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INEQUALITY AND PARALLEL IMPORTS IN INTERNATIONAL TRADE

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Abstract

The aim of this paper is to develop a model of international trade in which an importer's distribution of income determines the extensive margin of trade. Indivisibilities in consumption allow for a role of per capita income, and distribution effects play a role on firms' incentives to introduce new products.

Distribution of income affects the prices that a monopolist can charge and therefore the product diversity available in the economy. This paper investigates whether and how international trade patterns change when countries with different income distributions engage in trade, and whether the level of inequality helps in explaining which patterns of trade materialize, and which do not. I also observe how gains from trade of different consumers respond when countries allow parallel trade.

Keywords: Income Inequality; International Trade; Parallel Imports

JEL classification: F10; F12; D63

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Contents

List of Tables

List of Figures

List of abbreviations

b.c. budget constraint
i.e. id est
iff. if and only if
l.h.s. left hand side
r.c. resource constraint
r.h.s. right hand side
s.t. such that
w.r.t. with respect to
z.p.c. zero profit condition

1 Introduction

In this paper I develop a model of international trade to investigate how a country's income distribution impacts consumption on the extensive margin. Standard international trade theory assumes homothetic preferences across individuals[2], which imply that the proportion in which consumers buy goods only depends on relative prices and not on income. This specification has allowed theorists to concentrate on the supply-side explanations of international patterns of trade. Although homothetic preferences are a convenient modeling technique because of their nice aggregation properties, they do not allow to investigate demand factors. The latter are however always complementary to supply-side considerations. Linder (1961) was the first one to hypothesize that demand conditions could be a major determinant of patterns of trade. In the well known Linder Hypothesis, he asserts that countries trade more intensively with one another, the more similar their demand structure is. Mitra & Trindade (2005) show that income distribution and income per capita are important determinants of international trade flows. They classify goods as either luxuries or necessities and use a gravity model to illustrate how higher inequality increases imports of the former and decreases imports of the latter. Their result stems from the fact that in reality tastes cannot be considered strictly homothetic. Hunter & Markusen (1988) and Hunter (1991) estimate that the intercept of the linear income expansion path is significantly different from zero, which implies that income elasticities are not constant across goods and that income is a determinant of aggregate demand. Following these key results, a new strand of literature has emerged investigating the relationship between per-capita income, inequality and patterns of trade[3]. This paper belongs to this strand of literature insofar as it aims at disentangling the impact of income inequality on trade flows.

More recent research has focused instead on the importance of zeroes in trade flows[4]. Also in this case, supply-side considerations have been preponderant. Haveman & Hummels (2001) link zero bilateral trade values to incomplete specialization specifications in a gravity model. Melitz (2003) concentrates on the role of productivity on heterogeneous firms' incentives to trade. Other standard explanations for export zeroes involve either too high cost in the exporting country or too small market size in the importing one. The bulk of research in this field completely overlooks possible demand-side effects since it assumes that individuals have constant elasticity of

[2]Krugman (1980), Melitz (2003), Chaney (2008)

[3]Recent papers on the topic include Bekkers et al. (2012), Dalgin et al. (2008), Foellmi et al. (2010), Bernasconi (2013)

[4]Evenett & Venables (2002), Haveman & Hummels (2004), Anderson & van Wincoop (2004), Helpman et al. (2008)

substitution among goods. A clear exception is made by Foellmi et al. (2013), who investigate how differences in per capita income across countries help in explaining which patterns of trade materialize and which do not. In their model, price discrimination arises due to demand-side effects: differences in per capita incomes across countries allow firms to charge different prices in different markets. The authors recognize that with large price differences across countries, threats of international arbitrage might influence firms' trade decisions. Consider a German firm willing to export its products to Romania. Suppose the firm decides to charge a price equal to the marginal willingness to pay of consumers in both countries, so that it sells the good at a higher price in Germany than in Romania.[5] If price differences are sufficiently large w.r.t. transportation cost, international arbitrage opportunities can arise: arbitrageurs can purchase the good in Romania, ship it and sell it on the German market at a slightly lower price than the one charged by the producer. The latter foresees the threat of parallel trade and has two available strategies: *i)* Export the product to Romania but sell at a lower (arbitrage-preventing) price in Germany, in order to avoid the emergence of international arbitrage opportunities; *ii)* Abstain from exporting, thus exploiting the higher marginal willingness to pay of German consumers but serving only the home market. The above discussion indicates that producers ultimately face a tradeoff between prices and market size: those that are active on the global market must charge lower prices in rich countries, whereas those that are only active in rich countries can charge higher price but serve a smaller market. This paper extends the simple trade model developed by Foellmi et al. (2013) by introducing income inequality in one country. Firms are homogeneous and market entry cost are fixed, which restricts the scope of the model to demand-side considerations. Instead of investigating trade-flows between a rich and a poor country, I concentrate on the patterns of trade between an unequal and an egalitarian country having equal aggregate endowments. I assume goods are indivisible and consumed in unit quantities only. This allows me to concentrate on the extensive margin of consumption only, in a setting that is both specular and complementary to the one of standard CES assumption. I introduce inequality in a very stylized way: a fraction of individuals in the population is poor, the remaining being rich. This setting might seem overly simplistic, but it provides a tractable framework to disentangle the effects of inequality on both trade flows and parallel trade incentives. Indeed, even these small deviations from the standard model bring about quite some unexpected dynamics.

First, I show that when an egalitarian and an unequal country engage in trade,

[5] As a result of the utility maximization problem, the higher the income of consumers is, the higher is the willingness to pay for goods. This means that consumers in a rich country are willing to pay higher prices for a good than consumers in a poorer country.

all firms decide to export their product but they do not exploit all strategies available to them. Different equilibria arise depending on the level of income inequality in the unequal country.

Second, I solve the general equilibrium allowing for parallel trade. In this setting, firms no longer all decide to export: different partial equilibria arise, which still depend on the level of inequality in the unequal country. I show that considerations on welfare effects of trade change when I allow for the presence of parallel trade threats. When countries allow parallel trade, the distribution of gains from trade across types of consumers changes.

2 Autarky

I first investigate the autarky equilibrium in countries with an equal distribution of income. I then allow for differences in income distribution and investigate the autarky equilibrium in this case. This first step allows to disentangle the effect of distribution on product variety and mark-ups. The economy setting is the same in both kind of countries. Consumers either purchase one unit of a particular product or do not purchase it at all. This specification implies that any good j can only be consumed in discrete amounts so that $c(j) \epsilon \{0,1\}$. Every firm produces one good only, all goods are symmetric s.t. $p(j) = p$ and all firms are identical and use an homogeneous technology for every good. Labor is the only production factor, its market is competitive and the wage is W. Production requires fixed cost F and variables cost $1/a$ per unit of labor. The total cost of producing good j is $F + q(j)/a$. A consumer with marginal utility of income equal to λ will purchase a good if and only if the utility derived from purchasing it, $u(1) - u(0)$, exceeds the utility-adjusted price λp.

2.1 Equal income distribution

All individuals are identical and supply $l = 1$ units of labor. The aggregate labor supply is equal to the population size L. The marginal utility of income is λ s.t. people only buy a unit of the good if $1 \geq \lambda p$. Since all firms and households are identical, the equilibrium is symmetric and characterized by the following conditions:

Zero Profit Condition $FW = (p - \frac{W}{a})L$ which implies that the wage is going to be equal to:

$$W = p\frac{aL}{aF + L}$$

If, without loss of generality, I set price as the numéraire s.t. $p = 1$, the wage is:

$$W = \frac{aL}{aF + L}$$

Resource Constraint $L = N(F + \frac{L}{a})$, which ensures that there is full employment and implies that the number of active firms (and hence the number of available varieties in the market) is given by:

$$N = \frac{aL}{aF + L}$$

Budget Constraint $W = Np$, which is trivially satisfied $\forall$ W, p s.t. the other two constraints above hold.

The ratio of price over marginal cost is called mark up. Given marginal costs of W/a, the mark up in this economy is:

$$\mu = \frac{aF + L}{L}$$

2.2 Unequal income distribution

I introduce inequality as arising from different labor endowments. As before, aggregate labor supply is equal to the population size L. This time though, a fraction β of the population is poor and each poor individual is endowed with $l_P = \theta < 1$ units of labor. The remaining $1 - \beta$ individuals are rich and endowed with $l_R = \frac{1-\beta\theta}{1-\beta} > 1$ units of labor each. We can verify that the aggregate supply of labor is: $\theta\beta L + (1 - \theta\beta L) = L$. I plot the Lorenz curve for a country with $\beta = 0.5$ and $\theta = 0.6$ in figure 1. The lower dotted line shows what happens when beta increases to 0.7, the upper one shows the result of an increase in θ to 0.8. The higher the β and/or the lower the θ, the higher the level of income inequality in the country.

I define mass consumption goods the ones that both rich and poor individuals consume. In contrast, exclusive (or élite) goods are those that only the rich individuals can afford. The size of the mass consumption sector and the structure of prices characterize the asymmetry in the economy. Given lower labor endowments, poor individuals have a higher marginal utility of income, and hence a lower willingness to pay, than rich individuals. This result implies that every firm can chose among two courses of action:
1. Serve the entire population at the (lower) price $p_P = 1/\lambda_P$ and make profits equal to $\pi = L(\frac{1}{\lambda_P} - \frac{W}{a})$.
2. Serve the rich only at the (higher) price $p_R = 1/\lambda_R$ and make profits equal to $\pi = (1 - \beta)L(\frac{1}{\lambda_R} - \frac{W}{a})$.

Firms face a tradeoff between a high price and a greater market size. In equilibrium however, both types of goods must be produced. If this was not the case and all firms charged the lower price p_P and sold their goods to the whole population, all goods would be priced below rich individuals' willingness to pay. In such a case, the rich would not spend their whole income and they would have an infinitely large willingness to pay for additional products. An incentive would arise for firms to enter the market for exclusive goods and charge very high prices. As a result, in equilibrium, the following conditions must be satisfied:

Zero Profit Condition together with the assumption of free entry, this condition implies that the profit from the two above-specified strategies coincide:

$$(1-\beta)L\Big(\frac{1}{\lambda_R}-\frac{W}{a}\Big)=L\Big(\frac{1}{\lambda_P}-\frac{W}{a}\Big)=WF \tag{1}$$

hence $p_R=\frac{1}{\lambda_R}=\frac{WF}{(1-\beta)L}+\frac{W}{a}$.

Budget Constraint must hold for both poor and rich individuals and it is respectively:

$$N_P\frac{1}{\lambda_P}=W\theta \tag{2}$$

and

$$N_P\frac{1}{\lambda_P}+(N_R-N_P)\frac{1}{\lambda_R}=W\frac{(1-\beta\theta)}{(1-\beta)}$$

Where N_P is the number of mass consumption goods and N_R is the total number of goods the rich purchase.

Resource Constraint ensures full employment and is trivially satisfied when the two conditions above are met:

$$L=(N_R-N_P)\Big(F+\frac{(1-\beta)L}{a}\Big)+N_P\Big(F+\frac{L}{a}\Big)$$

Without loss of generality, I choose the price of mass consumption goods $p_P=1/\lambda_P$ as numéraire. From equation 1 and 2 I get respectively $L\Big(1-\frac{W}{a}\Big)=WF\Rightarrow W=\frac{aL}{L+aF}$ and $N_P=W\theta$. Putting the two together I obtain the number of firms active in the production of mass consumption goods:

$$N_P=\frac{aL\theta}{L+aF}$$

The r.c. can be rearranged in the following way:

$$aL=N_R(aF+(1-\beta)L)+\frac{aL\theta}{L+aF}\beta L$$

$$aL\Big(\frac{L+aF-\theta\beta L}{L+aF}\Big)=N_R\big(aF+(1-\beta)L\big)$$

$$N_R=\frac{aL}{aF+L}\frac{aF+(1-\theta\beta)L}{aF+(1-\beta)L}$$

The last equation shows the equilibrium product diversity, i.e. the number of goods purchased by rich people in equilibrium. The number of firms engaged in the production of exclusive goods is N_R-N_P.

The mark ups for mass-consumption and exclusive goods are calculated in the same way as above and they are respectively:

$$\mu_P = \frac{1}{\frac{W}{a}} = a\frac{aF+L}{aL} = \frac{aF+L}{L}$$

$$\mu_R = \frac{p_R}{\frac{W}{a}} = \Big[\frac{WF}{(1-\beta)L} + \frac{W}{a}\Big]\frac{a}{W} = \frac{aF+(1-\beta)L}{(1-\beta)L}$$

μ_P only depends on market size and technology. Since both elements are the same as for the equal income distribution case of the previous paragraph, the μ_P is equal to the mark up calculated in that case. Instead, μ_R also depends on the share of rich individuals in the economy. In all cases, the bigger the market in which a good is sold, the lower the mark-up for the firm.

The analysis above shows that the structure of prices is affected by the presence of inequality ($p_R > p > p_P$) and the same holds true for equilibrium product diversity ($N_R > N > N_P$).

3 Trade between a country with equal and one with unequal income distribution

I can now proceed to analyze the equilibrium in a world economy in which two countries with equal population size L and aggregate income Y but different income distributions trade with each other. On the wake of the previous discussion, I consider that country U has an unequal income distribution deriving from differing labor endowments, whereas country E does not display any income inequality. Superscript letters are used to denote country-specific variables. Since at equilibrium prices are equal to the willingness to pay of individuals, in each of the two countries the price of imported goods must equal the one of home-produced goods. When the two countries open up to trade, firms in U can decide to serve either the rich or the poor at home and export to all individuals in E. Firms in E instead serve all individuals at home and can decide to export either to the rich or to everybody in U. Shipping the goods from the country of origin to the country of destination costs $\tau \geq 1$ per unit of consumption goods, hence marginal costs are higher for goods sold abroad.[6] In order for full-trade to be an equilibrium, the following conditions must be satisfied:

Resource Constraint must be satisfied in both countries in order ensure full employment and balance of trade equilibrium. For country U:

$$\beta\theta L + (1-\beta\theta)L = (N_R^U - N_P^U)\Big(\frac{aF+(1-\beta)L+\tau L}{a}\Big) + N_P^U\Big(\frac{aF+\tau L+L}{a}\Big)$$

[6]From now on I refer to τ as either iceberg cost or transportation cost.

$$aL = N_R^U[aF + (1-\beta)L + \tau L] + N_P^U \beta L$$

for country E:

$$L = (N_R^E - N_P^E)\Big(\frac{aF + L + (1-\beta)L\tau}{a}\Big) + N_P^E\Big(\frac{aF + L + \tau L}{a}\Big)$$

$$aL = N_R^E[aF + (1-\beta)L + \tau L] + N_P^E \beta L$$

Where N_P^U is the number of firms that serve everybody in U, $N_R^U - N_P^U$ ($N_R^E - N_P^E$) is the number of firms that serve the rich only in U. The same specification holds for variables referring to country E. The number of goods available to poor individuals in U is $N_P^U + N_P^E$; the number of goods available to rich individuals in U is $N_R^U + N_R^E$ and the number of goods available for consumers in E is also $N_R^U + N_R^E$. On this setting, all firms that produce a good at home also export it.

Budget Constraint hold for every type of individuals. Poor individuals in U earn income equal to $W^U\theta$ and purchase $N_P^U + N_P^E$ products at price $p_P = 1/\lambda_P$. Rich individuals in U earn income $W^U\frac{1-\beta\theta}{1-\beta}$ and purchase $N_P^U + N_P^E$ products at price $p_R = 1/\lambda_R$. Finally, E individuals earn[7] $W^E 1$ and purchase $N_P^U + N_P^E$ goods at price $p = 1/\lambda$.

Zero Profit Conditions must hold for each firm, whatever the strategy it decides to pursue. In this case, 4 categories must be distinguished:

1. Firms in U serving the rich only and exporting:

$$\frac{1}{\lambda_R}(1-\beta)L + \frac{1}{\lambda}L = W^U F + W^U\Big[\frac{(1-\beta)L + \tau L}{a}\Big]$$

2. Firms in U serving everybody and exporting:

$$\frac{1}{\lambda_P}L + \frac{1}{\lambda}L = W^U F + W^U\Big[\frac{L + \tau L}{a}\Big]$$

3. Firms in E exporting to the rich in U only:

$$\frac{1}{\lambda_R}(1-\beta)L + \frac{1}{\lambda}L = W^E F + W^E\Big[\frac{(1-\beta)L\tau + L}{a}\Big]$$

4. Firms in E exporting to everybody in U:

$$\frac{1}{\lambda_P}L + \frac{1}{\lambda}L = W^E F + W^E\Big[\frac{L\tau + L}{a}\Big]$$

Both countries serve both the rich and everybody in U as long as firms derive the same profit whether they produce mass consumption or exclusive goods. This translates into the following conditions for country U and country E respectively:

$$\frac{1}{\lambda_R}(1-\beta)L + \frac{1}{\lambda}L - W^U\Big[\frac{(1-\beta)L + \tau L}{a}\Big] = W^U F = \frac{1}{\lambda_P}L + \frac{1}{\lambda}L - W^U\Big[\frac{L + \tau L}{a}\Big]$$

[7] In Eall individuals are endowed with one unit of labor

and

$$\frac{1}{\lambda_R}(1-\beta)L+\frac{1}{\lambda}L-W^E\Big[\frac{(1-\beta)L\tau+L}{a}\Big]=W^EF=\frac{1}{\lambda_P}L+\frac{1}{\lambda}L-W^E\Big[\frac{L\tau+L}{a}\Big]$$

The r.h.s. of the two equations coincide (for a given wage level) whereas the l.h.s. differs: these two conditions cannot coexist. Solving the last two equation for p_R we get respectively:

$$\frac{1}{\lambda_R}=\frac{1}{1-\beta}-\frac{\beta}{1-\beta}\frac{W^U}{a}$$

and

$$\frac{1}{\lambda_R}=\frac{1}{1-\beta}-\frac{\beta}{1-\beta}\frac{\tau W^E}{a}$$

In equilibrium the price for exclusive goods must be the same, whether they are produced at home or imported, which implies the following condition for the wage in the two countries: $W^U=W^E\tau$. This relation however cannot hold at equilibrium: if we substitute for W^U in the z.p.c.s above, not all of them are satisfied. In this model, there is no full trade equlibrium in which firms pursue all strategies available to them. At this point, there are four candidate equilibria in this economy:

3.1 Firms in *U* only produce mass consumption goods and export, firms in *E* export both exclusive and mass consumption goods

The equilibrium conditions are the same as before, and they take the following form:

Resource Constraint Must be satisfied for both countries. I denote by N^U the number of firms in U that produce mass consumption goods in U and export, N_P^E the number of firms in E that export mass consumption goods and N_R^E the number of imported goods consumed by rich individuals in U. For country U we have:

$$L=N^U\Big(\frac{aF+L+\tau L}{a}\Big)$$

For country E the condition is:

$$L=(N_R^E-N_P^E)\Big(\frac{aF+L+(1-\beta)L\tau}{a}\Big)+N_P^E\Big(\frac{aF+L+L\tau}{a}\Big)$$

Budget Constraint For poor and rich individuals in U it must be that $W^U\theta=1/\lambda_P(N^U+N_P^E)$ and $W^U\frac{1-\beta\theta}{1-\beta}=\frac{1}{\lambda_R}(N_R^E-N_P^E)+\frac{1}{\lambda_P}(N^U+N_P^E)$ respectively. For individuals in E, $W^E=1/\lambda(N_R^E+N^U)$ holds.

