Gender Wage Gaps across Skills and Trade Openness

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Abstract

This paper investigates how international trade impacts the gender wage gap at different points of the skill distribution, by integrating statistical discrimination and job assignment into a model of trade. Workers differ by their skills and their job commitment. Firms decide whether they invest in technology upgrading and which workers they hire. The inability to observe workers' job commitment induces employers to discriminate against women who have on average lower commitment. When skills and technology upgrading are complements, technological change is skill-biased. Technological change is gender-biased when job commitment and technology upgrading are complements. The widening of the gender wage gap occurs all along the skill distribution if skills and job commitment are complement as well. As a result of trade integration, the gender wage gap widens among skilled workers and decreases among unskilled workers. The theory can explain the increase in both within-group and between-group wage inequality following the adoption of new technologies, here induced by trade liberalization.

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1 Introduction

The effect of international trade on gender wage inequality depends on workers' skill levels. Recent empirical evidence has found that international trade contributes to a reduction in both the gender employment gap and the gender wage gap only in low-skill jobs (see Ozler (2000), Ederington et al. (2009), Fafchamps (2009), Fafchamps et al. (2009), Juhn et al. (in press) which we discuss below). Despite a strong empirical interest, the channels through which trade can have such an effect have not been formalized. This paper develops a model where international trade affects the gender wage gap differently at different points of the skill distribution. It shows that under general assumptions the gender wage gap widens at the top of the skill distribution following trade integration, while the gap is reduced at the bottom of the skill distribution.

The model features two groups of workers, men and women, whose characteristics vary in two dimensions, skills and job commitment which corresponds to workers' availability and willingness to maintain a intense and continuous working life. We assume that men and women have the same skill distribution that is perfectly observable. Commitment, however, is unobservable by the employer which generates statistical discrimination. In particular, employers discriminate against women because they have *on average* a lower labour market commitment[1]. As employers pay worker-specific wages, a woman is hired at a lower wage compared to a man with identical skills to compensate for the loss in case of lower commitment. This setting has been inspired by Lazear and Rosen's (1990) dynamic model of statistical discrimination where women face a lower promotion probability along the job ladder of a given firm because of learning in top jobs and a lower propensity for women to remain on the job. In our model, workers are sorted across firms rather than types of jobs; moreover, the matching is not determined by learning but by technology differences that result from firms' endogenous investment decisions. Firms make a simultaneous decision on technology investment and hiring as in Yeaple (2005). They calculate

[1] Gender differences in labour market commitment stem from work interruptions typically due to maternity and child rearing. They also result from the impossibility to work overtime as well as lower energy on the job due to greater time spent on housework and childcare. See Gauthier et al. (2002) among others for empirical evidence on gender differences in time allocation following child birth. Budig and England (2001) shows that the existing gender differences in the allocation of time between paid work and housework affect mothers over and above the reduction in real labour market experience. Changes in time spent on the job cannot explain the lower wages received, even controlling for fine characteristics of the job. Boye (2010) shows that total time spent on both paid work and housework is higher for women and that the resulting gender difference in leisure time causes gender differences in psychological distress. This might in turn affect energy on the job.

the expected workers' productivity conditional on the technology, their observation of the workers' skill levels and their expectation about the workers' degree of commitment. We assume that high-skill workers are more productive than low-skill workers (absolute advantage) but they are even more so in firms with sophisticated technology (comparative advantage). Similarly, strongly committed workers are always more productive than less-committed workers (absolute advantage) but they are more so in firms with sophisticated technology with sophisticated technology (comparative advantage).

The model gives the sorting of men and women across firms and the wage gap distribution in a closed economy setting. The most skilled workers are employed in high-technology firms where the reward to skills and expected commitment are higher. Yet, the skill threshold for women to be hired by a high-technology firm is higher than men to compensate for the uncertainty on their level of commitment. The higher female skill-threshold generates higher gender wage gaps in the upper part than in the lower part of the distribution.

We then shed light on the implications of international trade for the gender wage gap. We consider a monopolistic competition framework (Krugman, 1980) where two identical countries trade different varieties of a differentiated good. Trade is costly, generating both fixed and variable costs, so that only the most productive firms, i.e. those using the "high" technology, engage in exporting along the lines of Melitz (2003). A reduction in trade costs spurs firms to adopt "high" technology to benefit from larger markets. The new investments in technology increase the demand for skilled and strongly committed workers. Because demand for commitment is now higher, trade liberalization increases the gender wage gap in the upper tail of the distribution. Because of general equilibrium effects, the non-traded good sector expands which generates a reduction in the gender wage gap at the bottom of the distribution. However, the effect on the mean gender wage gap is ambiguous.

The model offers an explanation to several empirical findings. First, empirical studies that analyze the distribution of the gender wage gap show the gap is higher in the upper part of the wage distribution in both developed and developing countries[2]. This paper provides a simple explanation for that finding based on statistical discrimination and strong but general assumptions on the production function. The empirical literature highlights three phenomena that contribute to the increase in the gender wage gap along the wage distribution. Firstly, women are less often promoted to top jobs, the glass-ceiling effect[3].

[2]Recent examples in this literature include Nopo et al. (2010) for Latin American countries, de la Rica et al. (2008) for Spain and Albrecht et al. (2009) for the Netherlands.

[3]Gobillon et al. (2012) show that, in France, women have lower probability to be hired at all ranks of

Secondly, within jobs, women are paid less than their male counterparts, especially in high-responsibility high-wage jobs[4]. Thirdly, the sorting of women into low-wage firms, the glass-door effect, also contributes to higher adjusted wage gaps at the top of the wage distribution[5]. The model presented in this paper does not feature jobs specifically. It generates a gender wage gap conditional on skills that increases along the distribution for two reasons: the sorting of workers across heterogeneous firms differs for men and women, and within firms, women receive lower-wages.

A second empirical fact with which the model is consistent concerns the effect of trade openness on wage inequality. A large body of empirical evidence shows that exporters differ from non-exporters. They are bigger, more productive, more skill-intensive and pay higher wages, the exporter wage premium[6]. These differences help understand wage inequality. The skill premium is found to increase with trade exposure in both developed and developing countries[7]. Looking at both men and women, Klein (2010) finds that the export-wage premium increases with the skill level within gender groups. The model presented in this paper provides a mechanism that explains the increase in skill rewards with trade integration for both men and women.

The third body of empirical literature closely related to this paper highlights that international trade impacts the relative demand for female labour differently among skilled and unskilled workers. Following trade integration and the expansion of exporting sectors,

positions compared to men and that the gender gap in the probability to access high-wage positions is substantially larger than gap in the probability to enter low-wage positions.

[4]See all studies that control for precise occupation cells such as Jurajda and Harmgart (2007) for Germany, Oostendorp (2004) for a cross-country analysis.

[5]Meyersson Milgrom et al. (2001) show that segregation into low-wage occupations and low-wage establishments explains part of the wage gap in Sweden while Amuedo-Dorantes and De la Rica (2006) and Woodcock (2008) come to similar conclusions for Spain and the United-States. Javdani (2012) applies the methodology of Pendakur and Woodcock (2010) on Canadian data to decompose gender wage gaps along the distribution into a within-firm glass-ceiling effect and a glass-door effect i.e. the under-representation of women in high-wage firms.

[6]See the influential paper by Bernard and Jensen (1997) for evidence on the exporter wage premium and skill-biased shift in labour demand in U.S. manufacturing during the 1980s. Bustos (2011) gives evidence based on Argentinian firm data that corroborates her theoretical predictions: by increasing potential export revenues and making costly investment worthy, reduction in trade costs favours investment in new technologies which increases the demand for skilled workers. Helpman et al. (2012) use firm-level data for Brazil and show that there is significant wage inequality among workers with similar observable characteristics, holding similar jobs in the same sector. This wage differences are explained in part by the trade orientation of the firm.

[7]Empirical assessments of the impact of trade openness on wage inequality are for instance, Bernard and Jensen (1997) on the US, Pavcnik et al. (2004) on Brazil, Brambilla et al. (2012) on several Latin-American countries. See also Goldberg and Pavcnik (2007) for a literature review.

women benefit from employment gains but mostly in unskilled occupations. There is an increase in female relative wages in unskilled occupations, but not in skilled occupations (Oostendorp, 2004, Joekes, 1995, Nicita, 2008, Paul-Majumder and Begum, 2000). Other papers investigate firm-level changes following trade liberalization and confirm the pattern observed at the sectoral level. Exporting firms, that invest more in capital-intensive technologies, employ a lower share of skilled women (Ozler, 2000, Ederington et al., 2009). The exporter wage premium is lower for women (Klein et al., 2010, Fafchamps, 2009). Juhn et al. (in press) find that following trade liberalization in Mexico, firms have adopted new technologies and the gender wage gap has been reduced in blue-collar occupations but not in white-collar occupations. Our model is consistent with the fact that international trade affects differently men and women depending on their skill levels.

