Multi-product Firms, Export-only Products and Extensive Margin of Export*

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Abstract

The present paper develops a general equilibrium model of heterogeneous multi-product firms in international trade. The model allows for heterogeneity in productivity and fixed export cost across products. Firms endogenously choose the number of products produced and exported. High productive firms can manage more products in export than in domestic market due to export-only products. Exposure to trade induces reallocation across firms and within firms: Least productive firms exit; Least productive products are dropped from domestic market, however some (or all) of them are switched to export-only products for high productive firms; The most productive firms introduce new products as export-only products. This paper also explores the effect of trade liberalization. In particular, more firms manage export-only products and low productive firms reduce the number of products but high productive firms otherwise.

Keywords: Multi-product firms; Export-only products; Fixed export cost; Extensive margin; Trade liberalization

JEL: F12, F13, L1

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1 Introduction

Multi-product firms are of growing interest in international trade both in empirical and theoretical research. Existing theoretical literatures conclude that firms produce multiple products and a portion of firms export a subset of their products. This implies that the extensive margin (number of products) of export is lower than of domestic market. In fact, there are exporters which export the full set of their products and there are also exporters which manage more products in export than in domestic market. On the other hand, assumption in current literatures that products within a firm only differs in terms of productivity ignores the 'learning-by-exporting' effect. Actually, firms benefit from existing exported products to export a new product. This paper develops a general equilibrium model of multi-product firms, which allows for heterogeneity in productivity and fixed export cost across products. The paper takes into account 'learning-by-exporting' effect by assuming a continually decreased fixed export cost across exported products. Under this framework, firms endogenously decide the number of products produced and exported. We find that high productive firms export, while low productive firms serve domestic market only. Across exporters, low productive firms export a subset of their domestic-sold products while high productive firms export all their domestic-sold products and introduce new products as export-only products. The extensive margin of export is higher than of domestic market for high productive exporters.

This paper examines the reallocation across firms and within firms from autarky to trade. Exposure to trade forces the least productive firms out of the market, and induces high productive firms to export. All firms drop the least productive products from domestic market after trade. Across exporters, low productive exporters export a subset of left products. Middle productive exporters export all left products and switch domestic-dropped products to export-only products. All these firms manage less products after exposure to trade. However, high productive exporters export all left products, switch all domestic-dropped products to export-only products and introduce new products as export-only products. High productive exporters manage more products from autarky to trade. In general,

trade decreases the number of products managed by low productive firms but increases the number of products managed by high productive firms.

This paper explores the impact of trade liberalization in terms of decreases in fixed and variable export cost on the number of products produced and exported. Both decreases in trade cost have similar effects. Least productive firms are forced out of the market while more firms are induced to export. Firms shrink in domestic market along both intensive margin (how much of a given product) and extensive margin, but expand in export market along both margins. More firms manage export-only products, which means more firms have higher extensive margin of export than of domestic market. Similar to the effect from autarky to trade, for low productive firms, trade liberalization decreases the number of products managed; but for high productive firms otherwise.

This paper contributes to the limited theoretical literatures about multi-products firms in international trade (e.g. Feenstra and Ma, 2007; Nocke and Yeaple, 2008; Baldwin and Gu, 2009; Arkolakis and Muendler, 2010; Eckel and Neary, 2010; Bernard *et. al.*, 2011; Mayer *et. al.*, 2011). Baldwin and Gu (2009) develop a theoretical model to analyze the impact of trade on firm diversification, but their model assumes that multi-product firms are homogeneous and products are identical within firms. Feenstra & Ma(2007) and Nocke & Yeaple (2008) develop models with heterogeneous multi-product firms. The former paper relaxes the constant aggregate price index assumption of conventional CES utility to show that adding a new product will decrease the demand of other products within firms. And this competition effect is more severe as the market share of the firm is larger. So more productive the firm is, the more products it manages, but the more or less of how much per product it produces is uncertain due to this competition effect. Nocke and Yeaple introduce a firm-level heterogeneity in terms of organizational capability. The higher the organizational capability the firm has, the more products it produces, but the marginal cost for each product becomes higher. The paper gives a positive relationship between extensive margin and intensive margin. However, both two papers assume that the products within firms are identical. This assumption is not consistent with the empirical findings about the prevalence of product churning within firms (e.g. Goldberg *et. al.*,

2008; Bernard *et. al.*, 2010; Iacovone and Javorcik, 2010). This assumption papers over the novelty of multi-product firms that they can reallocate resources across products within firms.

Eckel and Neary (2010) build a model of multi-product firms with heterogeneous products. They assume that the firm produces its core competence product with most efficiency. The core product incurs the highest productivity, while for added products, the further away from core competence, the lower productivity the added product has. They include the competition effect between products, so that adding products decreases demand of existing products. Their paper shows a negative relationship between extensive margin and intensive margin, and demonstrates that trade induces a reallocation of resource within firms from fringe product to core competence product. However their framework assumes that the number of multi-product firms are exogenous and multi-product firms are homogeneous[1].

This paper is close to Arkolakis and Muendler (2010), Bernard *et. al.* (2011) and Mayer *et. al.* (2011), which develop models simultaneously considering heterogeneous multi-product firms, heterogeneous products within firms and firms' free entry. In the model of Arkolakis & Muendler, firms incur higher marginal cost for every added exported product and firms stop adding products until the they get zero profit for marginal product. Higher productive firms manage more products and sell more per products, which implies a positive relationship between extensive and intensive margins. However, their model is specific to multi-product exporters, so they get equilibrium from zero profit of export. They do not consider the domestic market.

In the model of Mayer *et. al.*, every mew added product has higher marginal cost than existing products, and firms stop adding products until the marginal cost of the last product is higher than the cut-off level. In Bernard *et. al.* model, products are assumed stochastic and continuous, and firms produce all products with product 'expertise' higher than cut-off level. Both papers shows that firms skew to their more productive products due to either competition of export market

[1]Though they can relax each assumption, they do not give an equilibrium with both assumptions relaxed.

or trade liberalization, and that extensive margin is correlated to intensive margin.[2] However, in these two papers, export firms can only export a subset of all products sold in domestic market and Bernard *et. al.* model hardly incorporates single product firms. This paper introduces a decreasing fixed export cost to show that high productive exporters export all their products and even export more products than in domestic market due to export-only products.

This paper relates to a recent research by Gao and Tvede (2012) on export-only firms. They introduce heterogeneous fixed export cost to explain export-only firms. In their paper, firms with a combination of low productivity and fixed export cost export only. We apply this finding into products. There are two dimensions of heterogeneity in productivity and fixed export cost across products within firms. So the high productive products are exported and sold in domestic market. If firms can decrease the fixed export cost to some extent, the added products are exported only due to the combination of low productivity and fixed export cost.

The rest of this paper is organized as follows. Section 2 is the set up of the model. Section 3 describes equilibrium of closed economy. Section 4 and 5 studies the open economy and explores the equilibrium of open economy. Section 6 discusses the impact of trade liberalization. Section 7 concludes the paper.

2 The Model

Our model assumes a CES-type utility function, so there is no competition between products within firms. Firm randomly draws a marginal cost (in terms of productivity) when entering the market and then select to exit, manage one product or multi-product. Products within firms are heterogeneous in terms of marginal cost and fixed export cost. Free entry and exit drives overall profit to zero.