Zero Profit Conditions Must hold for each firm and for all strategies available to it in this equilibrium.

1. Firms in U serving everybody at home and exporting:

$$\frac{1}{\lambda_P}L + \frac{1}{\lambda}L = W^U F + W^U \Big(\frac{L+\tau L}{a}\Big) \tag{3}$$

2. Firms in E exporting to everybody in country U:

$$\frac{1}{\lambda_P}L + \frac{1}{\lambda}L = W^E F + W^E \Big(\frac{\tau L + L}{a}\Big) \tag{4}$$

3. Firms in E exporting to the rich in U only:

$$\frac{1}{\lambda_R}(1-\beta)L + \frac{1}{\lambda}L = W^E F + W^E \Big(\frac{(1-\beta)\tau L + L}{a}\Big) \tag{5}$$

Equations 3 and 4 taken together imply that $W^U = W^E$. Moreover, firms in E must make the same (zero) profits, whatever the production strategy they choose. This consideration and the z.p.c.s above imply that: $\frac{1}{\lambda_P}L + \frac{1}{\lambda}L - W^E\Big(\frac{\tau L+L}{a}\Big) = \frac{1}{\lambda_R}(1-\beta)L + \frac{1}{\lambda}L - W^E\Big(\frac{(1-\beta)\tau L+L}{a}\Big)$ and hence:

$$\frac{1}{\lambda_R} = \frac{1}{1-\beta} - \frac{W^E\tau}{a}\frac{\beta}{1-\beta} = \frac{1}{1-\beta} - \frac{W^U\tau}{a}\frac{\beta}{1-\beta}$$

At this point I must check whether this setting constitutes an equilibrium and verify that for firms in U, profits from the production of mass goods are higher than the ones they would earn by producing exclusive goods.[8] Mathematically, the following condition must hold:

$$\frac{1}{\lambda_R}(1-\beta) + \frac{1}{\lambda} - W^U\Big(\frac{(1-\beta)L+\tau L}{aL}\Big) \geq \frac{1}{\lambda_P} + \frac{1}{\lambda} - W^U\Big(\frac{(1+\tau)L}{aL}\Big)$$

Rearranging yields:

$$\frac{1}{\lambda_R} \leq \frac{1}{1-\beta} - \frac{W^U}{a}\frac{\beta}{1-\beta}$$

Since $W^E = W^U$, the equation for $\frac{1}{\lambda_R}$ indicates that this last condition is always satisfied. It is always optimal for firms in U to produce mass consumption goods only)when firms in E produce and export both exclusive and mass consumption goods. We define this setting as equilibrium 1.

From equation 3 (or 4) we get: $\frac{1}{\lambda} = W^U\frac{aF+L+\tau L}{aL} - 1$. The r.c. for country U also implies that at the equilibrium, the number of goods produced in country U is equal to: $N^U = \frac{aL}{aF+L+\tau L}$. The number of mass consumption goods exported by country E can then be calculated by putting this last result into the b.c. of poor individuals in U: $N_P^E = W^U\theta - \frac{aL}{aF+L+\tau L}$. Likewise, one can derive the total number of goods imported

[8]In this setting, no firm in U is producing exclusive goods and exporting. This means that serving everybody in U and exporting must be a dominant strategy for them.

by country U from the b.c. of the individuals in E: $N_R^E = \frac{1}{\frac{aF+L+\tau L}{aL} - \frac{1}{W^U}} - \frac{aL}{aF+L+\tau L}$. To find the equilibrium wage level I substitute for N_R^E and N_P^E into the r.s. condition of country E, which can be rearranged to: $1 = N_R^E \frac{aF+L+(1-\beta)L\tau}{aL} + N_P^E \frac{\beta\tau L}{aL}$. This yields:

$$1 - \frac{\beta\tau L}{aL} W^U \theta + \frac{\beta\tau L}{aL} \frac{aL}{aF+L+\tau L} = \frac{aF+L+(1-\beta)L\tau}{aL} \Big(\frac{1}{\frac{aF+L+\tau L}{aL} - \frac{1}{W^U}} - \frac{aL}{aF+L+\tau L} \Big)$$

which is clearly quadratic in W^U. Rearrangement gives:

$$2 - \frac{\beta\tau L W^U \theta}{aL} = \frac{aF+(1+\tau)L-\beta\tau L}{(aF+L+\tau L) - \frac{aL}{W^U}} \tag{6}$$

Figure 2 plots the equation. The l.h.s. is decreasing in W^U and so is the r.h.s. Whereas the l.h.s. has a positive value at zero and eventually crosses the x-axis for high values of W^U, the r.h.s. is an hyperbola: it crosses the y-axis at $-\frac{aF+(1+\tau)L-\beta\tau L}{aF+(1+\tau)L}$ and has asymptote at $W^U = \frac{aL}{aF+(1+\tau)L}$ and $W^U = +\infty$. The two sides cross twice, giving rise to two possible equilibrium points, both decreasing in τ. Both a higher β and a lower θ decrease the wage, indicating that a higher inequality has a negative impact on the wage. Production costs also have a negative impact on wages, since both higher F and lower a decrease the wage equilibrium value.

3.2 Firms in U produce both exclusive and mass consumption goods and export, firms in E export exclusive goods only

The equilibrium conditions are the same as above, and they take the following form:

Resource Constraint Must be satisfied for both countries. I denote by N_P^U the number of firms that produce mass consumption goods in U and export, N_R^U the number of home-produced goods consumed by rich individuals in U and N^E the number of firms that produce in E and export to the rich only. For country U we have:

$$L = (N_R^U - N_P^U)\Big(\frac{aF+\tau L+(1-\beta)L}{a}\Big) + N_P^U\Big(\frac{aF+\tau L+L}{a}\Big)$$

For country E the condition is:

$$L = N^E \frac{aF+L+(1-\beta)L\tau}{a}$$

Budget Constraint For poor and rich individuals in U it must be that $W^U\theta = \frac{1}{\lambda_P} N_P^U$ and $W^U \frac{1-\beta\theta}{1-\beta} = \frac{1}{\lambda_R}(N_R^U - N_P^U + N^E) + \frac{1}{\lambda_P} N_P^U$ respectively. For individuals in E, $W^E = \frac{1}{\lambda}(N^E + N_R^U)$ holds.

Zero Profit Conditions Must hold for each firm and for all strategies available to it in this equilibrium.

1. Firms in U serving everybody at home and exporting:

$$\frac{1}{\lambda_P}L + \frac{1}{\lambda}L = W^U F + W^U\Big(\frac{L+\tau L}{a}\Big) \tag{7}$$

2. Firms in U serving rich only at home and exporting:

$$\frac{1}{\lambda_R}(1-\beta)L + \frac{1}{\lambda}L = W^U F + W^U\Big(\frac{(1-\beta)L+\tau L}{a}\Big) \tag{8}$$

3. Firms in E exporting to the rich in U only:

$$\frac{1}{\lambda_R}(1-\beta)L + \frac{1}{\lambda}L = W^E F + W^E\Big(\frac{(1-\beta)\tau L+L}{a}\Big) \tag{9}$$

From equation 7 we see that $\frac{1}{\lambda} = W^U\frac{aF+(1+\tau)L}{aL} - 1$. Since producers in U must make the same (zero) profit, whether they serve the rich or the poor, the following must also hold: $\frac{1}{\lambda_R} = \frac{1}{1-\beta} - \frac{W^U}{a}\frac{\beta}{1-\beta}$. Moreover, the b.c. of the poor in U yields $N_P^U = W^U\theta$. Equations 8 and 9 taken together imply that

$$W^U\Big(\frac{aF+(1-\beta)L+\tau L}{aL}\Big) = W^E\Big(\frac{aF+(1-\beta)\tau L+L}{aL}\Big)$$

Hence the relative wages are $\omega = \frac{W^U}{W^E} = \frac{aF+(1-\beta)L\tau+L}{aF+(1-\beta)L+\tau L}$. We notice that $W^U \geq W^E$ if and only if $aF+(1-\beta)L\tau+L \geq aF+(1-\beta)L+\tau L$. This is the case only if $\tau \leq 1$. Since by definition I imposed that $\tau \geq 1$, $W^E \geq W^U$ $\forall\beta > 0$. The wage in the egalitarian country is higher than the one in the unequal country, and the more so for higher values of β. The reason behind this finding is that in such an equilibrium, country E's producers only export to $(1-\beta)$ individuals and thus incur in less transportation costs than producers in U, who are all exporting to L individuals in E. Put differently, producers in E face less iceberg cost losses in transportation. Wages in the two countries are only equal if *i)* $\tau = 1$ (i.e. there are no trade costs) or *ii)* $\beta = 0$ (i.e. there is no inequality in U and producers in both countries face the same transportation losses).

At this point we can consider whether this setting constitutes an equilibrium. Specifically, one needs to check that for firms in E, profits from exporting to the rich only in U are higher than the ones they would earn by exporting to everybody in U.[9] Mathematically, the following condition must hold:

$$\frac{1}{\lambda_R}(1-\beta) + \frac{1}{\lambda} - W^E\Big(\frac{(1-\beta)L\tau+L}{aL}\Big) \geq \frac{1}{\lambda_P} + \frac{1}{\lambda} - W^E\Big(\frac{(1+\tau)L}{aL}\Big)$$

[9]Since in this settng firms in E only export to the rich, this strategy must be the dominant one, otherwise the setting cannot be an equilibrium.

Rearranging yields:

$$\frac{1}{\lambda_R} \geq \frac{1}{1-\beta} - \frac{W^E\tau}{a}\frac{\beta}{1-\beta}$$

Since, as specified above, $W^E \geq W^U$, this condition is always satisfied.It is always optimal for firms in E to export to the rich only when firms in U produce both exclusive and mass consumption goods and export. We define this setting as equilibrium 2.

Rearranging the resource constraint of country E we get:

$$N^E = \frac{aL}{aF + (1+\tau)L - \beta\tau L} \tag{10}$$

Substituting for N_P^U into the resource constraint of country U we obtain:

$$1 = N_R^U \frac{aF + \tau L + (1-\beta)L}{aL} + W^U\theta\frac{\beta L}{aL}$$

which, after rearranging, yields:

$$N_R^U = \frac{aL}{aF + \tau L + (1-\beta)L} - \frac{W^U\theta\beta L}{aF + \tau L + (1-\beta)L} \tag{11}$$

We can then substitute $1/\lambda, N_P^U, N_R^U$ and N^E into the equation of the budget constraint of rich individuals in U. This yield the following result:

$$W^U(1-\theta) = (1 - \frac{W^U\beta L}{aL})\Big(\frac{aL - W^U\theta(aF + (1+\tau)L)}{aF + (1+\tau)L - \beta L} + \frac{aL}{aF + (1+\tau)L - \beta\tau L}\Big) \tag{12}$$

which is clearly quadratic in W^U. Figure 3 plots the graph of equation 12. The l.h.s is an increasing function of W^U, with slope $(1-\theta)$. The r.h.s. is instead a parabola, having positive value at $W^U = 0$ and crossing the x-axis in 2 points. This implies that the two lines intersect at two points, which represent two possible candidate equilibrium points.

3.3 Firms in *U* serve only the rich and export, firms in *E* export both exclusive and mass consumption goods

Intuitively, this setting is not going to be an equilibrium because when we confront the z.p.c. derived before for country U and country E we see that when serving the rich and for a given wage, firms in E face lower costs in the biggest market, whereas this is not the case for mass consumption goods. It seems unlikely that firms in U would engage in the production of exclusive goods but not of mass consumption ones. In order to make sure that this line of reasoning is meaningful, I mathematically prove the statement. To do so, it is enough to work out the zero profit conditions implied by this setting.

1. Firms in U serving rich only at home and exporting:

$$\frac{1}{\lambda_R}(1-\beta)L + \frac{1}{\lambda}L = W^U F + W^U\Big(\frac{(1-\beta)L+\tau L}{a}\Big)$$

2. Firms in E exporting to the rich in U only:

$$\frac{1}{\lambda_R}(1-\beta)L + \frac{1}{\lambda}L = W^E F + W^E\Big(\frac{(1-\beta)\tau L+L}{a}\Big)$$

3. Firms in E exporting to everybody in country U:

$$\frac{1}{\lambda_P}L + \frac{1}{\lambda}L = W^E F + W^E\Big(\frac{\tau L+L}{a}\Big)$$

Equalizing the profits of mass consumption and exclusive goods producers in E we obtain

$$\frac{1}{\lambda_R} = \frac{1}{1-\beta} - \frac{W^E\tau}{a}\frac{\beta}{1-\beta}$$

Taking the first two z.p.c. above, we can derive that $W^U\left(\frac{aF+(1-\beta+\tau)L}{aL}\right) = W^E\left(\frac{aF+(1-\beta)\tau L+L}{aL}\right)$. The relative wages are then $\omega = \frac{W^U}{W^E} = \frac{aF+(1-\beta)\tau L+L}{aF+(1-\beta+\tau)L}$. We notice that $W^U \geq W^E$ iff. $aF+(1-\beta)L\tau+L \geq aF+(1-\beta)L+\tau L$, which is never true for $\tau \geq 1$, $\forall\beta > 0$. Serving only the rich and exporting is an optimal strategy for firms in U if and only if the profits that this strategy delivers are higher than the ones delivered by concurring strategies, i.e. serving everybody and exporting. This translates into the following condition for producers in U:

$$\frac{1}{\lambda_R}(1-\beta)L + \frac{1}{\lambda}L - W^U\Big(\frac{(1-\beta)L+\tau L}{a}\Big) \geq \frac{1}{\lambda_P}L + \frac{1}{\lambda}L - W^U\Big(\frac{L+\tau L}{a}\Big)$$

Rearranging the equation, we get the following lower boundary on the price of exclusive goods:

$$\frac{1}{\lambda_R} \geq \frac{1}{1-\beta} - \frac{W^U}{a}\frac{\beta}{1-\beta}$$

Since $W^U < W^E$ and $\tau \geq 1$, the value of $1/\lambda_R$ derived above is below the boundary. This result means that the condition is not fulfilled and profits earned from serving the rich are not greater (but lower) than the ones from serving the poor; which implies that this strategy is not an equilibrium.

3.4 Firms in *U* produce both exclusive and mass consumption goods and export, firms in *E* only export mass consumption goods

Intuitively, this setting is unlikely to be an equilibrium. Following the same line of reasoning as above, when confronting the z.p.c. for firms in U and in E we see that when serving the rich and for a given wage, firms in E face lower costs in the biggest market, whereas this is not the case for mass consumption goods. It seems unlikely then that firms in E would engage in the production of mass consumption goods but not of exclusive goods, given the possible advantage they have in that market. Again, a formal proof validates this reasoning. I start by defining the zero profit conditions for this setting:

1. Firms in U serving rich only at home and exporting:

$$\frac{1}{\lambda_R}(1-\beta)L + \frac{1}{\lambda}L = W^U F + W^U\Big(\frac{(1-\beta)L + \tau L}{a}\Big)$$

2. Firms in U serving everybody at home and exporting:

$$\frac{1}{\lambda_P}L + \frac{1}{\lambda}L = W^U F + W^U\Big(\frac{L + \tau L}{a}\Big)$$

3. Firms in E exporting to everybody in country U:

$$\frac{1}{\lambda_P}L + \frac{1}{\lambda}L = W^E F + W^E\Big(\frac{\tau L + L}{a}\Big)$$

Equalizing the profits producers in E earn by selling mass consumption and exclusive goods I obtain

$$\frac{1}{\lambda_R} = \frac{1}{1-\beta} - \frac{W^U}{a}\frac{\beta}{1-\beta}$$

Taking the last two z.p.c. above I derive that $W^E = W^U$. Exporting to everybody in U is only an optimal strategy for firms in E if the profits they get are greater than the ones they would earn from exporting to the rich only. This means that:

$$\frac{1}{\lambda_P}L + \frac{1}{\lambda}L - W^E\Big(\frac{L+\tau L}{a}\Big) \leq \frac{1}{\lambda_R}(1-\beta)L + \frac{1}{\lambda}L - W^E\Big(\frac{(1-\beta)\tau L + L}{a}\Big)$$

Rearranging the equation, we get the following upper boundary on $1/\lambda_R$:

$$\frac{1}{\lambda_R} \leq \frac{1}{1-\beta} - \frac{W^E\tau}{a}\frac{\beta}{1-\beta}$$

Given that $W^E = W^U$, the r.h.s. of the last equation is lower than the value of $1/\lambda_R$ calculated above. The boundary condition is not met: profits from producing and exporting mass consumption goods are not higher (but lower) than the ones earned from the production and export of exclusive goods. Exporting mass consumption goods is not an optimal strategy for producers in E when firms in U produce both exclusive and mass consumption goods. This setting does not constitute an equilibrium.

3.5 Cutoff point for the prevailing equilibrium

Whereas I proved that equilibria 1 and 2 are internally consistent, I have not identified the conditions under which either one of the two prevails. Country U's firms only produce and export mass consumption goods in equilibrium 1 but serve both types of consumers and export in equilibrium 2. The latter will then prevail only as long as the number of goods produced in U and consumed by poor individuals does not exceed the number of goods produced in U and consumed by rich individuals, i.e. $N_R^U \geq N_P^U$. We derived N_R^U in equation 11 and $N_P^U = W^U\theta$. Equilibrium 2 will prevail as long as:

$$\frac{aL}{aF+\tau L+(1-\beta)L} - \frac{W^U\theta\beta L}{aF+\tau L+(1-\beta)L} > W^U\theta$$

$$\frac{aL}{aF+\tau L+(1-\beta)L} > W^U\theta\Big(\frac{aF+\tau L+(1-\beta)L+\beta)L}{aF+\tau L+(1-\beta)L}\Big)$$

$$W^U\theta < \frac{aL}{aF+(1+\tau)L} \tag{13}$$

Equation 13 shows disposable income of poor individuals as a function of exogenous variables. If the disposable income is larger than the critical value $W^U\theta < \frac{aL}{aF+(1+\tau)L}$, poor individuals can afford to buy all goods produced in U[10]. The critical value of $W^U\theta$ can be substituted in the final equation of equilibrium 2 to obtain the value of W^U at the critical θ:

$$W^U(1-\theta) = (1-\beta\frac{W^U}{a})\frac{aL}{aF+(1+\tau)L-\beta\tau L}$$

$$W^U = \frac{aL}{aF+(1+\tau)L-\beta\tau L}\Big/\Big(1-\theta+\frac{\beta L}{aF+(1+\tau)L-\beta\tau L}\Big)$$

Substituting this value back into the boundary condition for θ we obtain:

$$\frac{\theta}{1-\theta+\frac{\beta L}{aF+(1+\tau)L-\beta\tau L}} < \frac{aL}{aF+(1+\tau)L}\frac{aF+(1+\tau)L-\beta\tau L}{aL}$$

$$\theta < \frac{aF+(1+\tau)L-\beta\tau L}{aF+(1+\tau)L}\Big[1-\theta+\frac{\beta L}{aF+(1+\tau)L-\beta\tau L}\Big]$$

$$\frac{2[aF+(1+\tau)L]-\beta\theta L}{aF+(1+\tau)L}\theta < \frac{aF+(1+\tau)L-\beta\tau L+\beta L}{aF+(1+\tau)L}$$

and hence:

$$\theta < \frac{aF+(1+\tau)L-\beta\tau L+\beta L}{2[aF+(1+\tau)L]-\beta\tau L} > \frac{1}{2} \tag{14}$$

Below the value identified above, equilibrium 2 will prevail, above it, equilibrium 1 will. We can now proceed to observe the outcomes of the two equilibria.[11]

[10] One should not forget that the disposable income of poor individuals in U is $W^U\theta$ whereas the one of rich individuals is $W^U\frac{1-\beta\theta}{1-\beta}$, so the effect of θ on the gap between the two is both direct and indirect through W^U.