This paper is related to several strands of literature. First, it contributes to a large body of work dealing with how trade openness, associated with firm heterogeneity, influences wage inequality. We can distinguish two groups of theoretical models where international trade contributes to wage inequality. One group of papers focuses on the impact of trade on the skill-premium through the investment in new technologies[8]. Neary (2002) develops an oligopolistic model where the threat of foreign product entry spurs firms to invest in new technologies in order to reduce production costs. Since innovation is skill intensive, trade openness increases the relative demand for skilled workers thereby increasing the skill-premium. Yeaple (2005), Bustos (2011) and Sampson (in press) develop models with monopolistic competition. They also show that trade liberalization contributes to rising wage inequality between high-skill and lower-skill workers through differences in firm technology. The present paper follows this approach and adds another dimension of worker heterogeneity that is unobservable.

Another group of papers investigates the effect of international trade on within-group inequality. In Helpman et al. (2010) workers differ in their unobservable abilities. Following trade integration, the most productive firms start exporting. Exporters are able to hire the best workers to whom they pay higher wages because they invest in better screening technologies. In Egger and Kreickemeier (2009, 2012), similar workers demand fair wages to their employers. The exporter wage premium is explained by exporters' higher profits. Trade liberalization increases profit differences between exporters and non-exporters and thus increases wage inequality among similar individuals at the sectoral level. Those models

[8]The mechanism of skill-biased technological change has received recent empirical support (Leonardi, 2007, Abowd et al., 2007, Bustos, 2011).

imply a uniform wage within firms. The model presented in this paper differs in that it features not only between-firm but also within-firm wage dispersion[9]. This setting enables us to draw a link between the rise in the skill-premium and the rise in the male wage premium.

This work is also related to recent trade models using labour assignment to provide insights about the impact of globalization on labour markets (see for example Yeaple, 2005; Ohnsorge and Trefler, 2007; Costinot and Vogel, 2010). The novelty of the present paper is to introduce statistical discrimination. We are then able to show that the wage dispersion induced by trade occurs both within gender groups, along the skill distribution, and between gender groups, generating changes in the gender wage gap adjusted for observable skills. In doing so, the paper contributes to the literature that investigate the linkages between the overall wage structure and the gender wage gaps[10]. This paper shows that trade openness affects simultaneously the overall wage structure through an increase in the skill premium and the gender wage gap.

Finally, this paper belongs to the small literature that studies the effect of trade openness on the adjusted gender wage gap. One set of of papers focuses on the competition effect of international trade in a taste-based discrimination framework. According to Becker's theory, prejudiced employers can engage in discrimination only if they earn enough profits and an increase in competition pressure reduces the gender wage gap. Artecona and Cunningham (2002) and Black and Brainerd (2004) provide empirical appraisals of the foreign competition effect on the average gender wage gap at the sectoral level. They show that higher import penetration reduces the gender wage gap at the sectoral level, in accordance with the prediction of Becker's theory. Ederington et al. (2009) develop a model of taste-based discrimination where an increase in import penetration raises the female share of employment within firms. In this model, the gender wage gap is exogenously fixed and only an exogenous increase in imports is considered. Ben Yahmed (2012) presents a trade model with taste-based discrimination and oligopolistic competition to draw attention to

[9]Within-firm dispersion is consistent with empirical evidence. In a study covering the United States from the 1960s to the late 1980s, Davis et al. (1991) investigate the rise in wage inequality within groups defined by their education, experience and gender. They show that, first, within-firm wage dispersion accounts for 35 to 40 percent of the wage dispersion in each group. Second, within-firm wage inequality is stronger among non-production workers than among production workers. Similarly, Iranzo et al. (2008) explore firm-level data for Italy and find considerable within-firm variation in the individual worker's effect of the wage equation.

[10]The wage structure is the values the labour market attaches to skills and other productive characteristics. For studies of the US labour market, see Blau and Kahn (1992, 1994, 2003)

both pro and anti-competition effects of international trade. The predictions of the model are confirmed by the empirical analysis of trade liberalization in Uruguay. When foreign firms benefit from an easier access to the Uruguayan market, the gender wage gap shrinks in sectors that were concentrated prior liberalization. This result is a validation of the prediction: “competition eliminates discrimination”. When trade liberalization facilitates the access of Uruguayan firms to foreign markets, however, the gender wage gap increases, only in sectors that were concentrated prior liberalization. This is a validation of the prediction: “profit-enhancing opportunities reinforce discrimination”.

The present study investigates a different mechanism. It focuses on how biased technological change induced by trade openness impacts men and women at different skill levels. Closely related to this paper, a few empirical studies have pointed out the heterogeneous effects of trade on the relative position of women depending on the skill intensity of the occupation. Oostendorp (2004) looks at the impact of sectoral trade shares on the gender wage gap within narrowly defined occupations for more than 80 countries. Exploiting the changes in trade intensity within a given occupation-sector-country cell over time, he finds that an increase in the sectoral trade share narrows the occupational wage gap for unskilled labour only and the occupational gender wage gap is lower in unskilled occupations compared to skilled occupations, the difference being bigger in developing countries. Joekes (1995) highlights that the expansion of the export manufacturing sectors in Morocco and Bangladesh created new sources of employment for women but in unskilled occupations, mainly in the textile and clothing industries. This result is confirmed by Fafchamps (2009) who finds that Moroccan exporters, concentrated in light industries such as textile and apparel where the workforce is mainly unskilled, employ significantly more women and pay them on average lower wages controlling for education.

Ozler (2000) uses plant-level data from Turkey and shows that trade liberalization in the 1980s led to employment gains for women relative to men in the manufacturing sector. Women, however, continued to be employed in low-skill and low-pay jobs within plants. Furthermore, among plants with a high female share, as well as among large establishments, investments in machinery and equipment brought about a decline in the female share of employment. This finding supports the argument that employment gains for women following trade liberalization might be reversed as a consequence of technological upgrading. Ederington et al. (2009) use plant-level data for Columbia to study changes in employment within firms over the period 1985-1991. They show that plants that have the highest female share of employment are less intensive in capital and pay lower wages compared to the industry average. As for the role of openness, the share of exports in the total production of the plant is positively associated with the female share the plant which implies that openness can be good for women's employment opportunities. In the same direction, a reduction in tariffs has a positive effect on female share of employment at the plant level. However, when they distinguish between the share of females among skilled workers and among unskilled workers, they reveal that those employment gains benefit mostly unskilled women while trade openness has been detrimental to skilled women. Indeed, an increase in the export share of the plant reduces the female share of skilled workers. As for a reduction in tariffs, it increases the demand for unskilled female labour but not the demand for skilled women among exporting plants, while it increases the demand for both types of female labour among non-exporting plants.

Two other papers investigate the impact of trade on gender inequality at the firm level. Klein et al. (2010) investigate how the trade orientation of firms impact wage inequality between and within male and female groups of workers for the German manufacturing industries. They find that women face a wage penalty compared to men and that this penalty increases with the skill level. The wage gap is smaller in exporting plants for low-skilled individuals but not for college educated individuals. They additionally find that the export-wage premium increases with the skill level within groups. To sum up, trade openness contributes to wage dispersion among men and among women, but the effect on the wage gap depends on the skill level of workers. Juhn et al. (in press) provide an explanation for narrowing gender wage gaps among blue-collars but not among white-collars, a pattern they observe in Mexico in the aftermath of NAFTA. In their model, all productive characteristics are observable; the gender difference is biological: physical strength. They assume that the new technology reduces the need for physical strength in blue collar occupations so that the relative demand for female labour increases in those occupations. We depart from their setting by proposing a model with worker heterogeneity in two dimensions, an observable characteristic and an unobservable characteristic unequally distributed among men and women. Because workers' productivity depends on the unobservable characteristics, employers engage in statistical discrimination. Because different technologies value differently the observable and the unobservable characteristics, the gender wage gap varies with technology and with skill levels. This paper gives general conditions on the production technology under which we can generate non-monotonic effects of trade on the gender wage gap.

The rest of the paper is organized as follows. Next section describes the setup of the model. Section 3 provides the equilibrium in a closed economy where the distribution the gender wage gap across fits within-country evidence. In section 4, we characterize the open economy equilibrium and derive the implications of international trade and further reductions in trading costs for the distribution of the gender wage gap. The final section concludes.

