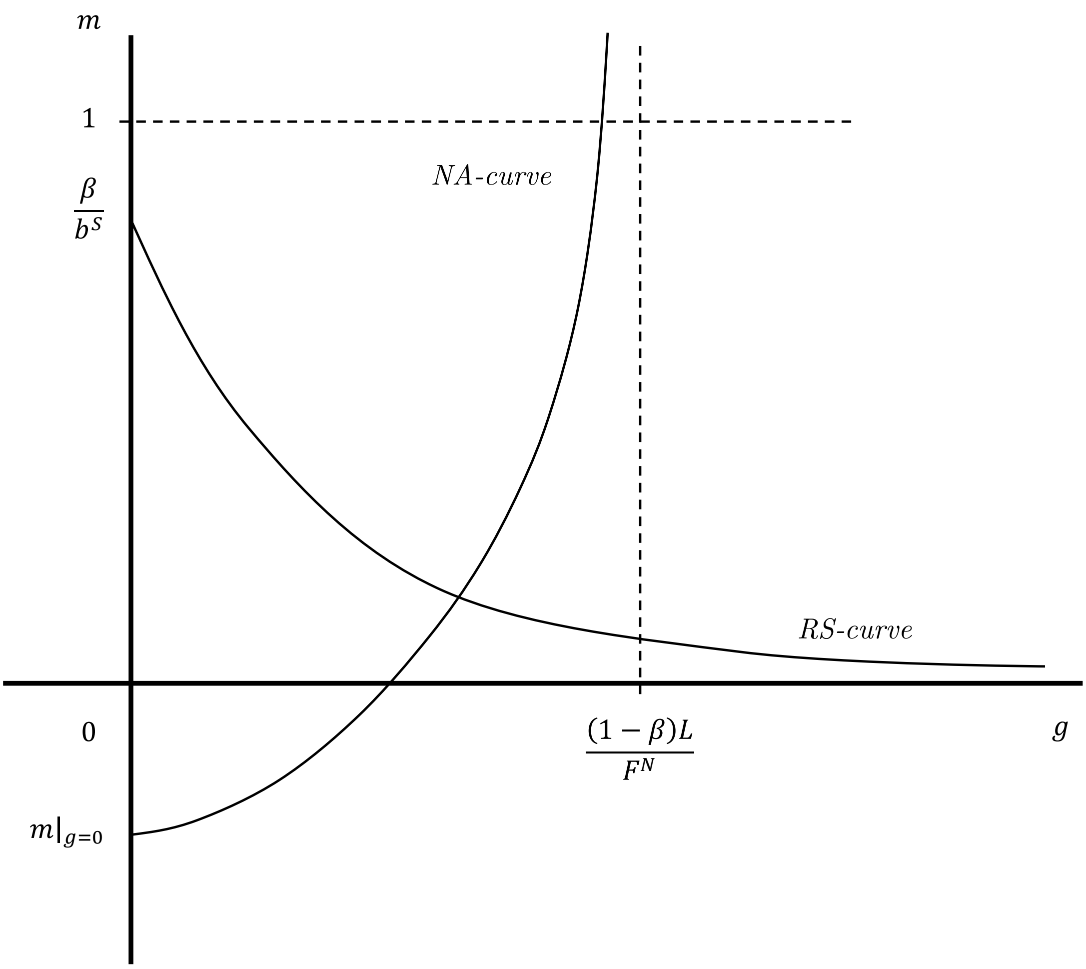

Figure 7: Steady state

4.6 Transitional dynamics

The transitional dynamics are easy to characterize. The full derivation of the transitional dynamics including a phase plane illustrating the dynamics is given in Appendix A.2. If we replace g with g^S at the axis in Figure 7 above, the *RS-curve* now determined by equation (8), representing the Southern full employment condition only, must hold also outside the steady

[16]Note that the $m|_{g=0}$ implied by $NA|_{g=0}$ is given by $m = -\left[\left(1 + \rho\frac{F^N}{b^N L}\right)(1-\beta)\left(\frac{1}{b^N} - 1\right) - \beta T\right] / \rho(1-\beta)\frac{F^N}{b^N L} < 0$ due to Assumption 1. Furthermore, evaluating the *NA-curve* at $m \to 1$ yields

$$1 + \rho\frac{F^N}{b^N L} + \frac{g}{1-m}\frac{F^N}{b^N L} = \frac{(1-\beta) + \beta T}{(1-\beta)\frac{1}{b^N} - g\frac{F^N}{b^N L}}$$

where the left-hand side goes to infinity as $m \to 1$ for any $0 < g < \infty$. Hence, the right-hand side goes to infinity as $g \to (1-\beta)L/F^N$.

state. Hence, along a transition path, m and g^S move along the *RS-curve*. The *NA-curve* (20), instead, is a steady state condition. Appendix A.2 further demonstrates that the steady state is sattlepath stable. When the number of industries in the South is below its steady state value, $m(0) < m$, then $\dot{m}/m = g^S - g > 0$, i.e. the growth rate of imitation is higher than the growth rate of innovation during the transition process. Thus, m converges monotonically to its steady state value.

5 Comparative statics

In this section we explore the steady state implications on the growth rate and the length of the product cycle for the following changes: An increase in imitation productivity in the South and an exogenous change in inequality across regions. We simulate comparative statics results. The simulation results are depicted in Appendix A.5. Note that the wage rate of North relative to South is determined by $w^N(t)/w^S(t) = \left(\omega^S b^N\right)^{-1}$, where $\omega^S = (b^S + (\rho + g)F^S/L)^{-1}$ is pinned down by (17). Using the zero-profit condition (16) and the arbitrage condition in the North, we get an expression for the terms of trade of the North, $z_S = 1 + [\rho + g/(1-m)]F^N/b^N L$.

5.1 Increase in Southern labor productivity

Proposition 2. *An increase in Southern productivity, i.e. a decrease in b^S or F^S, results in a higher growth rate g, Southern imitation share m and imitation rate μ. Hence, the average time span a product is being manufactured in the North $1/\mu$ becomes shorter. While the terms of trade move in favor of the North (z_S increases), two opposing effects move relative wages and the consumption share. Higher Southern productivity tends to increase Southern relative wages while the higher growth rate g tends to decrease them. A higher imitation rate expands the Southern consumption share whereas the higher growth rate dampens it.*

Proof. A decrease in b^S shifts the *RS-curve* upwards, whereas a decrease in F^S rotates the *RS-curve* upwards, both leaving the *NA-curve* unaffected. Hence, both a decrease in b^S and F^S lead to a higher growth rate g and Southern imitation and consumption share m. The imitation rate increases, as μ depends positively on g and m. According to the Northern zero profit condition (16) z_N and z_S increase. Using the Southern zero profit condition (17) we see that ω^S increases with higher productivity in the South but decreases in g. This implies that relative wages $w^N(t)/w^S(t)$ decrease due to the direct productivity effect and increase because of a higher growth rate. Using the Northern resource constraint, we see that a higher g reduces n while the higher m raises n. □