[11] The boundary for θ can be equivalently found by considering the conditions under which producers in E export mass consumption goods to U.

3.6 Equilibrium 1

3.6.1 Existence of equilibrium and welfare effects

So far I have assumed that trade would simply take place, but I need to check under what conditions trade actually arises in this setting. Firms must have an incentive to sell their product abroad, i.e. that they do not make losses when selling their product on the foreign market. Firms in U export goods to E only insofar as the price of goods in E exceeds the marginal cost of producing them. This translates into $\frac{1}{\lambda} \geq \frac{W^U \tau}{a}$. Substituting for the value of $\frac{1}{\lambda}$ that I found above I obtain:

$$W^U \frac{aF + (1+\tau)L}{aL} - 1 \geq \frac{W^U \tau L}{aL}$$

$$W^U \geq \frac{aL}{aF + L} \tag{15}$$

Vice versa, firms in E export mass consumption goods [12] if and only if $\frac{1}{\lambda_P} \geq \frac{W^E \tau}{a} = \frac{W^U \tau}{a}$, i.e.

$$W^U \leq \frac{a}{\tau} \tag{16}$$

Equation 6, implicitly gives the equilibrium wage level for equilibrium 1. Substituting first condition 15 and then equation 16 into equation 6, I obtain the following:

Proposition 1. *For values of θ s.t. equilibrium 1 occurs, all firms have an incentive to trade as long as the following holds:*

$$\frac{-(1+\beta) + 2\sqrt{(1+\beta)^2 - 4\beta\theta}}{4\beta\theta} \leq \frac{\tau}{\frac{aF}{L} + 1} \leq \frac{1-\beta\theta}{1-\beta}$$

Proof. **IN APPENDIX** □

Else being equal, the lower τ and β and the higher production cost, the more likely it is that the two countries engage in trade.

At this point I can make some considerations on what happens under trade compared to the autarky equilibrium. Specifically, the focus is on the number of goods produced in each country and purchased by each consumer type.

Proposition 2. *If firms have an incentive to trade and θ is s.t. equilibrium 1 arises, the number of active firms in each country is lower in trade than in autarky, whereas the number of goods available to consumers of all types is higher.*

Proof. IN APPENDIX □

[12]Note that since $\frac{1}{\lambda_R} > \frac{1}{\lambda_P}$, if firms in E find it profitable to export mass consumption goods, then it is also profitable for them to export exclusive goods because marginal cost is the same but prices are higher. Since the market for exclusive goods is smaller thoguh, it turns out that the two conditions actually exactly coincide

3.6.2 Uniqueness of the equilibrium point

I can now consider whether both of the equilibrium points originating from equilibrium 1 satisfy the constraint derived from my last argumentation. Equilibrium 1 only prevails over equilibrium 2 for high values of θ. I can substitute the critical value of $\theta = \frac{aL}{aF+(1+\tau)L}\frac{1}{W^U}$ into equation 6 in order to observe the behavior of the l.h.s. and the r.h.s. at the critical value of the wage. Figure 2 plots the graph of equation 6. A higher wage decreases θ, which at the critical value means shifting from equilibrium 1 to equilibrium 2. If the graph of the r.h.s. is below the one of the l.h.s. at the critical value of W^U, then only one equilibrium point will sustain equilibrium 2. If instead the graph of the l.h.s. lies below the one of the r.h.s. at the critical value of W^U, both equilibrium points are candidate for equilibrium scenario 2. Calculation shows that the lower equilibrium point is excluded for low values of θ:

$$\frac{aF+(1+\tau)L-(\tau-1)\beta L}{2[aF+(1+\tau)L]-\beta\tau L} < \theta \leq \frac{aF+(1+\tau)L}{2[aF+(1+\tau)L]-\beta\tau L} \tag{17}$$

whereas 2 equilibrium points emerge for high levels of θ:

$$\frac{aF+(1+\tau)L}{2[aF+(1+\tau)L]-\beta\tau L} < \theta \leq 1$$

At the lower wage intersection point, the price of goods in country E is higher than the one of mass consumption goods in U, but this is in line with the general finding that inequality increases mark-ups in the economy.[13] At the higher equilibrium point instead, the price of exclusive goods is always negative. Although mathematically meaningful, the highest equilibrium point is not economically valid: Equilibrium 1 can only exist when the wage equals the lowest of the two equilibrium points. However, in the interval $\frac{aF+(1+\tau)L-(\tau-1)\beta L}{2[aF+(1+\tau)L]-\beta\tau L} < \theta \leq \frac{aF+(1+\tau)L}{2[aF+(1+\tau)L]-\beta\tau L}$ this does not happen. When θ falls in this interval, the condition of equation 13 is not met, making us believe that the economy would be in equilibrium 2 instead. However, for values of θ falling in such interval, condition expressed in equation 13 is not met for equilibrium 2 either. I calculated the cutoff point of theta considering the disposable income of poor individuals in U. For values of theta falling in between the interval above, the disposable income available to poor consumers seems to be too low to purchase all the goods available to them in equilibrium one, and so high that they purchase as many goods produced in U as the rich. It appears that in this interval there should arise an equilibrium s.t. the number of goods purchased by the poor is lower than the one of equilibrium 1 and higher than the one of equilibrium 2.

[13] See Foellmi & Zweimüller (2004)

3.7 Equilibrium in the critical interval for θ

I make a guess for the equilibrium arising at values of theta falling in the interval shown in equation 17. All firms in U serve everybody and export whereas all firms in E serve everybody at home and export to the rich in U only. In this kind of setting, poor individuals purchase the same amount of goods produced in U as the rich, like in equilibrium 1, but they purchase in total less goods than the rich because they do not purchase any good imported from E, like in equilibrium 2. To verify whether such setting constitutes an equilibrium and, specifically, whether it holds for values of theta in the boundary, the following conditions must hold:

Resource Constraint Resources in country U are employed in the production of N^U goods that everybody consumes. The constraint is $L = N^U \frac{aF+(1+\tau)L}{a}$, which translates into:

$$N^U = \frac{aL}{aF + (1+\tau)L}$$

As expected, the number of goods consumed by poor individuals in this setting is higher than in equilibrium 2 but lower than in equilibrium 1. Resources in country E are all devoted to the production of N^E goods exported to the rich only in U. The constraint is $L = N^E \frac{aF+(1-\beta)\tau L+L}{a}$, which gives:

$$N^E = \frac{aL}{aF + (1+\tau)L - \beta\tau L}$$

Budget Constraint For poor individuals in U the following must be true: $W^U\theta = N^U \frac{1}{\lambda_P}$. As before, I set the marginal willingness to pay of the poor as numéraire. Substituting for the value of N^U found above, the b.c. of the poor yields the following:

$$W^U\theta = \frac{aL}{aF + (1+\tau)L}$$

which exactly coincides with the boundary condition of theta given in equation 13. This finding confirms that this setting is indeed a candidate equilibrium at the critical values of theta. The budget constraint for rich individuals in U and consumers in E is respectively $\frac{1-\beta\theta}{1-\beta}W^U = N^U\frac{1}{\lambda_P} + N^E\frac{1}{\lambda}$ and $W^E = (N^U + N^E)\frac{1}{\lambda}$.

Zero Profit Conditions In this case there is only one strategy available for firms in each of the two countries:

1. Firms in U serving everybody at home and exporting:

$$\frac{1}{\lambda_P}L + \frac{1}{\lambda}L = W^U F + W^U \frac{(1+\tau)L}{a} \tag{18}$$

2. Firms in E serving everybody at home and exporting to the rich only:

$$\frac{1}{\lambda_R}(1-\beta)L + \frac{1}{\lambda}L = W^E F + W^E \frac{(1-\beta)\tau L + L}{a} \tag{19}$$

Equation 18 yields $\frac{1}{\lambda} = \frac{1-\theta}{\theta}$. Substituting the number of goods and the value of the wage into the budget constraint of the rich, the marginal willingness to pay of the rich becomes: $\frac{1}{\lambda_R} = \frac{aF+(1+\tau)L-\beta\tau L}{aF+(1+\tau)L}\frac{(1-\theta)}{\theta(1-\beta)}$. Finally, substituting for the variables and rearranging in the budget constraint of individuals in E, the wage in country E becomes

$$W^E = \frac{aL}{aF+(1+\tau)L}\frac{2[aF+(1+\tau)L]-\beta\tau L}{aF+(1+\tau)L-\beta\tau L}$$

All the equilibrium conditions hold, indicating that this setting represents an equilibrium and that it holds exactly for those values of $W^U\theta$ at the boundary between equilibrium 1 and equilibrium 2. In equilibrium 1, the wages in the two countries are equalized, whereas in equilibrium 2 the wage in the equal country is higher. It is worth investigating how the wage ratio behaves in this interval. The wage rate in U is higher than the one in E if $\frac{1}{\theta} > \frac{2[aF+(1+\tau)L]-\beta\tau L}{aF+(1+\tau)L-\beta\tau L}$, i.e. if

$$\theta \leq \frac{aF+(1+\tau)L-\beta\tau L}{2[aF+(1+\tau)L]-\beta\tau L}$$

However, for those values of theta equilibrium 2 prevails, indicating that in this interval equilibrium, like in equilibrium 2, $W^E > W^U$.

Given this last discussion, it is clear that there exist an interval of values of θ in which a third equilibrium arises. The cutoff points among the three equilibria are as follows:

$$\underbrace{\theta_2}_{\text{Equilibrium2}} < \underbrace{\theta_l \leq \theta_h}_{\text{Interval Equilibrium}} < \underbrace{\theta_1}_{\text{Equilibrium 1}}$$

Where: $\theta_l = \frac{aF+(1+\tau)L-\beta\tau L+\beta L}{2(aF+(1+\tau)L)-\beta\tau L}$ and $\theta_h = \frac{aF+(1+\tau)L}{2[aF+(1+\tau)L]-\beta\tau L}$. Figure 4 shows how big the interval is for different values of the inequality parameters and $\tau = 1.2$. Numerical exercises show that the interval of values of theta for which the above equilibrium holds gets smaller as transportation costs or beta decrease, and is at its minimum for values of β equal to 0 or iceberg costs equal to unity. In either of the two cases, the interval reduces to a point, and the cutoff between equilibrium 1 and equilibrium 2 is at the value of theta defined in equation 14:

$$\theta = \frac{aF+(1+\tau)L-\beta\tau L+\beta L}{2[aF+(1+\tau)L]-\beta\tau L}$$

3.8 Equilibrium 2

3.8.1 Existence of the equilibrium and welfare effects

The two countries do not engage in trade unless firms have an incentive to export their product. This means that the critical value of τ at which autarky is preferred to trade depends on the incentives to trade that firms have. More specifically, firms in U will export as long as the price of goods in E exceeds marginal costs: $\frac{1}{\lambda} > \frac{W^U \tau}{a}$. Substituting for the value of $1/\lambda$, the condition becomes:

$$W^U \frac{aF + (1+\tau)L}{aL} - 1 > W^U \frac{\tau}{a}$$

$$W^U > \frac{aL}{aF + L} \tag{20}$$

i.e. trading cost must be s.t. the wage is at least the one given in the equation above. Vice versa, firms in E export to the rich in U only if the price of exclusive goods exceeds the marginal cost of producing them: $\frac{1}{\lambda_R} > \frac{W^E \tau}{a}$. Substituting again for the value of $\frac{1}{\lambda_R}$ I get:

$$\frac{1}{1-\beta} - \frac{W^U}{a}\frac{\beta}{1-\beta} > \frac{W^E \tau}{a}$$

Substituting for W^E and rearranging I obtain:

$$\frac{1}{1-\beta} - \frac{W^U}{a}\frac{\beta}{1-\beta} > W^U \Big(\frac{aF + (1-\beta)L + \tau L}{aF + (1-\beta)\tau L + L}\Big)\frac{\tau}{a}$$

Rearrangement yields:

$$W^U \leq \frac{a}{(aF + L)(\beta + \tau - \beta\tau) + \tau^2 L(1+\beta)} \tag{21}$$

In order for trade to occur as specified in equilibrium 2, both conditions must be fulfilled at the same time. As before, high production cost and low transportation cost make trade more likely.

Having determined that equilibrium 2 exists and is unique, it is of interest to evaluate how the welfare of consumers is affected by trade. The following holds:

Proposition 3. *If firms have incentives to trade and θ is s.t. equilibrium 2 arises, the number of active firms in each country is lower in trade than in autarky, whereas the number of goods available to consumers of all types is higher.*

Proof. IN APPENDIX □

3.8.2 Uniqueness of the equilibrium point

In this setting as well there are two candidate equilibrium points, and we need to verify whether they both represent valid equilibria. Remembering that the boundary condition on wage for equilibrium 2 to exist is $W^U \leq \frac{aL}{aF+(1+\tau)L}\frac{1}{\theta}$, we can substitute this maximum value into the l.h.s. and r.h.s. of equation 12 plotted in figure 3. Calculations show that at the critical maximum level of wage, the graph of the l.h.s. is above the one of the r.h.s. if $\theta < \frac{aF+(1+\tau)L-\beta\tau L+\beta L}{2(aF+(1+\tau)L)-\beta\tau L}$, which coincides with the condition s.t. equilibrium 2 prevails. This result means that if we are in equilibrium 2, the rightmost equilibrium point does not represent an equilibrium, since at that point the r.h.s. is at least equal to the l.h.s.. We understand that only one equilibrium arises in this setting, and it is the one associated with the lower wage equilibrium point. Furthermore, at the equilibrium wage level the price of goods in E is lower than the price of mass consumption goods in U. As argued in the context of equilibrium 1, this is in line with findings about the fact that inequality increases mark-ups.

3.9 Welfare Effects of Trade

As outlined above, in both equilibria trade increases the welfare of all types of consumers, although the number of active firms in each country is lower than under autarky. I am however interested in understanding what the impact of inequality on welfare is, considering that different equilibria do arise for different levels of inequality in U. Figures 5, 6 and 7 illustrate the number of goods consumed by each type of individuals for different values of θ. The upper line shows the number of goods consumed in the trade equilibrium prevailing for the value of θ considered, the lower line shows the amount of goods consumed under autarky. As expected, welfare of poor individuals increases in both cases with decreasing inequality, and the welfare gains are also higher for lower levels of inequality. The opposite is true for rich individuals and for consumers in E in the trade equilibrium, however for them both the number of goods consumed is less sensitive to changes in θ than for poor individuals.

4 Introducing Arbitrage

If we allow for large enough per-capita income differences among poor in U, individuals in E and rich in U, arbitrage opportunities may arise and it might no longer be optimal for a producer to export its products. Consider a firm in U serving both individuals in E and rich individuals in U and charging in each country a price equal to the marginal willingness to pay of consumers. Then if $1/\lambda_R$ is much higher than $1/\lambda$, arbitrageurs can buy the good cheaply in E's market, ship it to U and sell it there at a lower price than the prevailing one and make a profit. Producers who want to export must reduce their price in order to avoid parallel trade. Alternatively, since only firms operating on the global market are constrained by a threat of parallel trade, firms can decide not to engage in trade and keep prices equal to home's consumers' willingness to pay. Ultimately, firms face a tradeoff between serving a bigger market (home and foreign) at a lower price and serving only the smaller home market but at a higher price, fully exploiting the higher willingness to pay of its consumers. In equilibrium there must exist some firms that serve rich individuals at home and do not export. If everybody was charging the arbitrage-preventing price, all goods would be priced below the willingness to pay of rich consumers, who would then have an infinite willingness to pay for an additional unit. A profit opportunity would arise and there would be an incentive for firms to enter the home market and serve the rich only at a very high price. This observation implies that in equilibrium firms must be indifferent between selling at home at a lower price and exporting and serving only the home market at a higher price. The two strategies must yield equal (zero) profits. This condition must hold true for each of the two equilibria calculated above.

Let's now evaluate how the arbitrage constraints affect the equilibria derived above.

4.1 Equilibrium 1

Firms in U only produce mass consumption goods and export, firms in E export both exclusive and mass consumption goods. Given that in equilibrium 1 the price of goods in E is lower than the one of mass-consumed goods in U, arbitrage opportunities for mass consumption goods arise when $\frac{1}{\lambda}\tau \leq \frac{1}{\lambda_P}$, i.e.

$$\tau\Big[W^U \frac{aF + (1+\tau)L}{aL} - 1\Big] \leq 1 \tag{22}$$

For exclusive goods, a threat of parallel trade emerges any time $\frac{1}{\lambda_R} > \tau\frac{1}{\lambda}$. Substituting for the prices found in equilibrium 1 above, the condition becomes:

$$\tau[W^U \frac{aF + (1+\tau)L}{aL} - 1] < \frac{1}{1-\beta} - \frac{W^U\tau}{a}\frac{\beta}{1-\beta} \tag{23}$$

$$W^U < \frac{aL(1+\tau-\beta\tau)}{\tau(1-\beta)(aF+\tau L)+\tau L} \tag{24}$$

If $p\tau < p_R$, firms in E that export to rich in U must charge price $p_R^{Trade} = \tau p$ or not export at all. Since p_R^{Trade} is still greater than marginal cost from exporting, firms in E will all decide to export at the arbitrage-preventing price. However, if no firm in U produces exclusive goods the rich buy all their goods at a price below their willingness to pay. This means that they do not spend all of their income and they have an infinite willingness to pay for an additional good. Given the profit opportunity, firms in U start serving the rich without exporting, giving rise to export zeroes. The two equations above give us insight on how different parameters might affect the presence of a threat of arbitrage. Since wages are equal in the two countries, an increase in wages makes parallel trade more likely. In fact, whereas consumers in E see their disposable income increase 1 to 1 with wage, rich consumers in U appreciate an increase of $\frac{1-\beta\theta}{1-\beta}$ in disposable income for each additional unit increase in wage. This means that the willingness to pay of individuals in E increases less than the one of rich individuals in U, so that prices diverge and the threat of arbitrage becomes more pressing. Higher fixed (higher F) and variable (lower a) cost increase the threat of arbitrage as well. The share of transportation cost in total cost decreases so firms have an incentive to export and expand their market even if the price in the foreign country is lower. This means that higher price differences can be sustained with higher production costs and arbitrage opportunities are more likely to appear.