2 Setup of the model

2.1 Demand

Preferences are identical across all consumers who choose a quantity of a homogeneous good and a quantity of varieties of a differentiated good. The utility function is Cobb Douglas between the differentiated good X and the homogeneous good Y and presents a CES sub-utility over the varieties i of X. This function expresses a love of variety of consumers. Then

$$U = Y^{1-\beta} X^{\beta}$$
$$X = \left(\sum_{i}^{N} x_i^{\alpha} \right)^{\frac{1}{\alpha}}$$

where the elasticity of substitution across varieties of X is given by $\sigma = \frac{1}{1-\alpha}$. The price index of the differentiated good X is : $P_X = \left(\sum_i p_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$. If all prices are equal, the price index is $P_X = pN^{\frac{1}{1-\sigma}}$. It decreases with N the number of varieties produced and the elasticity of substitution σ. Consumers choose the share of their income M they will devote to the differentiated good by maximizing their utility subject to their revenue constraint. The price of the homogeneous good is normalized to one.

$$X = (\beta M)/P_X$$
$$Y = (1-\beta)M \tag{1}$$

Let us denote by $E = \beta M$ is the portion of income spent on the differentiated good. Consumers decide also how much of each variety they consume. As they value diversity, they consume a positive amount of each symmetric variety and spend a larger share of their budget on lower-price varieties:

$$x_i = \frac{E}{P_X}\left(\frac{p_i}{P_X}\right)^{-\sigma} \tag{2}$$

The demand for variety i takes into account the average price of good X. The term $\frac{E}{P_X}$ corresponds to the aggregate demand for X while the price differential $\frac{p_i}{P_X}$ models the competition effect between variety i and the other varieties.

2.2 Worker heterogeneity in observable and unobservable characteristics

The workforce is heterogeneous in both skills and job commitment. There is a continuum of skills s distributed among the population according to a distribution function L over the support $[0, \bar{s}]$. $L(s)$ is the inelastic supply of labour with skill no greater than s. We assume that men and women have the same exogenous skill distribution $L_f(s) = L_m(s) = L(s)$ where f and m denote, respectively, female and male. The mass of workers per group is normalized to one. As for the differences in job commitment, let us assume that there are two types of individuals, the highly-committed that spend the maximum time and effort in the firm over the period $e = \bar{e}$ and the low-committed ones for which $e = \underline{e}$ [11]. We simplify the model by assuming that men always exhibit a high level of commitment $Pr_m(e = \bar{e}) = 1$ while women have a probability to favour labour market activity over their domestic activities equal to $Pr_f(e = \bar{e}) = \eta$ with $0 < \eta < 1$ [12]. There is no correlation between s and e which means that the probability of being highly committed to one's job is independent of one's skill level[13].

The skill of a worker can be perfectly observed by the employers. However the level of job commitment is unobservable : employers cannot anticipate the time and energy a particular worker is going to put in the job. Even though employers expect some women to be highly committed, they are unable to know which ones at the time of hiring. As a

[11] In sociology, the preference theory developed in Hakim (2000) argues that differences in women's preferences for combination of domestic activities and paid employment explain differences in labour market attachment among women. She sorts women into three categories: home-centered, adaptative and work-centered. Only women belonging to the last two categories participate to the labour market. We model the difference between these two groups by an exogenous difference in job commitment. Another study that documents the heterogeneity in women's decisions over work and family life balance is Blair-Loy (2003).

[12] This amounts to a normalization of male probability of commitment. We could allow for heterogeneity among men too. The crucial assumption is that men are more likely than women to be work-centered and thus have a higher probability to be highly committed.

[13] Skill investment is exogenous in this model. Our results will hold if we allowed for a correlation between skills and commitment sufficiently low compared to the degree of complementarity in the production function as we will see below.

consequence, employers expect a lower female productivity because of the *average* female labour market attachment. Labour productivity is increasing in both s and e and depends on the technology j in use, as we will see in the next sub-section.

2.3 Production

The productivity of a worker endowed with skills s and a level of commitment e when working with technology j is noted $\varphi_j(s,e)$. Because employers cannot observe e, they form expectations based on observables, i.e. the skill and the sex of the worker. We denote by $\tilde{\varphi}_{jg}(s)$ the *expected* productivity of a worker with skill s from group g as viewed by the employer prior hiring when technology j is used. As men's productivity is perfectly observable, $\tilde{\varphi}_{jm}(s) = \varphi_j(s,\bar{e})$. For women, employers form identical expectations given by: $\tilde{\varphi}_{jf}(s) = E(\varphi_j(s,e)|\eta) = \eta\varphi_j(s,\bar{e}) + (1-\eta)\varphi_j(s,\underline{e})$. Employers anticipate different productivities for a man and a woman endowed with the same skill level and working with the same technology : $\tilde{\varphi}_{jm}(s) > \tilde{\varphi}_{jf}(s) \quad$ for all $j \in \{l,h\}$ and for all $s \in]0;\bar{s}]$.

In sector Y, the homogeneous good sector, firms produce under constant returns to scale and perfect competition using labour only. We assume that labour productivity does not depend on either workers' skills or effort in this sector and we set $\varphi_Y = 1$. We denote by c_Y the unit cost of production equal to the wage per efficiency unit of labour : $c_Y = \frac{w_Y}{\varphi_Y}$. Under perfect competition in both product and labour markets, firms set prices equal to their unit cost of production $p_Y = \frac{w_Y}{\varphi_Y}$. We choose the price of good Y as the numeraire $p_Y = 1$, consequently we have $c_Y = w_Y = \varphi_Y = 1$.

In sector X, the differentiated good sector, firms operate under imperfect competition and increasing returns to scale. We assume that the sector is characterized by horizontal product differentiation and monopolistic competition where N firms produce each a variety of the differentiated product. Firms have to pay a fixed investment cost to produce one variety. This innovation cost F acts as an entry barrier which ensures that each variety is produced by only one firm. As varieties are not perfect substitutes, firms enjoy some market power that enable them to make positive operating profits and pay the fixed cost. After choosing its technology, the firm can produce a variety of good X hiring labour. The following assumptions characterize the technology and the productivity function.

Assumption A1. ***Fixed and variable costs***

Firms can invest in two different technologies indexed by $j = \{l, h\}$. To acquire the high-technology firms bear a higher fixed cost $F_h > F_l$ but benefit from a higher productivity of labour for a given skill level and commitment, $\varphi_h(s, e) > \varphi_l(s, e)$. If a worker has no skill, $s = 0$, his/her productivity is the same in sector X and Y : $\varphi_h(0, e) = \varphi_l(0, e) = 1$.

Firms choose the type of investment they make considering both its cost and the resulting gain in productivity. This specification is consistent with R&D being positively correlated with firm productivity (see Klette and Kortum (2004) for example).

Assumption A2. ***Log-supermodularity in skills and technology***
Skill acts as a strategic complement to technology upgrading:

$$\frac{\varphi_h(s', e)}{\varphi_h(s, e)} > \frac{\varphi_l(s', e)}{\varphi_l(s, e)} \quad \textit{for any } s' > s \quad \forall e$$

The productivity gain derived from hiring a worker with a higher skill is greater under technology h. This means that workers with higher skill levels have a comparative advantage in the sophisticated technology.

Assumption A3. ***Log-supermodularity in commitment and technology***
Job commitment and technology upgrading are complementary:

$$\frac{\varphi_h(s, \bar{e})}{\varphi_h(s, e)} > \frac{\varphi_l(s, \bar{e})}{\varphi_l(s, e)} \qquad \textit{for any } \bar{e} > e \quad \forall s$$

Assumption A3 implies that strongly committed workers have a comparative advantage in the high technology.

Assumption A4. ***Log-supermodularity in skills and commitment***
Job commitment and skills are complementary:

$$\frac{\varphi_j(s', \bar{e})}{\varphi_j(s, \bar{e})} > \frac{\varphi_j(s', e)}{\varphi_j(s, e)} \qquad \textit{for any } \bar{e} > e \quad \textit{and } s' > s \quad \forall j = \{l, h\}$$

A high skill level is more valuable when the worker's job commitment is high.

Assumptions A2 to A4 require the productivity function to be non separable in s, e and j.

3 The closed economy Equilibrium

3.1 Profit maximization under monopolistic competition

Firms operating with the same technology j are symmetric. In particular a worker with a given level of skill s and sex g has the same expected productivity, denoted by $\tilde{\varphi}_{jg}$, in any firm of type j. As a result, the technology $j = \{h, l\}$ specifies all relevant firm's variable. We can thus solve firms' problem in sector X using two representative firms h and l.