Intuitively, a reduction in F^S or b^S triggers more imitation because it is cheaper to produce imitated goods. An increases in μ leads to a higher risk-adjusted interest rate and hence to a lower present discounted value of profits earned from innovation, reducing the incentives to innovate. At the same time, households in the South become relatively richer (ω^S increases). Their willingness to pay increases and hence Northern terms of trade z_S improve. Moreover, they can afford to purchase a broader range of products (n rises). Higher Northern prices raises profits from developing new products and therefore, the incentives to innovate increase. This latter effect dominates such that the innovation rate g rises. However, the imitation rate μ increases more than the innovation rate g, leading to an increase in the fraction of imitated goods m in equilibrium. The increase in Southern labor productivity tends to directly increase the wage rate ω^S, holding g constant. However, there is also an indirect effect through the increase in g, which leads to an increase in the interest rate r and therefore, to a decrease in the present discounted value of profits. The indirect effect induces less firms to enter the market in the South, which tends to depress labor demand in the South and hence the wage rate ω^S. In our simulations the direct effect dominates such that the Northern relative wage rate $w^N(t)/w^S(t)$ decreases. As Southern households grow richer, not only their willingness to pay for Northern products increases but also the range of consumed goods. This increases the consumption share of the South. At the same time, a higher innovation rate dampens the consumption share. In our simulations the first effect dominates, i.e. the consumption share increases.

The effect on the product life-cycle

The time length Δ where products are exclusively sold in the North becomes shorter due to two reasons: households in the South are relatively richer (n rises) and the overall growth rate g is higher. Since the imitation rate μ increases, the average time span a product is being manufactured in the North $1/\mu$ becomes shorter as well. The third stage during which the North imports a product clearly increases. The time period during which the North exports a product $(1/\mu - \Delta)$ decreases according to our simulations.

Limit case: costless reverse engineering

Suppose, less realistically, that reverse engineering is costless, i.e. $F^S \to 0$. As $F^S \to 0$ the *RA-curve* rotates upwards and becomes a horizontal line at $= \beta/b^S < 1$. Since the *NA-curve* is independent of F^S, the growth rate g and the Southern imitation share m increase as $F^S \to 0$. The intuition is that if imitation becomes costless, the South can take over the maximal share of production from the North. This releases resources in the North that can be allocated to the research and development of new products. At the same time, the imitation activity is at its maximum in the South (only restricted through limited resources), which means that the risk-adjusted interest rate peaks, depressing the present discounted value of profits earned from creating new products. In equilibrium, the first effect dominates the second one, and g reaches a maximum.

5.2 Changes in income inequality across regions

Proposition 3. *An increase in T, i.e. lowering world income inequality, leads to a new steady state where the growth rate g is lower and the share of imitated and consumed products, m and* *n, are higher. Northern relative wages deteriorate. There are opposing effects on the terms of trade and on the three stages of the product life cycle.*

Proof. An increase in T leads to an upward shift of the *NA-curve* since $NA|_{g=0}$ is a positive function of T. Notice that g implied by the *NA-curve* as $m \to 1$ is given by $g = (1-\beta)L/F^N$, independent of T. As the *RS-curve* is unaffected by a change in T, the new steady state has a lower g and higher m. Using the Northern resource constraint, we see that a lower g together with a higher m increases n. Since ω^S is a decreasing function of g, Northern relative wages $w^N(t)/w^S(t) = \left(\omega^S b^N\right)^{-1}$ are lower. A lower g tends to decrease the terms of trade z_S, whereas a higher m tends to increase them. □

A higher transfer leads to higher incomes in the South and lower incomes in the North, ceteris paribus. Lower incomes in the North depress the incentives to develop a new product, which decreases the innovation rate g. As Southern resources are fixed, the fraction of imitated products increase. At the same time, higher incomes in the South translate into a higher willingness to pay for older products produced in the North. This implies that profits of innovating firms in the North from selling only to Northern households fall short of profits from selling to all households, creating a disequilibrium in the North. This induces some Northern firms to start exporting. As Southern households consume more products, i.e. $N_S(t)$ increases, their marginal willingness to pay, ceteris paribus, decreases until the equilibrium in the North is restored. In the new equilibrium, households in the South consume a higher fraction of all products n, and their (marginal) willingness to pay is lower. In our simulations, the North's export prices z_S decrease, and as the North's import prices are equal to one, the terms of trade move in favor of the South.[17]

Effect on product life-cycle

There are two opposing effects on the the first stage of the product cycle (the demand lag Δ). On the one hand, households in the South are richer so that the Northern firm producing the latest product would like to export sooner (effect of higher n). On the other hand, even though the level of income for Southern households is higher, their income grows at a lower rate. This induces the Northern producer of the latest product to export later (effect of lower g). The simulations show that the first effect dominates so that the first stage, where new products are exclusively sold in the North, becomes shorter. There are two opposing effects on the second stage of the product cycle. On the one hand, the imitation rate μ decreases because of a lower growth rate g. On the other hand, the higher share of imitation increases μ. In our simulations the effect of a lower growth rate dominates. Hence, the average time span a product is being manufactured in the North $1/\mu$ becomes longer so that the third stage during which the North imports a product decreases. Moreover, the time period during which the North exports a product $(1/\mu - \Delta)$ becomes longer.

[17] Totally differentiating the Northern zero-profit condition (16) and the definition of the imitation rate (15) shows that $dz_N > 0$ and $d\mu < 0$ if and only if $\beta/\left(\beta - b^S m\right) > m/(1-m) > b^S L/F^S$, where $\beta/b^S > m$ and we used that along the *RS-curve* $dm/dg < 0$. Sufficient conditions are $m < 0.5$ and $b^S L/F^S < 1$.

5.3 Comparison to CES utility case

An important advantage of the non-homothetic utility function assumption is the possibility to analyze the effect of the demand side on the average time span of the three product cycle stages. With CES preferences there is no first stage where the product is exclusively produced and consumed in the North as all consumers consume all goods. This is a main difference of this chapter compared to Grossman and Helpman (1991). Besides being able to discuss effects on the first stage, the model presented in this paper is similar to the narrow-gap case in Grossman and Helpman (1991). In the following, we are going to elaborate some differences and similarities in the comparative statics results.

A change in Southern productivity has similar effects. An improvement in Southern productivity in imitation leads in both models to higher growth and imitation rates. Relative wage rates move in favor of the South. In Grossman and Helpman (1991) the Northern terms of trade are connected to the change in relative wage rates (prices are a constant markup over marginal costs), and hence deteriorate. In this paper, the terms of trade depend on the willingness to pay of households and hence move in favor of the North. As Southern relative wages increase, Southern households' willingness to pay for Northern products increase.

Next, we have a closer look at changes in inequality. In Grossman and Helpman (1991), a transfer from rich North to poor South has no effects on innovation incentives because of CES preferences. Instead, let us compare country size effects for which we provide simulation results in Appendix A.5. In Grossman and Helpman (1991), an increase of the Southern labor force leads to an acceleration of innovation and imitation. Moreover, relative Southern wage rates increase. In our paper, the effect of a larger Southern population share leads also to a higher imitation rate. The innovation rate, however, decreases if South is not too productive in producing and imitating. This can be easily seen using the two equilibrium conditions, the *RS-curve* and *NA-curve*. Both curves shift upwards with an increase in β. However, the *RS-curve* shifts up more than the *NA-curve*, and hence the positive effect on g becomes larger, the smaller b^S is. A higher population share in the South implies a higher production share in the South which releases resources in the North for innovation. This channel tends to dominate with a higher productivity in the production sector of the South. Northern relative wage rates decrease here, too. Northern terms of trade, however, increase as the Southern willingness to pay for Northern products increases with the rise in relative wages.