The l.h.s. of equation 23 coincides with the l.h.s. of equation 22. If the former condition is satisfied when the l.h.s. is equal to 1, it is also satisfied for all other values of τ that imply arbitrage for mass consumption goods. This means that whenever a threat of parallel trade arises in the mass consumption sector, it is also present in the exclusive goods sector. In order to verify this, we check whether the r.h.s. of equation 24 is always greater than 1.

$$1 \leq \frac{1}{1-\beta} - \frac{W^U \tau}{a} \frac{\beta}{1-\beta}$$

$$W^U \leq \frac{a}{\tau}$$

Remembering that the two countries engage in trade as long as $W^U \leq \frac{a}{\tau}$, the inequality always holds: whenever there is a threat of parallel trade for goods produced in E, the threat is also present for exclusive goods.

Proposition 4. *The arbitrage constraint always becomes binding for exclusive goods first.*

Proof. In text. □

4.1.1 Threat of arbitrage for exclusive goods

When there is a threat of arbitrage for exclusive goods, firms that serve all individuals in both countries can always charge a price equal to the marginal willingness to pay of consumers, i.e. $\frac{1}{\lambda}$ in E and $\frac{1}{\lambda_P}$ in U. However, firms in E that export to the rich only must sell their goods in U at an arbitrage-preventing price $\tau\frac{1}{\lambda}$. This means that rich individuals can buy exclusive goods at a price lower than their willingness to pay. They have an infinite willingness to pay for an additional unit, which gives incentives to new firms in U to enter the market and serve the rich only at a price $\frac{1}{\lambda_R}$. These producers only serve the home market and do not export, giving rise to export zeroes between the two countries. In order to solve for the arbitrage equilibrium, I define again all the necessary conditions:

Resource Constraints For U the constraint is:

$$L = (N_R^U - N_P^U)\frac{aF + (1-\beta)L}{a} + N_P^U \frac{aF + (1+\tau)L}{a}$$

where N_R^U is the total number of goods (traded and non-traded) produced in U. For E it is:

$$L = (N_R^E - N_P^E)\frac{aF + (1+\tau)L - \beta\tau L}{a} + N_P^E\frac{aF + (1+\tau)L}{a}$$

Budget Constraints For poor and rich individuals in U it must be that $W^U\theta = \frac{1}{\lambda_P}(N_P^U + N_P^E)$ and $W^U\frac{1-\beta\theta}{1-\beta} = \frac{1}{\lambda_R}(N_R^U - N_P^U) + \frac{1}{\lambda}\tau(N_R^E - N_P^E) + \frac{1}{\lambda_P}(N_P^U + N_P^E)$ respectively. For individuals in E, $W^E = \frac{1}{\lambda}(N_R^E + N_P^U)$ holds.

Zero Profit Conditions Must hold for each firm and for all strategies available to it in this equilibrium.

1. Firms in U serving everybody at home and exporting:

$$\frac{1}{\lambda_P}L + \frac{1}{\lambda}L = W^U F + W^U\frac{(1+\tau)L}{a} \tag{25}$$

2. Firms in U serving the rich and not exporting:

$$\frac{1}{\lambda_R}(1-\beta)L = W^U F + W^U\frac{(1-\beta)L}{a} \tag{26}$$

3. Firms in E exporting to everybody in country U:

$$\frac{1}{\lambda_P}L + \frac{1}{\lambda}L = W^E F + W^E\frac{(1+\tau)L}{a} \tag{27}$$

4. Firms in E exporting to the rich in U only:

$$\frac{1}{\lambda}\tau(1-\beta)L + \frac{1}{\lambda}L = W^E F + W^E\Big(\frac{(1-\beta)\tau L + L}{a}\Big) \tag{28}$$

Looking at equations 25 and 27 we understand that $W^E = W^U$. Taking again $1/\lambda_P$ as numéraire, I can solve equation 25 to obtain: $\frac{1}{\lambda} = W^U\frac{aF+(1+\tau)L}{aL} - 1$. Substituting this value in equation 28 I obtain:

$$W^U = W^E = \frac{\tau - \beta\tau + 1}{\tau}\frac{aL}{aF + (1+\tau)L - \beta\tau L - \beta aF}$$

which, substituted back, yields:

$$\frac{1}{\lambda} = \frac{aF + (1+\tau)L - \beta\tau L}{\tau[aF + (1+\tau)L - \beta\tau L - \beta aF]}$$

The marginal willingness of the rich is found from the zero profit condition 26:

$$\frac{1}{\lambda_R} = \frac{\tau - \beta\tau + 1}{\tau}\frac{aF + (1-\beta)L}{aF + (1+\tau)L - \beta\tau L - \beta aF}$$

From the b.c. of the poor in U and of individuals in E, I obtain $N_P^U + N_P^E = W^U\theta = \frac{\tau-\beta\tau+1}{\tau}\frac{aL\theta}{aF+(1+\tau)L-\beta\tau L-\beta aF}$ and $N_R^E + N_P^U = \frac{W^E}{1/\lambda} = (\tau - \beta\tau + 1)\frac{aL}{aF+(1+\tau)L-\beta\tau L}$ respectively. Substituting these values in the b.c. of the rich in U and rearranging I obtain: $N_R^U -$

$N_P^U = \frac{aL(1-\theta)}{aF+(1-\beta)L} - \frac{aL(1-\beta)}{aF+(1-\beta)L}\frac{[(\tau-\theta)(aF+(1+\tau)L-\beta\tau L)-\beta\tau aF]}{[aF+(1+\tau)L-\beta\tau L-\beta aF]}$. Once having substituted for all the variables above, the r.c. of country U yields:

$$N_P^U = \frac{aL\theta}{aF+(1+\tau)L} + \frac{aL(1-\beta)}{aF+(1+\tau)L}\frac{[(\tau-\theta)(aF+(1+\tau)L-\beta\tau L)-\beta\tau aF]}{[aF+(1+\tau)L-\beta\tau L-\beta aF]}$$

and I can finally substitute this value in the three b.c. above to obtain:

$$N_R^U = \frac{aL}{[aF+(1+\tau)L][aF+(1-\beta)L]}\Big\{aF+(1+\tau)L-\theta L(\beta+\tau)- \frac{(1-\beta)L(\beta+\tau)[(\tau-\theta)(aF+(1+\tau)L-\beta\tau L)-\beta\tau aF]}{[aF+(1+\tau)L-\beta\tau L-\beta aF]}\Big\}$$

$$N_R^E = \frac{aL(\tau-\beta\tau+1)}{aF+(1+\tau)L-\beta\tau L} - \frac{aL\theta}{aF+(1+\tau)L} - \frac{aL(1-\beta)}{aF+(1+\tau)L}\frac{(\tau-\theta)(aF+(1+\tau)L-\beta\tau L)-\beta\tau aF}{[aF+(1+\tau)L-\beta\tau L-\beta aF]}$$

$$N_P^E = \frac{\tau-\beta\tau+1}{\tau}\frac{aL\theta}{aF+(1+\tau)L-\beta\tau L-\beta aF} - \frac{aL\theta}{aF+(1+\tau)L} - \frac{aL(1-\beta)}{aF+(1+\tau)L}\frac{[(\tau-\theta)(aF+(1+\tau)L-\beta\tau L)-\beta\tau aF]}{[aF+(1+\tau)L-\beta\tau L-\beta aF]}$$

Having solved for the equilibrium, two main features stand out. First of all, the equilibrium wage is higher when I consider arbitrage than when I do not. Secondly, the marginal willingness to pay of E's consumers is here higher than the one of poor individuals in U, indicating that arbitrage opportunities could instead emerge for goods consumed in E. This possibility arises every time $\frac{1}{\lambda_P}\tau \leq \frac{1}{\lambda}$. Substituting for the values from the equilibrium above, this happens whenever

$$\tau \leq \frac{aF+(1+\tau)L-\beta\tau L}{\tau[aF+(1+\tau)L-\beta\tau L-\beta aF]}$$

It is useful for further analysis to express the same constraint in terms of the wage in U:

$$W^U \geq \frac{aL(1+\tau)}{aF+(1+\tau)L} \tag{29}$$

Depending on the values of the other variables, if iceberg cost are low enough, arbitrage opportunities can arise for goods consumed in E. This means that a second kind of arbitrage equilibrium can arise.

4.1.2 Threat of arbitrage for exclusive goods and goods consumed in *E*

When a threat of arbitrage arises for the two types of goods, firms that decide to export can charge a price equal to the marginal willingness to pay of consumers only when they serve poor individuals in U. Firms in E that export to all in U must charge an arbitrage-preventing price $\frac{1}{\lambda_P}\tau$ at home. Firms in U exporting to E must charge the same arbitrage-preventing price abroad, and firms in E exporting to the rich in U must charge the arbitrage-preventing price $\frac{1}{\lambda}\tau$ abroad. As before, there is a group of firms in U that do not export and serve the rich only at a price equal to their marginal willingness to pay. Once again, the necessary conditions allow me to solve for the equilibrium:

Resource Constraints For U the constraint is:

$$L = (N_R^U - N_P^U)\frac{aF + (1-\beta)L}{a} + N_P^U\frac{aF + (1+\tau)L}{a}$$

where N_R^U is the total number of goods (trade and non-traded) produced in U. For E it is:

$$L = (N_R^E - N_P^E)\frac{aF + (1+\tau)L - \beta\tau L}{a} + N_P^E\frac{aF + (1+\tau)L}{a}$$

Budget Constraints For poor and rich individuals in U it must be that $W^U\theta = \frac{1}{\lambda_P}(N_P^U + N_P^E)$ and $W^U\frac{1-\beta\theta}{1-\beta} = \frac{1}{\lambda_R}(N_R^U - N_P^U) + \frac{1}{\lambda}\tau(N_R^E - N_P^E) + \frac{1}{\lambda_P}(N_P^U + N_P^E)$ respectively. For individuals in E, $W^E = \frac{1}{\lambda}(N_R^E - N_P^E) + \tau\frac{1}{\lambda_P}(N_P^U + N_P^E)$ holds.

Zero Profit Conditions Must hold for each firm and for all strategies available to it in this equilibrium.

1. Firms in U serving everybody at home and exporting:

$$\frac{1}{\lambda_P}L + \frac{1}{\lambda_P}\tau L = W^U F + W^U\frac{(1+\tau)L}{a} \tag{30}$$

Simple rearrangement yields $W^U = \frac{aL(1+\tau)}{aF+(1+\tau)L}$.

2. Firms in U serving the rich and not exporting:

$$\frac{1}{\lambda_R}(1-\beta)L = W^U F + W^U\frac{(1-\beta)L}{a} \tag{31}$$

From which I obtain the price of exclusive goods after substituting for the wage found above: $\frac{1}{\lambda_R} = \frac{(1+\tau)}{(1-\beta}\frac{aF+(1-\beta)L}{aF+(1+\tau)L}$.

3. Firms in E exporting to everybody in country U:

$$\frac{1}{\lambda_P}L + \frac{1}{\lambda_P}\tau L = W^E F + W^E\frac{(1+\tau)L}{a} \tag{32}$$

Rearranging this condition one easily obtains the value of the wage in E. Consistently to the analysis for equilibrium 1 so far, this value is equal to the one of the wage in U.

4. Firms in E exporting to the rich in U only:

$$\frac{1}{\lambda}\tau(1-\beta)L + \frac{1}{\lambda}L = W^E F + W^E\Big(\frac{(1-\beta)\tau L + L}{a}\Big) \tag{33}$$

Here again I substitute for the wage level and obtain the price of goods in E: $\frac{1}{\lambda} = \frac{(1+\tau)}{(1+\tau-\beta\tau)}\frac{aF+(1+\tau-\beta\tau)L}{aF+(1+\tau)L}$. Given the wages and the prices, I obtain $N_P^U + N_P^E$ from the b.c. of poor individuals in U and substitute it into the b.c. of individuals in E to obtain $N_R^E - N_P^E = \frac{aL(1-\tau\theta)(1+\tau-\beta\tau)}{aF+(1+\tau-\beta\tau)L}$. Plugging this value into the b.c. of rich individuals in U and rearranging I obtain the number of goods produced in E and exported to everyone in U:

$$N_P^E = \frac{aL[(1-\tau\theta)(1+\tau-\beta\tau)+1]}{aF+(1+\tau)L}$$

I substitute this value back into the b.c. of the poor and of individuals in E and obtain respectively

$$N_P^U = \frac{aL[\theta+\tau\theta-1+(1-\tau\theta)(1+\tau-\beta\tau)]}{aF+(1+\tau)L}$$

and

$$N_R^E = \frac{aL(1-\tau\theta)(1+\tau-\beta\tau)}{aF+(1+\tau-\beta\tau)L} + \frac{aL[\theta+\tau\theta-1+(1-\tau\theta)(1+\tau-\beta\tau)]}{aF+(1+\tau)L}$$

Finally, I can substitute all variables above into the b.c. of the rich in U to obtain

$$N_R^U = \frac{aL[\theta+\tau\theta-1+(1-\tau\theta)(1+\tau-\beta\tau)]}{aF+(1+\tau)L} + \frac{aL(1-\theta-\tau(1-\beta)(1-\tau\theta)}{aF+(1-\beta)L}$$

4.1.3 Welfare consequences of arbitrage

When considering the threat of arbitrage, new equilibrium possibilities emerge, which reflect the effect of the constraint on producers. The number of goods produced and/or exported when a threat of parallel trade arises is different from the one calculated before, when I overlooked the role that parallel trade plays in the decision process of firms. It is therefore likely that factoring arbitrage constraints into the analysis yields different welfare considerations for different types of consumers. I showed before that all consumers gain from trade, to the extent that a higher number of goods is available to them than in autarky. I expect this general statement to remain true also when considering arbitrage, but it is possible that the size of the gain is not the same. The following proposition summarizes how arbitrage affects the gains from trade previously investigated:

Proposition 5. *When considering a threat of arbitrage, trade is still preferred to autarky by all types of consumers. If iceberg cost are high and threats of parallel trade only arise for exclusive goods, the gains from trade of the rich in* U *are lower than previously estimated. The same is true for consumers in* E, *unless β is very low, and the opposite is true for poor individuals, unless τ is low and inequality very high. If iceberg cost are low enough to trigger parallel trade for goods produced in* E *as well, the increase in welfare of consumers in* E *and rich in* U *is higher than previously estimated, whereas it is lower for poor consumers in* U.

Proof. IN APPENDIX □

In equilibrium , if arbitrage possibilities arise for exclusive goods only, some firms in U might decide to serve everybody in U and export instead of selling exclusively to the rich. As long as the market for exclusive goods is small enough, the welfare of poor individuals increases. As firms in E face threats on exclusive goods only, also some of them might decide to serve everybody in U instead of serving the rich only. Even if some firms in U serve the rich only without exporting, the welfare of rich individuals is lower than the one calculated without arbitrage considerations. Individuals in E also gain less than previously estimated. They have more goods available thanks to imports, but now some firms in U decide to serve the rich and not to export at all, making the gains of individuals in E smaller than previously calculated. If the market for exclusive goods is big enough though, more firms in E will keep on serving the rich in U and more firms in U will serve the rich only. All in all this makes the welfare of individuals in E higher and the one of the poor lower than previously estimated.

When arbitrage opportunities arise both for exclusive goods and goods consumed in E, welfare considerations are again different. The welfare of poor individuals is always lower than previously calculated, and vice versa is true for individuals in E. Welfare increases are also lower than previously estimated for rich consumers unless inequality is high. If inequality is low the difference among prices in U is not too high but the market for exclusive goods is smaller. With arbitrage threats arising for exclusive goods and mass goods alike, if inequality is low firms in E export more to everybody than to the rich only. When inequality is associated to very low iceberg transportation cost however, more firms in U decide to serve the rich only at home without exporting. This means that more goods are available to the rich, which compensates for the decrease in import of exclusive goods.

4.1.4 Numerical exercise for equilirbium 1

In order to render the discussion more pointed, I perform a numerical exercise to investigate the trade equilibrium and the welfare effects associated to it. I normalize population size to 1 and assume fixed and variable costs of $F = 1.7$ and $a = 1.2$ respectively. In equilibrium 1 inequality must be sufficiently low: values of $\theta = 0.7$ and $\beta = 0.7$ are compatible with the equilibrium and provide a starting point for some comparative statics. The numerical exercises comprise two different cases: (i) when parallel trade is not allowed, so that the arbitrage equilibrium is ruled out by assumption, and (ii) when parallel trade is allowed, so that trade costs limit the scope of price discrimination. A first task is observing the welfare effects of a reduction in iceberg cost from $\tau = 1.6$ to $\tau = 1.35$. The outcome of this exercise is presented in table 1. Without arbitrage, a reduction in τ increases the welfare of all consumers. The increase is of 3.95% for rich in U and individuals in E and of 7.16% for poor in U. If parallel trade can occur, reduction in transportation cost increases the welfare of rich and poor individuals in U by 3.02% and 16.7% respectively, but it reduces the welfare of individuals in E by 3%.

I repeat the same exercise for a decrease in inequality due to an increase in θ from 0.7 to 0.9 (with iceberg cost fixed at 1.35). The results are displayed in table 2. When parallel trade is prohibited, such increase enhances the welfare of poor individuals by 35.9% and decreases the one of rich individuals and consumers in E by 5.7%. The reason is that as θ increases the poor get richer so they can afford a higher amount of goods and the rich cannot afford as many goods in excess of what is consumed by the poor. Less firms in E export to the rich only; some of them start exporting to the poor, some others exit the market because the total number of goods consumed in the economy is lower. If parallel trade is allowed instead, an increase in θ increases the welfare of poor individuals by 28.57% and leaves the one of all other types unchanged. This is because, due to threats of arbitrage, the number of goods in the economy for $\theta = 0.7$ is already lower and less firms in E exporting to the rich only than when parallel trade is forbidden. The increase in θ then simply implies that firms serve everybody in U instead of serving the rich only but no firm exits the market.

If I consider a reduction in inequality due to a decrease in β, say from 0.7 to 0.5, when parallel trade is forbidden the outcome is similar to the one above: welfare decreases by 3.33% for rich in U and consumers in E and increases by 3% for poor individuals. As shown in table 3, changes in welfare are smaller than the ones that occur due to an increase in θ because the market for exclusive goods gets bigger. If parallel trade is allowed however, a reduction in inequality has a very different outcome when it is due to a reduction in β rather than an increase in θ. Welfare decreases for all individuals in U. The decrease is equal to 12.58% for the rich and 10.78% for the poor. Individuals in E instead see their welfare increase by 10.55%. The reason is that as the market for exclusive goods increases, more firms in E decide to export to the rich in U. The rich are now more but less wealthy: they purchase less goods but more of them are imported from E.

