NOT SO DEMANDING: PREFERENCE STRUCTURE, FIRM BEHAVIOR, AND WELFARE*

Monika Mrázová†
University of Geneva,
University of Surrey, and CEP, LSE

J. Peter Neary‡
University of Oxford
and CEPR

December 23, 2013
Revised June 15, 2014

Abstract

We introduce a new perspective on how assumptions about preferences and demand affect firm behavior and economic performance. Our key innovation is the "Demand Manifold," which links the elasticity and convexity of an arbitrary demand function. We show that it is a sufficient statistic for many comparative statics questions, leads naturally to some new families of demand functions, and relates demand structure to the behavior of monopoly firms and monopolistically competitive industries. Finally, we show how our approach can be extended to understand variable pass-through and to develop a quantitative framework for measuring the welfare effects of globalization.

Keywords: Heterogeneous Firms; Pass-Through; Quantifying Gains from Trade; Super- and Sub-Convexity; Supermodularity.

JEL Classification: F23, F15, F12

---

*We are grateful to Kevin Roberts for suggestions which substantially improved the paper, and to Jim Anderson, Pol Antràs, Arnaud Costinot, Simon Cowan, Dave Donaldson, Ron Jones, Arthur Lewbel, Paul Klemperer, Lars Mathiesen, Mark Melitz, Mathieu Parenti, Fred Schroyen, Glen Weyl, and participants at various seminars and conferences, for helpful comments. Monika Mrázová thanks the Fondation de Famille Sandoz for funding under the "Sandoz Family Foundation - Monique de Meuron" Programme for Academic Promotion. Peter Neary thanks the European Research Council for funding under the European Union's Seventh Framework Programme (FP7/2007-2013), ERC grant agreement no. 295669.

†Geneva School of Economics and Management (GSEM), University of Geneva, Bd. du Pont d'Arve 40, 1211 Geneva 4, Switzerland; e-mail: monika.mrazova@unige.ch.

‡Department of Economics, University of Oxford, Manor Road, Oxford OX1 3UQ, UK; e-mail: peter.neary@economics.ox.ac.uk.

1 Introduction

Assumptions about the structure of preferences and demand matter enormously for comparative statics in trade, industrial organization, and many other applied fields. Consider just a few examples of classic questions, and the answers to them, which have attracted recent attention:

1. Competition Effects: Does globalization reduce firms' markups? Yes, if and only if the elasticity of demand falls with sales.[1]

2. Pass-Through: Do firms pass on cost increases by more than dollar-for-dollar? Yes, if and only if the demand function is more than log-convex.[2]

3. Selection Effects: Do more productive firms select into FDI rather than exports? Yes, if and only if the elasticity and convexity of demand sum to more than three.[3]

4. Price Discrimination: Does it raise welfare? Yes, if demand convexity falls as price rises.[4]

5. Welfare: Is monopolistic competition efficient? Yes, if and only if preferences are CES.[5]

In each case, the answer to an important real-world question hinges on a feature of preferences or demand which seems at best arbitrary and in some cases esoteric. All bar specialists may have difficulty remembering these results, far less explicating them and relating them to each other.

There is an apparent paradox here. These applied questions are all supply-side puzzles: they concern the behavior of firms or the performance of industries. Why then should the answers to them hinge on the shape of demand functions, and in all but the last case

[1]See Krugman (1979) and Zhelobodko, Kokovin, Parenti, and Thisse (2012).

[2]See Bulow and Pfleiderer (1983), Fabinger and Weyl (2012), and Weyl and Fabinger (2013).

[3]See Helpman, Melitz, and Yeaple (2004) and Mrázová and Neary (2011). The result holds when exports incur iceberg trade costs. See Section 2.4 below.

[4]See Schmalensee (1981) and Aguirre, Cowan, and Vickers (2010), especially Proposition 2.

[5]See Dixit and Stiglitz (1977) and Dhingra and Morrow (2011).

on their second or even third derivatives? The paradox is only apparent, however. In perfectly competitive models, shifts in supply curves lead to movements along the demand curve, and so their effects hinge on the slope or elasticity of demand. When firms are monopolists or monopolistic competitors, as in this paper, they do not have a supply function as such; instead, exogenous supply-side shocks or differences between firms lead to more subtle differences in behavior, whose implications depend on the curvature as well as the slope of the demand function.

Different authors and even different sub-fields have adopted a variety of approaches to these issues. Weyl and Fabinger (2013) show that many results can be understood by taking the degree of pass-through of costs to prices as a unifying principle. Macroeconomists frequently work with the "superelasticity" of demand, due to Kimball (1995), to model more realistic patterns of price adjustment than allowed by CES preferences. In our previous work (Mrázová and Neary (2011)), we showed that, since monopoly firms adjust along their marginal revenue curve rather than the demand curve, the elasticity of marginal revenue itself pins down some results. Each of these approaches focuses on a single demand measure which is a sufficient statistic for particular results. This paper complements these by showing how the different measures are related and by providing a new perspective on how assumptions about the functional form of demand determine conclusions about comparative statics.

The key idea we explore is the value of taking a "firm's eye view" of demand functions. To understand a monopoly firm's responses to infinitesimal shocks it is enough to focus on the local properties of the demand function it faces, since these determine its choice of output: the slope of demand determines the firm's level of marginal revenue, which it wishes to equate to marginal cost, while the curvature of demand determines the slope of marginal revenue, which must be decreasing if the second-order condition for profit maximization is to be met. Measuring slope and curvature in unit-free ways leads us to focus on the elasticity and convexity of demand, following Seade (1980), and our major innovation is to show that for a given demand function these two parameters are related to each other. We call the implied relationship the "Demand Manifold", and show that it is a sufficient statistic linking the functional form of demand to many comparative statics properties. It thus allows us to illustrate existing results and develop new ones in a simple and compact way; and it accommodates a wide range of demand behavior, including some new demand functions which provide a parsimonious way of nesting better-known ones.

A "firm's-eye view" is partial-equilibrium by construction, of course. Nevertheless, it can provide the basis for understanding general-equilibrium behavior. To demonstrate this, we show how our approach allows us to characterize the responses of outputs, prices and firm numbers in the canonical model of international trade under monopolistic competition due to Krugman (1979). In addition, we are able to throw light on the welfare effects of globalization in this model by introducing a second illustrative device, the "Utility Manifold". Analogously to the demand manifold, this links the elasticity and the convexity of an arbitrary utility function, and is a key determinant of the efficiency properties of an imperfectly competitive equilibrium and the welfare effects of exogenous shocks. This is of interest both in itself, and in the light of Arkolakis, Costinot, Donaldson, and Rodríguez-Clare (2012), who provide a rare exception to the rule that the functional form of demand matters for comparative statics results. Extending earlier work by Arkolakis, Costinot, and Rodríguez-Clare (2012), they show that the gains from trade in a wide class of monopolistically competitive models are affected little by departing from CES assumptions. Our results do not contradict theirs, but they suggest some notes of caution when we wish to quantify the gains from trade in models that allow for a wide range of demand behavior.

The plan of the paper follows this route map. Section 2 introduces our new perspective on demand, showing how the elasticity and convexity of demand condition comparative statics results. Section 3 shows how the demand manifold can be located in the space of elasticity and convexity, and explores how a wide range of demand functions, both old and new, can be represented by their manifold in a parsimonious way. Section 4 illustrates the usefulness of our approach by applying it to a canonical general-equilibrium model of international trade under monopolistic competition, and characterizing the implications of assumptions about functional form for the quantitative effects of exogenous shocks. Section 5 concludes, while the Appendix gives proofs of all propositions and notes some more technical extensions.

2 Demand Functions and Comparative Statics

2.1 A Firm's-Eye View of Demand

Almost by definition, a perfectly competitive firm takes the price it faces as given. Our starting point is the fact that a monopolistic or monopolistically firm takes the demand function it faces as given. Observing economists will often wish to solve for the full general equilibrium of the economy, or to consider the implications of alternative assumptions about the structure of preferences (such as discrete choice, representative agent, homotheticity, separability, etc.). By contrast, the firm takes all these as given and is concerned only with maximizing profits subject to the demand function it perceives. For the most part we write this demand function in inverse form, $p = p(x)$, with the only restrictions that consumers' willingness to pay is continuous, twice differentiable, and strictly decreasing in sales: $p'(x) < 0$. It is sometimes convenient to switch to the corresponding direct demand function, $x = x(p)$, with $x'(p) < 0$, the inverse of $p(x)$.

Because we want to highlight the implications of alternative assumptions about demand, we assume throughout that marginal cost is constant.[6] Maximizing profits therefore requires that marginal revenue should equal marginal cost and should be decreasing with output. These conditions can be expressed in terms of the slope and curvature of demand, measured by two unit-free parameters, the elasticity ε and convexity ρ of the demand function:

$$\varepsilon(x) \equiv -\frac{p(x)}{xp'(x)} > 0 \quad \text{and} \quad \rho(x) \equiv -\frac{xp''(x)}{p'(x)} \tag{1}$$

[6] Zhelobodko, Kokovin, Parenti, and Thisse (2012) show that variable marginal costs make little difference to the properties of models with homogeneous firms. In models of heterogeneous firms it is standard to assume that marginal costs are constant.

These are not unique measures of slope and curvature, and our results could alternatively be presented in terms of other parameters, such as the convexity of the direct demand function, or the Kimball (1995) superelasticity of demand. (See Appendix A for more details and references.) If the first-order condition holds for a zero marginal cost, it implies that the elasticity cannot be less than one:

$$p + xp' = c \geq 0 \quad \Rightarrow \quad \varepsilon \geq 1 \tag{2}$$

As for the second-order condition, if marginal revenue decreases with output, then our measure of convexity must be less then two:

$$2p' + xp'' < 0 \quad \Rightarrow \quad \rho < 2 \tag{3}$$

These restrictions can be visualized in terms of an admissible region in $\{\varepsilon, \rho\}$ space, as shown by the shaded region in Figure 1.[7]

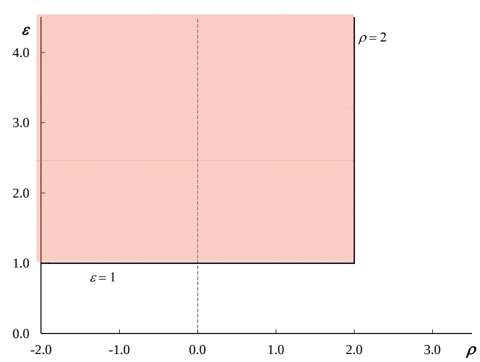


Figure 1: The Admissible Region

[7]The admissible region is $\{\varepsilon, \rho\} \in \{1 \leq \varepsilon \leq \infty, -\infty \leq \rho < 2\}$. We focus on the case where $\varepsilon \leq 4.5$ and $\rho \geq -2.0$, since this is where most interesting issues arise. Note that the admissible region is larger in oligopolistic markets, since both boundary conditions are less stringent than (2) and (3). See Appendix B for details.

2.2 Superconvexity

In general, both ε and ρ vary with sales. The only exception is the case of CES preferences or iso-elastic demands:[8]

$$p(x) = \beta x^{-1/\sigma} \quad \Rightarrow \quad \varepsilon = \sigma, \quad \rho = \rho^{CES} \equiv \frac{\sigma + 1}{\sigma} > 1 \tag{4}$$

Clearly this case is very special: both elasticity and convexity are determined by a single parameter. The curve labeled "SC" in Figure 2 illustrates the implied relationship between ε and ρ for all members of the CES family: $\varepsilon = \frac{1}{\rho - 1}$, or $\rho = \frac{\varepsilon+1}{\varepsilon}$. Every point on this curve corresponds to a different demand function: firms always operate at that point irrespective of the values of exogenous variables. In this respect too the CES is very special, as we will see. The Cobb-Douglas special case corresponds to the point $\{\varepsilon, \rho\} = \{1, 2\}$, and so has the dubious distinction of being just on the boundary of both the first- and second-order conditions.

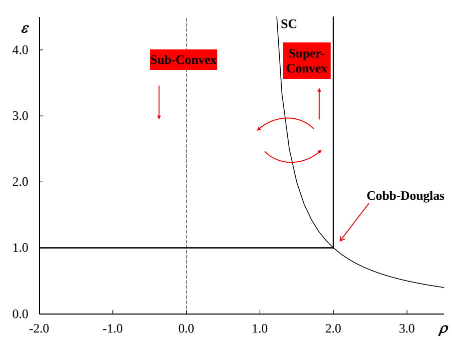


Figure 2: The Super- and Sub-Convex Regions

The CES case is important in itself but also because it is an important boundary for comparative statics results. Following Mrázová and Neary (2011), we can define a local property of any point on an arbitrary demand function as follows:

[8]That CES demands are sufficient for constant elasticity is obvious. That they are necessary follows from setting $-\frac{p(x)}{xp'(x)}$ equal to a constant σ and integrating.

Definition 1. A demand function $p(x)$ is superconvex at a point (p_0, x_0) if and only if $\log p(x)$ is convex in $\log x$ at (p_0, x_0).

As we show in Appendix C, superconvexity of a demand function at an arbitrary point is equivalent to the function being more convex at that point than a CES demand function with the same elasticity:

$$\frac{d^2 \log p}{d(\log x)^2} = \frac{1}{\varepsilon}\left(\rho - \frac{\varepsilon + 1}{\varepsilon}\right) = \frac{1}{\varepsilon}\left(\rho - \rho^{CES}\right) \geq 0 \tag{5}$$

Hence the SC curve in Figure 2 divides the admissible region in two: points to the right of the curve are strictly superconvex, points to the left are strictly subconvex, while all CES demand functions are both weakly superconvex and weakly subconvex. We also show in Appendix C that superconvexity determines the relationship between demand elasticity and sales: the elasticity of demand increases in sales (or, equivalently, decreases in price), $\varepsilon_x \geq 0$, if and only if the demand function $p(x)$ is superconvex. So, ε is independent of sales only along the SC locus, it increases with sales in the superconvex region to the right, and decreases with sales in the subconvex region to the left. These properties imply something like the comparative-statics analogue of a phase diagram: the arrows in Figure 2 indicate the direction of movement as sales rise.