6 Extensions

Due to our assumption of symmetric preferences and identical cost structures the product cycle of product j is indeterminate. In order to show that the product cycle we impose in our baseline model emerges from more complex models, without changing the basic channels through which the income distribution operates, we discuss two extensions. It is straightforward to either change the assumptions about preferences or about technology such that the indeterminacy vanishes.

6.1 Hierarchic Preferences

Following Foellmi and Zweimüller (2006), we assume that households have the following non-homothetic preferences

$$u(c(j,t)) = \int_{j=0}^{N(t)} j^{-\gamma} c(j,t) dj$$

where the parameter $\gamma \in (0,1)$ determines the "steepness" of the hierarchy, i.e. how fast marginal utility falls in the index j. One can view low-indexed products as satisfying more basic needs relative to higher-indexed products. It is straightforward to derive the willingness to pay for good j, which is given by $z(j,t) \equiv j^{-\gamma}[u(\cdot)\lambda(t)]^{-1}$, and decreases in the index j. In other words, households demand, and therefore Northern firms develop, products along the hierarchy, starting with low-indexed products and gradually moving up the hierarchy ladder. This implies that profit-maximizing prices for Northern products, and hence profits decrease in the index j, given all firms have the same cost structure.

We continue to assume that Southern households can afford to consume some products manufactured in the North. Which Northern firms do not export and which firms do? First, suppose that no firm in the North exports. In that case Southern households would not exhaust their budget constraints and their willingness to pay would become infinitely large. This implies that prices for the lowest-indexed products, which have not yet been imitated by the South, become infinitely high. Hence, the firms producing the lowest-indexed products have an incentive to start exporting their products. Second, consider the case where all Northern firms export. In that case, Northern households would not exhaust their budget constraints, and their willingness to pay for an additional product would become infinitely high. This implies that new firms enter the market along the consumption hierarchy, manufacturing products that Southern households cannot afford, and that are therefore not exported.

We keep our assumptions about technology in the North. However, instead of assuming that Southern firms target Northern products at random for imitation, we assume that patents expire at time $\widetilde{T} < \infty$, where $\widetilde{T}$ is now deterministic. Random imitation would imply that there might be "holes" in the hierarchy of products. Southern firms must still invest a fixed amount of resources, e.g. building local production facilities, reverse engineering or learning the production process, in order to manufacture products, whose patents have expired, at constant marginal costs. The fixed cost implies that it is never a dominant strategy for a Southern firm to copy a product, which has already been imitated by another Southern firm. After the patent expires the Southern firm imitating the product enters into price competition with the original Northern innovator, which leads to a limit price equal to marginal costs of the Northern firm, and the exit of the Northern firm.

In sum, this model would generate the following deterministic product cycle in a steady state. At some time $t \geq 0$, the Northern firm j introduces the lowest-indexed product that has not yet been invented. It starts selling its product to Northern households at the price $z_N(j,t)$ since only they can afford to purchase new products that satisfy relatively non-essential needs. The price $z_N(j,t)$ increases at rate γg until after Δ periods, which is still determined by $N(t) = N_S(t)\exp(g\Delta)$, the Northern firm finds it attractive to lower the price to $z_S(j,t)$ and starts exporting its product.[18] The price $z_S(j,t)$ increases at rate γg until after $\widetilde{T} > t+\Delta$ periods the patent expires, a Southern firm copies the product and price competition drives the Northern firm out of the market. The price drops to the marginal cost of production of Northern firms, and stays constant from then on. Hence, such a model would eliminate the indeterminacy of the product cycle. However, the analysis would be substantially more complicated without presumably adding much additional insight.

6.2 Learning-by-doing

In the following, we keep our assumptions from the basic model about preferences (Section 4.2) and technology in South (Section 4.3.2). However, we follow Matsuyama (2002) and assume that there is passive learning-by-doing (i.e. externality of the manufacturing process) in the production sector of the North. In particular, we assume that producing one unit of output requires $b^N(j,t) = b^N(Q(j,t))/N(t)$ units of labor, where $b^N(\cdot)$ is a decreasing function of the discounted cumulative output determined by

$$Q(j,t) = \delta \int_{-\infty}^{t} C(j,s) \exp(\delta(s-t))\, ds$$

where $\delta > 0$ can be interpreted as both the speed of learning as well as the rate of depreciation of the learning experience. Again, $C(j,t) \in \{0, (1-\beta)L, L\}$ denotes market demand. Due to depreciation the cumulative learning experience $Q(j,t)$ is bounded from above by $C(j,t)$, and can therefore not exceed L. We continue to assume that the creation of a new product requires $F^N(t)$ units of labor. As in the previous section, we assume that patents expire after $\widetilde{T} < \infty$ periods.

Again, consider a situation where Southern households can afford to purchase some of the products made in the North. Prices of Northern and Southern products are still determined as before. Our assumptions about technology imply that profits of Northern firms increase with production experience, ceteris paribus. In other words, firms which have been in the market for a longer time earn higher profits since their marginal costs are lower. In equilibrium, at any point in time some firms export and some sell exclusively to Northern households. Hence, there must be some threshold value $Q(N_S(t),t)$, implicitly defined by $\left[z_N(N_S(t),t) - w^N(t)b^N(Q(N_S(t),t))/N(t)\right](1-\beta) = \left[z_S(N_S(t),t) - w^N b^N(Q(N_S(t),t))/N(t)\right]$, at which a Northern firm is indifferent between exporting or not. Below this threshold value the profits from excluding Southern households

[18] This follows from taking the time derivate of the willingness to pay for the most recently innovated product $N(t)$, which is given in the steady state by $\dot{z}_N(N(t),t)/z_N(N(t),t) = r - \rho - g$. In a steady state where the allocation of resources in the North is constant across sectors the price of the newest product must be constant, i.e. $r = \rho + g$. In the steady state $n = N_S(t)/N(t)$ must be constant too, so that the price of any product j evolves over time as follows $\dot{z}_i(j,t)/z_i(j,t) = r - \rho - (1-\gamma)g$ for $i \in \{N,S\}$. Hence, using $r = \rho + g$ yields $\dot{z}_i(j,t)/z_i(j,t) = \gamma g$. Note that the firm selling the newest product must be indifferent in equilibrium whether to export or not, i.e. $[z_N(N_S(t),t) - 1](1-\beta) = [z_S(N_S(t),t) - 1]$, where $z_N(N_S(t),t) = n^{-\gamma} z_N(N(t),t)$.

exceed the profits from exporting, and vice versa. In other words, below the threshold value $Q(N_S(t), t)$ the price effect dominates the market size effect, and vice versa.