[2]Though Mayer *et. al.* (2011) use linear-quadratic preference, their competition only comes from market, which is different from Feenstra and Ma(2007) and Eckel and Neary (2010)

2.1 Consumers

There are a continuum of products which are normalized to the interval $[0, 1]$.For each product, there is a continuum of firms produce differentiated varieties of the product. The utility form is:

$$U = [\int_0^1 \int_{\omega \epsilon \Omega} q(\omega)^{\rho} \mathrm{d}\omega \mathrm{d}i]^{\frac{1}{\rho}} \tag{1}$$

where i indexes products, ω indexes varieties and Ω is a set of varieties. The elasticity of substitution between any two varieties is denoted by $\sigma = 1/(1-\rho)$, and $\sigma > 1$ since $0 < \rho < 1$. The aggregate quantity index and price index are denoted as:

$$Q = [\int_0^1 \int_{\omega \epsilon \Omega} q(\omega)^{\frac{\sigma-1}{\sigma}} \mathrm{d}\omega \mathrm{d}i]^{\frac{\sigma}{\sigma-1}} \qquad P = [\int_0^1 \int_{\omega \epsilon \Omega} p(\omega)^{1-\sigma} \mathrm{d}\omega \mathrm{d}i]^{\frac{1}{1-\sigma}} \tag{2}$$

Consumers maximize utility under limited budget, so the demand is denoted as:

$$q(\omega) = Q[\frac{p(\omega)}{P}]^{-\sigma} \tag{3}$$

The total expenditure R is determined by $R = \int_0^1 \int_{\omega \epsilon \Omega} q(\omega) p(\omega) \mathrm{d}\omega \mathrm{d}i$.

2.2 Firms

The dynamics of firm entry and exit follows Melitz(2003). Prior to entry, firms are identical. Firms pay a sunk cost f_e to enter the market and draw their parameters. Firm draws a random productivity φ from distribution $g(\varphi)$ for the production of first product. If the firm cannot get profit from this first product, it exits the market; otherwise the firm stays in the market and tries to add the second product. The firm will not produce the second product if the firm cannot get profit, otherwise the firm produce the second product and try to add the third

product. Firm proceeds this product ladder until it cannot get profit from adding a new product. I assume that the first product uses the firm's core competence, so the first product incurs the highest productivity. The second product incurs a lower productivity. The further way the added product is from the core product, the lower productivity it incurs. For each product, firms pay a marginal cost in terms of productivity to produce, and a identical fixed cost f to sell. Labor is the only input factor.

Product Ladder

In closed economy, the fixed cost to sell in domestic market is denoted as f_d. The output of the first product is denoted as:

$$l_1(\varphi) = f_d + q/\varphi \tag{4}$$

where l_1 is labor used for the first product. With demand from equation (3), firm maximizes the profit to set the price of first product as:

$$p_1(\varphi) = \frac{w}{\rho\varphi} \tag{5}$$

where w denotes wage, which we normalize to one. So the revenue and the profit for the first product are denoted as:

$$r_1(\varphi) = \frac{R}{(\rho P)^{1-\sigma}}\varphi^{\sigma-1} \qquad \pi_1(\varphi) = \frac{r(\varphi)}{\sigma} - f_d \tag{6}$$

Equation (6) shows that the ratio of revenue for any two varieties is solely determined by their relative productivity:

$$\frac{r(\varphi)}{r(\varphi')} = (\frac{\varphi}{\varphi'})^{\sigma-1} \tag{7}$$

If $\pi_1(\varphi) < 0$, firm exits from the market immediately; otherwise firm is active and try to add the second product with lower productivity φ_2. The output of

second product is determined as $l_2(\varphi) = f_d + q/\varphi_2$. The price of second product is $p_2(\varphi) = \frac{1}{\rho\varphi_2}$. So the profit of second product is denoted as:

$$\pi_2(\varphi) = \frac{r(\varphi_2)}{\sigma} - f_d = \frac{{\varphi_2}^{\sigma-1}}{\varphi^{\sigma-1}}\frac{r_1(\varphi)}{\sigma} - f_d \tag{8}$$

If $\pi(\varphi_2) < 0$, firm will not produce the second product; otherwise firm produces the second product and try to add the third product with productivity φ_3. Firm proceeds this product ladder until it cannot get profit for the added product. To simplify the situation, we assume that the ratio of productivity for two neighbor products is fixed as α, where $\alpha < 1$:

$$\alpha = \frac{\varphi_{i+1}}{\varphi_i} \tag{9}$$

The profit for the $i - th$ product is determined as:

$$\pi_i(\varphi) = \frac{r(\varphi_i)}{\sigma} - f_d = \beta^{i-1}\frac{r_1(\varphi)}{\sigma} - f_d \tag{10}$$

where $\beta = \alpha^{\sigma-1}$. The profit is decreasing as the number increases. The higher productivity first drawn is, the more products can the firm manage. The number of products $N_d(\varphi)$ is determined as:

$$N_d(\varphi) = \begin{cases} 0 & \text{if } \pi_1(\varphi) < 0 \\ max\,\{i|\pi_i(\varphi) \geq 0 \quad and \quad \pi_{i+1}(\varphi) < 0\} & \text{if } \pi_1(\varphi) \geq 0 \end{cases} \tag{11}$$

The productivity cut-off φ^* is determined by $\pi_1(\varphi) = 0$ (So φ^* also can be denoted as φ_1^*, hereafter $\varphi^* = \varphi_1^*$). If the firms draw a productivity lower than φ^*, they immediately exit. The productivity cut-off for $i-products$ firm φ_i^* is determined by $\pi_i(\varphi) = 0$. As we know $\pi_1(\varphi^*) = 0$, so $r_1(\varphi^*) = \sigma f_d$. $\pi_i(\varphi_i^*) = 0$, so $r_1(\varphi_i^*) = \frac{\sigma f_d}{\beta^{i-1}}$. So the relationship between productivity cut-off for $i-products$ ($i \geq 2$) firms and productivity cut-off is denoted as:

$$\frac{\varphi_i^*}{\varphi^*} = (\frac{1}{\beta^{i-1}})^{\frac{1}{\sigma-1}} = \alpha^{1-i} \tag{12}$$

Fig 1 gives an example of how the number of products is determined and what product ladder is like. If a firm draws a productivity φ_0 between φ_5^* and φ_6^*, it will produce 5 products. The firm produces the first product with productivity φ_0 and produces the fifth product with productivity $\beta^4\varphi_0$.