Superconvexity also matters for competition effects and for relative pass-through: the effects of globalization and of cost changes respectively on firms' proportional profit margins.[9] From the first-order condition, the relative markup $\frac{p-c}{p}$ equals $-\frac{xp'}{p}$, which is just the inverse of the elasticity ε. Hence, if globalization reduces incumbent firms' sales in their home markets, it is associated with a higher elasticity and so a lower markup if and only if demand is subconvex. Similarly, an increase in marginal cost c, which other things equal must lower sales, is associated with a higher elasticity and so a lower proportional profit margin, implying

[9] This is the sense in which the term "pass-through" is used in international macroeconomics. See for example Gopinath and Itskhoki (2010).

less than 100% pass-through, if and only if demands are subconvex:[10]

$$\frac{d\log p}{d\log c} = \frac{\varepsilon - 1}{\varepsilon}\frac{1}{2-\rho} > 0 \quad \Rightarrow \quad \frac{d\log p}{d\log c} - 1 = -\frac{\varepsilon + 1 - \varepsilon\rho}{\varepsilon(2-\rho)} \gtreqless 0 \tag{6}$$

However, for *absolute pass-through*, a different criterion applies, to which we turn next.

2.3 Super-Pass-Through

The criterion for absolute pass-through from cost to price has been known since Bulow and Pfleiderer (1983). Differentiating the first-order condition $p + xp' = c$, we see that an increase in cost must raise price provided only that the second-order condition holds, which implies a different expression for the effect of an increase in marginal cost on the absolute profit margin:

$$\frac{dp}{dc} = \frac{1}{2-\rho} > 0 \quad \Rightarrow \quad \frac{dp}{dc} - 1 = \frac{\rho - 1}{2-\rho} \gtreqless 0 \tag{7}$$

Hence we have what we call "Super-Pass-Through", whereby the equilibrium price rises by more than the increase in marginal cost, if and only if ρ is greater than one. Figure 3 shows how the admissible region in $\{\varepsilon, \rho\}$ space is divided into sub-regions corresponding to super- and sub-pass-through. The boundary between the sub-regions corresponds to a log-convex direct demand function, which is less convex than the CES.[11] It is immediately obvious that superconvexity implies super-pass-through, but not the converse.

We have already seen in the introduction that some comparative statics results hinge on whether convexity increases with sales, or, equivalently, decreases with price. It is clear from (7) that pass-through and convexity must increase together, so this is equivalent to asking whether absolute pass-through increases with sales, as highlighted by Weyl and Fabinger

[10] More generally, loci corresponding to $100k\%$ pass-through are defined by $\rho = \frac{2k-1}{k} + \frac{1}{k\varepsilon}$: a family of rectangular hyperbolas, all asymptotic to $\rho = \frac{2k-1}{k}$ and $\varepsilon = 0$, and all passing through the Cobb-Douglas point $\{\varepsilon, \rho\} = \{1, 2\}$.

[11] Setting $\rho = 1$ implies a second-order ordinary differential equation $xp''(x) + p'(x) = 0$. Integrating this yields $p(x) = c_1 + c_2 \log x$, where c_1 and c_2 are constants of integration, which is equivalent to a log-convex direct demand function, $\log x(p) = \gamma + \delta p$.

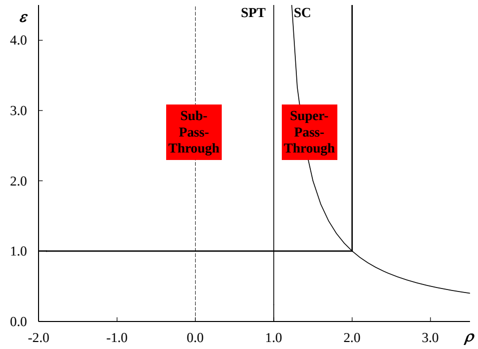


Figure 3: The Super- and Sub-Pass-Through Regions

(2013). As we show in Appendix C, a necessary and sufficient condition for this is the following:

Lemma 1. *Pass-through and convexity increase with sales if and only if* $\rho(1+\rho-\chi)>0$*, where* $\chi \equiv -\frac{xp'''}{p''}$*.*

The parameter χ is a unit-free measure of the third derivative of the demand function, which, following Kimball (1992) and Eeckhoudt, Gollier, and Schneider (1995), we call the "Coefficient of Relative Temperance," or simply "temperance." The result in Lemma 1 that the change in convexity as sales rise depends only on temperance and convexity itself parallels that in the previous section that the change in elasticity as sales rise depends only on convexity and elasticity itself: see Appendix C.

Our diagram in $\{\varepsilon, \rho\}$ space provides a convenient way of checking how pass-through varies with sales for a given demand function. Except for CES demand functions, both elasticity and convexity vary with sales; and we have already seen that the variation of elasticity with sales depends on whether demand is sub- or super-convex. From this it is easy to infer how convexity must vary with sales using the fact that:

$$\frac{d\varepsilon}{d\rho} = \varepsilon_x \frac{dx}{d\rho} = \frac{\varepsilon_x}{\rho_x} \tag{8}$$

Recalling that the direction of change of pass-through depends only on the sign of ρ_x, we can conclude that pass-through increases with sales if and only if $\frac{d\varepsilon}{d\rho}$ and ε_x have the same sign. Since, as we have seen in Section 2.2, ε_x is positive if and only if the demand function is superconvex, we can conclude the following:

Lemma 2. *Pass-through increases with sales if and only if* <u>*either*</u>*: elasticity and convexity move together in the subconvex region;* <u>*or*</u> *elasticity and convexity move in opposite directions in the superconvex region.*

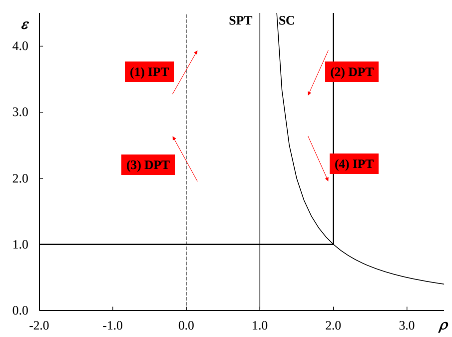


Figure 4: Increasing and Decreasing Pass-Through

This result is illustrated in Figure 4, where cases (1) and (4) represent the two configurations that are consistent with increasing pass-through, while cases (2) and (3) imply decreasing pass-through.[12]

2.4 Supermodularity

The third criterion for comparative statics responses that we can locate in our diagram arises in models with heterogeneous firms. Consider a choice between two ways of serving a market, one of which incurs lower variable costs but higher fixed costs than the other.[13] Let $\pi(c, t)$ denote the maximum operating profits which a firm with marginal production

[12]For an alternative approach to variable pass-through, see Appendix H.2.

[13]Mrázová and Neary (2011) call the resulting selection effects "second-order," contrasting them with "first-order selection effects" which arise from the decision whether to serve a market or not, as in Melitz (2003). First-order selection effects depend only on the first derivative of the profit function with respect to marginal cost, and are much more robust than second-order selection effects.

costs c can earn facing a marginal cost of accessing the market equal to t. Mrázová and Neary (2011) show that a sufficient (and, with additional restrictions, necessary) condition for more efficient firms (i.e., firms with lower c) to select into the activity with lower relative marginal cost (i.e., lower t) is that the firm's *ex post* profit function is *supermodular* in c and t. This criterion is very general, but in an important class of models it takes a relatively simple form. This class is where the profit function is twice differentiable, and depends on c and t multiplicatively:

$$\pi(c,t) \equiv \max_{x} \tilde{\pi}(x,c,t) \qquad \tilde{\pi}(x,c,t) = [p(x) - tc]\, x \tag{9}$$

This specification encompasses a number of important models. Interpreting t as an iceberg transport cost, it represents the canonical model of horizontal foreign direct investment (FDI) as in Helpman, Melitz, and Yeaple (2004): firms have to choose between proximity to foreign consumers - FDI incurs zero access costs - or "concentration", exporting from their home plant; the latter saves on fixed costs but requires that they produce tx units in order to sell x in the foreign market. Alternatively, interpreting t as the wage that must be paid to c workers to produce a unit of output, it represents the canonical model of *vertical* FDI, as in Antràs and Helpman (2004): firms in the high-income "North" wishing to serve their home market face a choice between producing at home and paying a high wage, or building a new plant in the low-wage "South." Finally, interpreting t as the premium over the marginal cost c which a firm will incur if it fails to invest in superior technology, equation (9) represents a model of choice of technique as in Bustos (2011).

When the profit function is twice differentiable in c and t, supermodularity is equivalent to a positive value of π_{ct} over the relevant range. Moreover, when the profit function is given by (9), Mrázová and Neary (2011) show that π_{ct} is positive if and only if the elasticity of marginal revenue with respect to sales is less than one.[14] When this condition holds, a 10%

[14]By the envelope theorem, $\pi_c = \tilde{\pi}_c = -tx$. Hence, $\pi_{ct} = -x - t\frac{dx}{dt} = -x - \frac{tc}{2p' + xp''} = -x + \frac{\varepsilon - 1}{2-\rho}x$. Writing revenue as $R(x) = xp(x)$, so marginal revenue is $R' = p + xp'$, the elasticity of marginal revenue (in absolute

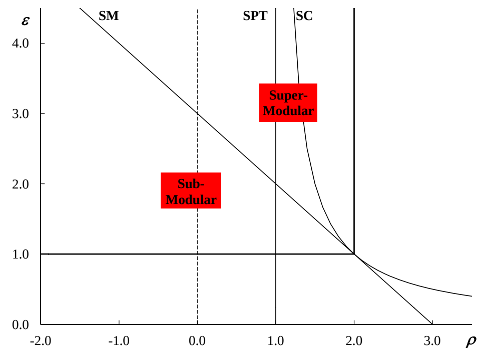


Figure 5: The Super- and Sub-Modularity Regions

reduction in the marginal cost of serving a market raises sales by more than 10%, so more productive firms have a bigger incentive to engage in FDI than in exports. The elasticity of marginal revenue depends on turn on the elasticity and convexity of demand:

$$\pi_{ct} = \frac{\varepsilon + \rho - 3}{2 - \rho} x \tag{10}$$

It follows that supermodularity holds in models described by (9) if and only if $\varepsilon + \rho > 3$. This criterion defines a third locus in $\{\varepsilon, \rho\}$ space, as shown in Figure 5. Once again it divides the admissible region into two sub-regions, one where either the elasticity or convexity or both are high, so supermodularity prevails, and the other where the profit function is submodular. The locus lies everywhere below the superconvex locus, and is tangential to it at the Cobb-Douglas point. Hence, supermodularity always holds with CES demands, the case assumed in Helpman, Melitz, and Yeaple (2004), Antràs and Helpman (2004) and Bustos (2011), among many others. However, when demands are subconvex and firms are large (operating at a point on their demand curve with relatively low elasticity), submodularity prevails, and so the standard comparative statics results may be reversed.

value) is seen to be: $-\frac{xR''}{R'} = \frac{2-\rho}{\varepsilon-1}$. Combining these results gives (10).

2.5 Summary

Figure 6 summarizes the implications of this section. The three loci, corresponding to constant elasticity (SC), unit convexity (SPT), and unit elasticity of marginal revenue (SM), place bounds on the combinations of elasticity and convexity consistent with particular comparative statics outcomes. Of eight logically possible sub-regions within the admissible region, three can be ruled out because superconvexity implies both super-pass-through and supermodularity. From the figure it is clear that knowing the values of the elasticity and convexity of demand which a firm faces is sufficient to predict its responses to a very wide range of exogenous shocks, including four of the five classic questions posed in the introduction.

Region	SC	SPT	SM
1			
2			✓
3		✓	
4		✓	✓
5	✓	✓	✓

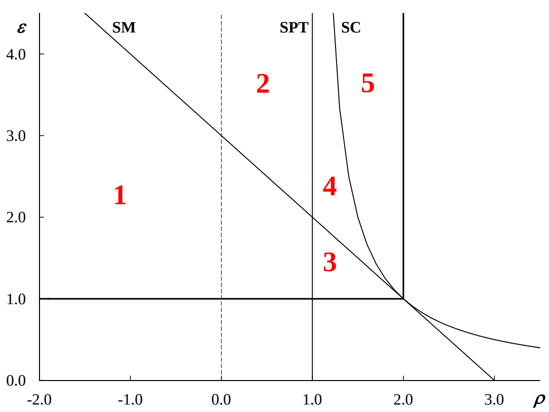


Figure 6: Regions of Comparative Statics

3 The Demand Manifold

3.1 Introduction

So far, we have shown how a very wide range of comparative statics responses can be signed just by knowing the values of ε and ρ which a firm faces. Next we want to see how different assumptions about the form of demand determine these responses. Formally, we seek to characterize the set of ε and ρ consistent with any given demand function $p = p_0(x)$:

$$\Omega[p_0(x)] \equiv \left\{\varepsilon, \rho : \varepsilon = -\frac{p_0(x)}{xp_0'(x)},\ \rho = -\frac{xp_0''(x)}{p_0'(x)}\right\} \tag{11}$$

We have already seen that this set and hence the comparative statics responses implied by particular demand functions are pinned down in two special cases: with CES demands the firm is always at a particular point in $\{\varepsilon, \rho\}$ space, while with linear demands it must lie along the $\rho = 0$ locus. Can anything be said more generally? The answer is "yes", as the following result shows:

Proposition 1. *For every continuous, three-times differentiable, strictly-decreasing demand function, $p_0(x)$, other than the CES, the set $\Omega[p_0(x)]$ corresponds to a smooth curve in $\{\varepsilon, \rho\}$ space.*

The proof is in Appendix D. It proceeds by showing that, at any point on every demand function other than the CES, at least one of the functions $\varepsilon = \varepsilon(x)$ and $\rho = \rho(x)$ can be inverted to solve for x, and the resulting expression, denoted $X^{\varepsilon}(\varepsilon)$ and denoted $X^{\rho}(\rho)$ respectively, substituted into the other function to give a relationship between ε and ρ:

$$\varepsilon = \bar{\varepsilon}(\rho) \equiv \varepsilon\left[X^{\rho}(\rho)\right] \quad \text{or} \quad \rho = \bar{\rho}(\varepsilon) \equiv \rho\left[X^{\varepsilon}(\varepsilon)\right] \tag{12}$$

We write this in two alternative ways, since at any given point only one may be well-defined, and, even when both are well-defined, one or the other may be more convenient depending on the context. The relationship between ε and ρ defined implicitly by (11) is not in general a function, since it need not be globally single-valued; but neither is it a correspondence, since it is locally single-valued. So we call it the "Demand Manifold" corresponding to the demand function $p_0(x)$.[15] In the CES case, not covered by Proposition 1, we follow the convention that, corresponding to each value of the elasticity of substitution σ, there is a one-dimensional point-manifold lying along the SC or CES locus.