Hence, this model would imply that products go through the following cycle in steady state. A new product introduced by a Northern firm is first sold at high prices z_N only in the domestic market since this firm has a relatively low productivity level at which the price effect dominates the market size effect. The Northern firm finds it optimal to lower the price to z_S and start exporting its product after Δ periods (still determined as before) since incomes in the South grow and the Northern firm becomes more productive. At time $\widetilde{T} > t + \Delta$ the patent of the product expires, and it is imitated by a Southern firm. Price competition implies that the limit price drops to marginal costs of Northern firms, and the Northern firm exits the market. From then onwards the product is imported by the North from the South.

7 Conclusion

Vernon's (1966) celebrated product cycle theory hypothesizes that new products go through the following stages. In the first stage, new products are developed and introduced in high-income countries. Later in the cycle, incomes in the poorer countries have grown sufficiently such that demand for these products appears there. Thus, products that were only consumed in high-income countries before are now exported. In the third stage, production moves from high-income countries to low-income countries because they have learned the technology to produce these goods and are able to produce them at lower costs.

In this paper, we provide suggestive evidence for the different stages of the product cycle hypothesis. We show that 6 major consumer durables appear to have gone (or still go) through a "typical" product cycle. In particular, new products are not introduced simultaneously across countries and the lag in introduction depends negatively on relative GDP per capita, i.e. relative to the first country where a product is introduced. In other words, new products are introduced in affluent countries before they are introduced in less prosperous countries.

The chapter contributes to the literature by building a dynamic general-equilibrium model that is able to generate the three stages of the product cycle described by Vernon (1966). In this model, a wealthy North develops new products, which a poor South randomly attempts to copy. Besides technology, the incentives to innovate and imitate are determined by the distribution of income across regions. In other words, the demand side is an important determinant of the product cycle stages. Aside from analyzing changes in Southern labor productivity, we elaborate the effects of a redistribution of income between North and South such that inequality across regions decreases. We show that a decrease in inequality across regions leads to a decline in the innovation rate and hence a slowdown of imitation activity in the South (for a given share of the South in total production). Since Southern households are wealthier after the redistribution of income, they can afford to purchase a higher share of goods available in the world market (in particular, they can afford more newer goods produced in the North). Since Southern households are wealthier (even though their incomes grow at a lower rate), firms in the North want to export their products sooner. Therefore, the first stage of the cycle becomes shorter. At the same time the average duration new products are manufactured in the North increases because imitation activity in the South has slowed down. Firms in the South master the technology to produce a good later so that on average it takes longer for the production to move there (because of the cost advantage). Hence, the second stage of the product cycle where new goods are exported by the North to the South becomes longer. Therefore, the third stage of the cycle where the products are imitated and exported to the North becomes shorter.

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A Appendix

A.1 Household problem

Households maximize logarithmic intertemporal utility, where consumption $c(j,t)$ is its control, and asset holdings $a(t)$ its (endogenous) state variable

$$\max_{\{c(j,t)\}_{t=0}^{\infty}} U(0) = \int_0^{\infty} \exp(-\rho t) \log u\left(\{c(j,t)\}_{j=0}^{N(t)}\right) dt$$

where $\rho > 0$ denotes the time preference rate, subject to the non-negativity constraint $c(j,t) \geq 0$, and the flow budget constraint

$$\dot{a}(t) = r(t)a(t) + w(t) + T(t) - e(t)$$

with $a(0) \geq 0$, $c(j,t) \in \{0,1\}$, and $e(t) = \int_0^{N(t)} p(j,t)c(j,t)dj$. Furthermore, households face a no-Ponzi game condition of the following form

$$\lim_{t\to\infty} \exp\left(-\int_0^t r(s)ds\right) a(t) = 0$$

where $r(t)$ denotes the risk-free interest rate. Households take the time paths of the interest rate, the wage rate, prices for all goods j, as well as the set of differentiated products in the economy $\{r(t), w(t), p(j,t), N(t)\}_{t=0}^{\infty}$ as given. The current value Hamiltonian is given by

$$H(t, c(j), a, \lambda, \mu) = \log u(\cdot) + \lambda(t)\left[r(t)a(t) + w(t) + T(t) - e(t)\right] + \sum_j^{\infty} \xi(j,t)c(j,t)$$

where $\lambda(t)$ denotes the costate variable on the flow budget constraint and $\xi(j,t)$ the one on the non-negativity constraints. The maximum principle conditions are

$$\max_{\{c(j,t)\}_{j=0}^{N(t)}} H(t, c(j), a, \lambda, \mu) \quad \text{for all } t \in [0,\infty], j \in [0, N(t)]:$$

$$\begin{aligned} u(\cdot)^{-1} - \lambda(t)p(j,t) &= 0, \quad c(j,t) = 1 \\ u(\cdot)^{-1} - \lambda(t)p(j,t) &\leq 0, \quad c(j,t) = 0 \end{aligned}$$

$$\begin{aligned} \lambda(t)r(t) &= -\dot{\lambda}(t) + \rho\lambda(t) \\ \dot{a}(t) &= r(t)a(t) + w(t) + T(t) - e(t) \\ \lim_{t\to\infty} \exp(-\rho t)\lambda(t)a(t) &= 0 \end{aligned}$$

A.2 Derivation of transitional dynamics

Using the resource constraint of the South, the relationship between g and g^S, the resource constraint of the North to substitute for g, and the balance of payments to substitute for n (assuming that it is balanced period by period), we obtain the $\dot{m}$ - schedule

$$\frac{\dot{m}}{m}=\left(\frac{1}{F^N/b^N L}\right)\left\{\frac{\lambda\beta z_N}{\beta+(1-\beta)z_N}-\left[(1-\beta)\left(\frac{1}{b^N}-1\right)+m-F\left(\frac{\beta/b^S}{m}-1\right)\right]\right\}$$

where $\lambda=\lambda_N/\lambda_S$ which is constant and equal to its steady state value, and $F=\frac{F^N/b^N L}{F^S/b^S L}$. The $\dot{m}=0$ locus is determined by

$$\frac{\beta\lambda z_N}{\beta+(1-\beta)z_N}=(1-\beta)\left(\frac{1}{b^N}-1\right)+m-F\left(\frac{\beta/b^S}{m}-1\right).$$

It is straightforward to show that $dz_N/dm>0$, $z_N(m)\to-\infty$ as $m\to 0$, and $z_N(m)$ equals a positive constant larger than one as $m\to\beta/b^S$ if and only if $(1-\beta)\left(1/b^N-1\right)+\beta/b^S>\lambda$ (this simply requires inequality between North and South to be sufficiently high). Thus, the $\dot{m}=0$ locus is increasing in the (z_N,m)-space. We deduce the following dynamics for m:

$$\frac{\dot{m}}{m}\begin{cases}>0, & z_N<z_N^*\\ =0, & z_N=z_N^*\\ <0, & z_N>z_N^*\end{cases}$$

where z_N^* denotes the steady state value.