Risk-neutral employers hire workers in a perfectly competitive labour market. The wage paid by a firm j to a worker g with skill s is denoted by $w_{jg}(s)$. Total production cost for a firm j can be written as :

$$TC_j = \frac{1}{N_j} \sum_g \left(\int_{s \in S_{jg}} w_{jg}(s) l(s) ds \right)$$

where S_{jg} is the set of skills of workers belonging to group g employed by a firm of type j. N_j is the endogenous number of j-firms. We assume that the fixed component of production uses labour in the same way as the variable component of production. The fixed cost is denominated in units of firm output. In other words, a firm produces consumption goods as well as a specific investment good using the same mix of workers[14].

$$q_j + F_j = \frac{1}{N_j} \sum_g \left(\int_{s \in S_j} \varphi(s) l(s) ds \right)$$

We denote by $\tilde{c}_j$ the *expected* cost per efficiency unit of labour under technology j. It is equal the ratio of worker's wage over worker's expected productivity, $\tilde{c}_j = \frac{w_g(s)}{\tilde{\varphi}_{jg}(s)}$. The expected profit of a firm using technology j can be written:

$$\pi_j = p_j q_j - \tilde{c}_j (q_j + F_j)$$

Firms maximize their expected profits with respect to quantities[15] and take the wage

[14]The alternative option is to denominate the fixed cost in unites of final good instead of firm output. Whether the firms use or not labour in the fixed component of production does not alter the results on wage inequality, cf. Sampson (in press) for a proof in a similar setting. Other papers explicitly model innovation and investment in R&D, which is part of the fixed component of production, as another activity that requires its specific mix of workers, usually using more intensively skilled labour, as in Neary (2002) for example.

[15]Under monopolistic competition without any strategic interactions, competition on prices or on quan-

rate per efficiency unit of labour as given.

$$\pi_j = max_{q_j}\{p_j q_j - \tilde{c}_j(q_j + F_j)\}$$

The first-order condition for equilibrium is :

$$p_j = \frac{\sigma}{\sigma - 1}\tilde{c}_j \tag{3}$$

Under competitive labour markets and monopolistic competition, firms with technology j hire workers up to the point where the wage per efficiency unit of labour, $\tilde{c_j} = w_j(s,\eta)/\varphi_j(s,\eta)$, equals the marginal revenue product $p_j\frac{\sigma-1}{\sigma}$. Hence, employees working with the same technology are paid the same fraction of their respective expected productivity:

$$w_{jg}(s) = p_j \frac{\sigma - 1}{\sigma}\tilde{\varphi}_{jg}(s)$$

with $0 < \frac{\sigma-1}{\sigma} < 1$.

Firms offer wages that are specific to workers' attributes. This specification is consistent with empirical evidence on within-firm wage dispersion[16] and on within-firm gender wage gaps.

3.2 The wage distribution for men and women

We follow Yeaple (2005) where workers with different skills sort across h and l type firms. In this paper, workers not only differ in their observable skill s but also in their unobservable degree of job commitment e. Following the literature on job assignment, we assume that workers know the wage they can earn if they are matched to a given firm j and go to the firm that offers the highest wage.

Proposition 1. ***Sorting of workers***
If higher skill workers have a comparative advantage in h-type firms then h-type firms hire the most skilled workers of each group g

We prove this result in the appendix by showing that if positive assortative matching is not followed, the value of the output and wages can increase by switching the assignment of

tities lead to the same equilibrium result.

[16] Davis et al. (1991) on the United-States and Iranzo et al. (2008) on Italy, for example. Meng (2004), among others, shows that men and women receive different wage within firms in Germany

workers to firms[17]. Positive assortative matching between firms and workers has received empirical support: several papers find that most productive firms employ more skilled workers[18].

The wage distribution for men and women is given by the function $w_{jg}(s) = \tilde{c_j}.\tilde{\varphi}_{jg}(s)$ where the wage of a worker is equal to the cost per unit of efficient labour times the expected productivity of the worker. We can give an expression for the wage that depends on firms' technologies and the skill thresholds s_{jg} below which a worker from group $g = \{f, m\}$ is not hired by a firm $j = \{l, h\}$.

$$w_g(s) = \begin{cases} c_Y \varphi_Y = 1 & \text{if } s < s_{lg} \\ \tilde{c}_l \tilde{\varphi}_{lg}(s) & \text{if } s_{lg} \leq s < s_{hg} \\ \tilde{c}_h \tilde{\varphi}_{hg}(s) & \text{if } s_{hg} \leq s \end{cases} \tag{4}$$

Among each group $g = \{m, f\}$, workers with a skill level equal to the threshold s_{lg} is indifferent between working in sector Y and working in a firm l in sector X. Similarly a worker with a skill level s_{hg} is indifferent between working in a firm using either technology h or l : $\tilde{c}_l \tilde{\varphi}_{lg}(s_{hg}) = \tilde{c}_h \tilde{\varphi}_{hg}(s_{hg})$. Consequently, we can rank the unit cost of production :

$$\frac{\tilde{c}_l}{c_Y} = \frac{\varphi_Y(s_{lg})}{\tilde{\varphi}_{lg}(s_{lg})} = \frac{1}{\tilde{\varphi}_{lg}(s_{lg})} < 1 \qquad \text{and} \qquad \frac{\tilde{c}_h}{\tilde{c}_l} = \frac{\tilde{\varphi}_{lg}(s_{hg})}{\tilde{\varphi}_{hg}(s_{hg})} < 1 \tag{5}$$

Firms in the diversified sector have lower unit cost of production than firms in sector Y. Within sector X, firms using the low technology have higher unit cost than firms using the high technology.

Using the indifference condition for both groups, we can rank the skill threshold required to men and women.

$$\frac{\varphi_Y(s_{lf})}{\tilde{\varphi}_{lf}(s_{lf})} = \frac{\varphi_Y(s_{lm})}{\tilde{\varphi}_{lg}(s_{lm})} \Leftrightarrow \frac{1}{\tilde{\varphi}_{lf}(s_{lf})} = \frac{1}{\tilde{\varphi}_{lm}(s_{lm})}$$

and

$$\frac{\tilde{\varphi}_{lm}(s_{hm})}{\tilde{\varphi}_{hm}(s_{hm})} = \frac{\tilde{\varphi}_{lf}(s_{hf})}{\tilde{\varphi}_{hf}(s_{hf})} \Leftrightarrow \frac{\tilde{\varphi}_{hf}(s_{hf})}{\tilde{\varphi}_{hm}(s_{hm})} = \frac{\tilde{\varphi}_{lf}(s_{hf})}{\tilde{\varphi}_{lm}(s_{hm})}$$

[17] This sorting mechanism has been first suggested by Roy (1951) where workers self-select into the occupation that gives them the highest expected earnings.

[18] Abowd et al. (1999) for France, Haltiwanger, Lane and Spletzer (1999, 2007) and Woodcock (2008) for the U.S., Haskel et al. (2005) for the UK, Iranzo et al. (2008) and Mion and Naticchioni (2009) for Italy and Lopes de Melo (2013) for Brazil

These two equations show that s_{jf} is a function of s_{jm} and η. Using both indifference conditions we can give the following proposition on the different sorting of men and women into heterogeneous firms:

Proposition 2. ***Ranking of male and female skill requirements***

i) Under the assumptions that $Pr_f(e = \bar{e}) < Pr_m(e = \bar{e})$ and φ_l is increasing in e, employers using the technology l require from women a higher skill level $s_{lf} > s_{lm}$

ii) Under the assumptions A2 and A3, the skill threshold to work for a firm h is higher for women $s_{hf} > s_{hm}$

The proof of proposition 2 is developed in the appendix.

Consequently, there are women working in sector Y who have a greater skill level than men working in a firm l; this holds for workers with skills comprised between the male and female threshold for entering sector X, $s_{lm} \leq s \leq s_{lf}$. Similarly, a female worker employed in a firm l can have a greater skill level than a men working in a firm h; this holds for workers with skills comprised between the male and female threshold for entering a firm h, $s_{hm} \leq s \leq s_{hf}$.

We can now describe the wage distribution for both men and women. The slope of the wage profile becomes steeper at each group-specific skill threshold s_{lg} and s_{hg} because the technology l enhances worker productivity compared to the technology used in sector Y and the technology h features stronger skill complementarity than the technology l. Within groups, the skill of any worker using the technology l is lower than the skill of a worker using the technology h[19].

We can further give the distribution of the wage gap along the skill distribution where $WG(s) = \frac{w_m(s)}{w_f(s)}$ is the gap between a man and a woman of skill s.

[19] It would be possible to smooth out the breaks in the wage distribution of both men and women, and thus in the gender wage gap by introducing a continuum of production technologies instead of just two, as in Sampson (in press).