[15] We are grateful to Kevin Roberts for pointing out that it is indeed a manifold.

The first advantage of working with the demand manifold rather than the demand function itself is that it is located in $\{\varepsilon, \rho\}$ space, and so it immediately reveals the implications of assumptions made about demand for comparative statics. A second advantage, departing from the "firm's-eye-view" that we have adopted so far, is that the manifold is often independent of exogenous parameters even though the demand function itself is typically not. Expressing this in the language of Chamberlin (1933), exogenous shocks typically shift the perceived demand curve, but they need not shift the corresponding demand manifold. When this property of "manifold invariance" holds, exogenous shocks lead only to movements along the manifold, not to shifts in it. As a result, it is particularly easy to make comparative statics predictions. Clearly, the manifold cannot in most cases be invariant to changes in all parameters: even in the CES case, the point-manifold is not independent of the value of σ.[16] However, it is invariant to changes in any parameter ϕ which affects the level term only; for ease of comparison with later functions, we write this in terms of both the direct and inverse CES demand functions:

$$x(p, \phi) = \delta(\phi)\, p^{-\sigma} \quad \Leftrightarrow \quad p(x, \phi) = \beta(\phi) x^{-1/\sigma} \tag{13}$$

In the same way, for many demand functions, including some of the most widely-used, the manifold turns out to be invariant with respect to some of their parameters, so it provides a parsimonious summary of their implications for comparative statics.[17] In the remainder of this section, we show that manifold invariance provides a fruitful organizing principle for a wide range of demand functions. Section 3.2 extends (13) in a non-parametric way, whereas Sections 3.3 and 3.4 extend it parametrically by adding an additional power-law term to the inverse and direct CES demand functions respectively.

[16] The manifold corresponding to the linear demand function is a relatively rare example which is invariant with respect to all demand parameters.

[17] Formally, the manifold can be written in full as either $\bar{\varepsilon}(\rho, \phi) = \varepsilon\left[X^{\rho}(\rho, \phi), \phi\right]$ or $\bar{\rho}(\varepsilon, \phi) \equiv \rho\left[X^{\varepsilon}(\varepsilon, \phi), \phi\right]$. Manifold invariance requires that either $\bar{\varepsilon}_{\phi} = \varepsilon_x X^{\rho}_{\phi} + \varepsilon_{\phi} = -\varepsilon_x \frac{\rho_{\phi}}{\rho_x} + \varepsilon_{\phi} = 0$ or $\bar{\rho}_{\phi} = \rho_x X^{\varepsilon}_{\phi} + \rho_{\phi} = -\rho_x \frac{\varepsilon_{\phi}}{\varepsilon_x} + \rho_{\phi} = 0$.

3.2 Multiplicatively Separable Demand Functions

Our first result is that manifold invariance holds when the demand function is multiplicatively separable in ϕ:

Proposition 2. *The Demand Manifold is invariant to shocks in a parameter ϕ if either the direct or inverse demand function is multiplicatively separable in ϕ:*

$$\text{(a) } x(p,\phi) = \delta(\phi)\,\tilde{x}(p)\,; \quad \text{or} \quad \text{(b) } p(x,\phi) = \beta(\phi)\tilde{p}(x) \tag{14}$$

The proof is in Appendix E, and relies on the convenient property that with separability of this kind both the elasticity and convexity are themselves invariant with respect to ϕ.

This result has some important corollaries. First, when utility is additively separable, the inverse demand function for any good equals the marginal utility of that good times the inverse of the marginal utility of income. The latter is a sufficient statistic for all economy-wide variables which affect the demand in an individual market, such as aggregate income or the price index. A similar property holds for the direct demand function if the indirect utility function is additively separable, with the qualification that the indirect sub-utility functions depend on prices relative to income. (See Appendix E for details.) Summarizing:

Corollary 1. *If preferences are additively separable, whether directly or indirectly, the demand manifold for any good is invariant to changes in aggregate variables.*

Given the pervasiveness of additive separability in theoretical models of monopolistic competition, this is an important result, which implies that in many models the manifold is invariant to economy-wide shocks. We will see a specific application in Section 4, where we apply our approach to the Krugman (1979) model of international trade with monopolistic competition.

A second corollary of Proposition 2 comes by noting that, setting $\delta(\phi)$ in (14)(a) equal to market size s, yields the following:

Corollary 2. *The Demand Manifold is invariant to neutral changes in market size:* $x(p, s) = s\tilde{x}(p)$.

This corollary is particularly useful since it does not depend on the functional form of the individual demand function. An example which illustrates this is the logistic direct demand function, equivalent to a logit inverse demand function (see Cowan (2012)):

$$x(p, s) = \left(1 + e^{p-a}\right)^{-1} s \quad \Leftrightarrow \quad p(x, s) = a - \log \frac{x}{s - x} \tag{15}$$

Here x/s is the share of the market served: $x \in [0, s]$; and a is the price which induces a 50% market share: $p = a$ implies $x = \frac{s}{2}$. The elasticity equals $\varepsilon = p\frac{s-x}{s}$, while the convexity equals $\rho = \frac{s-2x}{s-x}$, which must be less than one. Eliminating x and p yields a closed-form expression for the manifold:

$$\bar{\varepsilon}(\rho) = \frac{a - \log(1 - \rho)}{2 - \rho} \tag{16}$$

which is invariant with respect to market size s though not with respect to a. Figure 7 illustrates this for values of a equal to 2 and 5.[18]

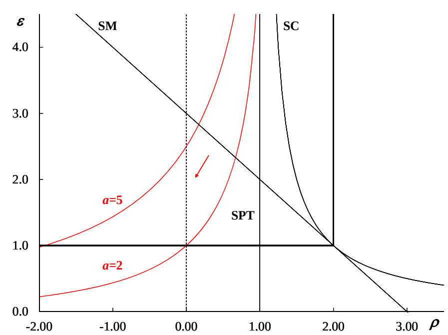


Figure 7: The Demand Manifold for the Logistic Demand Function

The logistic is just one example of a whole family of demand functions, many of which can be derived from log-concave distribution functions: Bergstrom and Bagnoli (2005) give

[18] The value of ρ determines market share and the level of price relative to a: $x = \frac{1-\rho}{2-\rho}s$ and $p = a - \log(1-\rho)$. In particular, when the function switches from convex to concave (i.e., ρ is zero), the elasticity equals $\frac{a}{2}$, market share is 50%, and $p = a$.

a comprehensive review of these. The power of the approach introduced in the last section is that we can immediately state the properties of all these functions: they imply sub-pass-through and, *a fortiori*, subconvexity, while they are typically supermodular for low values of output and submodular for high values. Any shock, such as a partial-equilibrium increase in market size, which raises the output of a monopoly firm, implies an adjustment as shown by the arrow in the figure.

Finally, a third corollary of Proposition 2 is the dual of Corollary 2, and comes from setting $\beta(\phi)$ in (14)(b) equal to quality q:

Corollary 3. *The Demand Manifold is invariant to neutral changes in quality:* $p(x,q) = q\tilde{p}(x)$.

Baldwin and Harrigan (2011) call this assumption "box-size quality": the consumer's willingness to pay for a single box of a good with quality level q is the same as their willingness to pay for q boxes of the same good with unit quality. Though special, it is a very convenient assumption, widely used in international trade theory, so it is useful that the comparative statics predictions of any such demand function are independent of the level of quality.

3.3 Bipower Inverse Demand Functions

A different approach to exploring the relationship between demand functions and the corresponding demand manifolds is to ask which demand functions correspond to particular forms of the manifold itself. The first result of this kind characterizes the demand functions that are consistent with a linear manifold:

Proposition 3. *The Demand Manifold is linear in* ε *and* ρ *if and only if the inverse demand function takes a bipower form:*

$$p(x) = \alpha x^{-\eta} + \beta x^{-\theta} \quad \Leftrightarrow \quad \bar{\rho}(\varepsilon) = \eta + \theta + 1 - \eta\theta\varepsilon \tag{17}$$

Sufficiency follows by differentiating $p(x)$ and calculating the manifold directly. Necessity follows by setting $\rho(x) = a + b\varepsilon(x)$, where a and b are constants, and solving the resulting Euler-Cauchy differential equation. Details are in Appendix F.1. Clearly, this manifold is invariant with respect to the parameters α and β, so changes in exogenous variables such as income or market size which only affect α or β do not shift the manifold. Putting this differently, we need four parameters to characterize the demand function, but only two to characterize the manifold, and therefore to place bounds on the comparative statics responses reviewed in Section 2. However, the other parameters in (17), α and β, are also qualitatively important, as the following proposition shows:

Proposition 4. *The bipower inverse demand functions in* (17) *are superconvex if and only if both* α *and* β *are positive.*

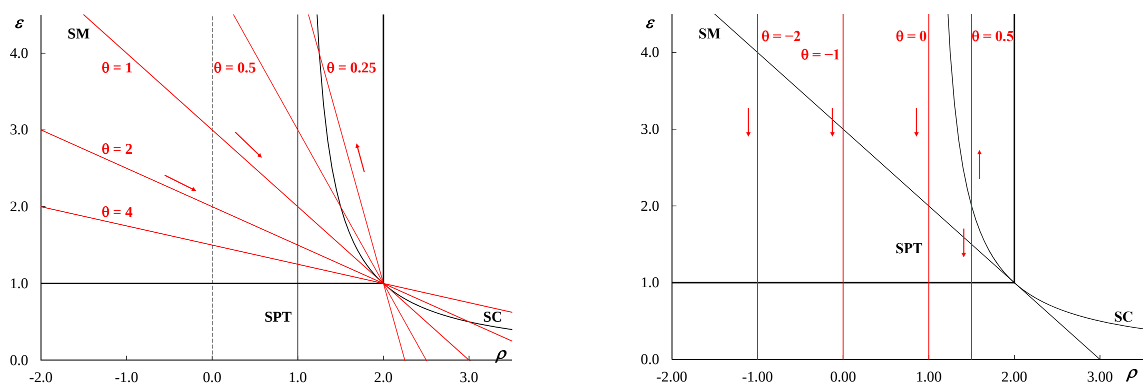
The two sets of parameters thus play very different roles. η and θ determine the location of the manifold, whereas α and β determine which "branch" of a particular manifold is relevant: the superconvex branch if they are both positive, the subconvex one if either of them is negative. (They cannot both be negative since the price is nonnegative.) How this works is best understood by considering some special cases.

The first sub-case of the demand functions in (17) we consider comes from setting η equal to one.[19] This gives the "inverse PIGL" ("price-independent generalized linear") system, which is dual to the direct PIGL system of Muellbauer (1975), to be considered in the next sub-section. Setting η equal to one expresses expenditure $p(x)x$ as a "translated-CES" function of sales: $p(x) = \frac{1}{x}(\alpha + \beta x^{1-\theta})$. This system implies that the elasticity of marginal revenue defined in footnote 14 is constant and equal to θ: $\eta = 1$ implies from (17) that $-\frac{xR''}{R'} = \frac{2-\rho}{\varepsilon-1} = \theta$. The limiting case as $\theta \to 1$ is the inverse "PIGLOG" ("price-independent generalized logarithmic") or inverse translog, $p(x) = \frac{1}{x}(\alpha' + \beta' \log x)$.[20] This implies that the elasticity of marginal revenue is unity, and so, as noted in Mrázová and Neary (2011),

[19] Because η and θ enter symmetrically into (17), it is arbitrary which we set equal to one. For concreteness and without loss of generality we assume $\beta \neq 0$ and $\theta \neq 0$ throughout.

[20] To show this, rewrite the constants as $\alpha = \alpha' - \frac{\beta'}{1-\theta}$ and $\beta = \frac{\beta'}{1-\theta}$, and apply l'Hôpital's Rule.

it coincides with the supermodularity locus: $\eta = \theta = 1$ implies from (17) that $\bar{\rho}(\varepsilon) = 3 - \varepsilon$. Figure 8(a) shows the demand manifolds for some members of this family. Manifolds with θ less than one have two branches, one each in the sub- and superconvex regions, implying different directions of adjustment with sales, as indicated by the arrows. Details are given in Appendix F.3.