The $\dot{z}_N$ - schedule is obtained by using the balance of payments, the Northern and Southern resource constraints, the definition of the hazard rate, the Euler equation, and the North's zero-profit condition

$$\begin{aligned}\frac{\dot{z}_N}{z_N} &= \left(\frac{1}{F^N/b^N L}\right)\left\{(z_N-1)(1-\beta)+\frac{\lambda\beta z_N}{\beta+(1-\beta)z_N}\right.\\ &- \left.\left[(1-\beta)\left(\frac{1}{b^N}-1\right)+m+\frac{\rho F^N}{b^N L}+F\left(\frac{m}{1-m}\right)\left(\frac{\beta/b^S}{m}-1\right)\right]\right\}.\end{aligned}$$

The $\dot{z}_N=0$ locus is determined by

$$\begin{aligned}(1-\beta)(z_N-1)+\frac{\beta\lambda z_N}{\beta+(1-\beta)z_N} &= (1-\beta)\left(\frac{1}{b^N}-1\right)\\ &+ m+\frac{\rho F^N}{b^N L}+F\left(\frac{m}{1-m}\right)\left(\frac{\beta/b^S}{m}-1\right).\end{aligned}$$

The slope of the $\dot{z}_N=0$ locus is given by

$$\frac{dz_N}{dm}=\frac{[\beta+(1-\beta)z_N]^2}{\beta^2\lambda+(1-\beta)[\beta+(1-\beta)z_N]^2}\left[1-\frac{F}{(1-m)^2}(1-\beta/b^S)\right].$$

We define $\widetilde{m} \equiv 1 - \sqrt{F(1-\beta/b^S)} > 0$ with $\beta/b^S < 1$, and where $\widetilde{m} > 0$ requires that $(1-\beta/b^S)^{-1} > F$, which holds e.g. in the case of identical technology, i.e. $F = 1$. It follows that $dz_N/dm > 0$ if $m < \widetilde{m}$, and vice versa. In other words, the $\dot{z}_N = 0$ locus is decreasing for $m \in (\widetilde{m}, \beta/b^S)$, and increasing for $m \in (0, \widetilde{m})$ in the (z_N, m)-space. We note that $z_N(m) \to -\infty$ as $m \to 1$ and $z_N(m)$ converges to a constant as $m \to 0$. Eventually, it follows that the dynamics for z_N are given by

$$\frac{\dot{z}_N}{z_N}\begin{cases} > 0, & z_N > z_N^* \\ = 0, & z_N = z_N^* \\ < 0, & z_N < z_N^*. \end{cases}$$

Hence, we have a system of two differential equations in m (state variable) and z_N (choice variable), whose solution is saddle-path stable. Figure 8 below shows the phase diagram. We see that if m is below (above) its steady state value m^* it converges monotonically towards the steady state along the saddle-path.

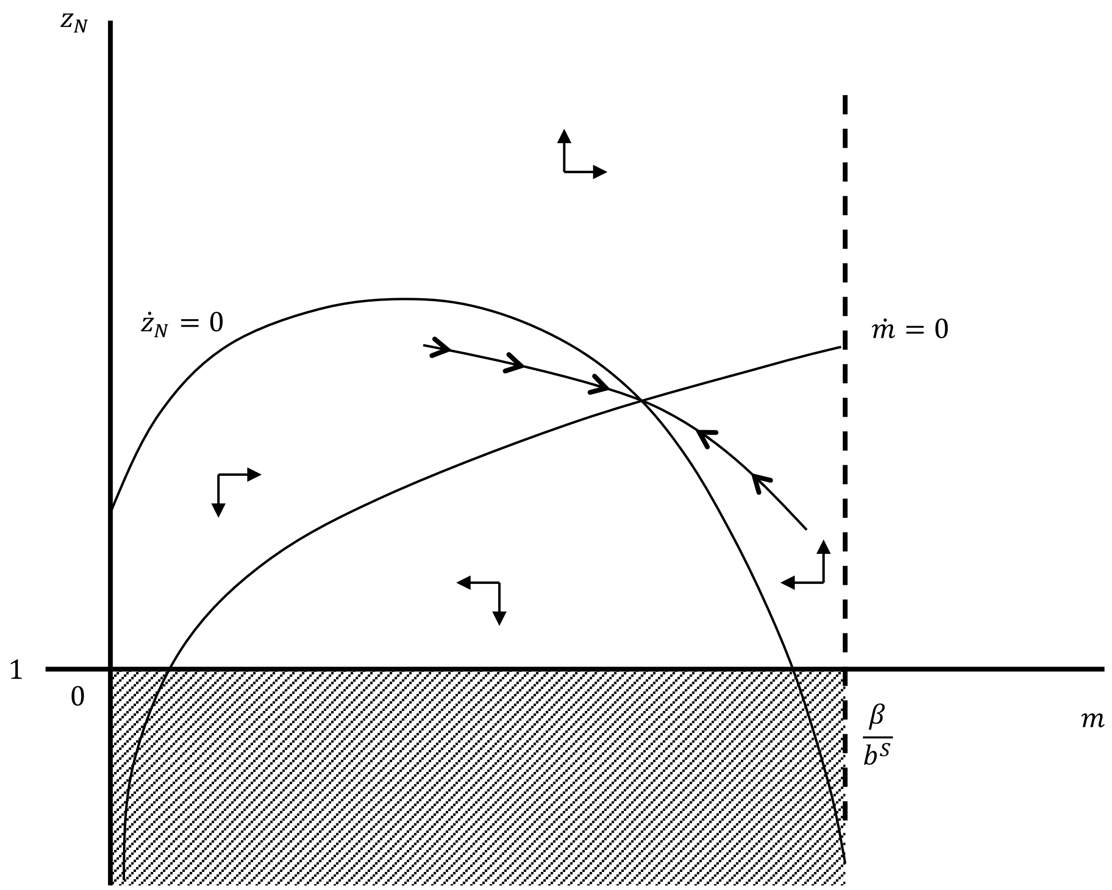

Figure 8: Phase diagram

A.3 Budget constraints

The intertemporal budget constraint of households in the North is in the steady state given by

$$N(t)\left\{m+(n-m)\left[\beta+(1-\beta)\,z_N\right]+(1-n)\,z_N\right\}=(r-g)a_N(t)+w^N(t)-T_N(t)$$

where $y_N(t)=a_N(t)+w^N(t)/(r-g)-T_N(t)/(r-g)$ denotes the lifetime income of a Northern household. We observe that Northern households save only out of their capital income (note that $r-g=\rho$), and consume all their labor income (and possible transfer income). In other words, the marginal propensity to consume out of labor and transfer income is one. Similarly, in the steady state the intertemporal budget constraint of households in the South becomes

$$N(t)\left\{m+(n-m)\left[\beta+(1-\beta)\,z_N\right]\right\}=(r-g)a_S(t)+w^S(t)+T_S(t)$$

where $y_S(t)=a_S(t)+w^S(t)/(r-g)+T_S(t)/(r-g)$ denotes the lifetime income of a household in the South. Similarly to Northern households, Southern households save only out of capital income and consume all labor income. Hence, relative lifetime incomes per capita in the steady state are (endogenously) determined by

$$\frac{y_S(t)}{y_N(t)}=\frac{\rho a_S(t)+w^S(t)+T_S(t)}{\rho a_N(t)+w^N(t)-T_N(t)}.$$

Note that in the simulations we measure inequality with the Gini coefficient. If the South's income share equals its population share the Lorenz curve lies on the 45 degree line of perfect equality, and the Gini coefficient is zero.