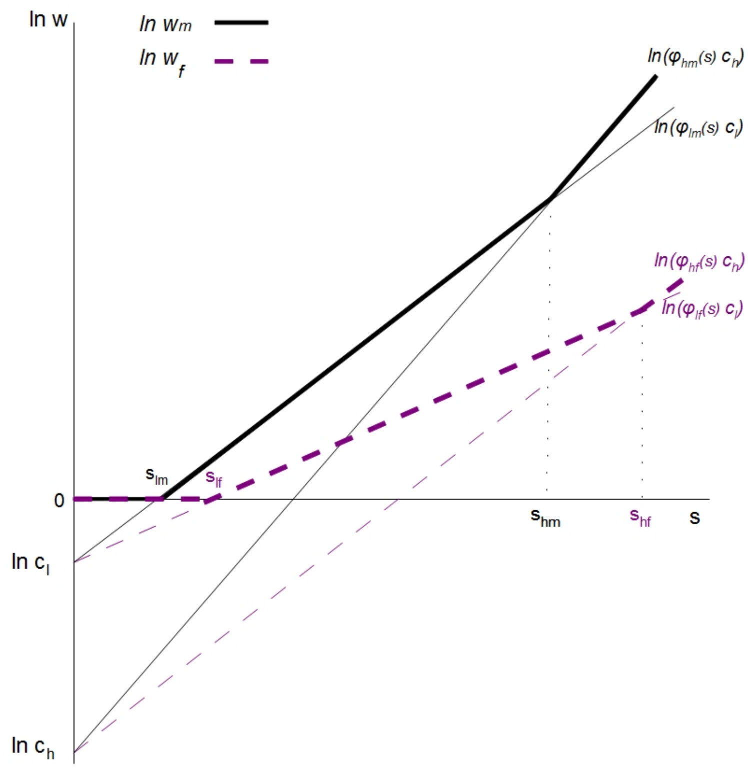

Figure 1: The wage distribution for men and women

$$WG(s) = \begin{cases} 1 & \text{if } s \leq s_{lm} \\ \tilde{c}_l \frac{\tilde{\varphi}_{lm}(s)}{\varphi_Y} & \text{if } s_{lm} \leq s \leq s_{lf} \\ \frac{\tilde{\varphi}_{lm}(s)}{\tilde{\varphi}_{lf}(s)} & \text{if } s_{lf} \leq s \leq s_{hm} \\ \frac{\tilde{c}_h}{\tilde{c}_l} \frac{\tilde{\varphi}_{hm}(s)}{\tilde{\varphi}_{lf}(s)} & \text{if } s_{hm} \leq s \leq s_{hf} \\ \frac{\tilde{\varphi}_{hm}(s)}{\tilde{\varphi}_{hf}(s)} & \text{if } s_{hf} \leq s \end{cases}$$

Proposition 3. *Under assumptions A1, A2 and A4, the gender wage gap is increasing in* *the skill level.*

The increase in the gender wage gap with skill, as depicted in figure 1, can be explained as follows. There is no wage gap between men and women in sector Y because of the assumption that labour productivity in Y does not depend on either skills or job commitment. We do not need such a strong assumption to generate an increasing gap further on but it simplifies the exposition. Production in sector Y can be thought of as involving mainly routine tasks. For workers with skill levels comprised between s_{lm} and s_{lf}, the gender wage gap is equal to $\tilde{c}_l \frac{\tilde{\varphi}_{lm}(s)}{\varphi_Y}$ which increases with s from (A1) and is greater than the gender wage gap at the bottom of the distribution. For the latter to be true, we need that technology l features stronger complementarities with skills compared to the technology used in sector Y. The supermodularity of φ in skills and technology upgrading (A2), ensures that the gender wage gap is increasing in skills when men and women work with different technologies. In particular, (A2) ensures that $\frac{\tilde{c}_h}{\tilde{c}_l} \frac{\tilde{\varphi}_{hm}(s)}{\tilde{\varphi}_{lf}(s)}$ increases in s. The supermodularity of φ in skills and commitment (A4) ensures that the the gender wage gap increases when men and women work with the same technology. In particular, (A4) ensures that the ratios $\frac{\tilde{\varphi}_{lm}(s)}{\tilde{\varphi}_{lf}(s)}$ and $\frac{\tilde{\varphi}_{hm}(s)}{\tilde{\varphi}_{hf}(s)}$ are increasing in s.

3.3 Free entry and market clearing

Investment in technology is unrestricted so that the number of firms adjusts until profits using either technology are zero. For each type of technology, the unit cost under which total revenues equal total (labour) costs is :

$$\tilde{c_j} = \frac{\sigma - 1}{\sigma}(EP_X^{\sigma-1})^{\frac{1}{\sigma}}((\sigma - 1)F_j)^{\frac{-1}{\sigma}} \tag{6}$$

The different fixed costs generate two productivity cutoffs. Producing with the technology h requires a higher productivity $\tilde{c}_h < \tilde{c}_l$ to be able to make higher operating profits to pay for the higher fixed cost. Firms make their investment and human resources decisions jointly as the unit cost of producing with a given technology depends on the skill level of the workforce.

Using equation (5) and the zero profit conditions for both types of firms, we obtain:

$$\frac{\tilde{\varphi}_{hg}(s_{hg})}{\tilde{\varphi}_{lg}(s_{hg})} = \left(\frac{F_h}{F_l}\right)^{\frac{1}{\sigma}} \tag{7}$$

This expression pins down the skill threshold to enter a firm h for both men and women as a function of the technologies' parameters. An increase in the fixed cost to invest in the high technology increase the skill threshold required to workers.

Female and male total labour supply is assumed to be fixed and is divided across the tree types of firms. The numbers of high-technology and low-technology firms in sector X are given by :

$$N_h(q_h + F_h) = \int_{s \in S_{hf}} \tilde{\varphi}_{hf}(s)l(s)ds + \int_{s \in S_{hm}} \tilde{\varphi}_{hm}(s)l(s)ds$$

$$N_l(q_l + F_l) = \int_{s \in S_{lf}} \tilde{\varphi}_{lf}(s)l(s)ds + \int_{s \in S_{lm}} \tilde{\varphi}_{lm}(s)l(s)ds$$

Using the free entry condition :

$$N_h = \frac{1}{\sigma F_h}(\int_{s \in S_{hf}} \tilde{\varphi}_{hf}(s)l(s)ds + \int_{s \in S_{hm}} \tilde{\varphi}_{hm}(s)l(s)ds) \tag{8}$$

$$N_l = \frac{1}{\sigma F_l}(\int_{s \in S_{lf}} \tilde{\varphi}_{lf}(s)l(s)ds + \int_{s \in S_{lm}} \tilde{\varphi}_{lm}(s)l(s)ds) \tag{9}$$

The number of firm j depends on the four skill thresholds s_{jg} with $j = \{h, l\}$ and $g = \{m, f\}$. The threshold s_{hm} is pinned down by the free entry condition in sector X while the sorting of workers across the two types of firms relates s_{hm} to s_{hf}. The market clearing condition for good Y determines the skill threshold s_{lm}.

To close the model, we finally use the market clearing condition in sector Y where the level of production is $Y = \sum_g \int_0^{s_{lg}} l(s)ds$.
The demand for good Y, given by the Cobb-Douglas preferences, must equal the production of the good. Since Y is the numeraire $p_Y = 1$, the market clearing condition is :

$$Y = (1 - \beta)M$$

where M is total revenue which equals total wages (firms make no positive profits in equilibrium) : $M = \sum_g(\int_{s \in S_{Y_g}} l(s)ds + \int_{s \in S_{lg}} w_{lg}(s)l(s)ds + \int_{s \in S_{hg}} w_{hg}(s)l(s)ds)$. Consumption of good Y is a function of the cost thresholds s_{lg} and s_{hg}.

Using equations (13) and (5), replacing M in the equation for the demand of good Y and equalizing demand and production for good Y we obtain :

$$\frac{\beta}{1-\beta}\tilde{\varphi}_l(s_{lm})\sum_g\int_0^{s_{lg}} l(s)ds = \sum_g\left(\int_{s_{lg}}^{s_{hg}}\tilde{\varphi}_{lg}(s)l(s)ds + \frac{\tilde{\varphi}_l(s_{hm})}{\tilde{\varphi}_h(s_{hm})}\int_{s_{hg}}^{\bar{s}}\tilde{\varphi}_{hg}(s)l(s)ds\right) \qquad (10)$$

Equation (10) defines the skill threshold below which individuals are working in sector Y.

4 The open economy

4.1 Profit maximization and export patterns in the open economy

We assume that the domestic country trades with an identical foreign country so that we need to define the allocations and equilibrium in one country only[20]. Markets are segmented because of a variable trade cost τ which includes freight and insurance costs along with tariffs. As a result, a firm may charge different prices on the domestic and foreign market. Besides, a firm incurs a fixed export cost F^t to start exporting as in Melitz (2003). F^t covers fixed market access costs such as setting up new distribution channels, shipping requirements as well as ensuring that the firm's goods conforms to foreign standards and regulatory environment. The fixed cost generates a selection of firms into exporting as established by the empirical literature. Regardless of the export decision, a firm always incurs the investment cost F_j . Because this overhead production cost is already incurred, a firm would not export and not produce for its own domestic market. Indeed domestic sales yield always strictly higher operating profits compared to sales to foreign markets because of the additional fixed and variable costs.