(a) The Inverse PIGL Family

(b) The Bulow-Pfleiderer Family

Figure 8: Demand Manifolds for Bipower Inverse Demand Functions

A second important special case of (17) comes from setting $\eta = 0$, giving the demand function $p(x) = \alpha + \beta x^{-\theta}$. This is the iso-convex or "constant pass-through" family of Bulow and Pfleiderer (1983): from (17) with $\eta = 0$, convexity ρ equals a constant $\theta + 1$, so from (7) $\frac{1}{1-\theta}$ measures the degree of absolute pass-through for this system. Pass-through can be more than 100%, as in the CES case ($\alpha = 0$, $\theta = \frac{1}{\sigma} > 0$); equal to 100%, as in the log-linear direct demand case ($\theta \to 0$, so $p(x) = \alpha' + \beta' \log x$, implying that $\log x(p) = \gamma + \delta p$); or less than 100%, as in the case of linear demand ($\theta = -1$ so pass-through is 50%). This family has many other attractive properties. It is necessary and sufficient for marginal revenue to be affine in price.[21] It can be given a discrete choice interpretation: it equals the cumulative demand that would be generated by a population of consumers if their preferences followed a Generalized Pareto Distribution.[22] Finally, as shown by Weyl and Fabinger (2013) and

[21]See Appendix F.4 for more details.
[22]See Bulow and Klemperer (2012).

empirically implemented by Atkin and Donaldson (2012), it allows the division of surplus between consumers and producers to be calculated without knowledge of quantities. Figure 8(b) shows the demand manifolds for some members of this family; see Appendix F.4 for details.

3.4 Bipower Direct Demand Functions

A second characterization result linking a family of demand functions to a particular functional form for the manifold is where the direct demand functions have the same form as the inverse demand functions in Proposition 3:

Proposition 5. *The Demand Manifold is such that ρ is linear in the inverse and squared inverse of ε if and only if the direct demand function takes a bipower form:*

$$x\left(p\right)=\gamma p^{-\nu}+\delta p^{-\sigma} \quad \Leftrightarrow \quad \bar{\rho}(\varepsilon)=\frac{\nu+\sigma+1}{\varepsilon}-\frac{\nu\sigma}{\varepsilon^{2}} \tag{18}$$

Formally, this proposition follows immediately from Proposition 3 by exploiting the duality between direct and inverse demand functions: see Appendix G.1 for details. The condition for superconvexity of these demand functions is also directly analogous to Proposition 4:

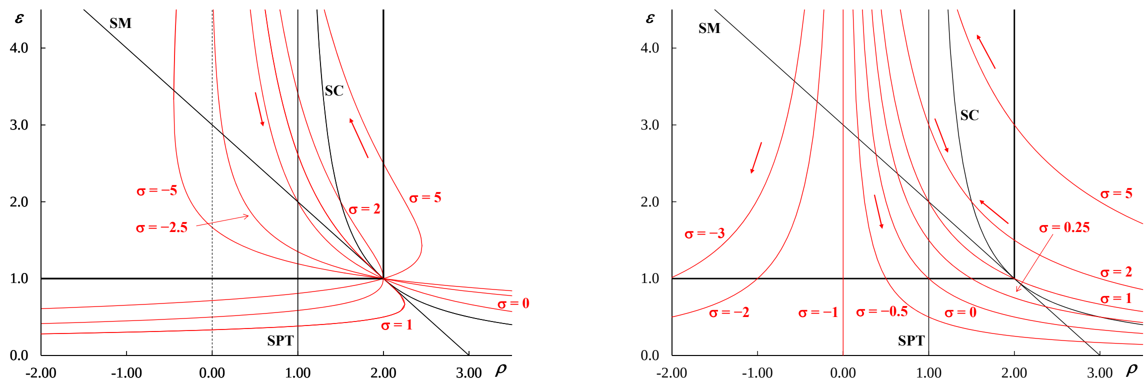
Proposition 6. *The bipower direct demand functions in* (18) *are superconvex if and only if both γ and δ are positive.*

The proof is in Appendix G.2. Substantively, the demand functions in (18) nest some of the most widely-used demand functions in applied economics.

Two special cases of (18), dual to the special cases of (17) already considered, are of particular interest.[23] The first, where $\nu=1$, is the direct PIGL system of Muellbauer (1975): $x\left(p\right)=\frac{1}{p}\left[\gamma+\delta p^{1-\sigma}\right]$, which implies that expenditure $px(p)$ is a translated-CES function of

[23]A third special case is the family of demand functions implied by the quadratic mean of order r expenditure function introduced by Diewert (1976) and extended to monopolistic competition by Feenstra (2014). See Appendix G.3 for details.

price. From (18), the manifold is given by: $\bar{\rho}(\varepsilon) = \frac{(\sigma+2)\varepsilon-\sigma}{\varepsilon^2}$. The translog is the limiting case as σ approaches 1 so $\bar{\rho}(\varepsilon) = \frac{3\varepsilon-1}{\varepsilon^2}$.[24] From the firm's perspective, this is consistent with both the AIDS model of Deaton and Muellbauer (1980), which is not in general homothetic, and with the homothetic translog of Feenstra (2003). In both cases, expenditure $px(p)$ is affine and decreasing in $\log p$: $x\left(p\right) = \frac{1}{p}\left(\gamma' + \delta' \log p\right)$, $\delta' < 0$.



(a) The PIGL Family

(b) The Pollak Family

Figure 9: Demand Manifolds for Bipower Direct Demand Functions

Figure 9(a) illustrates some members of the PIGL family. Details are given in Appendix G.4. A notable feature of the translog is that it is both subconcave and supermodular throughout the admissible region: see the curve labeled $\sigma = 1$. No other bipower demand function exhibits this property:[25]

Lemma 3. *The translog demand function is the only member of the bipower inverse or direct families which is always both strictly subconvex and strictly supermodular in the interior of the admissible region.*

This is an attractive feature: it implies that the translog is the only demand function from these two broad families which allows for competition effects (so mark-ups fall with globalization) but also implies that larger firms always serve foreign markets via FDI rather than

[24] To see this, take the limit as in footnote 20.

[25] The proof is in Appendix G.5.

exports.

A second important special case of (18) comes from setting $\nu = 0$. This gives the family of demand functions due to Pollak (1971): $x(p) = \gamma + \delta p^{-\sigma}$. This family includes many widely-used demand functions, including the CES ($\gamma = 0$), quadratic ($\sigma = -1$), Stone-Geary (or "LES" for "Linear Expenditure System": $\sigma = 1$), and "CARA" ("constant absolute risk aversion": $\sigma = 0$). Pollak showed that these are the only demand functions that are consistent with both additive separability and quasi-homotheticity (so the expenditure function exhibits the "Gorman Polar Form"). Just as (18) is dual to (17), so the Pollak family of direct demand functions is dual to the Bulow-Pfleiderer family of inverse demand functions. An implication of this is that, corresponding to the property of Bulow-Pfleiderer demands that marginal revenue is linear in price, Pollak demands exhibit the property that the marginal *loss* in revenue from a small increase in price is linear in sales.[26] This implies that the coefficient of absolute risk aversion for these demands is hyperbolic in sales, which is why, in the theory of choice under uncertainty, they are known as "HARA" ("hyperbolic absolute risk aversion") demands following Merton (1971).[27] Not surprisingly, the demand manifold is also a rectangular hyperbola: when $\nu = 0$, the left-hand side of (18) becomes $\bar{\rho}(\varepsilon) = \frac{\sigma+1}{\varepsilon}$. Figure 9(b) illustrates some members of the Pollak family. The CARA and LES cases correspond to σ equal to zero and one respectively. Further details are in Appendix G.6.[28]

[26] Recall from footnote 21 that Bulow-Pfleiderer demands $p(x) = \alpha + \beta x^{-\theta}$ satisfy the property: $p + xp' = \theta\alpha + (1-\theta)p$. Switching variables, we can conclude that Pollak demands $x(p) = \gamma + \delta p^{-\sigma}$ satisfy the property: $x + px' = \sigma\gamma + (1 - \sigma)x$.

[27] The Arrow-Pratt coefficient of absolute risk aversion is $A(x) \equiv -\frac{u''(x)}{u'(x)}$. With additive separability this becomes $A(x) = -\frac{p'(x)}{p(x)} = -\frac{1}{px'(p)}$. Using the result from footnote 26, this implies: $A(x) = \frac{1}{\sigma(x-\gamma)}$, which is hyperbolic in x.

[28] Note how these differ from the Bulow-Pfleiderer case in Figure 8, especially in the super-pass-through region. With Bulow-Pfleiderer demands, firms diverge from the CES benchmark along the SC locus as sales increase, whereas with Pollak demands they converge towards it; and both these statements hold whether demands are super- or subconvex.

3.5 Demand Functions that are Not Manifold-Invariant

In the rest of the paper we concentrate on the demand functions introduced here which have invariant manifolds. Appendix H presents two examples of demand functions which have non-invariant manifolds, and which nest some important cases, such as the "Adjustable Pass-Through" (Apt) demand function of Fabinger and Weyl (2012). These have manifolds with the same number of parameters as the demand function, four or more, which implies a clear trade-off. On the one hand, they are more flexible and so have greater potential to match any desired relationship between ε and ρ. On the other hand, they are more difficult to work with theoretically or to estimate empirically.

4 Monopolistic Competition in General Equilibrium

4.1 The Positive Effects of Globalization

To illustrate the power of the approach we have developed in previous sections, we turn in the remainder of the paper to apply it to a canonical model of international trade, a one-sector, one-factor, multi-country, general-equilibrium model of monopolistic competition. To highlight the new features of our approach, we focus on the case considered by Krugman (1979), where countries are symmetric, trade is unrestricted, and firms are homogeneous. Following Krugman (1979) and a large subsequent literature, we model globalization as an increase in the number of countries in the world economy.[29] For the present we assume only that preferences are symmetric, and that the elasticity of demand depends only on consumption levels, which in symmetric equilibrium means on the amount consumed of a typical variety, denoted by x. We do not need to make explicit our assumptions about preferences until we consider welfare in Section 4.2.

Symmetric demands and homogeneous firms imply that we can dispense with firm sub-

[29] As shown in Mrázová and Neary (2014), the aggregate welfare effect of this shock is similar that of a reduction in trade costs in the neighborhood of free trade.

scripts from the outset. Industry equilibrium requires that firms maximize profits by setting marginal revenue MR equal to marginal cost MC, and that profits are driven to zero by free entry (so average revenue AR equals average cost AC):

$$\text{Profit Maximization (MR=MC):} \quad p = \frac{\varepsilon(x)}{\varepsilon(x) - 1} c \tag{19}$$

$$\text{Free Entry (AR=AC):} \quad p = \frac{f}{y} + c \tag{20}$$

The model is completed by market-clearing conditions for the goods and labor markets:

$$\text{Goods-Market Equilibrium (GME):} \quad y = kLx \tag{21}$$

$$\text{Labor-Market Equilibrium (LME):} \quad L = n(f + cy) \tag{22}$$

Here L is the number of worker/consumers in each country, each of whom supplies one unit of labor and consumes an amount x of every variety; k is the number of identical countries; and n is the number of identical firms in each, all with total output y, so $N = kn$ is the total number of firms in the world. Since all firms are single-product by assumption, N is also the total number of varieties available to all consumers.

Equations (19) to (22) comprise a system of four equations in four endogenous variables, p, x, y and n, with the wage rate set equal to one by choice of numéraire. To solve for the effects of globalization, an increase in the number of countries k, we totally differentiate the equations, using "hats" to denote logarithmic derivatives, so $\hat{x} \equiv d \log x$, $x \neq 0$:

$$\text{MR=MC:} \quad \hat{p} = \frac{\varepsilon + 1 - \varepsilon\rho}{\varepsilon(\varepsilon - 1)} \hat{x} \tag{23}$$

$$\text{AR=AC:} \quad \hat{p} = -(1 - \omega)\hat{y} \tag{24}$$

$$\text{GME:} \quad \hat{y} = \hat{k} + \hat{x} \tag{25}$$

$$\text{LME:} \quad 0 = \hat{n} + \omega \hat{y} \tag{26}$$

Consider first the MR=MC equilibrium condition, equation (23). Clearly p and x move together if and only if $\varepsilon + 1 - \varepsilon\rho > 0$, i.e., if and only if demand is subconvex. This reflects the property noted in Section 2.2: higher sales are associated with a higher mark-up if and only if they imply a lower elasticity of demand. As for the free-entry condition, equation (24), it shows that the fall in price required to maintain zero profits following an increase in firm output is greater the smaller is $\omega \equiv \frac{cy}{f+cy}$, the share of variable in total costs, which is an inverse measure of returns to scale. This looks like a new parameter but in equilibrium it is not. It equals the ratio of marginal cost to price, $\frac{c}{p}$, which because of profit maximization equals the ratio of marginal revenue to price $\frac{p+xp'}{p}$, which in turn is a monotonically increasing transformation of the elasticity of demand ε: $\omega = \frac{c}{p} = \frac{p+xp'}{p} = \frac{\varepsilon-1}{\varepsilon}$. Similarly, equation (26) shows that the fall in the number of firms required to maintain full employment following an increase in firm output is greater the larger is ω. It follows by inspection that all four equations depend only on two parameters, which implies:

Lemma 4. *The local comparative statics properties of the symmetric monopolistic competition model with respect to a globalization shock depend only on ε and ρ.*