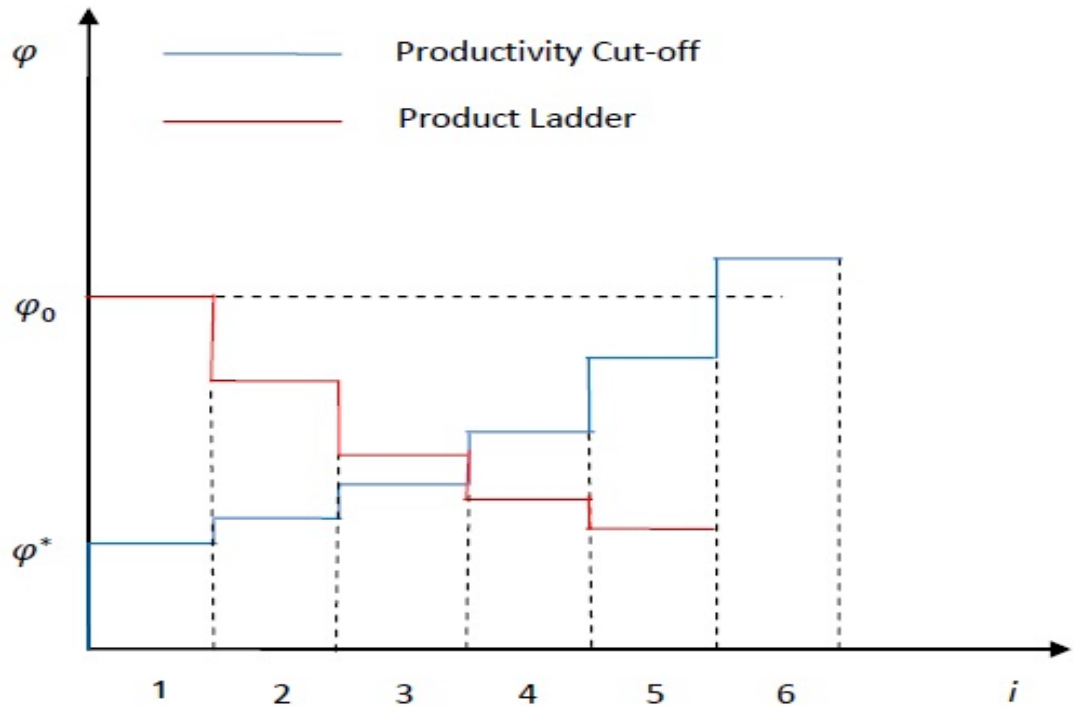

Fig 1: The number of products and product ladder

Aggregate Output and Profit

Firms choose the number of managed products based on the drawn productivity. The total output of the firm with the drawn productivity φ can be denoted as:

$$l(\varphi) = \sum_{j=1}^{i}(f_d + \frac{q_j}{\varphi_j}) = if_d + \frac{(\sum_{j=1}^{i} q_j^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}}}{(\sum_{j=1}^{i} \beta^{j-1})^{\frac{1}{\sigma-1}}\varphi} \qquad \varphi\epsilon[\varphi_i^*, \varphi_{i+1}^*) \tag{13}$$

If the output of the first product is assumed as q, $(\sum_{j=1}^{i} q_j^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}} = (\sum_{j=1}^{i} \beta^{j-1})^{\frac{\sigma}{\sigma-1}} q$. So the firm with drawn productivity φ managing i products can be represented by the firm with 'representative productivity' $\tilde{\varphi}(\varphi) = (\sum_{j=1}^{i} \beta^{j-1})^{\frac{1}{\sigma-1}}\varphi$ producing single product with i times of fixed domestic cost f_d.

The total profit by a firm with drawn productivity $\varphi\epsilon[\varphi_i^*, \varphi_{i+1}^*)$ is denoted as $\pi(\tilde{\varphi}) = \frac{r(\tilde{\varphi})}{\sigma} - if_d = (\frac{\tilde{\varphi}}{\varphi^*})^{\sigma-1}\frac{r(\varphi^*)}{\sigma} - if_d$. So the total profit by a firm is determined by:

$$\pi(\varphi) = \begin{cases} 0 & \text{if } \varphi < \varphi^* \\ \left[\sum_{j=1}^{i} \beta^{j-1} k(\varphi) - i\right] f_d & \text{if } \varphi\epsilon[\varphi_i^*, \varphi_{i+1}^*) \end{cases} \tag{14}$$

where $k(\varphi) = (\varphi/\varphi^*)^{\sigma-1}$.

Free entry

We assume that there is a probability δ that active firms are forced out of the market due to external shock. The average profit conditional on successful entry is denoted as $\overline{\pi}$, so the expected value of entry v_e is determined as:

$$v_e = \frac{1 - G(\varphi^*)}{\delta}\overline{\pi} \tag{15}$$

where $G(*)$ is the cumulative function of $g(\varphi)$ and $1 - G(\varphi^*)$ is the ex ante probability of successful entry. Under free entry, the expected value of entry equals the sunk cost, such that profit is zero.

$$v_e = f_e \tag{16}$$

3 Closed Economy Equilibrium

Equilibrium of closed economy is solely described as the set $\{\varphi^*, P, R\}$. φ_i^* is functions of φ^* and all other variables are functions of the vector of these elements.

The average profit is denoted as:

$$\overline{\pi} = \frac{1}{1 - G(\varphi^*)} \sum_{i=1}^{\infty} \int_{\varphi_i^*}^{\varphi_{i+1}^*} \pi(\varphi) g(\varphi) d\varphi \tag{17}$$

Substituting equation (14) into equation (17), the average profit is:

$$\overline{\pi} = \frac{f_d}{1 - G(\varphi^*)} \sum_{i=1}^{\infty} \int_{\varphi_i^*}^{\varphi_{i+1}^*} \big[\sum_{j=1}^{i} \beta^{j-1} k(\varphi) - i \big] g(\varphi) d\varphi \tag{18}$$

According to equation (15) and (16), equilibrium is determined by:

$$\frac{f_d}{\delta} \sum_{i=1}^{\infty} \int_{\varphi_i^*}^{\varphi_{i+1}^*} \big[\sum_{j=1}^{i} \beta^{j-1} k(\varphi) - i \big] g(\varphi) d\varphi = f_e \tag{19}$$

Theorem 3.1. There exists a unique equilibrium. See appendix 3.1 for proof.

The average revenue $\overline{r}$ is determined as:

$$\overline{r} = \frac{f_d}{1 - G(\varphi^*)} \sum_{i=1}^{\infty} \int_{\varphi_i^*}^{\varphi_{i+1}^*} \sum_{j=1}^{i} \beta^{j-1} k(\varphi) g(\varphi) d\varphi \tag{20}$$

So the number of incumbent firms M is determined by $M = R/\overline{r} = L/\overline{r}$, where L is the total labor in the market. If we assume the number of entrants is M_e, in equilibrium, the number of firms exit equals the number of successful entrants as $\delta M = (1 - G(\varphi^*))M_e$. The labor used by the incumbent firms is assumed as L_p and labor used by the entrants as L_e.

$$L_e = M_e f_e = \frac{\delta M}{1 - G(\varphi^*)} f_e = M\overline{\pi} = \Pi \tag{21}$$

So labor market clears as $L = R = L_p + L_e = L_p + \Pi$.

4 Open Economy

In this section, I explore the multi-products firms in open economy within a framework of two symmetric countries. As numerous literatures have assumed, there is a fixed cost and an 'iceberg' variable cost for export. Variable fixed cost τ is assumed to be identical across firms and products, where $\tau > 1$ units are shipped but only one unit arrives at destination. Fixed export cost f_x is assumed to be identical across firms but heterogeneous across products. Firms can learn from export (or simply share export distribution channel) to decrease the fixed export cost of added products.