A.4 Balance of Payments

The intertemporal budget constraint of households in the South, the resource constraint in the South, and the zero-profit condition in the South imply the balance of payments as stated in the text. Due to Walras' law, the intertemporal budget constraint of the North is redundant. We drop the time index t where no confusion arises. The balance of payments in present value form at $t=0$ is given by

$$\begin{aligned} 0 &= \left\{\int_0^\infty \left[(1-\beta)LN^S\omega^N b^N-\beta L\left(N_S-N^S\right)z_S\right]\exp\left(-\int_0^t r(s)ds\right)dt\right\}\\ &+ \int_0^\infty \beta L T_S\exp\left(-\int_0^t r(s)ds\right)dt\\ &+ \left\{\beta L a_S(0)-\int_0^\infty N^S\left[\pi^S-g^S v^S\right]\exp\left(-\int_0^t r(s)ds\right)dt\right\}\end{aligned}$$

where we used $\beta LN=\dot{N}^S F^S+N^S b^S L$ from the resource constraint, $v^S=\omega^S F^S$ from the zero-profit condition, and a no-Ponzi game condition. The first two lines denote the current account, which consists of the trade balance and net transfer payments. The third line denotes net foreign asset holdings. In the steady state, we have that r and π^S are constant, N^S grows at a constant rate $g^S = g$, and $v^S = \pi^S/r$. This implies that net foreign assets become $\left\{\beta La_S(0) - N^S(t)\pi^S/r\right\}$. Hence, the balance of payments in the steady state is determined by

$$0 = \left\{N^S(t)(1-\beta)L\omega^N b^N - \left(N_S(t) - N^S(t)\right) z_S(t)\beta L\right\} + \beta LT_S(t) + \left\{\beta La_S(t) - N^S(t)\pi^S/r\right\}$$

which holds for all t in steady state, in particular at $t = 0$. Hence, it becomes obvious that if we assume initial wealth at time $t = 0$ of households in the South $\beta La_S(t)$ to be exactly equal to the present discounted value of aggregate firm profits in the South $N^S(t)v^S(t)$, net foreign assets will remain zero in steady state. We see that if Southern households would inherit sufficiently large asset holdings they could run a permanent trade deficit (even in the absence of transfers from North).

A.5 Simulations

We choose the following parameter configuration for our baseline simulation: $L = 1$, $F^N = F^S = 5$, $b^N = b^S = 0.75$, $\beta = 0.5$, $\rho = 0.04$, and $T = 0$.

A.5.1 Increase in Southern labor productivity

Figures 9-11 show the comparative statics results of a change in labor productivity in production in the South.

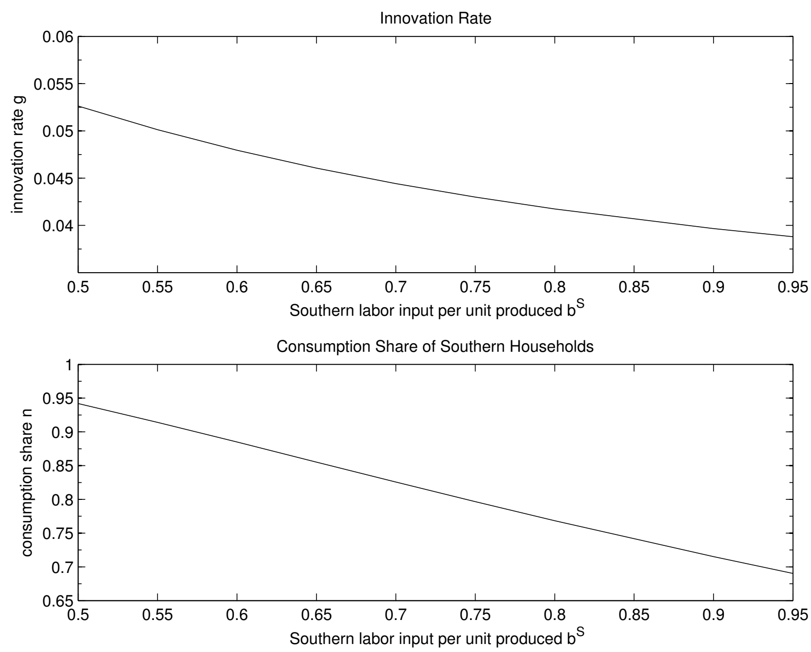

Figure 9: Effect on innovation rate and consumption share of the South

Figures 12-14 show the comparative statics results of a change in labor productivity in R&D in the South.