The demand for a variety i of the differentiated good comes now from both domestic and foreign consumers who are assumed to have the same preferences:

$$x_i = p_i^{-\sigma} E P_X^{\sigma-1}$$
$$x_i^t = (p_i^t)^{-\sigma} E P_X^{\sigma-1} \qquad (11)$$

where p_i is the price of variety i on the domestic market and p_i^t is the price of variety i when it is traded to a foreign market. E is the share of the income spent on goods X. The

[20] Assuming that the trading partner is identical allows us to consider only one set of skill thresholds, $s_{lf}, s_{lm}, s_{hf}, s_{hm}$, which are common to each country. Country differences, for example in the skill distribution or in the technology, would give rise to different thresholds and equilibrium conditions that vary across countries.

price index is now :

$$P_X = (\sum_i p_i^{1-\sigma} + \sum_k p_k^{t1-\sigma})^{\frac{1}{1-\sigma}}$$

where p_k^t is the price of variety k traded by a foreign firm and sold on the domestic market. Firms are subjected to per-unit iceberg trade cost τ. To address the foreign demand, a firm needs to produce $q^t = \tau x_i^t$ as a share τ of the production is required for transportation.

Firms maximize their profits with respect to either price or quantity :

$$\pi_j = \max_{p_j}\{p_j q_j + I^t.(p_j^t \frac{q_j^t}{\tau}) - \tilde{c}_j(q_j + F_j + I^t.(q_j^t + F^t))\}$$

where I^t equals 1 if the firm exports and $q_j^t = \tau(p_i^t)^{-\sigma} E P_X^{\sigma-1}$. As marginal costs are constant, we can separate the profits they earn on each market. The pricing rule in the domestic market implies, exactly as in the autarky case, that the marginal cost of production equates the marginal revenue.

$$p_j \frac{\sigma - 1}{\sigma} = \tilde{c}_j \tag{12}$$

Firms that export will set higher prices in the foreign markets that reflect the increased marginal cost due to the transportation cost τ that is completely supported by the consumer (the standard mill pricing strategy):

$$p_j^t \frac{\sigma - 1}{\sigma} = \tau \tilde{c}_j \Leftrightarrow p_j^t = \tau p_j$$

The marginal cost of production is still given by $\frac{w_{jg}(s)}{\tilde{\varphi}_{jg}(s)}$ so that the sorting of workers across firms stated in proposition 1 continues to hold, h-firms employ the workers with the highest skill level.

To know what are the firms that export, we need to compare the profits made when exporting with the profits made when selling only in the domestic market. Profits of a firm j are:

$$\pi_j = \begin{cases} \tilde{c_j}^{1-\sigma} \frac{\sigma^{-\sigma}}{(\sigma-1)^{1-\sigma}} E P_X^{\sigma-1} - \tilde{c}_j F_j & \text{if firm } j \text{ serves only the domestic market} \\ \tilde{c_j}^{1-\sigma} \frac{\sigma^{-\sigma}}{(\sigma-1)^{1-\sigma}} E P_X^{\sigma-1}(1 + \tau^{1-\sigma}) - \tilde{c}_j(F_j + F^t) & \text{if firm } j \text{ serves both markets} \end{cases}$$

Three cases arise :

i if $F^t\tau^{\sigma-1} \geq F_h$, no firm export

ii if $F_l \leq F^t\tau^{\sigma-1} \leq F_h$, h firms only export

iii if $F^t\tau^{\sigma-1} \leq F_l$, both l and h firms export

We can see directly that if there is no fixed export cost, $F^t = 0$, all firms that continue to be active are able to export and no level of variable cost $\tau > 1$ can generate the selection of the most productive firms into exporting. As the differences between exporters and non exporters -within sectors- are empirically pervasive, it is accepted that models with CES demand should assume a combination of fixed and variable trade costs to generate a sorting of firms according to their productivity.

From now on, we focus on the case 2 where only the high-technology firms are able to export. The free entry conditions determine which workers are employed by exporters. For h firms, the zero-profit conditions implies :

$$\tilde{c}_h = (\sigma(F_h + F^t))^{\frac{-1}{\sigma}}(\frac{\sigma}{\sigma-1})^{\frac{1-\sigma}{\sigma}}EP_X^{\sigma-1}(1+\tau^{1-\sigma})^{\frac{1}{\sigma}}$$

We denote $\tilde{c}_j^a$ the marginal cost of j firms under autarky. The expected cost per efficiency unit of labour that firms h can pay $\tilde{c}_h$ is larger under trade, $\tilde{c}_h > \tilde{c}_h^a$. This stems from the increase in market size that benefits exporting firms.

The zero profit condition for l-firms is :

$$\tilde{c}_l = (\sigma F_l)^{\frac{-1}{\sigma}}(\frac{\sigma}{\sigma-1})^{\frac{1-\sigma}{\sigma}}(EP_X^{\sigma-1})^{\frac{1}{\sigma}}$$

4.2 The wage distribution in the open economy

Trade openness has an impact on the skill-thresholds that define which type of men and women are hired by high-tech firms. As before, we relate the two zero-profit conditions for h and l firms to find the new skill-threshold s_{hg}:

$$\frac{\tilde{c}_h}{\tilde{c}_l} = \left(\frac{(F_h + F^t)}{F_l(1+\tau^{1-\sigma})}\right)^{\frac{1}{\sigma}}$$

From the above equation, we can see that the difference in marginal costs between the two types of firms is smaller under trade than under autarky, $1 > \frac{\tilde{c}_h}{\tilde{c}_l} > \frac{\tilde{c}_h^a}{\tilde{c}_l^a}$. Using the indifference conditions for the marginal workers of each group whose skill levels define the skill-threshold, $w_{lg}(s_{hg}) = w_{hg}(s_{hg}) \Leftrightarrow \frac{c_h^t}{c_l^t} = \frac{\tilde{\varphi}_{lg}(s_{hg})}{\tilde{\varphi}_{hg}(s_{hg})}$, we have :

Proposition 4. *When only h firms export,*

i) the skill threshold to enter a firm h is lower under trade compared to the autarky case for both groups, $s_{hg} < s_{hg}^a \quad$ for $g = \{l, h\}$. More workers are matched with a high technology firm under trade.

ii) the skill threshold to enter a firm h is still higher for women, $s_{hm} < s_{hf}$

iii) trade liberalization further reduces the skill requirement for both groups, $\frac{\partial(\tilde{c}_h/\tilde{c}_l)}{\partial\tau} > 0 \Rightarrow \frac{\partial s_{hg}}{\partial\tau} > 0 \quad \forall g$

Although the expression for $\tilde{c}_l$ does not change, its value changes with openness. The decrease in the skill threshold s_{hg} to enter h firms raises wages for the most skill workers; this in turn raises total income which corresponds to a higher demand for the non-trade good. This effect will be explicit when the general equilibrium effect is highlighted. Sector Y thus demands more labour. Consequently, we have a higher skill threshold to enter the manufacturing industry under trade $s_l > s_l^a$ and the marginal production cost of a low technology firm goes down $\tilde{c}_l < \tilde{c}_l^a$. Trade openness brings an increase in productivity in the manufacturing sector along with a higher demand for local services for instance, as a result some workers move from the manufacturing sector to the non-traded sectors; this is in line with general employment patterns.

Proposition 5. *When only h firms export,*

i) the skill threshold to enter a firm l is higher under trade than under autarky for both groups, $s_{lg}^t > s_{lg}^a$

ii) the skill threshold to enter a firm l remains higher for women under trade, $s_{lm}^t < s_{lf}^t$

iii) trade liberalization further increases the skill threshold above which workers are employed in the traded sector, $\frac{\partial(\tilde{c}_l/c_Y)}{\partial\tau} > 0 \Rightarrow \frac{\partial s_{lg}}{\partial\tau} > 0 \quad \forall g$

This is consistent with two stylized facts. First, the share of manufacturing employment in female employment is lower than the share of manufacturing employment in male employment. Second, trade openness does not reverse this pattern.