Product Ladder for Export

Firms set price of the first product for export as:

$$p_{1,x}(\varphi) = \tau p_{1,d}(\varphi) = \frac{\tau}{\rho\varphi} \tag{22}$$

where subscript x denotes 'export' and d denotes 'domestic'. Firms export from the core product to fringe product. If $\pi_{1,x}(\varphi) < 0$, firm will not export; otherwise firm will export the first product and try to export the second product. Firm proceeds the product ladder until the firm cannot get extra profit from export the new product. Fixed export cost of added product is decreasing as the number of products due to 'learning effect' and 'export sharing'. I assume fixed export cost for $i - th$ product is:

$$f_{i,x} = \begin{cases} f_x & \text{if } i = 1 \\ (\beta^{i-1} + \gamma) f_x & \text{if } i \geq 2 \end{cases} \tag{23}$$

where $\beta + \gamma < 1$. The boundary of fixed export cost is γf_x. According to equation

(6), the profit from export of the $i-th$ product is:

$$\pi_{i,x}(\varphi) = \begin{cases} \frac{r_{1,x}(\varphi)}{\sigma} - f_x & \text{if } i = 1 \\ \frac{r_{i,x}(\alpha^{i-1}\varphi)}{\sigma} - \beta^{i-1} f_x - \gamma f_x = \beta^{i-1}\pi_{1,x}(\varphi) - \gamma f_x & \text{if } i \geq 2 \end{cases} \tag{24}$$

If $\pi_{i,x}(\varphi) \geq 0$, the $i-th$ product is exported. The number of products exported is determined as:

$$N_x(\varphi) = \begin{cases} 0 & \text{if } \pi_{1,x}(\varphi) < 0 \\ max\left\{i|\pi_{i,x}(\varphi) \geq 0 and \pi_{i+1,x}(\varphi) < 0\right\} & \text{if } \pi_{1,x}(\varphi) \geq 0 \end{cases} \tag{25}$$

The productivity cut-off for export is assumed as φ_x^* (also can be denoted as $\varphi_{1,x}^*$, hereafter $\varphi_{1,x}^* = \varphi_x^*$), and the productivity cut-off for exporting i product is assumed as $\varphi_{i,x}^*$. φ_x^* is determined by:

$$\pi_{1,x}(\varphi_x^*) = \frac{r_{1,x}(\varphi_x^*)}{\sigma} - f_x = \tau^{1-\sigma}\frac{r_{1,d}(\varphi_x^*)}{\sigma} - f_x = 0 \tag{26}$$

The productivity cut-off for domestic is still determined by $\frac{r_{1,d}(\varphi^*)}{\sigma} - f_d = 0$. So the relationship between the productivity cut-off for domestic market and export is denoted as:

$$\frac{\varphi_x^*}{\varphi^*} = \tau(\frac{f_x}{f_d})^{\frac{1}{\sigma-1}} \tag{27}$$

We assume $f_x > \tau^{1-\sigma} f_d$ as Melitz(2003) to assure that $\varphi_x^* > \varphi^*$. According to equation (13), there must be a productivity cut-off for domestic market φ_λ^* to allow $\varphi_x^* \epsilon [\varphi_\lambda^*, \varphi_{\lambda+1}^*)$.

$$\lambda = int[1 + \frac{ln(\tau^{1-\sigma} f_d / f_x)}{ln\beta}] \tag{28}$$

The firms that draw productivity lower than φ_x^* do not export and can produce λ products at most.

$\varphi_{i,x}^*$ $(i \geq 2)$ is determined by:

$$\pi_{i,x}(\varphi_{i,x}^*) = \beta^{i-1}\pi_{1,x}(\varphi_{i,x}^*) - \gamma f_x = \beta^{i-1}\tau^{1-\sigma}\frac{r_{1,d}(\varphi_{i,x}^*)}{\sigma} - \beta^{i-1} f_x - \gamma f_x = 0 \tag{29}$$

The relationship between productivity cut-off for export $i-products$ $(i \geq 2)$ firms and productivity cut-off for export is denoted as:

$$\frac{\varphi_{i,x}^*}{\varphi_x^*} = (\frac{\gamma + \beta^{i-1}}{\beta^{i-1}})^{\frac{1}{\sigma-1}} \tag{30}$$

Export-only Products

Firms can export added product with a lower fixed export cost than existing products. If a firm first draws a high productivity and manges a big number of products, the fixed export cost of some products may even lower than fixed cost to sell in domestic market. For these products, firm cannot get profit from domestic market but export only. According to equation (12) and (27), we can get:

$$\frac{\varphi_{i,x}^*}{\varphi_i^*} = (\gamma + \beta^{i-1})^{\frac{1}{\sigma-1}} \tau (\frac{f_x}{f_d})^{\frac{1}{\sigma-1}} \tag{31}$$

There exists the number θ, such that $\varphi_{\theta,x}^* \geq \varphi_\theta^*$ and $\varphi_{\theta+1,x}^* < \varphi_{\theta+1}^*$:

$$\theta = int[1 + \frac{ln(\tau^{1-\sigma} f_d / f_x - \gamma)}{ln\beta}] \tag{32}$$

If firms first draw productivity higher than φ_θ^*, the number of exported products is no less than number of products sold in domestic market $(N_x \geq N_d)$, otherwise $N_x \leq N_d$. There are firms that export all their products $(N_x = N_d)$, which need the conditions:

$$\begin{aligned} & \varphi_\xi^* < \varphi_{\xi,x}^* < \varphi_{\xi+1}^* \quad \text{if } \xi < \theta \\ or \quad & \varphi_{\zeta,x}^* < \varphi_\zeta^* < \varphi_{\zeta+1,x}^* \quad \text{if } \zeta \geq \theta \end{aligned} \tag{33}$$

The firms which draw productivity between $\varphi_{\xi,x}^*$ and $\varphi_{\xi+1}^*$ or between φ_ζ^* and $\varphi_{\zeta+1,x}^*$ export all their products as $N_x = N_d$. Solving equation (33), we can get $\theta_1 \leq \xi < \theta$ and $\theta \leq \zeta \leq \theta_2$, where:

$$\theta_1 = int[1 + \frac{ln(\tau^{1-\sigma} f_d / f_x - \gamma\beta)}{ln\beta}] \tag{34}$$

$$\theta_2 = int[1 + \frac{ln(\tau^{1-\sigma} f_d/f_x - \gamma/\beta)}{ln\beta}] \tag{35}$$

We assume $\gamma/\beta < \tau^{1-\sigma} f_d/f_x$ to assure θ, θ_1 and θ_2 all exist.

The firms which draw productivity lower than φ^* exit. The firms which draw productivity between φ^* and φ_x^* only serve the domestic market and can at most produce λ products. The firms draw a productivity higher φ_x^* export. Across exporters, low productive exporters export only a subset of their products while high productive exporters manage export-only products, thereby managing more products in export than in domestic market. Fig 2 gives how the number of products produced and exported are determined.

Aggregate Profit

A firm with drawn productivity $\varphi\epsilon[\varphi_{i,x}^*, \varphi_{i+1,x}^*)$ can export i products. The export of this multi-product firm can be represented by a single product firm with a productivity $(\sum_{j=1}^{i} \beta^{j-1})^{\frac{1}{\sigma-1}}\varphi$ and fixed export cost $[(\sum_{j=1}^{i} \beta^{j-1})^{\frac{1}{\sigma-1}} + (i-1)\gamma]f_x$. As $\frac{r_{1,x}(\varphi_x^*)}{\sigma} - f_x = 0$, so the total profit by a firm from export is determined as:

$$\pi_x(\varphi) = \begin{cases} 0 & \text{if } \varphi < \varphi_x^* \\ \left[\sum_{j=1}^{i} \beta^{j-1} k_x(\varphi) - (i-1)\gamma\right] f_x & \text{if } \varphi\epsilon[\varphi_{i,x}^*, \varphi_{i+1,x}^*) \end{cases} \tag{36}$$

where $k_x(\varphi) = (\varphi/\varphi_x^*)^{\sigma-1} - 1$. The average profit for export is denoted as:

$$\overline{\pi}_x = \frac{1}{1 - G(\varphi_x^*)} \sum_{i=1}^{\infty} \int_{\varphi_{i,x}^*}^{\varphi_{i+1,x}^*} \pi_x(\varphi) g(\varphi) d\varphi \tag{37}$$