4.2 Equilibrium 2

Firms in *U* produce both exclusive and mass consumption goods and export, firms in *E* export exclusive goods only. Also in this case, the price of goods in E is higher than the price of mass-consumption goods in U. Arbitrage arises for mass-consumed goods in if $\tau\frac{1}{\lambda} < \frac{1}{\lambda_P}$, i.e. if

$$1 \geq \tau\Big[W^U \frac{aF + (1+\tau)L}{aL} - 1\Big] \tag{34}$$

which coincides with the condition for equilibrium 1. For what concerns exclusive goods instead, consumers can buy the good in E and ship it to U at cost τ. This strategy is cheaper than simply buying the good at home if $\frac{1}{\lambda_R} > \frac{1}{\lambda}\tau$. The condition for τ is:

$$\frac{1}{1-\beta} - \frac{W^U}{a}\frac{\beta}{1-\beta} \geq \tau\Big[W^U \frac{aF + (1+\tau)L}{aL} - 1\Big] \tag{35}$$

Following the same reasoning as for equilibrium 1, the r.h.s. of the two conditions above coincide. If the l.h.s. of the second one is always greater than 1, then the second condition holds every time the first condition is binding. Rearrangement yields $W^U \leq a$. Given that trade only arises when condition 21 holds, the wage is always smaller than a. Otherwise said, arbitrage always becomes binding for exclusive goods first and proposition 4 holds for equilibrium 2 as well. If arbitrage threats only arise for exclusive goods, the equilibrium will look as follows: firms in U that produce mass consumption goods all export and sell their product at price $\frac{1}{\lambda}$and $\frac{1}{\lambda_P}$ in E and U respectively; firms in E all export exclusive goods at price $\tau\frac{1}{\lambda}$; firms in U that produce exclusive goods do not all export: some do and charge an arbitrage preventing price $\tau\frac{1}{\lambda}$ on the home market, some don't and charge a higher price, equal to the marginal willingness to pay of the rich: $p_R = \frac{1}{\lambda_R}$. In this setting, the conditions to be fulfilled for the equilibrium to hold are the following:

Resource Constraint Denoting as N^U_{RT} the total number of traded goods for country U and by N^U_{RN} the total number of non-traded (exclusive) goods produced in U, the constraint for U is:

$$L = N^U_P \frac{aF + (1+\tau)L}{a} + (N^U_{RT} - N^U_P)\frac{aF + (1+\tau)L - \beta\tau L}{a} + N^U_{RN}\frac{aF + (1-\beta)L}{a}$$

whereas the constraint for E is:

$$L = N^E \frac{aF + (1+\tau)L - \beta\tau L}{a}$$

which translates into:

$$N^E = \frac{aL}{aF + (1+\tau)L - \beta\tau L}$$

Budget Constraint For poor and rich individuals in U the condition is respectively: $W^U\theta = \frac{1}{\lambda_P}N^U_P$ and $\frac{(1-\beta\theta)}{(1-\beta)}W^U = N^U_{RN}\frac{1}{\lambda_R} + (N^U_{RT} - N^U_P + N^E)\tau\frac{1}{\lambda} + N^U_P\frac{1}{\lambda_P}$. For individuals in E the condition is instead: $W^E = \frac{1}{\lambda}(N^U_{RT} + N^E)$[14].

[14]Being N^U_{RT} the total number of goods exported from U, they are all sold at the same price equal to the willingness to pay of individuals in E because in this setting there is no threat of parallel trade for goods produced in E.

Zero Profit Conditions Must hold for each firm and for all strategies available to it in this equilibrium.

1. Firms in U serving everybody at home and exporting:

$$\frac{1}{\lambda_P}L + \frac{1}{\lambda}L = W^U F + W^U \frac{(1+\tau)L}{a} \tag{36}$$

2. Firms in U serving rich only at home and exporting:

$$\tau\frac{1}{\lambda}(1-\beta)L + \frac{1}{\lambda}L = W^U F + W^U \frac{(1+\tau)L - \beta L}{a} \tag{37}$$

3. Firms in U serving rich only at home and not exporting:

$$\frac{1}{\lambda_R}(1-\beta)L = W^U F + W^U \frac{(1-\beta)L}{a} \tag{38}$$

4. Firms in E exporting to the rich in U only:

$$\tau\frac{1}{\lambda}(1-\beta)L + \frac{1}{\lambda}L = W^E F + W^E \frac{(1+\tau)L - \beta\tau L}{a} \tag{39}$$

Equations 37 and 39 yield the following:

$$\frac{W^U}{W^E} = \frac{aF + (1+\tau)L - \beta\tau L}{aF + (1+\tau)L - \beta L} < 1$$

After rearrangement, equation 37 becomes: $\frac{1}{\lambda} = W^U \frac{aF+(1+\tau)L-\beta L}{aL(1+\tau-\tau\beta)}$. Inserting this result into equation 36 and rearranging I get:

$$W^U = \frac{aL(1+\tau-\tau\beta)}{\tau(1-\beta)(aF+(1+\tau)L)+\beta L}$$

This result for the wage allows to obtain: $\frac{1}{\lambda} = \frac{aF+(1+\tau)L-\beta\tau L}{\tau(1-\beta)(aF+(1+\tau)L+\beta L}$ and

$$W^E = \frac{aL(1+\tau-\tau\beta)}{\tau(1-\beta)(aF+(1+\tau)L)+\beta L}\frac{aF+(1+\tau)L-\beta L}{aF+(1+\tau)L-\beta\tau L}$$

The marginal willingness to pay of rich individuals in U derives from equation 38:

$$\frac{1}{\lambda_R} = \frac{(1+\tau-\tau\beta)}{(1-\beta)}\frac{(aF+(1-\beta)L)}{\tau(1-\beta)(aF+(1+\tau)L)+\beta L}$$

The b.c. of poor individuals yields the number of goods produced in U and consumed by everybody in the country:

$$N_P^U = \frac{aL(1+\tau-\tau\beta)\theta}{\tau(1-\beta)(aF+(1+\tau)L)+\beta L}$$

After substitution and rearrangement, the b.c. of consumers in E shows the total number of goods produced in U and traded:

$$N_{RT}^U = \frac{W^E}{1/\lambda} - N^E = \frac{aL[\tau(1-\beta)(aF+(1+\tau)L)+\beta L(\beta\tau-1)]}{[aF+(1+\tau)L-\beta\tau L]^2}$$

Taking the b.c. of rich consumers in U and substituting for all other variables, I obtain the following value for non traded goods produced in U:

$$N_{RN}^U = \frac{aL}{aF+(1-\beta)L}\left\{\frac{\tau(1-\beta)[aF+(1+\tau)L-\theta\beta\tau L]+(1-\theta)\beta L}{\tau(1-\beta)(aF+(1+\tau)L)+\beta L}+ \right. \\ \left. -\frac{\tau(1-\beta)[aF+(1+\tau)L-\beta L]}{aF+(1+\tau)L-\beta\tau L}\right\}$$

Similarly to the results for arbitrage in equilibrium 1, the equilibrium wage is higher when I consider arbitrage than when I do not and the marginal willingness to pay of E's consumers is higher than the one of poor individuals in U. This implies that arbitrage opportunities could arise for goods consumed in E whenever $\frac{1}{\lambda_P}\tau \leq \frac{1}{\lambda}$. Substituting for the values from the equilibrium above, this happens whenever

$$\tau \leq \frac{aF+(1+\tau)L-\beta\tau L}{\tau(1-\beta)[aF+(1+\tau)L]+\beta L}$$

It is useful for further analysis to express the same constraint in terms of the wage in U:

$$W^U \geq \frac{\tau aL(1+\tau-\beta\tau)}{aF+(1+\tau)L-\beta L} \tag{40}$$

Depending on the values of the other variables, if iceberg cost are low enough, arbitrage opportunities arise for goods consumed in E. This means that another kind of arbitrage equilibrium would then arise.

4.2.1 Threat of arbitrage for exclusive goods and goods consumed in E

When arbitrage opportunities arise for both exclusive goods and goods consumed in E, firms in U that serve everybody at home and export must charge an arbitrage-preventing price $\frac{1}{\lambda_P}\tau$ abroad. Firms in E that export to the rich only must charge the arbitrage-preventing price $\frac{1}{\lambda}\tau$ in U, and the same must do U's firms that serve the rich only at home and export. As before, there are some firms in U that only sell to the rich at price $\frac{1}{\lambda_R}$ and do not export. One last time, I can find the equilibrium by solving the necessary conditions:

Resource Constraint Denoting as N_{RT}^U the total number of traded goods for country U and by N_{RN}^U the total number of non-traded (exclusive) goods produced in U, the constraint for U is:

$$L = N_P^U\frac{aF+(1+\tau)L}{a} + (N_{RT}^U - N_P^U)\frac{aF+(1+\tau)L-\beta\tau L}{a} + N_{RN}^U\frac{aF+(1-\beta)L}{a}$$

whereas the constraint for E is:

$$L = N^E\frac{aF+(1+\tau)L-\beta\tau L}{a}$$

which translates into:

$$N^E = \frac{aL}{aF+(1+\tau-\beta\tau)L}$$

Budget Constraint For poor and rich individuals in U the condition is respectively: $W^U\theta = \frac{1}{\lambda_P}N_P^U$ and $\frac{(1-\beta\theta)}{(1-\beta)}W^U = N_{RN}^U\frac{1}{\lambda_R}+(N_{RT}^U-N_P^U+N^E)\tau\frac{1}{\lambda}+N_P^U\frac{1}{\lambda_P}$. For individuals in E the condition is instead: $W^E = \frac{1}{\lambda}(N_{RT}^U - N_P^U + N^E) + \tau\frac{1}{\lambda_P}N_P^U$.

Zero Profit Conditions Must hold for each firm and for all strategies available to it in this equilibrium.

1. Firms in U serving everybody at home and exporting:

$$\frac{1}{\lambda_P}L + \frac{1}{\lambda_P}\tau L = W^U F + W^U\frac{(1+\tau)L}{a} \tag{41}$$

Which rearranged yields $W^U = \frac{aL(1+\tau)}{aF+(1+\tau)L}$ 2. Firms in U serving rich only at home and not exporting:

$$\frac{1}{\lambda_R}(1-\beta)L = W^U F + W^U\frac{(1-\beta)L}{a} \tag{42}$$

Substituting for the wage found above, the marginal willingness to pay of the rich equals $\frac{1}{\lambda_R} = \frac{(1+\tau)}{(1-\beta)}\frac{aF+(1-\beta)L}{aF+(1+\tau)L}$.

3. Firms in U serving rich only at home and exporting:

$$\tau\frac{1}{\lambda}(1-\beta)L + \frac{1}{\lambda}L = W^U F + W^U\frac{(1+\tau)L-\beta L}{a} \tag{43}$$

Here again, substituting for the wage yields the marginal willingness to pay of individuals in E $\frac{1}{\lambda} = \frac{(1+\tau)}{(1+\tau-\beta\tau)}\frac{aF+(1+\tau-\beta)L}{aF+(1+\tau)L}$.

4. Firms in E exporting to the rich in U only:

$$\tau\frac{1}{\lambda}(1-\beta)L + \frac{1}{\lambda}L = W^E F + W^E\frac{(1+\tau)L-\beta\tau L}{a} \tag{44}$$

Since I have already derived the marginal willingness to pay of individuals in E, I can substitute it above and find the wage in E $W^E = \frac{aL(1+\tau)}{aF+(1+\tau)L}\frac{aF+(1+\tau-\beta)L}{aF+(1+\tau-\beta\tau)L} > W^U$

The b.c. of the poor yields $N_P^U = \frac{aL(1+\tau)\theta}{aF+(1+\tau)L}$. I substitute this into the b.c. of individuals in E and obtain $N_{RT}^U - N_P^U + N^E = \frac{aL(1+\tau-\beta\tau)}{aF+(1+\tau-\beta)L}\left(\frac{aF+(1+\tau-\beta)L}{aF+(1+\tau-\beta\tau)L} - \tau\theta\right)$. I then plug this value into the b.c. of the rich in U and rearrange to get

$$N_{RN}^U = \frac{aL\tau(1-\beta)}{aF+(1+\tau-\beta\tau)L}\frac{aF+(1+\tau-\beta)L}{aF+(1-\beta)L} + \frac{aL(1-\theta+\tau^2\theta-\beta\tau^2\theta)}{aF+(1-\beta)L}$$

Finally, I substitute this value back into the b.c. of E's individuals to find

$$N_{RT}^U = \frac{aL(1-\beta)}{aF+(1+\tau-\beta\tau)L} + \frac{aL(1+\tau)\theta}{aF+(1+\tau)L} - \frac{aL(1+\tau-\beta\tau)\tau\theta}{aF+(1+\tau-\beta)L}$$

4.2.2 Welfare consequences of arbitrage

Similarly to the discussion developed in the context of equilibrium 1 under arbitrage constraints, it is important to understand how wealth gains are affected when arbitrage threats enter the picture. Again, I expect trade to be preferred to autarky even when parallel trade is allowed. The question is however how big the gain is for every type of consumer. It is in fact possible, as I showed in the case of equilibrium 1, that when one considers arbitrage possibilities, the welfare gains turn out to be higher or lower than previously calculated. In an effort to avoid parallel trade, firms may decide to export a lower amount of goods than the one calculated when parallel trade is forbidden. The welfare of consumers might be lower than expected if the decrease in imports is not compensated by an increase in domestic production.

Proposition 6. *When considering a threat of arbitrage, trade is still preferred to autarky by all types of consumers. A non-arbitrage equilibrium always overestimates the gains from trade of rich consumers in* U *and also of* E*'s consumers, unless transportation cost are very low. If arbitrage arises in both goods, the increase in welfare of poor consumers is higher than estimated in the non-arbitrage equilibrium iff.* $\tau \geq \frac{1}{\theta} - 1$. *If arbitrage arises in exclusive goods only, the welfare of poor consumers is higher than in the non-arbitrage equilibrium unless both iceberg cost and* β *are low.*

Proof. IN APPENDIX □

Once we allow for arbitrage threats in exclusive goods, firms in U that serve the rich only at home and export are forced to charge a lower, arbitrage-preventing price on the home market. Some of them might then decide to serve everybody at home and export, thus exploiting the bigger market, or to serve the rich only at home and not export, thus exploiting the higher prices. This means that poor individuals in U consume a greater amount of goods, whereas rich in U consume less and individuals in E face a reduction in imports and also consume less goods than previously calculated. When β is very low however, the market for exclusive goods is big. This means that more firms in U will decide to serve the rich only. In this case, the welfare of poor individuals is lower than previously calculated. These considerations also hold true when arbitrage threats arise for goods consumed in E as well, with the only difference being that when iceberg cost are very low, the arbitrage constraint becomes highly binding for firms in U serving everybody at home and exporting. Some of these firms might decide to serve the rich at home, thus decreasing the welfare of poor individuals, whilst the one of individuals in E increases and the more so for low levels of inequality.

4.2.3 Numerical exercise for equilirbium 2

Similarly to what I did for equilibrium 1, I now carry out a numerical exercise for equilibrium 2 in order to grasp some main features of the model. As before, I normalize population size to 1 and assume fixed and variable costs of $F = 1.7$ and $a = 1.2$

respectively. This time inequality must be sufficiently high: values of $\theta = 0.3$ and $\beta = 0.6$ are compatible with equilibrium 2 and provide a starting point for some comparative statics. The numerical exercises comprise two different cases: (i) when parallel trade is not allowed, so that the arbitrage equilibrium is ruled out by assumption, and (ii) when parallel trade is allowed, so that trade costs limit the scope of price discrimination. A first task is observing the welfare effects of a reduction in iceberg cost from $\tau = 1.6$ to $\tau = 1.3$. The outcome of this exercise is presented in table 4. Without arbitrage, a reduction in τ increases the welfare of all consumers. The increase is of 5.33% for rich in U and individuals in E and of 4.24% for poor in U. If parallel trade is allowed, reduction in transportation cost increases the welfare of the poor by 15.8%, but it reduces the welfare of the rich and of individuals in E by 10.75% and 8.3% respectively. This is because a reduction in τ is associated to a stricter arbitrage constraint.

I repeat the same exercise for a decrease in inequality due to an increase in θ from 0.3 to 0.5. Table 5 shows the results. When parallel trade is prohibited, such increase enhances the welfare of poor individuals by as much as 70.75% and decreases the one of rich individuals and consumers in E by 2.75%. The reason is that as θ increases from low levels, the poor get richer and can afford a much higher amount of goods whereas the rich can afford a lower amount of goods. They start purchasing more mass goods and less exclusive goods, with the final effect being negative and equal to the one for individuals in E. If parallel trade is allowed instead, an increase in θ increases the welfare of poor individuals by 66.67% and decreases the one of the rich by 2.08%. The size of the effect is smaller because there are some firms that serve the rich at home and do not export. The decrease of welfare of rich individuals is due to the reduction in the number of firms that serve the rich and export. The number of goods consumed by individuals in E remains unchanged because although less firms in U produce exclusive goods and export, more firms in U decide to serve everybody at home and export.

If I consider a reduction in inequality due to a decrease in β, say from 0.6 to 0.4, the outcome is similar to the one above when parallel trade is forbidden: welfare decreases by 4.83% for rich in U and consumers in E and increases by 2.76% for poor individuals. The changes in welfare are smaller than the ones that occur due to an increase in θ (although smaller for the poor) because the market for exclusive goods is increased. If parallel trade is allowed however, a reduction in inequality has a very different outcome when it is due to a reduction in β rather than an increase in θ. Welfare decreases by 11.62% for the poor in U. Individuals in E and the rich in U instead see their welfare increase by 7.15% and 13.57% respectively. The reason is that as the market for exclusive goods increases, the rich become more but less wealthy, so that the arbitrage constraint is less stringent. More firms in U decide to serve the rich only and export whereas more firms in E enter the market. Table 6 illustrates the results of a decrease in β.

5 Conclusion

In this paper I develop a model of international trade in which income inequality among individuals determines the equilibrium trade patterns. Specifically, I consider trade opportunities between an egalitarian country and a country with an unequal distribution of income. I illustrate how in this setting, different equilibria arise depending on the level of inequality. I consider indivisible goods and non-homothetic preferences in such a way that allows me to concentrate exclusively on the extensive margin of consumption and to emphasize demand-side effects. These aspects make the paper complementary to the existing literature on the effects of supply-related factors on international trade.