The wage function has the same form than under autarky but the values of the skill thresholds s_{lg} and s_{hg} as well as the cost thresholds $\tilde{c}_l$ and $\tilde{c}_h$ have changed:

$$w_g(s) = \begin{cases} c_Y \varphi_Y = 1 & \text{if } s \leq s_{lg} \\ \tilde{c}_l \tilde{\varphi}_{lg}(s) & \text{if } s_{lg} \leq s \leq s_{hg} \\ \tilde{c}_h \tilde{\varphi}_{hg}(s) & \text{if } s_{hg} \leq s \end{cases} \tag{13}$$

To measure how these changes affect the gender wage gap along the skill distribution, we compare the gap under autarky with the gap under openness. The gender wage gap $\frac{w_m(s)}{w_f(s)}$ is now given by :

$$WG(s) = \begin{cases} 1 & \text{if } s \leq s_{lm} \\ \tilde{c}_l \frac{\tilde{\varphi}_{lm}(s)}{\varphi_Y} & \text{if } s_{lm} \leq s \leq s_{lf} \\ \frac{\tilde{\varphi}_{lm}(s)}{\tilde{\varphi}_{lf}(s)} & \text{if } s_{lf} \leq s \leq s_{hm} \\ \frac{\tilde{c}_h}{\tilde{c}_l} \frac{\tilde{\varphi}_{hm}(s)}{\tilde{\varphi}_{lf}(s)} & \text{if } s_{hm} \leq s \leq s_{hf} \\ \frac{\tilde{\varphi}_{hm}(s)}{\tilde{\varphi}_{hf}(s)} & \text{if } s_{hf} \leq s \end{cases}$$

Comparing the gender wage gap in autarky and in the open economy for different skill segments, we can state the following proposition that is made explicit in appendix C.

Proposition 6. *Following trade integration,*

i) the gender wage gap is reduced at the bottom of the skill distribution as $s_{lm} > s^a_{lm}$

ii) the gender wage gap widens at the top of the distribution given that $s_{hm} < s^a_{hm}$ *and the wage profile is steeper under technology* h

Why do we observe those changes in the gender wage gap with international trade? Following trade integration, there is a reallocation of workers towards high-technology firms. Workers moving to h firms increase their productivity and earn higher wages compared to autarky. This has different effects on the gender wage gap at the top and at the bottom of the wage distribution. The increase in domestic income induces a rise in the demand of good Y (preferences are homothetic). The non-traded good sector expands and employs more workers. This movement induces a reduction in the gender wage gap up to the new male skill-threshold s_{lm}, i.e. in the segment $[s^a_{lm}; s_{lm}]$. The reallocation of workers toward h firms generates an increase in the gender wage gap from s_{hm} because the point at which men work with the best technology arrives sooner than under autarky. It increases the

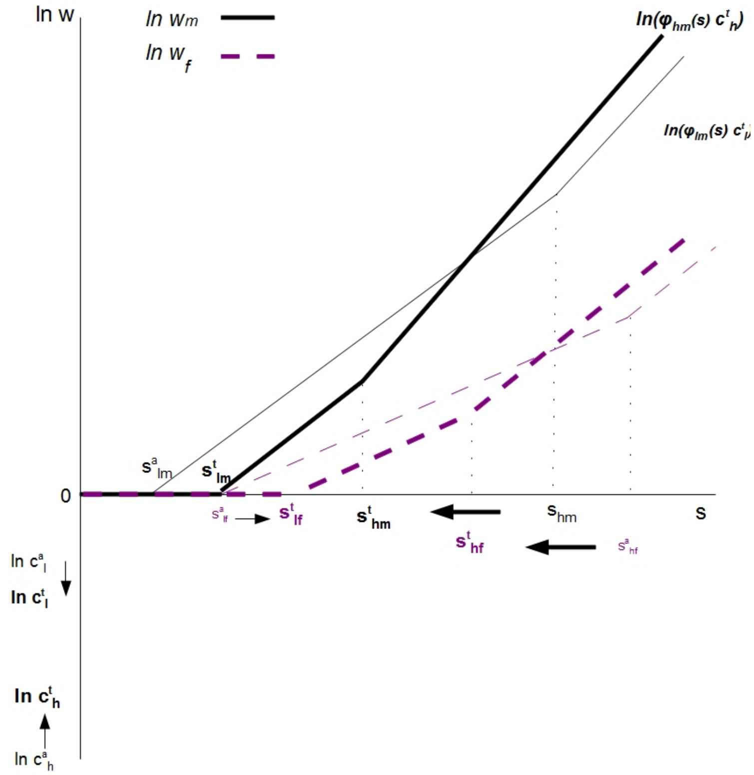

Figure 2: Changes in the gender wage gap with trade openness

gender wage gap up to the point s^a_{hf} because the high technology features stronger complementarity with job commitment than the low technology. Among workers with the highest skill level, above the autarky female threshold i.e. in the segment $[s^a_{hf}; \bar{s}]$, the gender wage gap remains unchanged.

4.3 Free-entry and market clearing

In the case where only h firms export and using the free entry condition, the number of firms is given by :

$$N_h = \frac{1}{\sigma(F_h + F^t)}(\int_{s \in S_{hf}} \tilde{\varphi}_{hf}(s)l(s)ds + \int_{s \in S_{hm}} \tilde{\varphi}_{hm}(s)l(s)ds) \tag{14}$$

$$N_l = \frac{1}{\sigma F_l}(\int_{s \in S_{lf}} \tilde{\varphi}_{lf}(s)l(s)ds + \int_{s \in S_{lm}} \tilde{\varphi}_{lm}(s)l(s)ds) \tag{15}$$

More workers are hired by h firms under trade as the skill threshold is lower $s_{hg} > s_{hg}^a \quad \forall g$.

Finally, the market clearing condition for good Y determines the new skill threshold s_{lm}. Good Y is not traded. The market clearing condition is still given by $Y = (1-\beta)M$,where M equals total wages in the open economy. Skilled workers' wages have increased following trade liberalization as more firms adopt the high-technology. $M = \sum_g(\int_{s \in S_{Y_g}} l(s)ds + \int_{s \in S_{lg}} w_{lg}(s)l(s)ds + \int_{s \in S_{hg}} w_{hg}(s)l(s)ds)$. Using (13) with the new skill thresholds,

$$M = \sum_g(\int_0^{s_{lg}} l(s)ds + \tilde{c}_l \int_{s_{lg}}^{s_{hg}} \tilde{\varphi}_{lg}(s)l(s)ds + \tilde{c}_h \int_{s_{hg}}^{\bar{s}} \tilde{\varphi}_{hg}(s)l(s)ds)$$

Using equation (5) and equalizing demand and production for good Y, we have :

$$\frac{\beta}{1-\beta}\tilde{\varphi}_l(s_{lm}) \sum_g \int_0^{s_{lg}} l(s)ds = \sum_g \left(\int_{s_{lg}}^{s_{hg}} \tilde{\varphi}_{lg}(s)l(s)ds + \frac{\tilde{\varphi}_l(s_{hm})}{\tilde{\varphi}_h(s_{hm})} \int_{s_{hg}}^{\bar{s}} \tilde{\varphi}_{hg}(s)l(s)ds \right) \tag{16}$$

This equation determines the skill threshold below which individuals are now working in sector Y and closes the model.

What are the effects of trade integration on the number of operating firms? Under the assumption that only h firms are able to export ($F_l \leq F^t\tau^{\sigma-1} \leq F_h$), there is a reallocation of labour towards h firms (s_{hg} falls) but the change in the number of h firms remains ambiguous. The increase in market size with trade integration leads to an increase in total output produced by h type firms. But the increase in the fixed operating costs ($F_h + F_t$ instead of F_h) puts a downward pressure on the number of firms investing in the high-technology. Each h firm must produce more to meet the fixed export cost. From equations (14) and (14), we cannot tell whether the increase in the quantity produced by each h firm fully compensate the increase in total output produced by h firms. What we can see, however, is that a reduction in trade costs τ increases N_h the number of firms adopting the high-technology and reduces N_l.

5 Conclusion

This paper offers a theoretical explanation for varying gender wage gaps along the skill distribution and for the heterogeneous impact of trade openness on the wage gap depending on the position along the skill distribution. We need three supermodularity assumptions on the labour productivity function to give general conditions under which we find the pattern observed in empirical studies. More precisely we show that if skills and job commitment are complements to technological upgrading and if skills and to each other, statistical discrimination based on job commitment expectations generates a higher gender wage gap at the upper part of the distribution. In a closed economy, the model puts forward one reason for the glass ceiling effect as well as the increase in residual wage disparity within gender groups as documented by numerous empirical studies.

The analysis provides insights into the impact of trade openness in a setting with intra-industry trade and monopolistic competition. First, we show that when trade openness induces technological change biased towards observable and unobservable productive characteristics, it increases the wage gap at the top of the skill distribution. Second, general equilibrium effects implies that the gender wage gap is reduced at the lower part of the distribution. As a result the average gender wage gap may increase or decrease following trade integration.