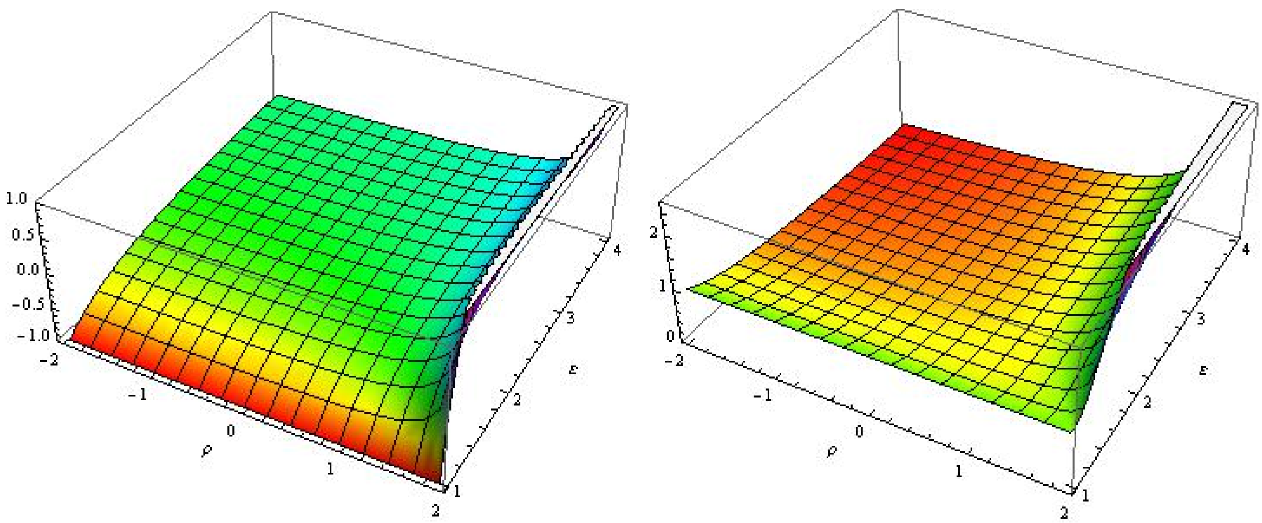
Solving for the effects of globalization on outputs, prices and the number of firms in each country gives:

$$\hat{y} = \frac{\varepsilon + 1 - \varepsilon\rho}{\varepsilon\,(2-\rho)}\hat{k}, \quad \hat{p} = -\frac{1}{\varepsilon}\hat{y}, \quad \hat{n} = -\frac{\varepsilon - 1}{\varepsilon}\hat{y} \tag{27}$$

(Details of the solution are given in Appendix I.) The signs of these depend solely on whether demands are sub- or superconvex, i.e., whether $\varepsilon + 1 - \varepsilon\rho$ is positive or negative. With subconvexity we get what Krugman (1979) called "sensible" results: globalization prompts a shift from the extensive to the intensive margin, with fewer but larger firms in each country, as firms move down their average cost curves and prices of all varieties fall. With superconvexity, as noted by Zhelobodko, Kokovin, Parenti, and Thisse (2012), all these results are reversed. (See also Neary (2009).) The CES case, where $\varepsilon + 1 - \varepsilon\rho = 0$, is the boundary one, with firm outputs, prices, and the number of firms per country unchanged. The only effects which hold irrespective of the form of demand are that consumption per head of each variety falls and the total number of varieties produced in the world and consumed in each country rises:

$$\hat{x} = -\frac{1}{2-\rho}\frac{\varepsilon-1}{\varepsilon}\hat{k} \ < \ 0, \qquad \hat{N} = \frac{(\varepsilon-1)^2 + (2-\rho)\,\varepsilon}{\varepsilon^2\,(2-\rho)}\hat{k} \ > \ 0 \tag{28}$$

In qualitative terms these results are not new. The new feature that our approach highlights is that their quantitative magnitudes depend only on two parameters, ε and ρ, the same ones on which we have focused throughout.



(a) Changes in Prices

(b) Changes in the Number of Varieties

Figure 10: The Effects of Globalization

Figure 10 gives the quantitative magnitudes of changes in the two variables that matter most for welfare, prices and the number of varieties. In each panel, the vertical axis measures the proportional change in either p or N relative to k as a function of the elasticity and convexity of demand. These three-dimensional surfaces are independent of the functional form of demand, so we can combine them with the results on demand manifolds from Section 3 to read off the quantitative effects of globalization implied by different assumptions about demand. We know already from equations (27) and (28) that prices fall if and only if demand is subconvex and that product variety always rises. The figures show in addition that lower values of the elasticity of demand are usually associated with greater falls in prices and larger increases in variety;[30] while more convex demand is always associated with greater increases (in absolute value) in both prices and the number of varieties.

To summarize this sub-section, Lemma 4 implies that the demand manifold is a sufficient statistic for the positive effects of globalization in the Krugman (1979) model, just as it is for the comparative statics results discussed in Section 2. However, to make quantitative predictions about how welfare is affected we need to know how consumers trade off the changes in prices and product variety illustrated in Figure 10. We turn to this next.

4.2 Welfare: Additive Separability and the Utility Manifold

To quantify the welfare effects of globalization, we follow Dixit and Stiglitz (1977) and assume that preferences are additively separable. Hence the overall utility function, denoted by U, is a monotonically increasing function of an integral of sub-utility functions, denoted by u, each defined over the consumption of a single variety: $U = F\left[\int_0^N u\{x(\omega)\}\,\mathrm{d}\omega\right]$, $F' > 0$. Marginal utility must be positive and decreasing in the consumption of each variety: $u'(x) > 0$ and $u''(x) < 0$. With symmetric preferences and no trade costs the overall utility function becomes: $U = F\,[Nu(x)]$. So, welfare depends on the extensive margin of consumption N times the utility of the intensive margin x.

Using the budget constraint to eliminate x, we write the change in utility in terms of its income equivalent $\hat{Y}$, which is independent of the function F (see Appendix J for details):

$$\hat{Y} = \frac{1-\xi}{\xi}\hat{N} - \hat{p} \tag{29}$$

Here $\xi(x) \equiv \frac{xu'(x)}{u(x)}$ is the elasticity of utility with respect to consumption. We thus have a

[30] Though these properties are reversed if demands are highly convex: $\hat{p}/\hat{k}$ is increasing in ε if and only if $\rho < 1 + \frac{2}{\varepsilon}$, and $\hat{N}/\hat{k}$ is decreasing in ε if and only if $\rho < \frac{2}{\varepsilon}$.

clear division of labor between three preference parameters: ε and ρ determine the positive effects of globalization as we have just seen, whereas ξ determines their implications for welfare. It is clear from (29) that ξ must lie between zero and one if preferences exhibit a taste for variety. (See also Vives (1999).) Moreover, since welfare rises more slowly with N the higher is ξ, it is an inverse measure of preference for variety. This parameter plays the key role of trading off changes in prices and number of varieties from a consumer's perspective. To understand this role, we relate $\xi(x)$ to the elasticity of demand, $\varepsilon(x) \equiv -\frac{p(x)}{xp'(x)}$, which is an inverse measure of the concavity of the sub-utility function: $\varepsilon(x) = -\frac{u'(x)}{xu''(x)}$.[31] Given these two parameters that are unit-free measures of the slope and curvature of u, we can proceed in a similar manner to Sections 2 and 3: we first show how different configurations of ξ and ε determine the properties of the model, and in particular the efficiency of equilibrium; we then solve for the relationship between the two parameters, which we call the "utility manifold", that is implied by specific assumptions about the form of u.

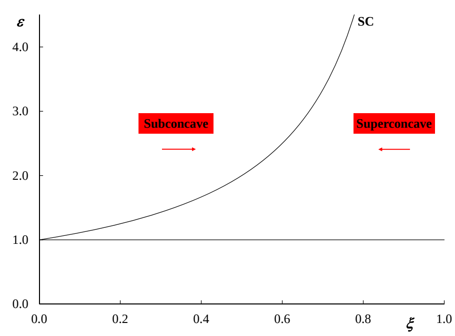


Figure 11: The Sub- and Superconcave Regions in $\{\varepsilon, \xi\}$ Space

Figure 11 illustrates part of the admissible region in $\{\varepsilon, \xi\}$ space. As in previous sections, the CES case is an important threshold. Corresponding to the CES inverse demand function

[31] A utility function $u_1(x)$ is more concave than a different utility function $u_2(x)$ at a common point if $\frac{xu_1''}{u_1'} < \frac{xu_2''}{u_2'}$. Switching both from negative to positive and inverting, we can conclude that $u_1(x)$ is more concave than $u_2(x)$ if $\varepsilon_1(x) < \varepsilon_2(x)$.

from (4), $p(x) = \beta x^{-1/\sigma}$, is the sub-utility function:

$$u(x) = \frac{\sigma}{\sigma - 1} \beta x^{\frac{\sigma-1}{\sigma}} \quad \Rightarrow \quad \xi(x) = \xi^{CES} \equiv \frac{\sigma - 1}{\sigma}, \;\; \varepsilon(x) = \varepsilon^{CES} \equiv \sigma \tag{30}$$

Both ε and ξ depend only on σ, so we can solve for the locus of $\{\varepsilon, \xi\}$ combinations, i.e., the locus of utility point-manifolds, consistent with CES preferences: $\varepsilon = \frac{1}{1-\xi}$, or $\xi = \frac{\varepsilon - 1}{\varepsilon}$. This locus is labeled SC in Figure 11. By analogy with our treatment of demand in Section 2.2, it defines a threshold degree of concavity of the sub-utility function, and we call a utility or sub-utility function "superconcave" if it is more concave than that threshold:

Definition 2. A utility function $u(x)$ is superconcave at a point (u_0, x_0) if and only if $\log u(x)$ is concave in $\log x$ at (u_0, x_0).

Analogously to Section 2.2, a utility function is superconcave at a point if and only if it is more concave than a CES function with the same elasticity of utility at that point:

$$\frac{d^2 \log u}{d(\log x)^2} = \xi \left(\frac{\varepsilon - 1}{\varepsilon} - \xi \right) = \xi \left(\frac{1}{\varepsilon^{CES}} - \frac{1}{\varepsilon} \right) \leq 0 \tag{31}$$

Since ε is an inverse measure of the concavity of u ($\frac{1}{\varepsilon} = -\frac{xu''}{u'}$), the region below and to the right of the SC locus in Figure 11 corresponds to superconcavity of utility. In addition, as we show in Appendix C, superconcavity of utility is equivalent to the elasticity of utility decreasing in consumption: $\xi_x = \frac{\xi}{x} \left(\frac{\varepsilon - 1}{\varepsilon} - \xi \right)$. It follows that the elasticity of utility is associated with higher levels of consumption as indicated by the arrows in Figure 11: rising in the subconcave region and falling in the superconcave region, in a similar manner to how the elasticity of demand varies with consumption in Figure 2.[32]

[32]Vives (1999), pages 170-1, argues that the latter case, preference for variety that is increasing in consumption, or superconcavity of utility, is intuitively plausible, though, as he notes, this is not the view of Dixit and Stiglitz (1977).

4.3 Superconcavity, Efficiency and Welfare

To relate superconcavity of utility to the efficiency of equilibrium, we return to the expression for welfare change in (29). The change in the extensive margin can be decomposed into two components, one exogenous and international, the other endogenous and intranational: $\hat{N} = \hat{k} + \hat{n}$. We can eliminate the latter using the full-employment and zero-profit conditions, equations (24) and (26): $\hat{n} = -\omega\hat{y} = \frac{\omega}{1-\omega}\hat{p}$. This allows us to decompose the change in real income into a *direct* and an *indirect* effect of globalization:

$$\hat{Y} = \frac{1-\xi}{\xi}\hat{k} + \frac{\omega - \xi}{\xi(1-\omega)}\hat{p} \tag{32}$$

The direct effect given by the first term is always positive, so we focus on the indirect effect, which works through the induced change in prices. A non-zero coefficient of $\hat{p}$ implies that there is scope for increasing real income even in the absence of globalization, in other words, that the initial equilibrium is *inefficient*. Hence the condition for efficiency in this model is that ω and ξ should be equal. To see why, note that the social optimum requires that ξ should equal one in a competitive economy: with no fixed costs (so $f = 0$ and $\omega = 1$), it is optimal to produce as many varieties as needed to indulge consumers' taste for variety. By contrast, when fixed costs are strictly positive ($f > 0$ so $\omega < 1$), efficiency requires that ξ be strictly less than one. Recalling that ω is an inverse measure of returns to scale, the intuition for this is that more strongly increasing returns to scale mandate substitution away from the extensive towards the intensive margin. The social optimum occurs when the gain to consumers from an additional variety, of which ξ is an inverse measure, exactly matches the cost to society of setting up an additional firm, of which ω is an inverse measure. Absent efficiency, we can say that varieties are *under-supplied* if and only if ξ is less than ω. Recalling Definition 2 and the fact that ω equals $\frac{\varepsilon-1}{\varepsilon}$ in equilibrium, we can conclude:

Corollary 4. *Relative to the efficient benchmark where $\xi = \omega$, varieties are under-supplied if and only if utility is subconcave, i.e., $\xi \leq \omega$.*

The intuition underlying this is that subconcavity of utility, $\xi \leq \frac{\varepsilon-1}{\varepsilon}$, which is a partial-equilibrium property of preferences, turns out in general equilibrium to imply that varieties are under-supplied: $\xi \leq \omega$. Note that there is not a unique efficient equilibrium in general: any equilibrium which lies along the SC locus in Figure 11 is efficient.

Quantifying the gains from globalization in this model is straightforward when the initial equilibrium is efficient, but this will occur in only two cases. The first is when preferences are CES. In that case, as we have already seen, both ω and ξ equal $\frac{\sigma-1}{\sigma}$. Quantitatively, the gains from globalization, $\hat{Y}/\hat{k}$, reduce to $\frac{1}{\sigma-1}$, exactly the expression found for the gains from trade in a range of CES-based models by Arkolakis, Costinot, and Rodríguez-Clare (2012). Qualitatively, this result is familiar from Dixit and Stiglitz (1977): only CES preferences ensure that the gains from greater variety are exactly matched by the losses from reducing the scale of production of existing varieties. A second case in which efficiency occurs is when it is brought about by government intervention: for given k, optimal competition policy chooses the welfare-maximizing level of n (which in turn determines p and x). This case requires strong assumptions: benevolent and cooperative governments, plus an institutional framework for anti-trust policy which does not impose any further inefficiencies of its own. We abstract from policy issues from now on, and focus on the implications of non-CES preferences.