Substituting equation (36) into (27), we can get:

$$\overline{\pi}_x = \frac{f_x}{1 - G(\varphi_x^*)} \sum_{i=1}^{\infty} \int_{\varphi_{i,x}^*}^{\varphi_{i+1,x}^*} \left[\sum_{j=1}^{i} \beta^{j-1} k_x(\varphi) - (i-1)\gamma\right] g(\varphi) d\varphi \tag{38}$$

If the average profit from domestic market is denoted as $\overline{\pi}_d$, which is determined by equation (18), the combined average profit from domestic market and export

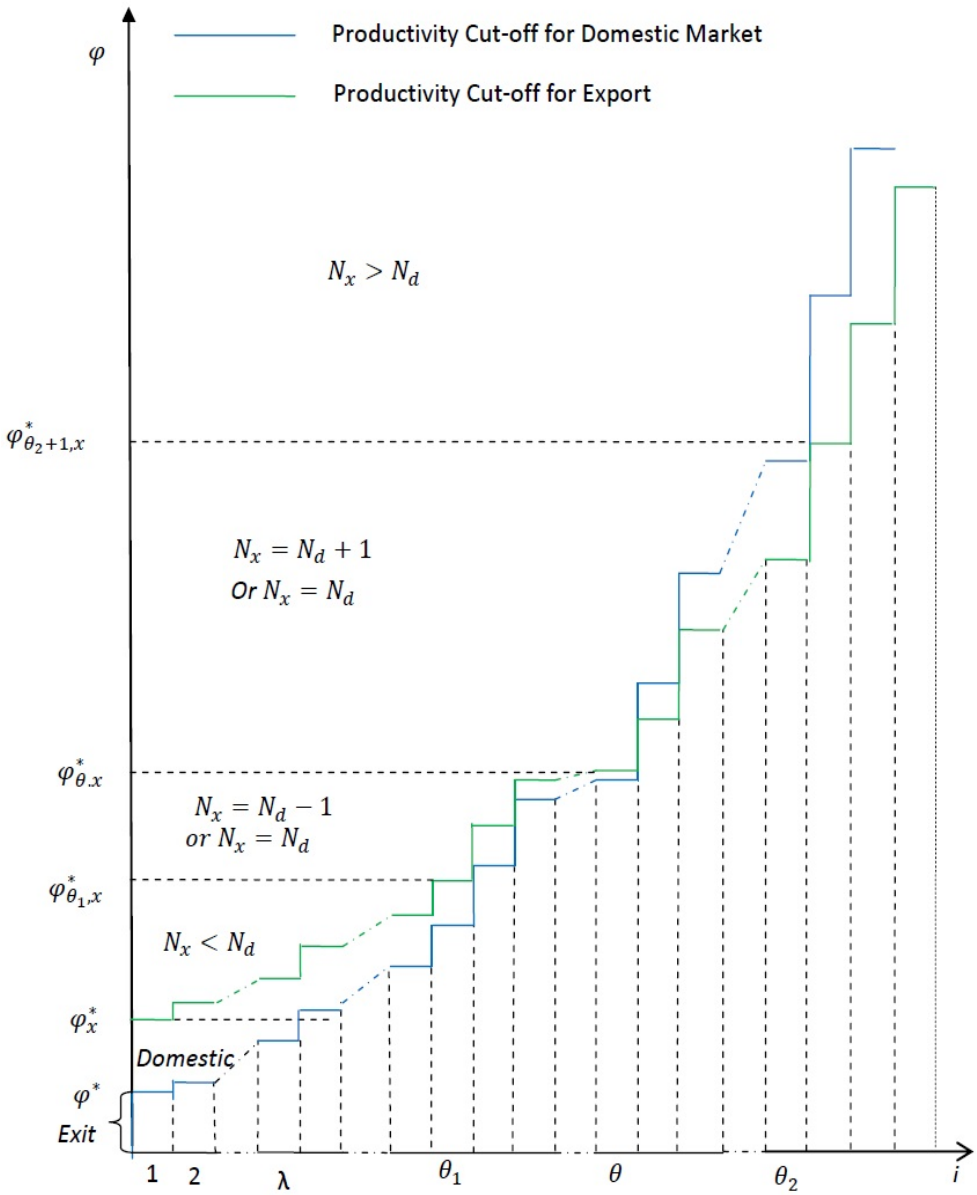

Fig 2: The number of products exported and product ladder

is determined as:

$$\overline{\pi} = \overline{\pi}_d + p_x \overline{\pi}_x \tag{39}$$

where $p_x = \frac{1-G(\varphi_x^*)}{1-G(\varphi^*)}$ is ex-post probability that active firms export, and it is also the fraction of export firms out of all active firms. Substituting equation (18) and (31) into (32), we can get the combined average profit as:

$$\overline{\pi} = \frac{f_d}{1-G(\varphi^*)} \sum_{i=1}^{\infty} \int_{\varphi_i^*}^{\varphi_{i+1}^*} \big[\sum_{j=1}^{i} \beta^{j-1} k(\varphi) - i\big] g(\varphi) d\varphi$$
$$+ \frac{f_x}{1-G(\varphi^*)} \sum_{i=1}^{\infty} \int_{\varphi_{i,x}^*}^{\varphi_{i+1,x}^*} \big[\sum_{j=1}^{i} \beta^{j-1} k_x(\varphi) - (i-1)\gamma\big] g(\varphi) d\varphi \tag{40}$$

5 Open Economy Equilibrium

In open economy, equilibrium is solely described as the set $\{\varphi^*, \varphi_x^*, P, R\}$. φ_i^* is functions of φ^* and $\varphi_{i,x}^*$ is functions of φ_x^*. All other variables are functions of the vector of these elements. The expected value of entry equals sunk cost. According to equation (16), we can get:

$$\frac{f_d}{\delta} \sum_{i=1}^{\infty} \int_{\varphi_i^*}^{\varphi_{i+1}^*} \big[\sum_{j=1}^{i} \beta^{j-1} k(\varphi) - i\big] g(\varphi) d\varphi$$
$$+ \frac{f_x}{\delta} \sum_{i=1}^{\infty} \int_{\varphi_{i,x}^*}^{\varphi_{i+1,x}^*} \big[\sum_{j=1}^{i} \beta^{j-1} k_x(\varphi) - (i-1)\gamma\big] g(\varphi) d\varphi = f_e \tag{41}$$

Theorem 5.1. There exists a unique equilibrium in open economy. See appendix 5.1 for proof.

The average revenue $\overline{r}$ is determined as:

$$\overline{r} = \frac{f_d}{1-G(\varphi^*)} \sum_{i=1}^{\infty} \int_{\varphi_i^*}^{\varphi_{i+1}^*} \sum_{j=1}^{i} \beta^{j-1} k(\varphi) g(\varphi) d\varphi$$
$$+ \frac{f_x}{1-G(\varphi^*)} \sum_{i=1}^{\infty} \int_{\varphi_{i,x}^*}^{\varphi_{i+1,x}^*} \sum_{j=1}^{i} \beta^{j-1} [k_x(\varphi) + 1] g(\varphi) d\varphi \tag{42}$$

So the number of incumbent firms M is determined by $M = R/\overline{r} = L/\overline{r}$. The

number of export firms is $M_x = p_x M$. In equilibrium, the number of firms that exit the market equals the number of successful entrants. $\delta M = (1 - G(\varphi^*))M_e$. Labor market clears as $L = R = L_p + L_e = L_p + \Pi$.