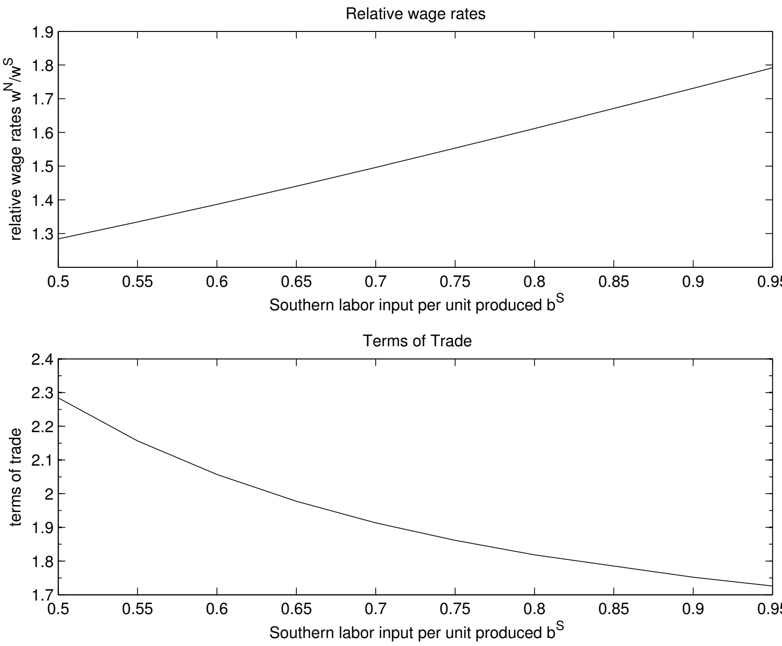

Figure 10: Effect on relative wages

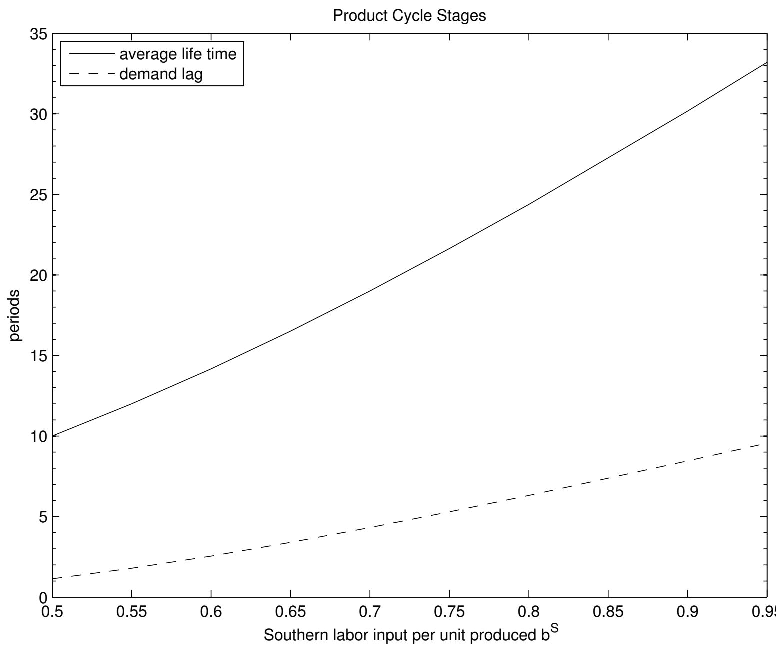

Figure 11: Effect on the stages of the product cycle

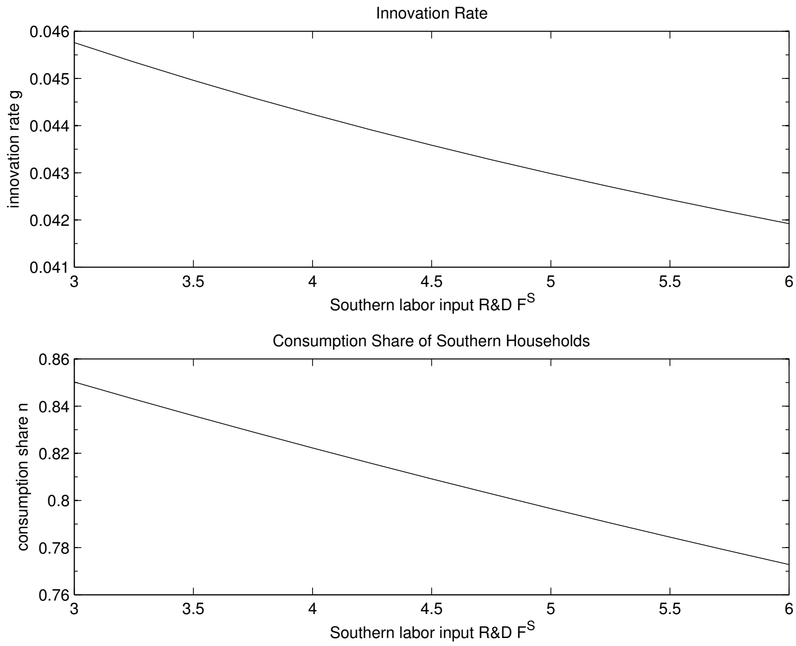

Figure 12: Effect on innovation rate and consumption share of the South

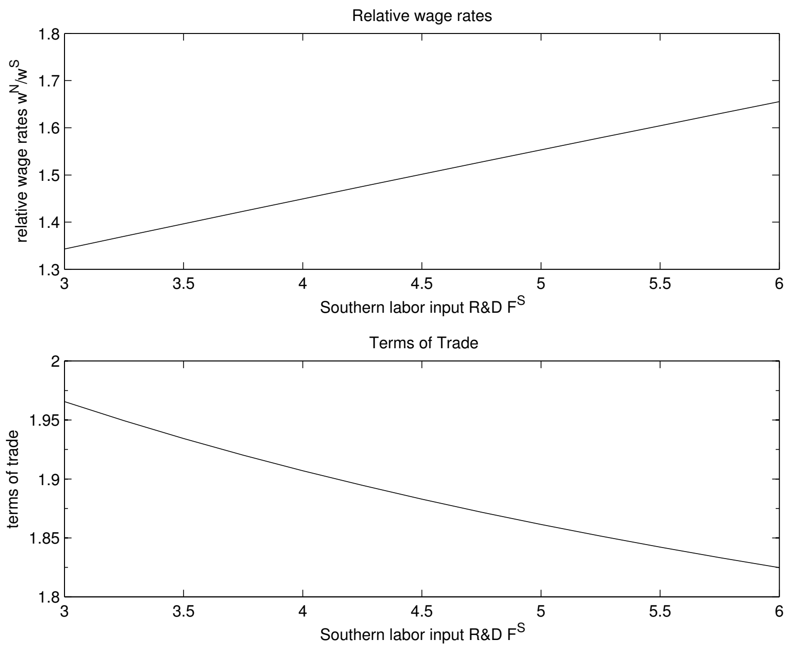

Figure 13: Effect on relative wages

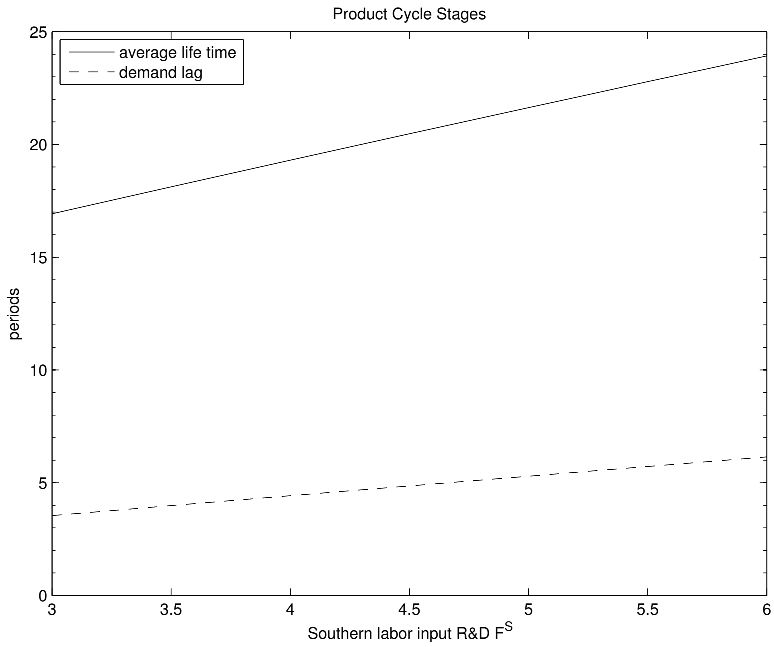

Figure 14: Effect on the stages of the product cycle

A.5.2 Changes in inequality across regions

Figures 15-17 depict the effects of an increase in inequality across regions due to a regressive transfer, i.e. a transfer from poor South to rich North.