Absent efficiency, we can assess the sign and magnitude of the gains from globalization with reference to the CES benchmark, which is also the efficient or "first-best" benchmark. Consider first the direct gain in (32), given by the coefficient of $\hat{k}$. It exceeds the CES benchmark $\frac{1}{\varepsilon-1}$ if and only if $\xi \leq \frac{\varepsilon-1}{\varepsilon}$, which gives our first result:

Proposition 7. *The direct gain from globalization exceeds the CES benchmark if and only if utility is subconcave.*

This makes sense: consumers always gain directly from more varieties, and gain by more when varieties are initially under-supplied.

Consider next the indirect effect, which will be positive if the exogenous shock brings the world economy closer to efficiency. Recalling from (27) that prices rise if and only if demand is superconvex, we can sign this as follows:

Proposition 8. *The indirect gain from globalization is positive if and only if either: (a) demand is superconvex and utility is subconcave; or (b) demand is subconvex and utility is superconcave.*

In case (a), prices rise, but varieties are under-supplied, so the loss at the intensive margin is offset by the gain to consumers of increased variety. This reasoning is reversed in case (b): now varieties are over-supplied, but efficiency is increased as consumers gain from the fall in prices.

Improving efficiency is sufficient for a globalization shock to raise welfare but it is not necessary. To assess the full effect, we have to combine the indirect or induced-efficiency effect with the direct effect. First we obtain two alternative sufficient conditions for the overall gains to be positive from the two equations for the change in real income, (29) and (32):

Proposition 9. *Gains from globalization are guaranteed if either: (a) demand is subconvex; or (b) utility is subconcave.*

Putting this differently, losses from globalization are possible only if demand is superconvex *and* utility is superconcave. The intuition for this proposition combines that from previous results. In case (a), demand is *subconvex*, i.e., $\frac{\varepsilon+1}{\varepsilon}$ is greater than ρ, so prices fall. As a result, from (29), consumers reap a double dividend: welfare rises at both the intensive and extensive margins. In this case, the elasticity of utility is irrelevant for the sign of welfare change. In case (b), utility is *subconcave*, i.e., $\frac{\varepsilon-1}{\varepsilon}$ is greater than ξ, so varieties are under-supplied in the initial equilibrium. Even if prices rise, consumers value variety sufficiently that the gain at the extensive margin from the increase in the number of varieties offsets the losses at the intensive margin due to higher prices.

Second, we can calculate a necessary and sufficient condition for gains from globalization. Substitute for $\omega = \frac{\varepsilon-1}{\varepsilon}$ and $\hat{p}$ into (32) to obtain an explicit expression for the gain in welfare:

$$\hat{Y} = \frac{1}{\xi\varepsilon}\left[1 - \left(\xi - \frac{\varepsilon-1}{\varepsilon}\right)\frac{\varepsilon-1}{2-\rho}\right]\hat{k} \tag{33}$$

Now there are three sufficient statistics for the change in welfare, only one of which has an unambiguous effect. The gains from globalization are always decreasing in ξ: unsurprisingly, consumers gain more from a proliferation of countries and hence of products, the greater their taste for variety. By contrast, the gains from globalization depend ambiguously on both ε and ρ. Of course, the values of the three key parameters do not in general vary independently of each other, which suggests how we should proceed. Equation (33) gives the change in real income as a function of ξ, ε, and ρ only. These in turn are related to each other and to ϕ, the vector of non-invariant parameters, via the demand manifold, $\varepsilon = \bar{\varepsilon}(\rho, \phi)$, and the utility manifold, $\xi = \bar{\xi}(\varepsilon, \phi)$. To get an explicit solution for the gains from globalization we thus have to solve three equations in $4 + m$ unknowns, where m is the dimension of ϕ. The solution can be visualized in three dimensions when m equals one, so we turn next to consider how ξ varies in the two families of demand functions with only a single non-invariant parameter discussed in Section 3.

4.4 Globalization and Welfare with Bipower Preferences

The first example we consider is that of bipower demands, given by the demand function (17) in Section 3.3. Integrating that function yields the corresponding utility function, which also takes a bipower form:[33]

$$u(x) = \frac{1}{1-\eta}\alpha x^{1-\eta} + \frac{1}{1-\theta}\beta x^{1-\theta} \tag{34}$$

[33] It is natural to set the constant of integration to zero, which implies that $u(0) = 0$. A non-zero value of $u(0)$ could be interpreted to imply that consumers gain or lose from the introduction of new varieties even if they do not consume them, though Dixit and Stiglitz (1979) argue to the contrary. We return to this issue in the next sub-section.

Proceeding as in Proposition 3, we can derive a closed-form expression for the utility manifold in this case:[34]

Proposition 10. *If and only if the utility function is as in* (34)*, the utility manifold is:*

$$\bar{\xi}(\varepsilon) = \frac{(1-\eta)(1-\theta)\varepsilon}{(1-\eta-\theta)\varepsilon+1} \tag{35}$$

Direct calculations now yield:

Proposition 11. *With bipower utility as in* (34)*, demand is subconvex if and only if* $(\eta\varepsilon - 1)(\theta\varepsilon - 1) \geq 0$*; utility is superconcave if and only if* $\frac{(\eta\varepsilon-1)(\theta\varepsilon-1)}{(1-\eta-\theta)\varepsilon+1} \geq 0$*.*

Recalling the sufficient conditions for gains from globalization given in Proposition 9, this implies:

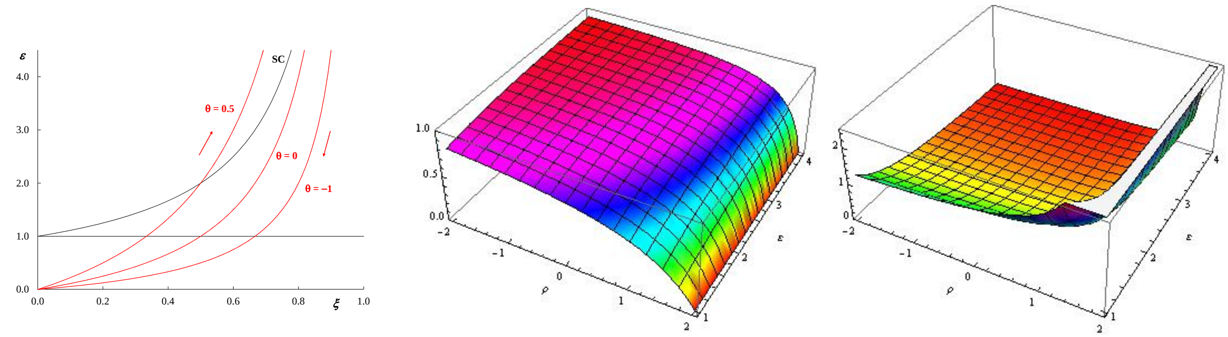
Corollary 5. *With bipower utility as in* (34)*, gains from globalization are guaranteed if* $(1-\eta-\theta)\varepsilon + 1 > 0$*.*

If this condition holds, Proposition 11 implies that the only possible combinations allowed by the utility function (34) are *either* subconvex demand and superconcave utility *or* superconvex demand and subconcave utility; either prices fall, or consumers don't lose too much when they rise because varieties are initially under-supplied. It follows from Proposition 9 that welfare must increase as the world economy expands.

In the Bulow-Pfleiderer special case, when η is zero and β and θ have the same sign, the sufficient condition from Corollary 5 holds, and we can characterize the possible outcomes as follows:

Corollary 6. *With Bulow-Pfleiderer preferences, so utility is given by* (34) *with* $\eta = 0$*, demand is superconvex and utility is subconcave if* $\theta > 0$ *and* $\alpha > 0$*; otherwise demand is subconvex and utility is superconcave. In both cases, globalization must raise welfare.*

[34] Proofs of all results in this sub-section are in Appendix K.



(a) Utility Manifold (b) Elasticity of Utility (c) Change in Real Income

Figure 12: Globalization and Welfare: Bulow-Pfleiderer Preferences

Figure 12(a) illustrates, showing three utility manifolds from the Bulow-Pfleiderer family. We follow the convention that the arrows represent the direction of movement as sales increase. Hence, recalling from (28) that globalization *reduces* consumption of each variety, the equilibrium moves in the opposite direction to the arrows. To the right of the SC locus, utility is superconcave and so, from Proposition 6, demand is subconvex. Globalization leads to a rightward movement, and so always moves the economy closer to efficiency, though the elasticity of utility changes in a paradoxical way. From Corollary 4, varieties are initially over-supplied, as the "efficiency gap" $\xi-\omega$ is positive. Globalization reduces this gap, though at the same time it *raises* ξ: consumers' preference for variety falls. A similar outcome follows, *mutatis mutandis*, to the left of the SC locus. Here too globalization raises efficiency, though by moving the efficiency gap, initially negative, closer to zero, while at the same time lowering ξ. In both cases, the global economy converges asymptotically towards efficiency, though in different ways depending on whether θ is greater or less than zero. From Figure 8 in Section 3.3, if θ is greater than zero, so demands are strictly log-convex, the equilibrium converges towards a CES equilibrium with positive profit margins. By contrast, if θ is equal to or less than zero, so demands are log-concave, the equilibrium converges towards a quasi-perfectly-competitive outcome in which price equals marginal cost and the elasticity of utility equals one. Products are still differentiated, but consumers no longer care about variety.

The remaining panels of Figure 12 illustrate how the initial value of ξ and the change in welfare vary with ε and ρ. As panel (b) shows, the elasticity of utility is always between zero and one, is increasing in the elasticity of demand, and decreasing in convexity: there is a greater taste for variety at high ρ. Recalling from equation (29) that ξ is an inverse measure of the weight that consumers attach to the variety changes shown in Figure 10, we find the implications for the gains from globalization shown in Panel (c). The gains are always positive, are decreasing in ε and increasing in ρ.

4.5 Globalization and Welfare with Pollak Preferences

The second example we consider is the Pollak demand function from Section 3.4. The welfare implications of this specification are sensitive to how we normalize the sub-utility function. To highlight the contrast with the Bulow-Pfleiderer case in the previous sub-section, we focus in the text on the case considered by Pollak (1971) and Dixit and Stiglitz (1977). (In Appendix L.4 we consider an alternative specification, due to Pettengill (1979), which yields different results.) This gives the following sub-utility function:

$$u(x) = \frac{\beta}{\sigma - 1}(\sigma x + \zeta)^{\frac{\sigma-1}{\sigma}} \tag{36}$$

Relative to the Pollak demand function in Section 3.4, it is convenient to redefine the constants as $\zeta \equiv -\gamma\sigma$ and $\beta \equiv (\delta/\sigma)^{1/\sigma}$. (See Appendix L.1 for details.)

Solving for the elasticity of utility, the utility manifold is:

$$\bar{\xi}(\varepsilon; \sigma) = \frac{\sigma - 1}{\varepsilon} \tag{37}$$

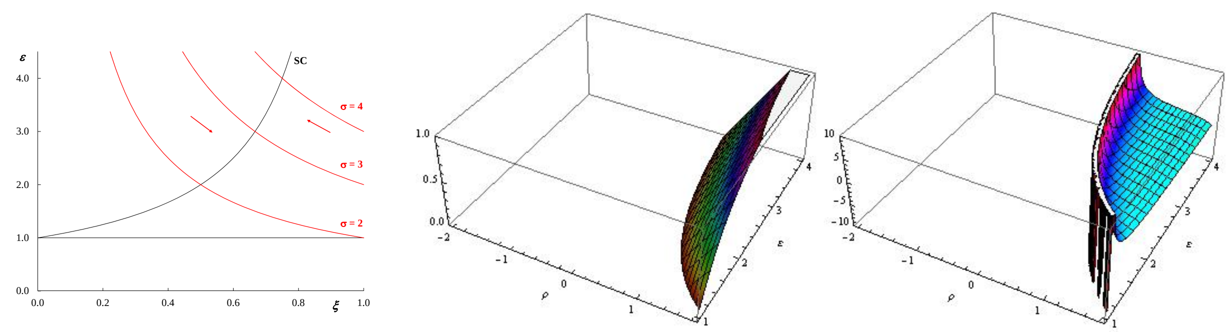
Combining this with the demand manifold from Section 3.4, we can express the elasticity of utility as a function of ε and ρ only:

$$\xi(\varepsilon, \rho) = \frac{\varepsilon\rho - 2}{\varepsilon} \tag{38}$$

Since only values of ξ between zero and one are consistent with a preference for variety, we restrict attention to the range $\frac{2}{\varepsilon} < \rho < 1 + \frac{2}{\varepsilon}$.[35] Within this range, the behavior of the elasticity of utility is the opposite to that in the Bulow-Pfleiderer case:[36]

Proposition 12. *With Pollak preferences as in* (36) *and* $0 < \xi < 1$*, demand is superconvex and utility is superconcave if and only if* ζ *is negative.*

Figure 13(a) illustrates. To the left of the SC locus, utility is subconcave and so, from Proposition 12, demand is subconvex. Hence globalization lowers prices and raises welfare, even though it *reduces* efficiency: the "efficiency gap" is initially negative, and is reduced further as the elasticity of utility falls: varieties are increasingly under-supplied. Efficiency also falls to the right of the locus, and now this indirect effect can dominate the direct effect, leading to a net loss in welfare, as prices rise and varieties are increasingly over-supplied. In both cases, the global economy converges asymptotically towards an inefficient equilibrium: in the former case profit margins are driven to zero and varieties are under-supplied, whereas in the latter case either the elasticity of demand or of utility converges to unity.