Theorem 5.2. There is a positive relationship between extensive margin (the number of exported products) and intensive margin of export (how much of a exported product).

This argument follows Bernard *et. al.* (2011). The more productive firms are larger exporters, not only because they can produce more in a given product but also because they can expand along extensive margin to export more products.

Theorem 5.3. Exposure to trade drives up the productivity cut-off(see appendix 5.3 for proof), thereby leading to :

1) least productive firms exit from the market;

2) low productive firms serve domestic market only, they drop least productive products and manage less products;

3) high productive firms become exporters and drop least productive products from domestic market, across exporters:

a) low productive exporters export a subset of the left products and manage less products;

b) middle productive exporters export all left products and switch domestic-dropped products to export-only products, and the higher productivity is, the more products are switched;

c) high productive exporters export all left products, switch all domestic-dropped products to export-only products, and further introduce new products as export-only products, they manage more products.

To simply and clearly show the effect from auturky to trade, I draw Fig 3 to depict all the above arguments. The curves N_d, N_d' and N_x denote the number of products sold in domestic market in autarky, the number of products sold in domestic market after trade and the number of products exported after trade. The

interaction circles denote the overlapping area as shown in Fig 2 between $\varphi^*_{\theta_1,x}$ and $\varphi^*_{\theta_2+1,x}$. The bold black curve depicts the number of products managed in autarky. The bold red curve depicts the number of products after exposure to trade.

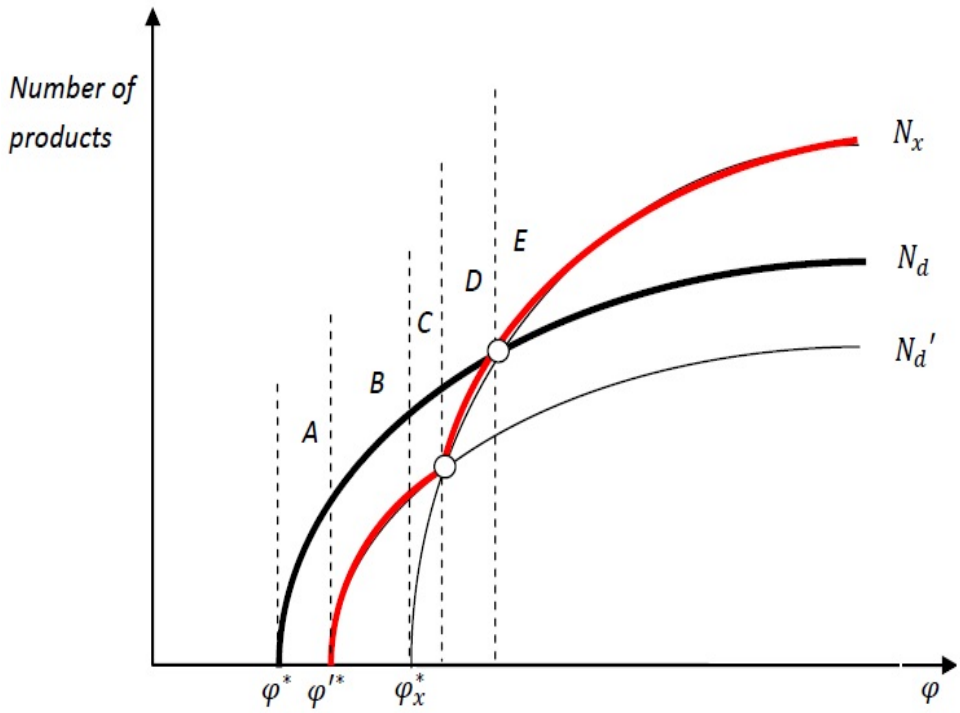

Fig 3: The extensive margins of firms

Exposure to trade gives an extra profit opportunity for high productive firms and increases the competition of labor market. The real wage rate is pushed up to force least productive firms out of market (A area). The higher real wage rate also force the least productive products exit from domestic market, shown as $N_d > N'_d$.. So low productive firms (B area) cannot afford the fixed export cost and serve domestic market only. All firms with productivity higher than φ^*_x become exporters. Low productive exporters (C area) drop least productive products from domestic market and export a subset of left products, shown as $N_d > N'_d > N_x$. Middle productive exporters (D area) drop the least productive products from domestic but some of these domestic-dropped products are exported. These middle productive exporters manage export-only products and have a higher extensive margin

of export than of domestic market, shown as $N_d > N_x > N'_d$. This is because that after trade firms can decrease the fixed export cost for added products. The fringe products of these middle productive exporters have a lower fixed export cost than fixed cost to sell in domestic market. High productive exporters (E area) drop least productive products from domestic market, but all of these domestic-dropped products are exported. These exporters even introduce new products as export-only products, shown as $N_x > N_d > N'_d$. High productive exporters can manage more products and can have a lower fixed export cost for the added fringe products. These products can not be sold in domestic market but only be exported. After trade, high productive exporters manage more products while other low productive firms manage less products, as shown by the bold black and red curves.

6 Trade Liberalization

The preceding analysis explores how firms decide the number of products produced and exported in equilibrium from autarky to trade. In this section, we try to study the impact of trade liberalization on extensive margin of export and of domestic market. Two such channels of trade liberalization are investigated: decrease in variable export cost τ and decrease in fixed export cost f_x. Both channels have similar effects on extensive margin of export.

Theorem 6.1. Decrease in fixed or variable export cost leads to :

1) least productive firms exit, and firms shrink from domestic market both along intensive margin and extensive margin;

2) more firms export, and exporters expand in export both along intensive margin and extensive margin;

3) more firms manage export-only products;

4) low productive firms manage less products but high productive firms manage more products.

Decease in fixed or variable export cost drives up productivity cut-off for domestic

market and reduces the productivity cut-off for export (see appendix 6.1 for proof). I use prime to denote the variables after decrease in fixed or variable export cost. Fig 4 depicts how the number of products in the equilibrium changes. N_d denotes the number of products exported and N_x denotes the number of products sold in domestic market. The black and red bold curves are the number of managed products.