The paper adds to the understanding of the interactions between the overall wage structure of an economy and the gender wage gap, and can be used to interpret more general shocks that affect the demand for observable and unobservable characteristics of workers. It provides a rationale for looking at the contribution of what we call employers' requirement for commitment in shaping gender inequalities. Constructing a empirical measure for "commitment requirement" at the job level would be the first step of an empirical analysis aiming at testing the predictions of the model. The new job classification could then be used to address the issue of the gender wage gaps differently than what has been done in the literature, and to explore whether the effects of trade openness on the gender wage gap depends on both skill and commitment.

An extension of the model would be to develop an asymmetric country case and to investigate whether changes in the wage gaps depends on the characteristics of the trade partner. We could explore the consequences of having different technologies, or different distribution of commitment among men and women, across countries. This extension would be useful in providing policy recommendations

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A Sorting of heterogeneous workers across firms

We can prove by contradiction, that a high-technology firm hires workers with higher skill level compared to the skill level of workers in a low-technology firm.

Consider two workers with skill $s_1 < s_2$. Let us assume that worker 1 is hired by a firm h and worker 2 is hired by a firm l.
Firm h pays worker 1 so that its profit is maximized :

$$\frac{\sigma - 1}{\sigma} p_h = \frac{w_h(s_1)}{\tilde{\varphi}_h(s_1)}$$

Firm l pays worker 2 so that its profit is maximized :

$$\frac{\sigma - 1}{\sigma} p_l = \frac{w_l(s_2)}{\tilde{\varphi}_l(s_2)}$$

Firm l would not increase its profit by hiring worker 1 at a wage just above the one paid by a firm l :

$$\frac{\sigma - 1}{\sigma} p_l \leq \frac{w_h(s_1)}{\tilde{\varphi}_l(s_1)}$$

Firm h would not increase its profit by hiring worker 2 at a wage just above the one paid by a firm h :

$$\frac{\sigma - 1}{\sigma} p_h \leq \frac{w_l(s_2)}{\tilde{\varphi}_h(s_2)}$$

Equations 1 and 3 implies that $\frac{p_l}{p_h} \leq \frac{\tilde{\varphi}_h(s_1)}{\tilde{\varphi}_l(s_1)}$
Equations 2 and 4 implies that $\frac{p_l}{p_h} \geq \frac{\tilde{\varphi}_h(s_2)}{\tilde{\varphi}_l(s_2)}$
Which implies that $\frac{\tilde{\varphi}_h(s_2)}{\tilde{\varphi}_l(s_2)} \leq \frac{\tilde{\varphi}_h(s_1)}{\tilde{\varphi}_l(s_1)}$. But this contradicts the assumption that more skilled workers have a comparative advantage in the high-technology.

B Ranking of of male and female skill requirements

The indifference condition states that :

$$\frac{\tilde{\varphi}_{hf}(s_{hf})}{\tilde{\varphi}_{hm}(s_{hm})} = \frac{\tilde{\varphi}_{lf}(s_{hf})}{\tilde{\varphi}_{lm}(s_{hm})}$$

$$\Leftrightarrow \frac{\eta\varphi_h(s_{hf}, \bar{e}) + (1-\eta)\varphi_h(s_h, \underline{e})}{\varphi_h(s_{hm}, \bar{e})} = \frac{\eta\varphi_l(s_{hf}, \bar{e}) + (1-\eta)\varphi_h(s_h, \underline{e})}{\varphi_l(s_{hm}, \bar{e})}$$

That we can rearrange

$$\Leftrightarrow \frac{\eta + (1-\eta)\frac{\varphi_h(s_{hf}, \underline{e})}{\varphi_h(s_{hf}, \bar{e})}}{\eta + (1-\eta)\frac{\varphi_l(s_{hf}, \underline{e})}{\varphi_l(s_{hf}, \bar{e})}} = \frac{\varphi_l(s_{hf}, \bar{e})}{\varphi_l(s_{hm}, \bar{e})}\frac{\varphi_h(s_{hm}, \bar{e})}{\varphi_h(s_{hf}, \bar{e})}$$

Let us prove by contradiction that $s_{hf} > s_{hm}$.

Suppose that $s_{hf} = s_{hm} = s_h$, the condition is now

$$\frac{\eta + (1-\eta)\frac{\varphi_h(s_h, \underline{e})}{\varphi_h(s_h, \bar{e})}}{\eta + (1-\eta)\frac{\varphi_l(s_h, \underline{e})}{\varphi_l(s_h, \bar{e})}} = \frac{\varphi_l(s_h, \bar{e})}{\varphi_l(s_h, \bar{e})}\frac{\varphi_h(s_h, \bar{e})}{\varphi_h(s_h, \bar{e})} \Leftrightarrow \frac{\varphi_h(s_h, \underline{e})}{\varphi_h(s_h, \bar{e})} = \frac{\varphi_l(s_h, \underline{e})}{\varphi_l(s_h, \bar{e})}$$

Which contradicts the supermodularity assumption between technology and commitment. So that the male and female skill requirements cannot be equal.

Suppose now that $s_{hf} < s_{hm}$. By supermodularity between technology upgrading and skills, we know that :

$$\frac{\eta + (1-\eta)\frac{\varphi_h(s_{hf}, \underline{e})}{\varphi_h(s_{hf}, \bar{e})}}{\eta + (1-\eta)\frac{\varphi_l(s_{hf}, \underline{e})}{\varphi_l(s_{hf}, \bar{e})}} < 1$$

By supermodularity between technology upgrading and skills and the fact that labour productivity is increasing in skills, we know that:

$$\frac{\varphi_h(s_{hm}, \bar{e})}{\varphi_h(s_{hf}, \bar{e})} > \frac{\varphi_l(s_{hm}, \bar{e})}{\varphi_l(s_{hf}, \bar{e})} > 1$$

Combining the two we have :

$$\frac{\eta+(1-\eta)\frac{\varphi_h(s_{hf},\underline{e})}{\varphi_h(s_{hf},\bar{e})}}{\eta+(1-\eta)\frac{\varphi_l(s_{hf},\underline{e})}{\varphi_l(s_{hf},\bar{e})}}\frac{\varphi_l(s_{hm},\bar{e})}{\varphi_l(s_{hf},\bar{e})}<\frac{\varphi_h(s_{hm},\bar{e})}{\varphi_h(s_{hf},\bar{e})}$$

which contradicts the indifference condition. The female skill requirement to be hired by a high-tech firm cannot be lower than the male skill requirement.

C Proof of proposition 5 on the changes in the gender wage gap with trade openness

How has the gender wage gap changed compared to the autarky case? Changes in the gender wage gap $\frac{WG(s)}{WG^a(s)}$ are non-linear in s.

For $s \in [0; s^a_{lm}]$, there is no wage gap under either trade or autarky.

For $s \in [s^a_{lm}; s_{lm}]$, $\frac{WG(s)}{WG^a(s)} = \frac{1}{\tilde{c}^a_l \tilde{\varphi}_{lm}(s)}$, the wage gap is *lower* under trade as more men are employed in sector Y where there is no wage gap.

For $s \in [s_{lm}; s^a_{lf}]$, $\frac{WG(s)}{WG^a(s)} = \frac{\tilde{c}_l}{\tilde{c}^a_l}$, the wage gap is *lower* under trade as the unit cost of l-firms has decreased.

For $s \in [s^a_{lf}; s_{lf}]$, $\frac{WG(s)}{WG^a(s)} = \tilde{c}_l \tilde{\varphi}_{lf}(s)$, the wage gap is *higher* under trade.

For $s \in [s_{lf}; s_{hm}]$, the wage gap is of the same magnitude under trade and autarky.

For $s \in [s_{hm}; s^a_{hm}]$, $\frac{WG(s)}{WG^a(s)} = \frac{\tilde{c}_h \tilde{\varphi}_{hm}(s)}{\tilde{c}_l \tilde{\varphi}_{lm}}$ which is greater than 1 as sorting implies that $\tilde{c}_h \tilde{\varphi}_{hm}(s) > \tilde{c}_l \tilde{\varphi}_{lm}$ for $s > s_{hm}$. The wage gap is *higher* under trade.

For $s \in [s^a_{hm}; s_{hf}]$, $\frac{WG(s)}{WG^a(s)} = \frac{\tilde{c}_h}{\tilde{c}^a_h}\frac{\tilde{c}_l}{\tilde{c}^a_l}$, the wage gap is *higher* under trade.

For $s \in [s_{hf}; s^a_{hf}]$, $\frac{WG(s)}{WG^a(s)} = \frac{\tilde{\varphi}_{hm}(s)}{\tilde{\varphi}_{lm}}$ which is greater than 1. The wage gap has increased with trade.

For $s \in [s^a_{hf}; \bar{s}]$, $\frac{WG(s)}{WG^a(s)} = 1$