(a) Utility Manifold (b) Elasticity of Utility (c) Change in Real Income

Figure 13: Globalization and Welfare: Pollak Preferences

The remaining panels of Figure 13 illustrate how the elasticity of utility and the gains from globalization vary with ε and ρ in this case. The contrast with the Bulow-Pfleiderer

[35] See Pettengill (1979) and Dixit and Stiglitz (1979). The restricted range excludes the Linear Expenditure System ($\rho = \frac{2}{\varepsilon}$) but includes demand functions that are both more and less convex than the CES ($\rho = \frac{\varepsilon+1}{\varepsilon}$).

[36] This extends a result of Vives (1999), p. 371, who shows (in our terminology) that, with Pollak preferences, utility is subconcave when demand is subconvex.

case in Figure 12 could hardly be more striking. Panel (b) shows that the elasticity of utility is now increasing in *both* ε and ρ: consumers have a lower taste for variety at high ρ, which we know from Figure 10(a) is when prices increase most. As a result, welfare can fall with globalization. As panel (c) shows, the gains from globalization are decreasing in both ε and ρ, and are *negative* for sufficiently convex demand: as shown in Appendix L.3, the exact condition for this is $\rho > \frac{\varepsilon^2+2\varepsilon-1}{\varepsilon^2}$. This provides, to our knowledge, the first concrete example of the "folk theorem" that globalization in the presence of monopolistic competition can be immiserizing if preferences are (in our terminology) sufficiently superconvex. Perhaps equally striking is that welfare rises by more for lower values of ε and ρ: estimates based on CES preferences grossly *underestimate* the gains from globalization in much of the subconvex region, just as they fail to predict losses from globalization in the superconvex region.

5 Conclusion

In this paper we have presented a new way of relating the structure of demand and utility functions to the positive and normative properties of monopolistic and monopolistically competitive markets. By adopting a "firms'-eye view" of demand, we have shown how the elasticity and convexity of demand determine many comparative statics responses. In turn, we have shown how the relationship between these two parameters, which we call the "demand manifold," provides a parsimonious representation of an arbitrary demand function, and a sufficient statistic for many comparative statics results. The manifold is particularly useful when it is unaffected by changes in exogenous variables, a property which we call "manifold invariance." We have introduced some new families of demand systems which exhibit manifold invariance, and have shown that they nest many of the most widely used functions in applied theory. For example, our "bipower direct" family provides a natural way of nesting translog, CES and linear demand functions.[37]

[37] Alternative ways of nesting translog and CES demands, though with considerably more complicated demand manifolds, appear in Novy (2013) and in Pollak, Sickles, and Wales (1984).

To illustrate the usefulness of our approach, we have shown how it allows a parsimonious way of understanding how monopolistically competitive economies adjust to external shocks, as well as a framework for quantifying the effects of globalization. The demand manifold turns out to be a sufficient statistic for the positive implications of globalization in general equilibrium. As for the normative implications, we have shown that the same approach can be applied to an arbitrary utility function. The relationship between the elasticity and concavity of such a function, which we call the "utility manifold," plays a similar role to the demand manifold. We have shown how to compute the gains from trade for an arbitrary utility function, and have illustrated how sensitive are estimates of these gains to alternative specifications of preferences. Our approach suggest that the CES case is an unreliable reference point for calibrating the effects of exogenous shocks. It has been known since Dixit and Stiglitz (1977) that CES preferences ensure efficiency, but the implications of this observation for calibration have not been brought out. When the initial equilibrium is not efficient, any shock has both a direct effect and an indirect effect whose implications hinge on whether it brings the economy closer to or further away from efficiency. As a result, while positive gains are guaranteed with CES preferences, losses from trade ("immiserizing globalization") are possible with demands that are more convex than CES, and gains from trade can be greater than in the CES case when demands are less convex. Quantifying the gains from trade *assuming* CES preferences is going to miss some important effects.

Many extensions of our approach naturally suggest themselves. There are many other topics where functional form plays a key role in determining the implications of a given set of assumptions: applications to choice under uncertainty and to oligopoly immediately come to mind. As for our application to the gains from globalization in monopolistic competition, the framework we have presented can be extended to allow for trade costs and heterogeneous firms.[38] Finally, the families of demand functions we have introduced provide a natural

[38]Models combining trade costs and/or heterogeneous firms with general non-CES preferences have been considered by Bertoletti and Epifani (2012), Arkolakis, Costinot, Donaldson, and Rodríguez-Clare (2012), and Mrázová and Neary (2014).

setting for estimating relatively flexible functional forms, and direct attention towards the parameters which matter for comparative statics predictions.

Appendices

A Alternative Measures of Slope and Curvature

As our measure of demand slope, we work throughout with the price elasticity of demand, which can be expressed in terms of the derivatives of either the inverse or the direct demand functions $p(x)$ and $x(p)$: $\varepsilon \equiv -\frac{p}{xp'} = -\frac{px'}{x}$. Many authors have used the inverse of this elasticity, $e \equiv -\frac{x}{px'} = \frac{1}{\varepsilon}$, under a variety of names: the elasticity of marginal utility: $e = -\frac{d \log u'(x)}{d \log x}$; the "relative love for variety" as in Zhelobodko, Kokovin, Parenti, and Thisse (2012); or (in monopoly equilibrium) the profit margin or Lerner Index of monopoly power: $e = \frac{p-c}{p}$. This has the advantage that its definition is symmetric with that of curvature ρ (and also with those of the elasticity and "temperance" of utility, ξ and χ, to be discussed below). Offsetting advantages of using ε include its greater intuitive appeal, and the fact that it focuses attention on the region of parameter space where comparative statics results are ambiguous.

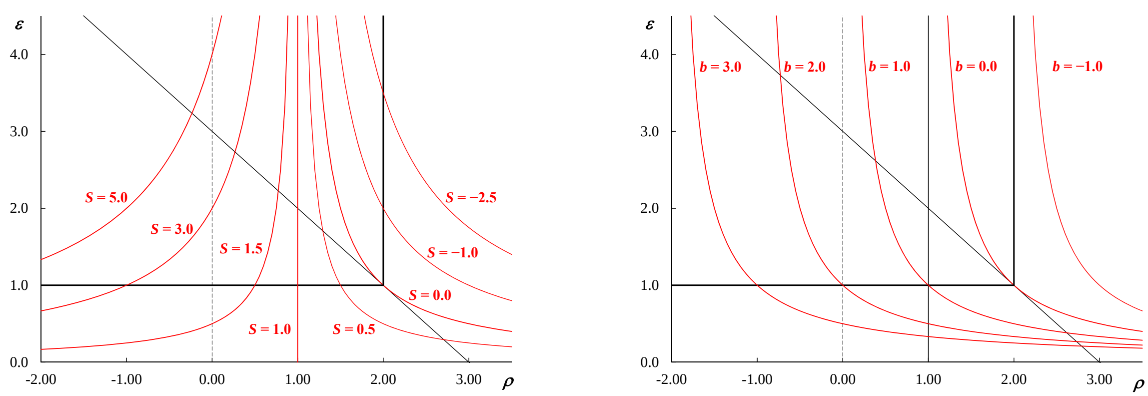
Turning to measures of curvature, the convexity of inverse demand which we use throughout equals the elasticity of the slope of inverse demand, $\rho \equiv -\frac{xp''}{p'} = -\frac{d \log p'(x)}{d \log x}$. Its importance for comparative statics results was highlighted by Seade (1980), and it is widely used in industrial organization, for example by Bulow, Geanakoplos, and Klemperer (1985) and Shapiro (1989). An alternative measure is the convexity of the direct demand function $x(p)$: $r(p) \equiv -\frac{px''(p)}{x'(p)}$. A convenient property is that e and r are dual to ε and ρ:

$$e \equiv -\frac{x}{px'} = \frac{1}{\varepsilon} \qquad r \equiv -\frac{px''}{x'} = \frac{pp''}{(p')^2} = \varepsilon\rho \tag{39}$$

$$\varepsilon \equiv -\frac{p}{xp'} = \frac{1}{e} \qquad \rho \equiv -\frac{xp''}{p'} = \frac{xx''}{(x')^2} = er \tag{40}$$

We use these properties in the proof of Proposition 5 below.

Yet another measure of demand curvature, widely used in macroeconomics, is the superelasticity of Kimball (1995), defined as the elasticity with respect to price of the elasticity of demand, $S \equiv \frac{d \log \varepsilon}{d \log p}$. Positive values of S allow for asymmetric price setting in monopolistic competition. It is related to our measures as follows: $S = \frac{d \log \varepsilon}{d \log x} \frac{d \log x}{d \log p} = \left(\frac{x \varepsilon_x}{\varepsilon}\right)(-\varepsilon) = \varepsilon + 1 - \varepsilon\rho$ (using (42)), so it is positive if and only if demand is subconvex. Figure 14(a) illustrates loci of constant superelasticity, $\rho = \frac{\varepsilon+1-S}{\varepsilon}$. Formally, they correspond to the family of Pollak manifolds, $\bar{\rho}(\varepsilon) = \frac{\sigma+1}{\varepsilon}$, displaced rightwards to be symmetric around the log-linear ($\rho = 1$) rather than the linear ($\rho = 0$) demand manifold.



(a) Iso-Superelasticity Loci

(b) Klenow-Willis Demand Manifolds

Figure 14: Kimball Superelasticity

Kimball himself did not present a parametric family of demand functions. Klenow and Willis (2006) introduce a parametric family which has the property that the superelasticity is a linear function of the elasticity: $S = b\varepsilon$. Substituting for S leads to the family of demand manifolds $\bar{\rho}(\varepsilon) = \frac{(1-b)\varepsilon+1}{\varepsilon}$, which are lateral displacements of the CES locus. Figure 14(b) illustrates some members of this family.

We note in footnotes some implications of these alternative measures. The choice between them is largely a matter of convenience. We express all our results in terms of ε and ρ, partly because this is standard in industrial organization, partly because (unlike e and r) the inverse demand functions are easily integrated to obtain the direct utility function, and partly because (unlike ε and S) they lead to simple restrictions on the shape of the demand manifold as shown in Proposition 3. However, our results could just as well be expressed in terms of e and r or of ε and S. Details of these alternative ways of presenting them are available on request.

B Oligopoly

We consider only monopoly and monopolistic competition in the text, but our approach can also be applied to oligopolistic markets. Even in the simplest case of Cournot competition between n firms producing an identical good, this leads to extra complications. Now we need to distinguish market demand X from the sales of a typical firm i, x_i, with the elasticity and convexity of the demand function $p(X)$ defined in terms of the former: $\varepsilon \equiv -\frac{p}{Xp'}$ and $\rho \equiv -\frac{Xp''}{p'}$. The first-order condition is now $p + x_i p' = c_i \geq 0$. Since this differs between firms, the restriction it implies for the admissible region must be expressed in terms of market shares $\left(\omega_i \equiv \frac{x_i}{X}\right)$: $\varepsilon \geq \max_i(\omega_i)$, which attains its lower bound of $\frac{1}{n}$ when firms are identical.[39] As for the second-order condition, it becomes $2p' + x_i p'' < 0$, implying that $\rho < 2\min_i\left(\frac{1}{\omega_i}\right)$, which attains its upper bound of $2n$ when firms are identical. A different restriction on convexity comes from the stability condition: $\rho < n+1$. This imposes a tighter bound than the second-order condition provided the largest firm is not "too" large: $\max_i(\omega_i) < \frac{2}{n+1}$. Relative to the monopoly case, the admissible region expands unambiguously, except in the boundary case of a dominant firm, where $\max_i(\omega_i) = 1$. Equally important in oligopoly, as we know from Bulow, Geanakoplos, and Klemperer (1985), is that many comparative statics results hinge on strategic substitutability: the marginal revenue of firm i is decreasing in the output of every other firm. This is equivalent to $p' + x_i p'' < 0$, $\forall i$, which in our notation implies a restriction on convexity that lies within the admissible region: $\rho < \min_i\left(\frac{1}{\omega_i}\right) \geq 1$, which attains its upper bound of n when firms are identical.

[39] See Mathiesen (2014) for further discussion.

C Preliminaries: A Key Lemma

We make repeated use of the following result:

Lemma 5. *Consider a twice-differentiable function $g(x)$. Both the double-log convexity of $g(x)$ and the rate of change of its elasticity can be expressed in terms of its first and second derivatives as follows:*

$$\frac{d^2 \log g}{d(\log x)^2} = x\frac{d}{dx}\left(\frac{xg'}{g}\right) = \frac{xg'}{g}\left(1 - \frac{xg'}{g} + \frac{xg''}{g'}\right) \tag{41}$$

In most of Sections 2 and 3, $g(x)$ is the inverse demand function $p(x)$, and the result can be expressed in terms of the demand elasticity and convexity:

$$\frac{d^2 \log p}{d(\log x)^2} = \frac{x\varepsilon_x}{\varepsilon^2} = -\frac{1}{\varepsilon}\left(1 + \frac{1}{\varepsilon} - \rho\right) \tag{42}$$

Qualitatively the same outcome comes from applying Lemma 5 to the direct demand function, replacing $g(x)$ by $x(p)$, and making use of (39) and (40):

$$\frac{d^2 \log x}{d(\log p)^2} = -p\frac{d\varepsilon}{dp} = -\varepsilon\left(1 + \varepsilon - \varepsilon\rho\right) \tag{43}$$

In Section 2.3, $g(x)$ is the absolute value of the demand slope $-p'(x)$, and the result can be expressed in terms of demand convexity and temperance:

$$\frac{d^2 \log(-p')}{d(\log x)^2} = -x\rho_x = -\rho\left(1 + \rho - \chi\right) \tag{44}$$

Finally, in Section 4.2 onwards, $g(x)$ is the sub-utility function $u(x)$, and the result can be expressed in terms of the utility and demand elasticities:

$$\frac{d^2 \log u}{d(\log x)^2} = x\xi_x = \xi\left(1 - \xi - \frac{1}{\varepsilon}\right) \tag{45}$$

All these expressions are zero in the CES case given by (4) and (30), when all four parameters depend only on the elasticity σ: $\{\xi,\, \varepsilon,\, \rho,\, \chi\}^{CES} = \left\{1 - \frac{1}{\sigma},\, \sigma,\, 1 + \frac{1}{\sigma},\, 2 + \frac{1}{\sigma}\right\}$.