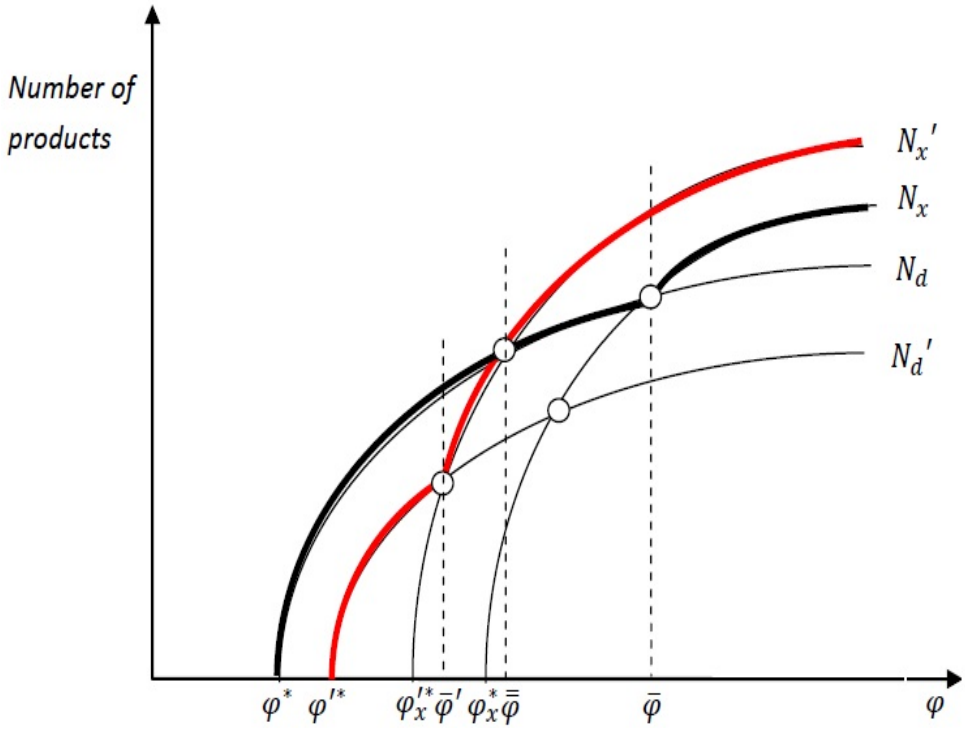

Fig 4: Decrease in trade cost

Due to opportunity for more profit from foreign market, the increased competition for labor drives up the real wage rate. The productivity cut-off for domestic market goes up from φ^* to $\varphi'*$, forcing firms with productivity between φ^* and $\varphi'*$ out of market. The higher real wage rate make firms sell less in domestic market and force firms drop the least productive products from domestic market (curve N_d' is below N_d). This means that firms shrink from domestic market both along intensive margin and extensive margin. For exporters, even though real wage rate is higher, they can still benefit from decreased fixed or variable export cost. The

productivity cut-off for export is lower, inducing more firms into export. Exporters export more of a given product and export more products (curve N_x' is above N_x).

Before trade liberalization, the exporters with productivity higher than $\overline{\varphi}$ manage export-only products ($N_x > N_d$). After trade liberalization, the lower productive exporters with productivity between $\overline{\varphi}'$ and $\overline{\varphi}$ can manage export-only products. Similar to the effect from autarky to trade, some of domestic-dropped products are switched to export-only products for firms with productivity from $\overline{\varphi}'$ to $\overline{\overline{\varphi}}$. All firms with productivity lower than $\overline{\overline{\varphi}}$ manage less products due to trade liberalization. For the exporters with productivity higher than $\overline{\overline{\varphi}}$, they switch all the domestic-dropped products to export-only products, and introduce new export-only products. These firms manage more products after trade liberalization. This argument is quite different from the previous literatures, which is derived from the assumption of decreasing fixed export cost for added products.

7 Conclusion

Multi-product firms are getting more and more attention in international trade. This paper builds a general equilibrium model to allow for heterogeneity in productivity and fixed export cost. Firms add a new product with a lower productivity and export a new product with a lower fixed export cost. The model takes account 'learning-by-export' effect with a decreasing fixed export cost across products. The model captures export-only products, managed by high productive exporters which has great potential to expand the product ladder and decrease fixed export cost. The export-only products are relevant especially in developing countries but not explained theoretically in trade under framework of multi-product firms. The paper gives an implication to try to model these products.

The model provides some interesting and even surprising results. Transition from autarky to trade and decrease in fixed or variable fixed cost both force the least productive firms out and kick the least productive products out of domestic market, however which can be switched to export by high productive firms. Firms shrink in

domestic market both along intensive and extensive margin, and expand in export both along intensive margin and extensive margin. The increase in export extensive margin is reflected in exporting more left products for low productive exporters, switching some domestic-dropped products to export-only products for middle productive exporters and not only switching all domestic-dropped products but inducing new products as export-only products. Similar to the effect from autarky to trade, for low productive firms, trade liberalization decreases the number of products managed; but for high productive firms otherwise.

Our framework where firms endogenously decide the number of products produced and exported is highly tractable. One further extension might be to introduce multiple countries with different fixed export cost, which would add another export margin for firms to adjust under exposure to trade and trade liberalization. Another extension might be to introduce a increasing productivity across products to include ' quality ladder' within firms. Multi-product firms climb quality ladder from low productive product to high productive product, which would be a total novelty in research in trade.

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Appendix

Appendix 3.1. Existence of Closed Economy Equilibrium

We assume a function $f(\varphi^*)$ such that:

$$f(\varphi^*) = \sum_{i=1}^{\infty} \int_{\varphi_i^*}^{\varphi_{i+1}^*} \big[\sum_{j=1}^{i} \beta^{j-1} k(\varphi) - i\big] g(\varphi) d\varphi$$

We prove the existence of equilibrium by showing that $f(\varphi^*)$ is monotonically decreasing from infinity to zero as φ^* approaches to infinity from zero. The derivative of $f(\varphi^*)$ is:

$$\begin{aligned} f'(\varphi^*) = & \sum_{i=1}^{\infty} \int_{\varphi_i^*}^{\varphi_{i+1}^*} \sum_{j=1}^{i} \beta^{j-1} \frac{dk(\varphi)}{d\varphi^*} g(\varphi) d\varphi + \\ & \sum_{i=1}^{\infty} \left\{ \big[\sum_{j=1}^{i} \beta^{j-1} k(\varphi_{i+1}^*) - i\big] g(\varphi_{i+1}^*) \frac{d\varphi_{i+1}^*}{d\varphi^*} - \big[\sum_{j=1}^{i} \beta^{j-1} k(\varphi_i^*) - i\big] g(\varphi_i^*) \frac{d\varphi_i^*}{d\varphi^*} \right\} \end{aligned}$$

To rewrite the above equation, we can get:

$$f'(\varphi^*) = \sum_{i=1}^{\infty} \int_{\varphi_i^*}^{\varphi_{i+1}^*} \sum_{j=1}^{i} \beta^{j-1} \frac{dk(\varphi)}{d\varphi^*} g(\varphi) d\varphi + \sum_{i=1}^{\infty} \big[1 - \beta^{i-1} k(\varphi_i^*)\big] g(\varphi_i^*) \frac{d\varphi_i^*}{d\varphi^*}$$

where $\beta^{j-1} k(\varphi_i^*) = \beta^{j-1} (\frac{\varphi_i^*}{\varphi^*})^{\sigma-1} = 1$. So The derivative of $f(\varphi^*)$ is:

$$f'(\varphi^*) = \sum_{i=1}^{\infty} \int_{\varphi_i^*}^{\varphi_{i+1}^*} \sum_{j=1}^{i} \beta^{j-1} \frac{dk(\varphi)}{d\varphi^*} g(\varphi) d\varphi$$

where $\frac{dk(\varphi)}{d\varphi^*} < 0$. So $f'(\varphi^*) < 0$. As φ^* approaches to zero (infinity), $k(\varphi)$ approaches to infinity (zero), as a result $f(\varphi^*)$ approaches to infinity (zero). $f(\varphi^*)$ is monotonically decreasing from infinity to zero. There must be a unique φ^* such that:

$$f(\varphi^*) = \frac{\delta f_e}{f_d}$$

There exists a unique equilibrium in closed economy.