Following the intuition of Foellmi et al. (2013), I investigate the relevance and the welfare-related consequences of international arbitrage opportunities in a setting in which price differentials arise due to differences in per capita income both within and across countries. When exporting to poorer individuals, firms face a tradeoff between higher prices and bigger markets. Ultimately some firms will decide to either serve rich consumers abroad instead of poor ones, or not to export at all and serve only the rich consumers in the home market. This paper shows that arbitrage opportunities give rise to export zeroes and reshape the patterns of trade among countries. When countries forbid parallel trade, the prices of all goods are higher in the unequal country than in the egalitarian country. Once parallel trade is allowed however, mass-consumed goods in the unequal country become cheaper than goods in the egalitarian country and the wage level is higher everywhere. The distribution of winners and losers also changes once arbitrage is allowed. The following general findings always hold, independently of whether countries allow parallel trade: *i)* wages and iceberg cost are negatively correlated and *ii)* all consumers always prefer trade to autarky.

This paper introduces a basic model that allows tractability of the subject matter but abstract from a number of issues. I take the total labor endowment as equal in the two countries but this restriction might reveal to be overly simplistic in a world in which both inequality levels and per capita incomes differ widely across countries. I also restrict my analysis to a two-country setting and it would be interesting to understand what happens when a multiplicity of countries having different inequality levels engage in trade. I also introduce inequality in a stylized way and although it could be of interest to observe the effect of a finer specification of the income distribution, this model still allows to grasp the general intuition and to obtain first results on the topic. For this reason I believe it is a good starting point for deepening the knowledge in future research.

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6 APPENDICES

A. Proof of proposition 1

Figure 2 shows equation 6, which represents the equilibrium wage for equilibrium 1: both the r.h.s. and the l.h.s. of the equation are decreasing in wage. I prove in section 3.6.2 that the rightmost crossing point is not an economically valid equilibrium. If I substitute the trade condition 15 into equation 6, I know that the graph of the r.h.s. has to lie above the one of the l.h.s., in order for the equilibrium to occur at the leftmost crossing point. Vice versa, if I substitute for the trade condition of equation 16, I want the graph of the l.h.s. to be above the one of the r.h.s. in order for the equilibrium to occur at the leftmost crossing point. Substituting in equation 6 the critical value of the wage found in equation 15 and setting the r.h.s. to be greater than the l.h.s. I obtain the following:

$$2 - \frac{\beta\tau L\theta}{aL}\frac{aL}{aF+L} < \frac{aF+(1+\tau)L-\beta\tau L}{aF+(1+\tau)L-aL\frac{aF+L}{aL}}$$

$$\frac{2(aF+L)-\beta\tau L\theta}{aF+L} < \frac{aF+(1+\tau)L-\beta\tau L}{\tau L}$$

$$2\tau L(aF+L)-\beta\theta(\tau L)^2 < (aF+L)^2+(aF+L)(1-\beta)\tau L$$

$$\beta\theta(\tau L)^2-(1+\beta)(aF+L)\tau L+(aF+L)^2 > 0$$

Solving the associated equation for τL we obtain:

$$(\tau L)_{1,2} = \frac{\frac{-(1+\beta)(aF+L)}{2} \pm \sqrt{(1+\beta)^2(aF+L)^2-4\beta\theta(aF+L)^2}}{2\beta\theta} =$$

$$= \frac{(aF+L)[-(1+\beta)\pm 2\sqrt{1+\beta^2+2\beta-4\beta\theta}]}{4\beta\theta}$$

This implies that the inequality is solved for values of τ s.t.:

$$\tau < \frac{(\frac{aF}{L}+1)[-(1+\beta)-2\sqrt{1+\beta^2+2\beta(1-2\theta)}]}{4\beta\theta}$$

or

$$\tau > \frac{(\frac{aF}{L}+1)[-(1+\beta)+2\sqrt{1+\beta^2+2\beta(1-2\theta)}]}{4\beta\theta}$$

The r.h.s. of the smallest solution is always negative, hence it is not binding in the model above because trading cost cannot be lower than 1. The second equation is the one that matters for equilibrium 1. Rearranging we obtain:

$$\frac{\tau}{\frac{aF}{L}+1} > \frac{-1-\beta+2\sqrt{1+\beta^2+2\beta(1-2\theta)}]}{4\beta\theta}$$

Applying the same reasoning as above, I can substitute the critical value of equation 16 into equation 6 and set the l.h.s. greater than the r.h.s. this time. It follows that:

$$2 - \frac{\beta\tau L\theta}{aL}\frac{a}{\tau} < \frac{aF + (1+\tau)L - \beta\tau L}{aF + (1+\tau)L - aL\frac{\tau}{a}}$$

$$2 - \beta\tau > \frac{aF + L + \tau L(1-\beta)}{aF + L}$$

$$2 - \beta\tau - 1 > \frac{\tau L(1-\beta)}{aF + L}$$

$$\frac{\tau}{\frac{aF}{L} + 1} < \frac{1 - \beta\theta}{1 - \beta}$$

The two conditions are exactly the ones of proposition 1.

B. Proof of proposition 2

For country E, the number of active firms is $N_R^E = \frac{1}{\frac{aF+L+\tau L}{aL} - \frac{1}{W^U}} - \frac{aL}{aF+L+\tau L}$ under trade and $N = \frac{aL}{aF+L}$ under autarky. The former is greater than or equal to the latter if and only if

$$\frac{1}{aF + (1+\tau)L - \frac{aL}{W^U}} \geq \frac{aF + (1+\tau)L + aF + L}{[aF + (1+\tau)L](aF + L)}$$

The greater the value of the wage, the less likely it is that this condition will be satisfied. The lowest admissible value for the wage under equilibrium 1 is given in equation 13. If the condition is not satisfied at this minimum value, then we know that it does not hold for any other value in this setting. Substituting for the critical value of W^U and rearranging we obtain:

$$\theta \geq \frac{aF + (1+\tau)L}{2(aF + L) + \tau L} = \frac{(aF + L) + \tau L}{2(aF + L) + \tau L}$$

This last value is obviously greater than $\frac{1}{2}$, but it is also greater than the critical value found in equation 14. The inequality above never holds true under equilibrium 1, indicating that the number of active firms in E under equilibrium 1 is lower than under autarky. The number of goods available to individuals in E is $N_R^E + N^U = \frac{aL}{aF+(1+\tau)L - \frac{aL}{W^U}} - \frac{aL}{aF+(1+\tau)L} + \frac{aL}{aF+(1+\tau)L}$ in equilibrium 1 and $N = \frac{aL}{aF+L}$ under autarky. After some rearrangement, we obtain that the former is greater than the latter as long as $W^U < \frac{a}{\tau}$, which coincides with the trade condition calculated above. The number of varieties available in E under equilibrium 1 is greater than the one available under autarky.

In country U, the number of active firms is $N^U = \frac{aL}{aF+(1+\tau)L}$ under trade and $N^R = \frac{aL}{aF+L}\frac{aF+(1-\theta\beta)L}{aF+(1-\beta)L}$ in autarky. Since $(1-\theta\beta) > (1-\beta)$ by construction and $\frac{aL}{aF+(1+\tau)L} < \frac{aL}{aF+L}$, it is always true that $N^R > N^U$ and that the number of active firms in U is greater under autarky than under equilibrium 1. Under equilibrium one, the number of goods consumed by the rich in U is equal to the number of goods consumed by individuals in E and is greater than the number of varieties available under autarky if

$$\frac{aL}{aF+(1+\tau)L-\frac{aL}{W^U}}-\frac{aL}{aF+(1+\tau)L}+\frac{aL}{aF+(1+\tau)L}>\frac{aL}{aF+L}\frac{aF+(1-\theta\beta)L}{aF+(1-\beta)L}$$

After rearrangement we obtain:

$$W^U(aF+L)(\beta(1-\theta)+\tau)<a(aF+L-\beta\theta L)+W^U\tau\theta\beta L$$

Simulations show that W^U is always lower than one and around a value of 0.5, indicating that the inequality always holds and the number of goods consumed by the rich under equilibrium one exceeds the number of goods consumed under autarky. The number of goods consumed by poor individuals in U under equilibrium 1 is $N^U+N^E_P=\frac{aL}{aF+(1+\tau)L}+W^U\theta-\frac{aL}{aF+(1+\tau)L}$, which is greater than the varieties available under autarky $N^P=\frac{aL\theta}{aF+L}$ if and only if $W^U>\frac{aL}{aF+L}$, which corresponds to the trade condition 15. The number of varieties consumed by poor individuals under equilibrium 1 exceeds the one of autarky.

C. Proof of proposition 3

In equilibrium 2, the number of goods produced in E is given by equation 10. This value is smaller than the one calculated for the autarky equilibrium: $N=\frac{aL}{aF+L}$, indicating that a lower number of firms is active under equilibrium 2 than in autarky. However, since the two countries engage in trade, the number of goods consumed by individuals in E is $N^E+N^U_R$, which is equal to the number of goods consumed by rich individuals in U. Equation 11 shows the total number of goods produced in U. The minimum value of N^U_R is attained at the (lower) boundary level of $W^U\theta$. The smallest number of active firms in U under equilibrium 2 is:

$$\begin{aligned}N^U_R &\geq \frac{aL}{aF+(1+\tau)L-\beta L}-\frac{\beta L\frac{aL}{aF+(1+\tau)L}}{aF+(1+\tau)L-\beta L}=\\ &=\frac{aL[aF+(1+\tau)L-\beta L]}{[aF+(1+\tau)L-\beta L][aF+(1+\tau)L]}=\frac{aL[aF+(1+\tau)L-\beta L]}{[aF+(1+\tau)L-\beta L][aF+(1+\tau)L]}=\\ &=\frac{aL}{aF+(1+\tau)L}\end{aligned}$$

which is smaller than the value calculated in the autarky equilibrium and which implies that the number of goods consumed by individuals in E and rich individuals in U is at least:

$$N^E+N^U_R=\frac{aL}{aF+(1+\tau)L-\beta\tau L}+\frac{aL}{aF+(1+\tau)L}$$

This value is greater than N^E as calculated for the autarky equilibrium if

$$\frac{aL}{aF+(1+\tau)L-\beta\tau L}+\frac{aL}{aF+(1+\tau)L}>\frac{aL}{aF+L}$$

rearrangement yields $(aF+L-\tau L)(aF+(1+\tau)L)+\beta(\tau L)^2>0$ which is always true, indicating that individuals in E can afford a higher amount of goods in a trade equilibrium. The same holds for rich individuals in U as long as

$$\frac{aL}{aF+(1+\tau)L-\beta\tau L}+\frac{aL}{aF+(1+\tau)L}>\frac{aL}{aF+L}\frac{aF+(1-\theta\beta)L}{aF+(1-\beta)L}$$

Rearranging again results in the following $(aF+(1-\beta)L)(aF+L)[2(aF+(1+\tau)L)-\beta\tau L]+(aF+(1+\tau)L)[\beta\tau L(aF+(1-\beta\tau)L)-(aF+(1-\beta\theta)L)(aF+(1+\tau)L)]>0$, which is also always true.

The number of goods consumed by poor individuals in U under equilibrium 2 is $N_P^U=W^U\theta\leq\frac{aL}{aF+(1+\tau)L}$. The lower wage equilibrium point is a decreasing function of trading cost. Consequently, the number of goods consumed by poor individuals decreases with increasing trading costs. Under an autarkic regime, poor individuals can afford to buy $\frac{aL\theta}{aF+L}$ goods. As proved in the text, trade only occurs if $W^U>\frac{aL}{aF+L}$: poor individuals can purchase at least $\frac{aL\theta}{aF+L}$, i.e. the amount they can afford in autarky.

D. Proof of proposition 5

In order to prove the statement, we must prove that for every type of consumer, the amount of goods consumed in the arbitrage equilibrium is higher than in autarky and higher/lower than in non-arbitrage trade equilibrium (depending on the type of consumer).

Poor individuals in *U* If arbitrage only arises for exclusive goods, poor individuals consume the following amount of goods: $N_P=N_P^U+N_P^E=\frac{\tau-\beta\tau+1}{\tau}\frac{aL\theta}{aF+(1+\tau)L-\beta\tau L-\beta aF}$. Under autarky, $N_P=\frac{aL\theta}{aF+L}$. Trade is preferred as long as:

$$\frac{\tau-\beta\tau+1}{\tau}\frac{aL\theta}{aF+(1+\tau)L-\beta\tau L-\beta aF}\geq\frac{aL\theta}{aF+L}$$

$$(1+\tau-\beta\tau)(aF+L)\geq\tau(aF+(1+\tau)L-\beta\tau L-\beta aF)$$

$$aF+L+\tau aF+\tau L-\beta\tau aF-\beta\tau L\geq\tau aF+\tau L+\tau^2L-\beta\tau aF-\beta\tau^2L$$

$$aF+L-\beta\tau L\geq\tau^2L(1-\beta)$$

$$aF+L\geq\tau L(\tau+\beta-\beta\tau)$$

$$\frac{\tau}{\frac{aF}{L}+1}\leq\frac{1}{\tau+\beta-\beta\tau}$$

Since, as specified in proposition 1, the r.h.s. cannot exceed $\frac{1-\beta\theta}{1-\beta}$, the inequality always holds. If arbitrage opportunities arise for both exclusive goods and goods consumed in E, poor individuals consume the following amount of goods: $N_P = N_P^U + N_P^E = \frac{aL(\theta+\theta\tau)}{aF+(1+\tau)L}$ which is preferred to autarky as long as it exceeds $\frac{aL\theta}{aF+L}$. Setting the inequality and rearranging yields $\tau\theta aF > 0$, which is always true and indicates that poor individuals always consume a higher number of goods in an arbitrage equilibrium than in autarky.

When arbitrage is forbidden, poor individuals consume $N_P = N^U + N_P^E = W^U\theta \geq \frac{aL}{aF+(1+\tau)L}$. They consume a higher number of goods when arbitrage opportunities only arise for exclusive goods[15] if

$$\frac{\tau-\beta\tau+1}{\tau}\frac{aL\theta}{aF+(1+\tau)L-\beta\tau L-\beta aF} \geq \frac{aL}{aF+(1+\tau)L}$$

$$\theta \geq \frac{\tau(1-\beta)(aF+\tau L)}{\tau(1-\beta)(aF+\tau L)+L+AF+\tau L+\tau(1-\beta)L} < \frac{1}{2}$$

which is always true for equilibrium 1 and indicates that poor individuals consume a higher amount of goods in the arbitrage equilibrium than in the non-arbitrage one when the wage is at its minimum. If the condition also holds at the maximum wage, then it always does. The maximum wage s.t. parallel trade only occurs for exclusive goods is $W^U = \frac{aL(1+\tau)}{aF+(1+\tau)L}$ (see equation 29). Substituting and rearranging yields:

$$\frac{\tau(1+\tau)(1-\beta)aF+\tau(1+\tau)(1+\tau-\beta\tau)L}{\tau(1-\beta)aF+\tau(1+\tau)(1-\beta)L+aF+(1+\tau)L} > 1$$

this condition always holds true unless tau is low (but not so low to trigger arbitrage in goods consumed in E) and β is very high.

If arbitrage arises in both types of goods,[16] the welfare of poor individuals is higher than previously estimated if

$$\frac{aL(\theta+\tau\theta)}{aF+(1+\tau)L} \geq W^U\theta$$

If this condition holds at the highest value of the wage, then it always does. The highest value of the wage s.t. arbitrage occurs is given in equation 24. Substituting and solving yields $\tau^2(1-\beta)(aF+\tau L)+\tau L(\tau+\beta-1)-(aF+L) \geq 0$. Numerical simulations show that this never holds true for values of iceberg cost for which parallel trade arises in both goods. If the same also holds for low values of the wage, then one knows it never holds true. The minimum value of the wage s.t. parallel trade occurs for goods consumed in$E$ is given in equation 29. Substituting and rearranging we obtain $0 \geq 0$, implying that the welfare in the arbitrage cannot exceed the one of the non-arbitrage equilibrium.

[15] From now on I shall refer to this kind of arbitrage as arbitrage setting 1.
[16] From now on I shall refer to this kind of arbitrage as arbitrage setting 2.

Rich individuals in U In arbitrage setting 1, rich individuals consume the following amount of goods:

$$N_R = N_R^E + N_R^U = \frac{aL(\tau-\beta\tau+1)}{aF+(1+\tau)L-\beta\tau L} - \frac{aL\theta}{aF+(1+\tau)L} -$$
$$\frac{aL(1-\beta)}{aF+(1+\tau)L}\frac{(\tau-\theta)[aF+(1+\tau)L]-\beta\tau aF}{aF+(1+\tau)L-\beta\tau L-\beta aF} + \frac{aL[aF+(1+\tau)L-(\beta+\tau)\theta L]}{(aF+(1+\tau)L)(aF+(1-\beta)L} -$$
$$\frac{aL(1-\beta)(\beta L+\tau L)\{(\tau-\theta)[aF+(1+\tau)L-\beta\tau L]-\beta\tau aF\}}{(aF+(1+\tau)L)(aF+(1-\beta)L)[aF+(1+\tau)L-\beta\tau L-\beta\tau aF]} =$$
$$= \frac{aL(\tau-\beta\tau+1)}{aF+(1+\tau)L-\beta\tau L} + \frac{aL[aF+L+\tau L-\beta\theta L-\tau\theta L-\theta aF-\theta L+\theta\beta L]}{(aF+(1+\tau)L)(aF+(1-\beta)L)} -$$
$$\frac{aL(1-\beta)}{aF+(1+\tau)L}\frac{(\tau-\theta)[aF+(1+\tau)L]-\beta\tau aF}{aF+(1+\tau)L-\beta\tau L-\beta aF}\frac{(aF+L-\beta L+\beta L+\tau L)}{aF+(1-\beta)L} =$$
$$= \frac{aL(\tau-\beta\tau+1)}{aF+(1+\tau)L-\beta\tau L} + \frac{aL(1-\theta)(aF+(1+\tau)L)}{(aF+(1+\tau)L)(aF+(1-\beta)L)} -$$
$$\frac{aL(1-\beta)\{(\tau-\theta)[aF+(1+\tau)L]-\beta\tau aF\}}{(aF+(1+\tau)L)(aF+(1-\beta)L)}\frac{aF+(1+\tau)L}{aF+(1+\tau)L-\beta\tau L-\beta aF} =$$
$$= \frac{aL(\tau-\beta\tau+1)}{aF+(1+\tau)L-\beta\tau L} + \frac{aL(1-\theta)}{aF+(1-\beta)L} -$$
$$\frac{aL(1-\beta)}{aF+(1-\beta)L}\frac{(\tau-\theta)[aF+(1+\tau)L-\beta\tau L]-\beta\tau aF}{aF+(1+\tau)L-\beta\tau L-\beta aF} =$$
$$= \frac{aL(\tau-\beta\tau+1)}{aF+(1+\tau)L-\beta\tau L} + \frac{aL}{aF+(1-\beta)L} *$$
$$* \frac{(1-\theta)[aF+(1+\tau)L-\beta\tau L-\beta aF]-(1-\beta)\{(\tau-\theta)[aF+(1+\tau)L]-\beta\tau aF\}}{aF+(1+\tau)L-\beta\tau L-\beta aF}$$