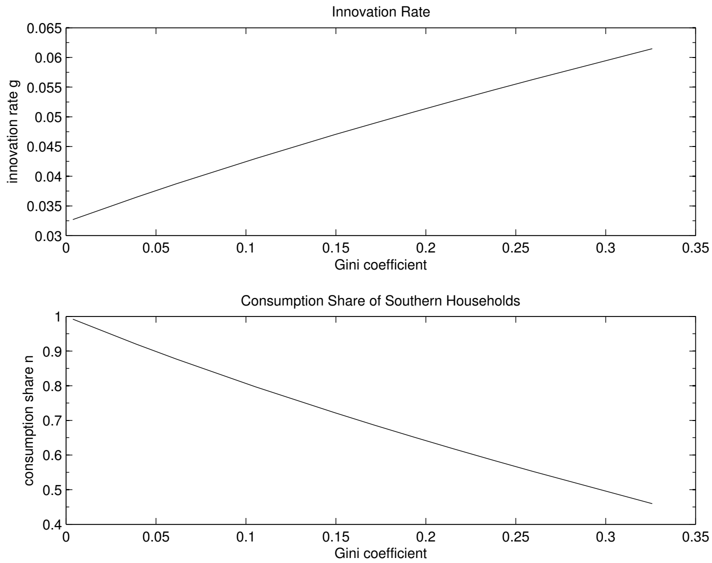

Figure 15: Effect on innovation rate and consumption share of the South

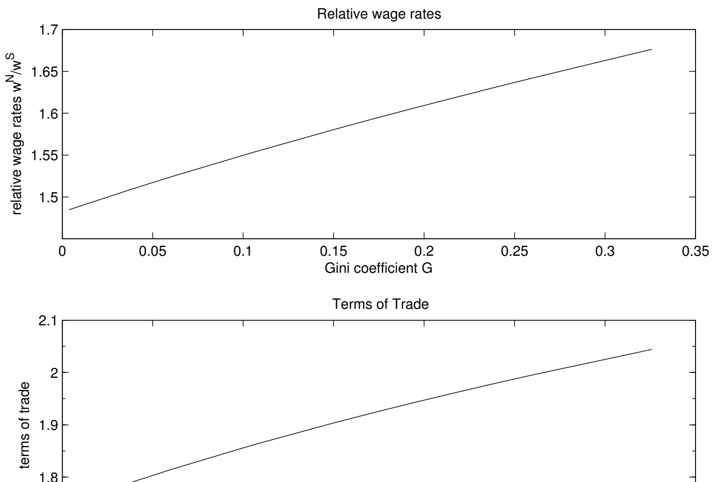

Figure 16: Effect on relative wage rate and terms of trade

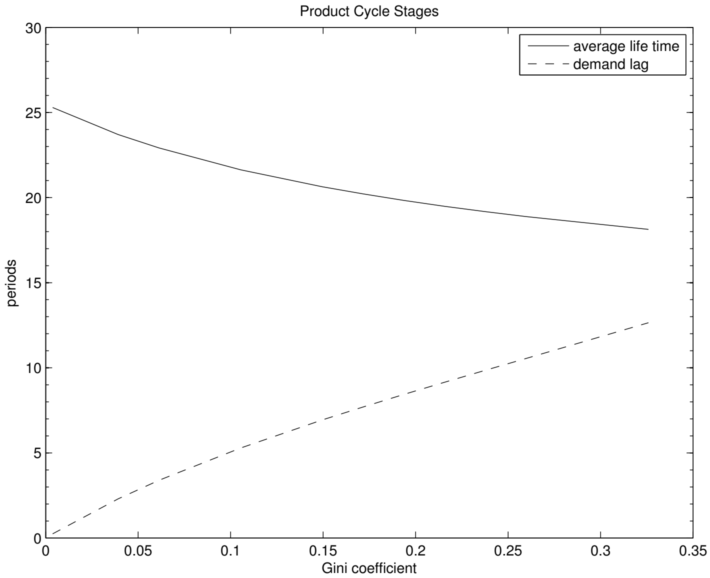

Figure 17: Effect on stages of the product cycle

A.5.3 Changes in population share

Figures 18-20 show the effects of an increase in Southern population share.

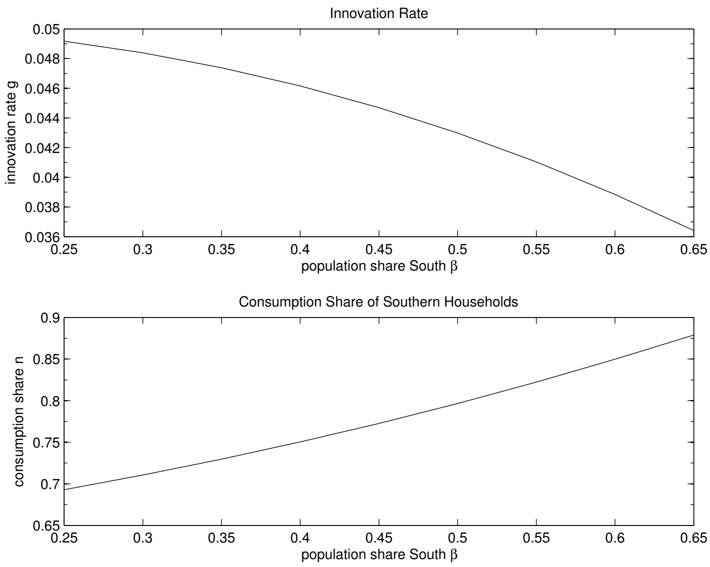

Figure 18: Effect on innovation rate and consumption share of the South

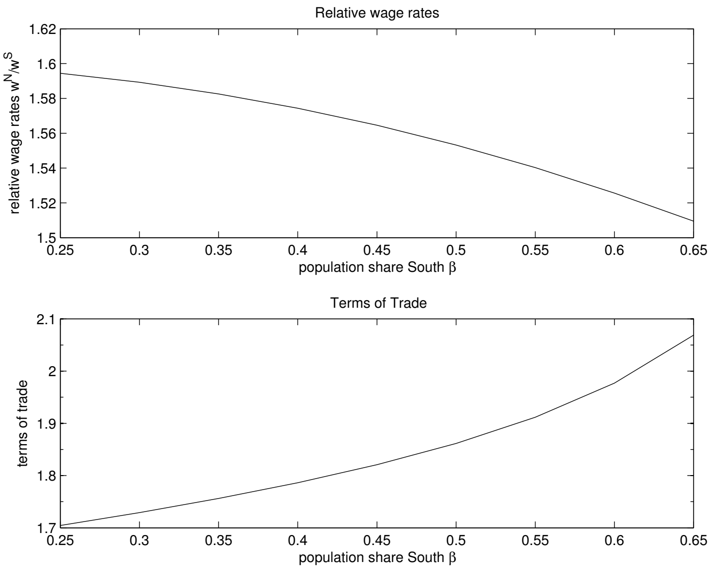

Figure 19: Effect on relative wage rate and terms of trade

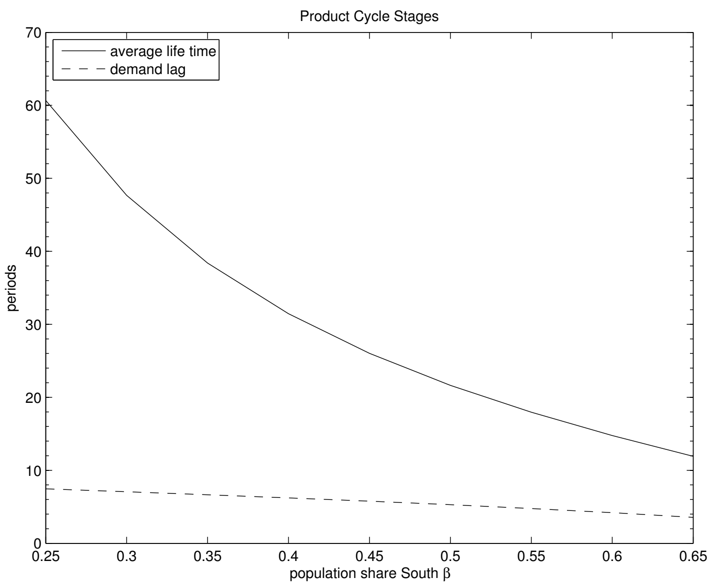

Figure 20: Effect on stages of the product cycle