Appendix 5.1. Existence of Open Economy Equilibrium

We assume a function $f(\varphi^*)$ such that:

$$f(\varphi^*) = f_1(\varphi^*) + f_2(\varphi_x^*)$$

where

$$f_1(\varphi^*) = f_d \sum_{i=1}^{\infty} \int_{\varphi_i^*}^{\varphi_{i+1}^*} \big[\sum_{j=1}^{i} \beta^{j-1} k(\varphi) - i\big] g(\varphi) d\varphi$$

$$f_2(\varphi_x^*) = f_x \sum_{i=1}^{\infty} \int_{\varphi_{i,x}^*}^{\varphi_{i+1,x}^*} \big[\sum_{j=1}^{i} \beta^{j-1} k_x(\varphi) - (i-1)\gamma\big] g(\varphi) d\varphi$$

We prove the existence of equilibrium by showing that $f(\varphi^*)$ is monotonically decreasing from infinity to zero as φ^* approaches to infinity from zero. According to appendix 3.1, $f_1'(\varphi^*) < 0$. The derivative of $f_2(\varphi_x^*)$ is:

$$f_2'(\varphi_x^*) = \sum_{i=1}^{\infty} \int_{\varphi_{i,x}^*}^{\varphi_{i+1,x}^*} \sum_{j=1}^{i} \beta^{j-1} \frac{dk_x(\varphi)}{d\varphi^*} g(\varphi) d\varphi +$$

$$\sum_{i=1}^{\infty} \left\{ \big[\sum_{j=1}^{i} \beta^{j-1} k_x(\varphi_{i+1,x}^*) - (i-1)\gamma\big] g(\varphi_{i+1,x}^*) \frac{d\varphi_{i+1,x}^*}{d\varphi^*} - \big[\sum_{j=1}^{i} \beta^{j-1} k(\varphi_{i,x}^*) - (i-1)\gamma\big] g(\varphi_{i,x}^*) \frac{d\varphi_{i,x}^*}{d\varphi^*} \right\}$$

To rewrite the above equation, we can get:

$$f_2'(\varphi_x^*) = \sum_{i=1}^{\infty} \int_{\varphi_{i,x}^*}^{\varphi_{i+1,x}^*} \sum_{j=1}^{i} \beta^{j-1} \frac{dk_x(\varphi)}{d\varphi^*} g(\varphi) d\varphi - k_x(\varphi_x^*) g(\varphi_x^*) \frac{dk_x(\varphi_x^*)}{d\varphi^*} +$$

$$\sum_{i=2}^{\infty} \big[\gamma - \beta^{i-1} k(\varphi_{i,x}^*)\big] g(\varphi_{i,x}^*) \frac{d\varphi_{i,x}^*}{d\varphi^*}$$

where $\frac{dk_x(\varphi_x^*)}{d\varphi^*} < 0$, $k_x(\varphi_x^*) = 0$, $\gamma - \beta^{i-1} k(\varphi_{i,x}^*) = (\gamma + \beta^{i-1})(1 - f_x/\tau^{1-\sigma} f_d) < 0$ and $\frac{d\varphi_{i,x}^*}{d\varphi^*} > 0$. So we can get $f_2'(\varphi_x^*) < 0$. Till now, we prove that:

$$f'(\varphi^*) < 0$$

As φ^* approaches to zero (infinity), $k(\varphi)$ approaches to infinity (zero), as a result $f(\varphi^*)$ approaches to infinity (zero). $f(\varphi^*)$ is monotonically decreasing from infinity to zero. There must be a unique φ^* such that:

$$f(\varphi^*) = \delta f_e$$

There exists a unique equilibrium in open economy.

Appendix 5.3. Effect of Exposure to Trade

We use prime to denote the state after exposure to trade. φ^* and φ_i^* denote productivity cut-off in autarky, and φ'^* and $\varphi_i'^*$ denote productivity cut-off for domestic in open economy. According to appendix 3.1 and 5.1, v_e and v_e' are decreasing functions and $v_e' > v_e$. The curve v_e is below e v_e'. So in equilibrium, $\varphi'^* > \varphi^*$ and $\varphi_i'^* > \varphi_i^*$.

Appendix 6.1. Effect of Trade liberalization

We use prime to denote the state after trade liberalization.

1)Decrease in variable export cost.

According to appendix 5.1, φ^* is determined by $F(\varphi^*, \tau) = f_1(\varphi^*) + f_2(\varphi_x^*) = \delta f_e$. So:

$$\frac{d\varphi^*}{d\tau} = -\frac{\partial F(\varphi^*, \tau)/\partial \tau}{\partial F(\varphi^*, \tau)/\partial \varphi^*}$$

From appendix 5.1, we can get

$$\frac{\partial F(\varphi^*, \tau)}{\partial \tau} = \sum_{i=1}^{\infty} \int_{\varphi_{i,x}^*}^{\varphi_{i+1,x}^*} \sum_{j=1}^{i} \beta^{j-1} \frac{dk_x(\varphi)}{d\tau} g(\varphi) d\varphi + \sum_{i=2}^{\infty} \left[\gamma - \beta^{i-1} k(\varphi_{i,x}^*)\right] g(\varphi_{i,x}^*) \frac{d\varphi_{i,x}^*}{d\tau}$$

where $\frac{dk_x(\varphi_x^*)}{d\tau}$ and $\frac{d\varphi_{i,x}^*}{d\tau}$ are partial derivatives in respect to τ. So $\frac{dk_x(\varphi_x^*)}{d\tau} < 0$ and $\frac{d\varphi_{i,x}^*}{d\tau} < 0$. We can get $\frac{\partial F(\varphi^*, \tau)}{\partial \tau} < 0$. $\frac{\partial F(\varphi^*, \tau)}{\partial \varphi^*} < 0$ as appendix 5.1. So

$$\frac{d\varphi^*}{d\tau} < 0$$

As variable export cost decreases, φ^* and φ_i^* increase.

φ_x^* is determined by $\Gamma(\varphi_x^*, \tau) = f_1(\varphi^*) + f_2(\varphi_x^*) = \delta f_e$. So:

$$\frac{d\varphi_x^*}{d\tau} = -\frac{\partial\Gamma(\varphi_x^*, \tau)/\partial\tau}{\partial\Gamma(\varphi_x^*, \tau)/\partial\varphi_x^*}$$

From appendix 3.1, we can get

$$\frac{\partial\Gamma(\varphi_x^*, \tau)}{\partial\tau} = \sum_{i=1}^{\infty} \int_{\varphi_i^*}^{\varphi_{i+1}^*} \sum_{j=1}^{i} \beta^{j-1} \frac{dk(\varphi)}{d\tau} g(\varphi) d\varphi$$

where $\frac{dk(\varphi)}{d\tau}$ is partial derivative in respect to τ. Partial derivative $\frac{dk(\varphi)}{d\tau} > 0$. So $\frac{\partial\Gamma(\varphi_x^*, \tau)}{\partial\tau} > 0$. $\frac{\partial F(\varphi_x^*, \tau)}{\partial\varphi_x^*} < 0$. So

$$\frac{d\varphi_x^*}{d\tau} > 0$$

As variable export cost decreases, φ_x^* and $\varphi_{i,x}^*$ decrease.

2)Decrease in fixed export cost. The proving process is similar to above. The result is:

$$\frac{d\varphi^*}{df_x} < 0 \qquad \frac{d\varphi_x^*}{df_x} > 0$$

As variable export cost decreases, φ^* and φ_i^* increase while φ_x^* and $\varphi_{i,x}^*$ decrease.