The numerator of the last term can be rearranged in the following way: $aF[1-\beta+\tau(2\beta-1-\beta^2)]+L[1+\tau^2(2\beta-1-\beta^2)]$, hence the final result is:

$$N_R = \frac{aL(\tau-\beta\tau+1)}{aF+(1+\tau)L-\beta\tau L} + \frac{aL\{aF[1-\beta+\tau(2\beta-1-\beta^2)]+L[1+\tau^2(2\beta-1-\beta^2)]\}}{(aF+(1-\beta)L)[aF+(1+\tau)L-\beta\tau L-\beta aF]}$$

In autarky rich individuals consume $\frac{aL}{aF+L}\frac{aF+(1-\beta\theta)L}{aF+(1-\beta)L}$, hereby trade is preferred to autarky as long as

$$\frac{aL(\tau-\beta\tau+1)}{aF+(1+\tau)L-\beta\tau L} + \frac{aL\{aF[1-\beta+\tau(2\beta-1-\beta^2)]+L[1+\tau^2(2\beta-1-\beta^2)]\}}{(aF+(1-\beta)L)[aF+(1+\tau)L-\beta\tau L-\beta aF]} \geq \frac{aL}{aF+L}\frac{aF+(1-\beta\theta)L}{aF+(1-\beta)L}$$

For simplicity, I define the last factor of the l.h.s. as B. After multiplying both sides by $aF+(1-\beta)L$ this yields:

$$\frac{(aF+(1-\beta)L)(\tau+1-\beta\tau)}{aF+(1+\tau)L-\beta\tau L} + \frac{B}{aL} \geq \frac{aF+(1-\beta\theta)L)}{aF+L}$$

$$\frac{B}{aL} + \frac{\tau(1-\beta)(aF+L-\beta L)(aF+L)+(aF+L)^2-\beta L(aF+L)}{(aF+L)(aF+L+\tau(1-\beta)L)} \geq \frac{(aF+L-\beta\theta L)(aF+L+\tau(1-\beta)L}{(aF+L)(aF+L+\tau(1-\beta)L)}$$

After some rearrangement, the inequality becomes:

$$\frac{B}{aL(1-\beta)}+\frac{\tau aF+\beta\tau L/(1-\beta)+\beta\tau\theta L/(aF+L)}{(aF+L)(aF+L+\tau(1-\beta)L)}\geq\frac{\beta\tau L}{(aF+L)(aF+L+\tau(1-\beta)L)}$$

which is always true, indicating that even when we consider the threat of arbitrage for exclusive goods, rich individuals can purchase more goods than in autarky. In arbitrage setting 2, the rich consume an amount of goods greater than in autarky if:

$$N_R=N_R^U+N_R^E=\frac{aL(1-\tau\theta)(1+\tau-\beta\tau)}{aF+(1+\tau-\beta\tau)L}+\frac{2aL[\theta+\tau\theta-1+(1-\tau\theta)(1+\tau-\beta\tau)]}{aF+(1+\tau)L}+$$
$$\frac{aL[1-\theta-\tau(1-\beta)(1-\tau\theta)]}{aF+(1-\beta)L}>\frac{aL}{aF+L}\frac{aF+(1-\beta\theta)L}{aF+(1-\beta)L}$$

Rearrangement yields:

$$\frac{aL(1-\tau\theta)(1+\tau-\beta\tau)}{aF+(1+\tau-\beta\tau)L}+\frac{2aL[\theta+\tau\theta-1+(1-\tau\theta)(1+\tau-\beta\tau)]}{aF+(1+\tau)L}-$$
$$\frac{(aF+L)[\theta+\tau(1-\beta)(1-\tau\theta)]+\theta\beta L}{(aF+L)(aF+(1-\beta)L)}>0$$

The inequality is always satisfied for values of τ s.t. arbitrage opportunities arise in both kinds of goods.

If parallel trade is not allowed, rich individuals purchase $\frac{aL}{aF+(1+\tau)L-\frac{aL}{W^U}}$ goods, which is lower than the amount calculated for arbitrage setting 1 iff.

$$\frac{aL(\tau-\beta\tau+1)}{aF+(1+\tau)L-\beta\tau L}+\frac{aL\{aF[1-\beta+\tau(2\beta-1-\beta^2)]+L[1+\tau^2(2\beta-1-\beta^2)]\}}{(aF+(1-\beta)L)[aF+(1+\tau)L-\beta\tau L-\beta aF]}$$
$$\geq\frac{aL}{aF+(1+\tau)L-\frac{aL}{W^U}}$$

A higher wage in U increases the denominator of the r.h.s., thereby making it more likely that the inequality is satisfied. The smaller wage s.t. trade occurs is given in equation 15; if the inequality is satisfied at this critical point, it is satisfied at all other wage levels. Substituting for the wage and rearranging yields:

$$\frac{B}{aL}+\frac{\tau L(\tau+\beta-\beta\tau)-(aF+L)}{\tau L(aF+(1+\tau)L-\beta\tau L)}\geq 0$$

which is never true for values of iceberg cost s.t. trade occurs. We deduce that for low values of wage rich people consume more in the non-arbitrage equilibrium than in the arbitrage one: Omitting the threat of arbitrage from the model means overestimating the gains from trade of rich individuals. One needs to verify what happens at high levels of wage as well though: if the condition is not satisfied at the upper critical level of wage, then it is never satisfied. The upper limit on wage that we consider is the one at which the threat of arbitrage does not arise for goods consumed in E, which is expressed in equation 29. Substituting and rearranging, the condition becomes:

$$\frac{\tau^2-1-\beta\tau^2}{aF+(1+\tau-\beta\tau)L}+\frac{(1+\tau)\beta\tau L}{(aF+(1+\tau-\beta\tau)L)(aF+(1+\tau)L)}+$$
$$\frac{\tau}{aF+(1-\beta)L}\frac{aF[1-\beta+\tau(2\beta-1-\beta^2)]+L[1+\tau^2(2\beta-1-\beta^2)]}{aF+(1+\tau-\beta\tau)L-\beta aF}\geq 0$$

which is never true. Only the second term on the l.h.s. is positive, and it is of smaller value than the other two terms together. This means that the gains from trade for rich individuals are lower than the ones expected when arbitrage is not considered.

The same proof has to be done for arbitrage setting 2, when parallel trade arises in both exclusive goods and goods consumed in E. If the amount of goods consumed in the arbitrage equilibrium exceed the one consumed in the non-arbitrage equilibrium for low values of wage, then it always does. This time, the lowest value of wage s.t. arbitrage occurs in both types of goods is $W^U=\frac{aL(1+\tau)}{aF+(1+\tau)L}$. Substituting and rearranging yields:

$$\frac{(1-\tau\theta)(1+\tau-\beta\tau)}{aF+(1+\tau-\beta\tau)L}+\frac{1-\theta-\tau(1-\beta)(1-\tau\theta)}{aF+(1-\beta)L}+$$
$$\frac{(2\theta-1)\tau+2\tau^2(1-\beta)-2\tau^3\theta(1-\beta)-1}{\tau[aF+(1+\tau)L]}\geq 0$$

The inequality always holds for values of tau s.t. parallel trade occurs, indicating that the non-arbitrage equilibrium underestimates the gains from trade of rich individuals when trade cost are sufficiently low to trigger parallel trade in both types of goods.

Individuals in *E* In the first arbitrage equilibrium, individuals in E consume the following amount of goods:

$$N^E=N_R^E+N_P^U=\frac{aL(\tau+1-\beta\tau)}{aF+(1+\tau)L-\beta\tau L}$$

The amount they consume in autarky is $\frac{aL}{aF+L}$. The former is greater than or equal to the latter as long as $\frac{aL(\tau+1-\beta\tau)}{aF+(1+\tau)L-\beta\tau L}\geq\frac{aL}{aF+L}$, which after rearrangement becomes:

$$\tau(1-\beta)aF\geq 0$$

The inequality always holds. In the second arbitrage equilibrium, individuals in E consume $\frac{aL(1-\tau\theta)(1+\tau-\beta\tau)}{aF+(1+\tau-\beta\tau)L}+\frac{2aL[\theta+\tau\theta-1+(1-\tau\theta)(1+\tau-\beta\tau)]}{aF+(1+\tau)L}$ goods, which is greater than the amount consumed in autarky as long as

$$\frac{\tau aF[1-\beta-\theta-\tau\theta(1-\beta)]-L\tau\theta(1+\tau-\beta\tau)}{(aF+L)(aF+(1+\tau)L)}+\frac{2[\tau(1-\beta)-\tau^2\theta(1-\beta)+\theta]}{aF+(1+\tau)L}\geq 0$$

The first term is negative, the second term is positive and greater than the first term, indicating that the inequality always holds.

The number of goods that individuals in E consume when arbitrage is forbidden is:

$$N^E = N^R + N^U = \frac{aL}{aF + (1+\tau)L - \frac{aL}{W^U}}$$

Which is lower than the amount consumed in arbitrage iff

$$\frac{aL(\tau + 1 - \beta\tau)}{aF + (1+\tau)L - \beta\tau L} \geq \frac{aL}{aF + (1+\tau)L - \frac{aL}{W^U}}$$

Similarly to the reasoning applied in the previous paragraph, a higher wage decreases the r.h.s. of the equation, making it more likely that the inequality holds. The minimum value of the wage for which arbitrage only arises in exclusive goods is given in equation 29. If the inequality is satisfied at this critical point, then it is satisfied for all other (higher) values of the wage (and iceberg transportation cost). Substituting for the critical value of the wage and rearranging we obtain $\tau(\beta+\tau)L \geq aF + (1+\beta\tau^2)L$, which is never true. We deduct that for high values of transportation cost (i.e. low values of wage), overlooking the threat of parallel trade means overestimating the gains from trade of individuals in E. To verify what happens at high wage levels, I can substitute the highest wage value s.t. arbitrage opportunities arise. The value was derived in equation 29. Substituting into the inequality above and rearranging yields $(\tau^2 - 1)(aF + (1+\tau-\beta\tau)L) \geq \beta\tau^2 aF$, which is only satisfied when β is very low, i.e. when the market for exclusive goods is big. The same reasoning can be applied to the second arbitrage equilibrium, but this time the lowest wage s.t. the equilibrium occurs is the minimum wage s.t. arbitrage arises for goods consumed in E as expressed in equation 29. Substituting and rearranging yields $\frac{\tau(2\tau-1)-1+2\tau^2(1-\beta)(1-\tau\theta)}{\tau[aF+(1+\tau)L]} \geq 0$ Which is always true: The gains from trade are higher if arbitrage is allowed and if it occurs in both kind of goods.

E. Proof of proposition 6

Similarly to what I did for equilibrium 1, I must prove that for every type of consumer the amount of goods consumed in the arbitrage equilibrium is higher than in autarky and higher/lower than in non-arbitrage trade equilibrium (depending on the type of consumer).

Poor individuals in U In arbitrage equilibrium 1, poor individuals consume the following amount of goods: $N_P = N_P^U = \frac{aL(1+\tau-\beta\tau)\theta}{\tau(1-\beta)(aF+(1+\tau)L)+\beta L}$. Under autarky, poor consumers can afford $\frac{aL\theta}{aF+L}$ goods. Their welfare is higher in the arbitrage equilibrium if:

$$\frac{aL(1+\tau-\beta\tau)\theta}{\tau(1-\beta)(aF+(1+\tau)L)+\beta L} \geq \frac{aL\theta}{aF+L}$$

After rearrangement we have:

$$\frac{aF}{L} + 1 + \beta\tau^2 \geq \beta + \tau^2$$

which always holds true. In arbitrage setting 2, poor individuals consume $N_P = \frac{aL(1+\tau)\theta}{aF+(1+\tau)L}$ goods. Following the same proceeding as before and rearranging, this exceed the autarky amount if $\tau aF \geq 0$, which is always true. Applying the same reasoning, the welfare of poor individuals is higher in arbitrage setting 1 than in the non-arbitrage trade equilibrium if:

$$\frac{aL(1+\tau-\beta\tau)\theta}{\tau(1-\beta)(aF+(1+\tau)L)+\beta L} \geq W^U\theta$$

i.e. if

$$W^U \leq \frac{aL(1+\tau)}{\tau(1-\beta)(aF+(1+\tau)L)+\beta L} - \frac{\beta\tau aL}{\tau(1-\beta)(aF+(1+\tau)L)+\beta L}$$

If the condition is satisfied at the higher level of the wage the it always is. Arbitrage arises only for exclusive goods as long as the wage is below the amount expressed in equation 29. Substituting and rearranging, the condition above becomes $1 \geq \frac{\beta L}{aF+(1+\tau)L} + \tau^2(1-\beta)$ which is true unless β is very low. To gain full understanding, one must check what happens at the lowest value of the wage. The minimum value of the wage s.t. trade arises is $W^U = \frac{aL}{aF+L}$. Substituting and rearranging yields $aF+L \geq (1-\beta)(\tau+\beta)\tau L$, which always holds true. This means that if transportation cost are high, the welfare is higher in the arbitrage setting than in non-arbitrage one. If iceberg cost are low, non-arbitrage welfare is almost always higher, although there is no arbitrage for goods consumed in E.

When τ is low enough to trigger parallel trade in both goods, poor individuals consume $\frac{aL(1+\tau)\theta}{aF+(1+\tau)L}$ goods. If this amount exceeds the one consumed in the non-arbitrage equilibrium at the maximum value of the wage, then it always does. The maximum value of the wage s.t. equilibrium 2 occurs is $\frac{aL}{aF+(1+\tau)L}\frac{1}{\theta}$. Substituting and rearranging I obtain: $(1+\tau)\theta > 1$, which only holds true if $\tau \geq \frac{1}{\theta} - 1$. The minimum value of W^U s.t. arbitrage setting 2 occurs is $W^U = \frac{\tau aL(1+\tau-\beta\tau)}{aF+(1+\tau)L-\beta L}$. Substituting and rearranging yields:

$$(aF+(1+\tau)L)(1-\tau^2(1-\beta)) > \beta L(1+\tau)$$

which is always satisfied for values of the variables s.t. arbitrage setting 2 occurs. Hence for low values of τ, poor individuals have higher gains in the arbitrage setting.

Rich individuals in *U* In the first arbitrage equilibrium, rich individuals consume the following amount of goods:

$$N_R = N^U_{RN} + N^U_{RT} + N^E = \frac{aL(aF+(1+\tau)L-\beta L)(1+\tau-\beta\tau)}{2(aF+(1+\tau)L-\beta\tau L)} + \frac{aL}{aF+(1-\beta)L} * \left\{\frac{\tau(1-\beta)(aF+(1+\tau)L-\theta\beta\tau L)+(1-\theta)\beta L}{\tau(1-\beta)(aF+(1+\tau)L)+\beta L} + \frac{\tau(1-\beta)(aF+(1+\tau)L-\beta L)}{aF+(1+\tau)L-\beta\tau L}\right\}$$

Under autarky, they consume instead $\frac{aL}{aF+L}\frac{aF+(1-\beta\theta)L}{af+(1-\beta)L}$ goods. They prefer an arbitrage equilibrium if they can consume more goods than under autarky. Setting the inequality and rearranging yields the following:

$$\frac{1}{(aF+(1+\tau)L-\beta\tau L)^2}\Big\{(aF+(1+\tau)L)[(aF+L)(aF+(1-\beta)L)(1+\tau-\beta\tau)+$$
$$(aF+(1+\tau)L)\tau(1-\beta)-2\beta\tau^2L(1-\beta)]+\beta^2\tau^3L^2(1-\beta)-\beta L(1+\tau-\beta\tau)(aF+L)*$$
$$*(aF+(1-\beta)L)\Big\}+\frac{\beta\theta L[\tau(1-\beta)(aF(1-\tau)+L)-(aF+(1-\beta)L]}{\tau(1-\beta)(aF+(1+\tau)L+\beta L}\geq 0$$

The first term is always greater than zero, whereas the second one can be greater or lower than zero, depending on the values of the variables. Intuitively, it seems that the first term exceeds zero by a greater amount than the second term is lower than zero. This implies that the inequality is satisfied. In order to check the intuition, I run simulations, which confirm my first guess. I must repeat the same exercise for the second arbitrage equilibrium. In this case rich individuals consume the following amount of goods:

$$N_R = N_{RN}^U + N_{RT}^U + N^E = \frac{aL(1+\tau-\beta\tau)}{aF+(1+\tau-\beta\tau)L}+\frac{aL(1+\tau)\theta}{aF+(1+\tau)L}-\frac{aL(1+\tau-\beta\tau)\tau\theta}{aF+(1+\tau-\beta)L}+$$
$$\frac{aL(1-\beta)\tau}{aF+(1+\tau-\beta\tau)L}\frac{aF+(1+\tau-\beta)L}{aF+(1-\beta)L}+\frac{aL(1-\theta+\tau^2\theta-\beta\tau^2\theta)}{aF+(1-\beta)L}$$

Setting this amount greater than the autarky amount, it is straightforward that the condition is always satisfied.

Rich individuals in U consume $\frac{aL}{aF+(1+\tau)L-\beta\tau L}+\frac{aL-W^U\theta\beta L}{aF+(1+\tau)L-\beta L}$ goods in a non-arbitrage trade equilibrium. A smaller wage in U increases the number of goods consumed, which is then minimum at the highest wage. Given that trade only arises if the wage is greater than $\frac{aL}{aF+L}$, if the arbitrage equilibrium is preferred at the minimum wage level, then it is also preferred at any other wage level. Substituting for the critical value of the wage and rearranging yields the following:

$$\frac{(aF+(1-\beta)L)[\tau aF(1-\beta)+L(\beta^2\tau+2\tau-2\beta\tau-\beta+2\tau^2+\beta^2\tau^2-3\beta\tau^2)]}{(aF+(1-\beta)L)(aF+(1+\tau)L-\beta\tau L)^2}-$$
$$\frac{\beta\tau L(1-\beta)(aF+(1+\tau)L-\beta\tau L)}{(aF+(1-\beta)L)(aF+(1+\tau)L-\beta\tau L)^2}+\frac{\tau(1-\beta)(aF+(1+\tau)L-\theta\beta\tau L)+(1-\theta)\beta L}{[\tau(1-\beta)(aF+(1+\tau)L+\beta L](aF+(1-\beta)L)}$$
$$-\frac{aF+(1-\beta\theta)L}{(aF+L)(aF+(1+\tau)L-\beta L)}\geq 0$$

The first term on the l.h.s. always exceeds the second one, and the third term is always positive. The whole l.h.s. is higher for higher values of τ and lower values of all other variables (β,θ,a,F). Simulations show that the term is never lower than zero. A trade equilibrium that does not consider arbitrage threats always underestimates the actual gains from trade of rich individuals.

I must repeat the same exercise for the case in which arbitrage arises in both types of goods. In this case, the minimum wage to consider is the critical one that gives rise to parallel trade in goods consumed in E as indicated in equation 40. If the amounts of goods consumed in the arbitrage equilibrium exceeds the amount consumed in the non-arbitrage equilibrium at this point, then it always does. Substituting and rearranging yields

$$\frac{(1-\beta)\tau}{aF+(1+\tau-\beta\tau)L}+\frac{(1-\beta)\tau}{aF+(1+\tau-\beta\tau)L}\frac{aF+(1+\tau-\beta)L}{aF+(1-\beta)L}-$$
$$\frac{(1+\tau-\beta\tau)\tau\theta(aF+(1+\tau-\beta)L)+aF+[1+\tau-\beta-\beta\theta\tau(1+\tau-\beta\tau]L}{[aF+(1+\tau-\beta)L]^2}+$$
$$\frac{1-\theta+\tau^2\theta-\beta\tau^2\theta}{aF+(1-\beta)L}+\frac{(1+\tau)\theta}{aF+(1+\tau)L}\geq 0$$

All positive terms exceed the one negative term in size, indicating that the condition is always satisfied.

Individuals in *E* In arbitrage setting 1, individuals in E consume the following amount of goods:

$$N^E=N^U_{RT}+N^E=\frac{aL(aF+(1+\tau)L-\beta L)(1+\tau-\beta\tau)}{(aF+(1+\tau)L-\beta\tau L)^2}$$

The amount they consume in autarky is $\frac{aL}{aF+L}$. The former is greater than or equal to the latter as long as $\frac{aL(aF+(1+\tau)L-\beta L)(1+\tau-\beta\tau)}{(aF+(1+\tau)L-\beta\tau L)^2}\geq\frac{aL}{aF+L}$, which after rearrangement becomes:

$$\tau(1-\beta)[(aF+(1+\tau)L-\beta L)(aF+L)-(aF+(1+\tau)L-\beta\tau L)L]+\beta L(aF+L)(\tau-1)\geq 0$$

The first term in the square bracket is always greater than the second, hence the inequality is always satisfied. In arbitrage setting 2, individuals in E consume the following amount of goods:

$$N^E=N^U_{RT}+N^E=\frac{aL(1+\tau-\beta\tau)}{(aF+(1+\tau-\beta\tau)L)}+\frac{aL(1+\tau)\theta}{aF+(1+\tau)L}-\frac{aL(1+\tau-\beta\tau)\tau\theta}{aF+(1+\tau-\beta)L}$$

To check whether this amount exceeds consumption under autarky one must set the inequality and solve. This yields:

$$\frac{(1+\tau-\beta\tau)}{(aF+(1+\tau-\beta\tau)L)}+\frac{(1+\tau)\theta}{aF+(1+\tau)L}-\frac{(1+\tau-\beta\tau)\tau\theta}{aF+(1+\tau-\beta)L}-\frac{1}{aF+L}\geq 0$$

The first term is greater than the third, whereas the second is smaller than the fourth. The first difference is however much greater than the second, indicating that the condition is always satisfied. Individuals in E can consume more goods in a trade equilibrium than in autarky.

In the non-arbitrage trade equilibrium, consumers in E buy $\frac{aL}{aF+(1+\tau)L-\beta\tau L}+\frac{aL-W^U\theta\beta L}{aF+(1+\tau)L-\beta L}$ goods. This quantity depends negatively on the wage, reaching its maximum at the lowest level of the wage, s.t. trade arises. If the number of goods consumed by individuals in E in arbitrage setting 1 is higher than the one consumed in the non-arbitrage equilibrium at the critical (lowest) wage, then it is also higher at any other level of the wage. When I consider the first arbitrage equilibrium, the critical level of the wage is given in equation 20. Substituting and rearranging gives:

$$\tau(1-\beta)(aF+(1+\tau)L)^2-\beta L(1+\tau)(1-\beta)(aF+(1+\tau)L)+(\beta L)^2(1-\beta)\geq$$
$$(aF+(1-\beta\theta)L)(aF+(1+\tau)L)^2-\beta L2\tau(aF+(1+\tau)L)(aF+(1-\beta\theta)L)+$$
$$(\beta L)^2\tau^2(af+(1-\beta\theta)L)$$

The first and third terms on the r.h.s. are greater than the first and third term on the l.h.s., whereas the opposite is true for the second term. The difference in the odd terms is however higher than the difference in the even term, indicating that the r.h.s. is always greater than the l.h.s. Simulations confirm this intuition, indicating that when the number of goods consumed in the non-arbitrage equilibrium is maximum, the inequality does not hold. In order to check whether the inequality ever holds, I must check what happens at the maximum critical level of the wage s.t. arbitrage occurs, i.e. $W^U=\frac{aL(1+\tau)}{\tau(1-\beta)(aF+(1+\tau)L)+\beta L}$. If the inequality is not satisfied at this point, then it never is. After substituting and rearranging I obtain:

$$-\tau(1-\beta)(aF+(1+\tau)L)^3(1+\beta\tau-\tau)+\beta L\tau(aF+(1+\tau)L)^2(1-\theta+\tau(1-4\beta-\theta-\beta^2))$$
$$(\beta L)^2(aF+(1+\tau)L)[\tau(\beta^2-3\beta-(1-\beta)\tau^2+2\theta(1+\tau)-1-\beta)]+$$
$$(\beta L)^3(1-\beta+\theta\tau^2(1+\tau)-\tau^2)\geq 0$$

All terms are lower than zero, hence the equality does not hold. This means that the number of goods consumed by individuals in E in a non-arbitrage equilibrium is always higher than the one calculated in arbitrage setting 1.

I must repeat the same exercise for arbitrage setting 2 as well. In this case, the lowest value of the wage is the one of equation 40. Substituting and rearranging I obtain: $\frac{(1-\beta)\tau}{aF+(1+\tau-\beta\tau)L}+\frac{(1+\tau)\theta}{aF+(1+\tau)L}-\frac{(aF+L)(\tau\theta+\tau^2\theta+1-\beta\tau^2\theta)+L(\tau^2\theta-2\beta\tau^2\theta+\tau^3\theta(1-\beta)-2\beta\tau\theta+\tau-\beta)}{[aF+(1+\tau-\beta)L]^2}>0$, which is always negative. At low values of the wage, individuals consume less goods in the arbitrage equilibrium. If this also holds true for high values of the wage, then it is generally true. The maximum value of the wage s.t. arbitrage in exclusive goods occurs in equilibrium 2 is implicitly expressed in equation 35, which can be rearranged to $W^U\leq\frac{aL(1+\tau-\beta\tau)}{\tau(1-\beta)(aF+(1+\tau)L)+\beta L}$. Substituting this value and rearranging yields:

$$\frac{1+\tau-\beta\tau}{aF+(1+\tau-\beta\tau)L}+\frac{(1+\tau)\theta}{aF+(1+\tau)L}+-\frac{(1+\tau-\beta\tau)\tau\theta+1}{aF+(1+\tau-\beta)L}$$

$$+ \frac{(1+\tau-\beta\tau)\theta\beta L}{[aF+(1+\tau-\beta)L][\tau(1-\beta)(aF+(1+\tau)L)+\beta L]} \geq 0$$

The inequality always holds, indicating that for high values of the wage (i.e. low values of transportation cost), the welfare of E's consumers is higher in the arbitrage equilibrium than in the non-arbitrage one.

	Rich	Poor	Individuals in E	
Goods Consumed - No Arbitrage	0.58	0.33	0.58	$\tau = 1.6$
	0.6	0.35	0.6	$\tau = 1.35$
Percentage change	3.95%	7.16%	3.95%	
Goods Consumed - Arbitrage	0.77	0.37	0.51	$\tau = 1.6$
	0.79	0.43	0.49	$\tau = 1.35$
Percentage change	3.02%	16.7%	-3%	

Table 1: Change in the number of goods consumed before and after a decrease in iceberg cost in equilibrium 1. $\theta = 0.7$ and $\beta = 0.7$

	Rich	Poor	Individuals in E	
Goods Consumed - No Arbitrage	0.6	0.35	0.6	$\theta = 0.7$
	0.57	0.48	0.57	$\theta = 0.9$
Percentage change	-5.7%	35.39%	-5.7%	
Goods Consumed - Arbitrage	0.79	0.43	0.49	$\theta = 0.7$
	0.79	0.56	0.49	$\theta = 0.9$
Percentage change	0%	28.57%	0%	

Table 2: Change in the number of goods consumed before and after an increase in θ in equilibrium 1. $\tau = 1.35$ and $\beta = 0.7$

	Rich	Poor	Individuals in E	
Goods Consumed - No Arbitrage	0.6	0.35	0.6	$\beta = 0.7$
	0.58	0.36	0.58	$\beta = 0.5$
Percentage change	-3.33%	3%	-3.33%	
Goods Consumed - Arbitrage	0.79	0.43	0.49	$\beta = 0.7$
	0.7	0.4	0.54	$\beta = 0.5$
Percentage change	-12.58%	-10.78%	10.55%	

Table 3: Change in the number of goods consumed before and after a decrease in β in equilibrium 1. $\tau = 1.35$ and $\theta = 0.7$

	Rich	Poor	Individuals in E	
Goods Consumed - No Arbitrage	0.6	0.146	0.6	$\tau = 1.6$
	0.63	0.153	0.63	$\tau = 1.3$
Percentage change	5.33%	4.24%	5.33%	
Goods Consumed - Arbitrage	1.87	0.17	0.59	$\tau = 1.6$
	1.67	0.19	0.54	$\tau = 1.3$
Percentage change	3.02%	16.7%	-3%	

Table 4: Change in the number of goods consumed before and after a decrease in iceberg cost in equilibrium 2. $\theta = 0.3$ and $\beta = 0.6$

	Rich	Poor	Individuals in E	
Goods Consumed - No Arbitrage	0.63	0.153	0.63	$\theta = 0.3$
	0.62	0.26	0.62	$\theta = 0.5$
Percentage change	-2.75%	70.75%	-2.75%	
Goods Consumed - Arbitrage	1.67	0.19	0.54	$\theta = 0.3$
	1.63	0.32	0.54	$\theta = 0.5$
Percentage change	-2.08%	66.67%	0%	

Table 5: Change in the number of goods consumed before and after an increase in θ in equilibrium 2. $\tau = 1.3$ and $\beta = 0.6$

	Rich	Poor	Individuals in E	
Goods Consumed - No Arbitrage	0.63	0.153	0.63	$\beta = 0.6$
	0.6	0.16	0.6	$\beta = 0.4$
Percentage change	-4.83%	2.76%	-4.83%	
Goods Consumed - Arbitrage	1.67	0.19	0.54	$\beta = 0.6$
	1.9	0.17	7.15	$\beta = 0.4$
Percentage change	13.57%	-11.62%	7.15%	

Table 6: Change in the number of goods consumed before and after a decrease in β in equilibrium 2. $\tau = 1.3$ and $\theta = 0.3$

Figure 1: Lorenz Curve for $\theta = \beta = 0.5$

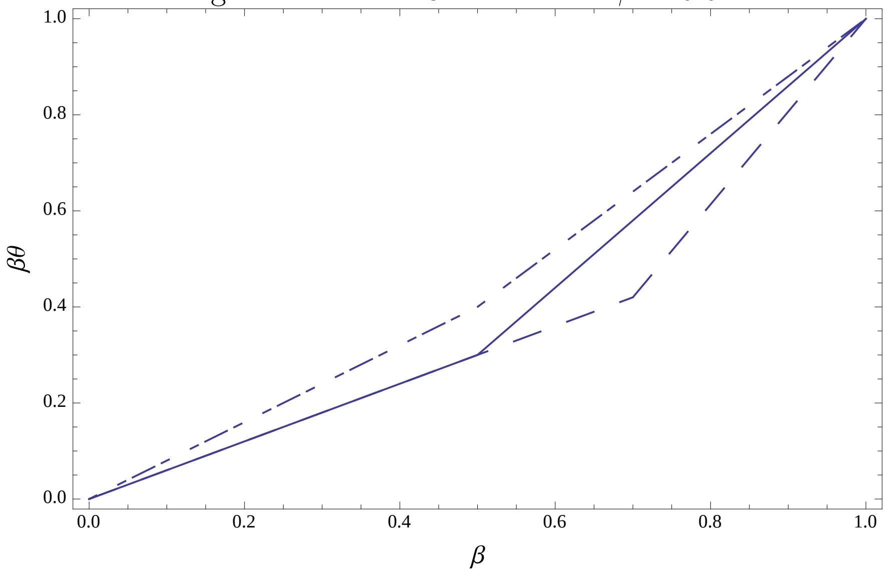

Figure 2: Final equation for equilibrium 1: l.h.s. and r.h.s.

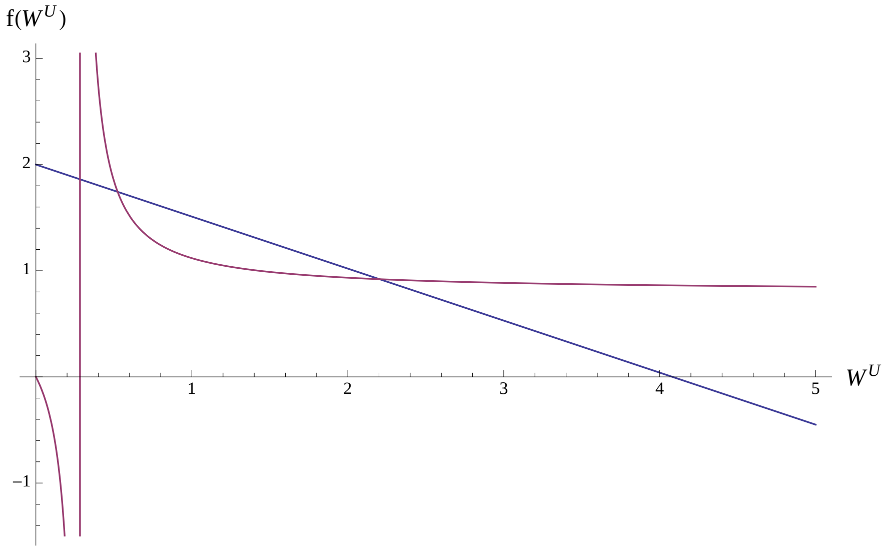

Figure 3: Final equation for equilibrium 2: l.h.s. and r.h.s.

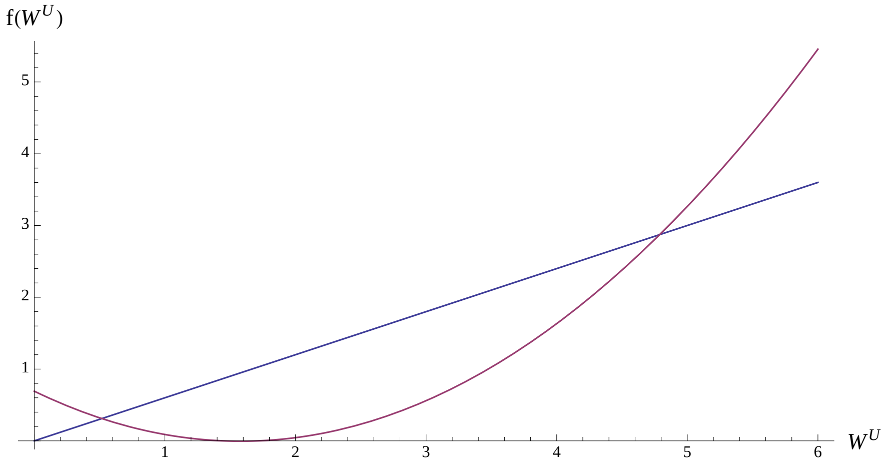

Figure 4: Cutoff Lines among equilibria for $\tau = 1.2$ and different values of inequality parameters

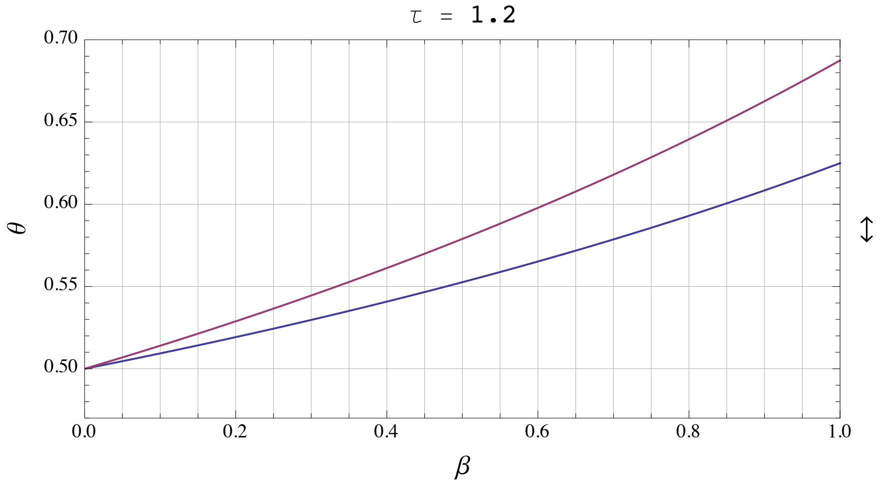

Figure 5: Number of goods consumed by poor individuals in each trade equilibrium and under autarky

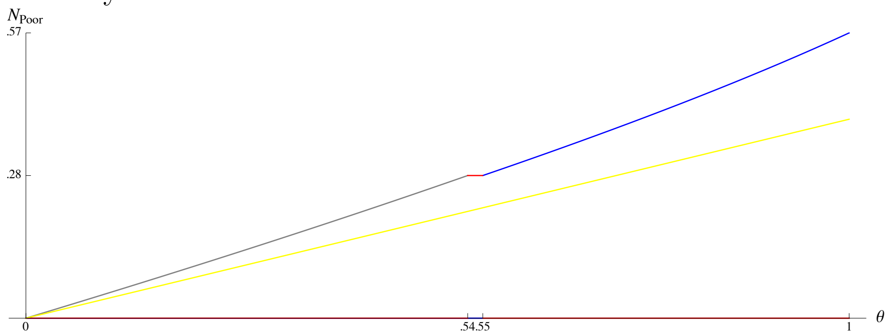

Figure 6: Number of goods consumed by rich individuals in each trade equilibrium and under autarky

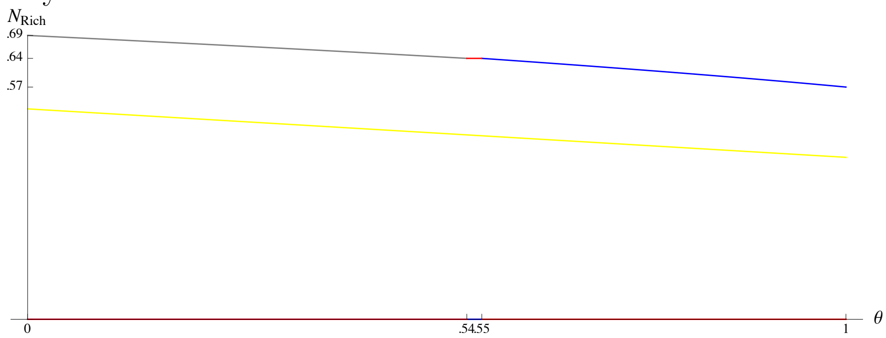

Figure 7: Number of goods consumed by individuals in E in each trade equilibrium and under autarky