D Proof of Proposition 1

We wish to prove that, except in the CES case, only one of ε_x and ρ_x can be zero at any x. Recall from equations 42 and 44 that $\varepsilon_x = \frac{\varepsilon}{x}\left[\rho - \frac{\varepsilon+1}{\varepsilon}\right]$ and $\rho_x = \frac{\rho}{x}(1 + \rho - \chi)$, where $\chi \equiv -\frac{xp'''}{p''}$. We have already seen that ε_x can be zero only along the CES locus. As for $\rho_x = 0$, there are two cases where it can equal zero. The first is where $\rho = 0$. From 44 , this implies that ε_x equals $-\frac{\varepsilon+1}{x}$ which is non-zero. The second is where $1 + \rho - \chi = 0$. As we show in Section 3.3 below, this implies that the demand function takes the Bulow-Pfleiderer form: $p(x) = \alpha + \beta x^{-\theta}$. The intersection of this with $\varepsilon_x = 0$ is the CES limiting case of Bulow-Pfleiderer as sales tend towards zero. Hence we can conclude that the only cases where both ε_x and ρ_x equal zero at a given x lie on a CES demand function.

E Proof of Proposition 2

If demands are multiplicatively separable in ϕ, both the elasticity and convexity are independent of ϕ. In the case of inverse demands, $p\,(x, \phi) = \beta(\phi)\tilde{p}(x)$ implies:

$$\varepsilon = -\frac{p(x,\phi)}{xp_x(x,\phi)} = -\frac{\tilde{p}(x)}{x\tilde{p}'(x)} \quad \text{and} \quad \rho = -\frac{xp_{xx}(x,\phi)}{p_x(x,\phi)} = -\frac{x\tilde{p}''(x)}{\tilde{p}'(x)} \tag{46}$$

A special case of this is additive preferences: $\int_{\omega\in\Omega} u\,[x(\omega)]\, d\omega$. The first-order condition is $u'\,[x(\omega)] = \lambda^{-1}p(\omega)$, which implies that the perceived indirect demand function can be written in multiplicative form: $p(x, \phi) = \lambda(\phi)\tilde{p}(x)$.

Similar derivations hold for direct demands. If $x\,(p, \phi) = \delta\,(\phi)\,\tilde{x}\,(p)$ then:

$$\varepsilon = -\frac{px_p(p,\phi)}{x(p,\phi)} = -\frac{p\tilde{x}'(p)}{\tilde{x}(p)} \quad \text{and} \quad \rho = \frac{x(p,\phi)x_{pp}(p,\phi)}{[x_p(p,\phi)]^2} = \frac{\tilde{x}(p)\tilde{x}''(p)}{[\tilde{x}'(p)]^2} \tag{47}$$

We also have a similar corollary, the case of indirect additivity, where the indirect utility function can be written as: $\int_{\omega\in\Omega} v\left[p(\omega)/I\right]d\omega$. Roy's Identity implies that: $v'\left[p(\omega)/I\right] = -\lambda x(\omega)$, where λ is the marginal utility of income, from which the direct demand function facing a firm can be written in multiplicative form: $x(p/I,\phi) = -\lambda^{-1}(\phi)\tilde{x}(p/I)$.

F Bipower Inverse Demands

F.1 Proof of Proposition 3

To prove sufficiency, we first define $A \equiv \alpha x^{-\eta}$ and $B \equiv \beta x^{-\theta}$, so the demand function can be written as $p(x) = A+B$. Calculating the first and second derivatives yields: $xp' = -\eta A - \theta B$ and $x^2 p'' = \eta\left(\eta+1\right)A + \theta\left(\theta+1\right)B$. Adding x^2p'' to $\eta\theta p$ yields:

$$x^2p'' + \eta\theta p = \left(\eta+\theta+1\right)\left(\eta A + \theta B\right). \tag{48}$$

Using the expression for xp', this implies:

$$x^2p'' + (\eta+\theta+1)xp' + \eta\theta p = 0. \tag{49}$$

Dividing by xp' gives the desired result: $\bar{\rho}\left(\varepsilon\right) = \eta+\theta+1-\eta\theta\varepsilon$.

To prove necessity, assume the manifold is linear, so $\rho(x) = a + b\varepsilon(x)$ where a and b are constants. Substituting for $\rho(x)$ and $\varepsilon(x)$ and collecting terms yields:

$$x^2p''(x) + axp'(x) - bp(x) = 0 \tag{50}$$

To solve this second-order Euler-Cauchy differential equation, we change variables as follows: $t = \log x$ and $p(x) = g(\log x) = g(t)$. Substituting for $p(x) = g(t)$, $p'(x) = \frac{1}{x}g'(t)$ and

$p''(x) = \frac{1}{x^2}\left[g''(t) - g'(t)\right]$ into (50) gives a linear differential equation:

$$g''(t) + (a-1)g'(t) - bg(t) = 0 \tag{51}$$

Assuming a trial solution $g(t) = e^{\lambda t}$ gives the characteristic polynomial: $\lambda^2 + (a-1)\lambda - b = 0$, whose roots are $\lambda = \frac{1}{2}\left[-(a-1) \pm \sqrt{(a-1)^2 + 4b}\right]$. Only real roots make sense, so we assume $(a-1)^2 + 4b \geq 0$. If the inequality is strict, the roots are distinct and the general solution is given by $g(t) = \alpha e^{\lambda_1 t} + \beta e^{\lambda_2 t}$, where α and β are constants of integration. If $(a-1)^2 = -4b$, the roots are equal and the general solution is given by $g(t) = (\alpha + \beta t)e^{\lambda t}$. In both cases, the solution may be found by switching back from t and $g(t)$ to $\log x$ and $p(x)$, recalling that $e^{\lambda \log x} = x^{\lambda}$. Hence, in the first case, $p(x) = \alpha x^{\lambda_1} + \beta x^{\lambda_2}$, and in the second case, $p(x) = (\alpha + \beta \log x)x^{\lambda}$.[40] The final step is to note that the sum of the roots is $\lambda_1 + \lambda_2 = 1 - a$ and their product is $\lambda_1 \lambda_2 = b$, which implies the relationship between the coefficients of the manifold and those of the implied demand function stated in the proposition. This completes the proof.

F.2 Proof of Proposition 4

Substituting from the bipower inverse demand manifold, $\rho = \eta + \theta + 1 - \eta\theta\varepsilon$, into the condition for superconvexity, $\rho \geq \frac{\varepsilon+1}{\varepsilon}$, and using the fact that the elasticity of demand equals $\varepsilon = \frac{\alpha x^{-\eta} + \beta x^{-\theta}}{\eta\alpha x^{-\eta} + \theta\beta x^{-\theta}}$, yields:

$$\rho - \frac{\varepsilon + 1}{\varepsilon} = -\frac{1}{\varepsilon}\left(\eta\varepsilon - 1\right)\left(\theta\varepsilon - 1\right) = \alpha\beta H \tag{52}$$

where $H \equiv \frac{1}{\varepsilon}\frac{(\eta-\theta)^2 x^{-\eta-\theta}}{\left(\eta\alpha x^{-\eta} + \theta\beta x^{-\theta}\right)^2}$, which is strictly positive. Hence, superconvexity requires that α and β must have the same sign, which implies (since at least one of them must be positive) that they must both be positive, which proves the Proposition.

[40] We do not present the case of equal roots separately in the statement of Proposition 3 in the text: the economic interpretation is more convenient if we view it as the limiting case of the general expression as η approaches zero.

F.3 Properties of Inverse PIGL Demands

With $\eta = 1$, so the elasticity of demand becomes $\varepsilon = \frac{\alpha x^{-1}+\beta x^{-\theta}}{\alpha x^{-1}+\theta\beta x^{-\theta}}$, its value less one can be written in two alternative ways: $\varepsilon - 1 = \frac{(1-\theta)\beta x^{1-\theta}}{\alpha+\theta\beta x^{1-\theta}} = (1-\theta)\frac{px-\alpha}{\theta px+(1-\theta)\alpha}$. It follows that $1-\theta$, β and $px - \alpha$ must have the same sign. (Recall that θ itself equals $\frac{2-\rho}{\varepsilon-1}$ and so must be positive in the admissible region.) The value of $1-\theta$ also determines whether the demand function is supermodular or not: substituting from the demand manifold $\bar{\rho}(\varepsilon) = 2 + (1-\varepsilon)\theta$ into the condition for supermodularity gives $\varepsilon + \rho > 3 \;\Leftrightarrow\; (\varepsilon - 1)(1-\theta) > 0 \;\Leftrightarrow\; \theta < 1$. Combining these results with Proposition 4 shows that there are three possible cases of this demand function:

	$\alpha > 0$	$\alpha < 0$
$\theta < 1$, $\beta > 0$	1. Superconvex; supermodular: $px > \alpha > 0$	2. Subconvex; supermodular
$\theta > 1$, $\beta < 0$	3. Subconvex; submodular: $\alpha > px > 0$	n/a

F.4 Properties of Bulow-Pfleiderer Demands

With $\eta = 0$, the elasticity of demand becomes: $\varepsilon = \frac{\alpha+\beta x^{-\theta}}{\theta\beta x^{-\theta}} = \frac{p}{\theta\beta x^{-\theta}} = \frac{p}{\theta(p-\alpha)}$. It follows that θ, β and $p - \alpha$ must have the same sign. The sign of θ also determines whether the direct demand function is logconvex (i.e., whether it exhibits super-pass-through) or not: recall that $\rho - 1 = \theta$. There are therefore three possible cases of this demand function:

	$\alpha > 0$	$\alpha < 0$
$\theta > 0$, $\beta > 0$	1. Superconvex; logconvex: $p > \alpha > 0$	2. Subconvex; logconvex
$\theta < 0$, $\beta < 0$	3. Subconvex; logconcave: $\alpha > p > 0$	n/a

Next, we wish to show that Bulow-Pfleiderer demands are necessary and sufficient for marginal revenue to be affine in price. Sufficiency is immediate: marginal revenue is $p + xp' = \theta\alpha + (1-\theta)p$. Necessity follows by solving the differential equation $p(x) + xp'(x) = a + bp(x)$, which yields $p(x) = \frac{a}{1-b} + c_1 x^{b-1}$, where c_1 is a constant of integration.

G Bipower Direct Demands

G.1 Proof of Proposition 5

The proof follows immediately by noting that the direct demand function in Proposition 5, $x(p) = \gamma p^{-\nu} + \delta p^{-\sigma}$, is the dual of the inverse demand function in Proposition 3, $p(x) = \alpha x^{-\eta} + \beta x^{-\theta}$. Hence Proposition 3 with appropriate relabeling implies that the direct demand function $x(p) = \gamma p^{-\nu} + \delta p^{-\sigma}$ is necessary and sufficient for a linear *dual* manifold, that is to say, an equation linking the dual parameters r and e: $\bar{r}(e) = \nu + \sigma + 1 - \nu\sigma e$. Recalling from (39) that $e = \frac{1}{\varepsilon}$ and $r = \varepsilon\rho$ gives the desired result.

G.2 Proof of Proposition 6

The proof follows the same steps as that of Proposition 4. The bipower direct demand manifold is given in equation (18), while the elasticity of demand ε equals $\frac{\nu\gamma p^{-\nu}+\sigma\delta p^{-\sigma}}{\gamma p^{-\nu}+\delta p^{-\sigma}}$. Substituting into the condition for superconvexity yields:

$$\rho - \frac{\varepsilon + 1}{\varepsilon} = -\frac{1}{\varepsilon^2}\left(\varepsilon - \nu\right)\left(\varepsilon - \sigma\right) = \gamma\delta H' \tag{53}$$

where $H' \equiv \frac{1}{\varepsilon^2}\frac{(\sigma-\nu)^2 p^{-\sigma}}{(\gamma p^{-\nu}+\delta p^{-\sigma})^2}$, which is positive. It follows that both γ and δ must be positive for superconvexity, which proves the Proposition.

G.3 QMOR Demand Functions

Diewert (1976) introduced the quadratic mean of order r expenditure function, which implies a general functional form for homothetic demand functions. Feenstra (2014) considers a symmetric special case and shows how it can be extended to allow for entry and exit of goods, so making it applicable to models of monopolistic competition. In our notation, the resulting family of demand functions, taking a "firm's eye view", is:

$$x(p) = \gamma p^{-(1-r)} + \delta p^{-\frac{2-r}{2}} \tag{54}$$

This is clearly a member of the bipower direct family, with $\nu = 1 - r$ and $\sigma = \frac{2-r}{2}$. Hence, from Proposition 5, its demand manifold is:

$$\bar{\rho}(\varepsilon) = \frac{(2-r)(3\varepsilon - 1 + r)}{2\varepsilon^2} \tag{55}$$

In the limit as $r \to 0$, this becomes $\bar{\rho}(\varepsilon) = \frac{3\varepsilon-1}{\varepsilon^2}$, which is the translog manifold discussed in the text. Figure 15 illustrates this demand manifold for a range of values of r. For $r = 2.0$ it coincides with the $\rho = 0$ vertical line: i.e., a linear demand function from the firm's perspective. For negative values of r (i.e., more convex than the translog), the manifolds extend into the superconvex region. However, this is for arbitrary values of γ and δ. Feenstra (2014) shows that these parameters, which depend on real income and on prices of other goods, must be of opposite sign when the demand function (54) is derived from expenditure minimization. Hence, from Proposition 6, QMOR demands are not consistent with superconvexity, though in other respects they allow for considerable flexibility in modeling homothetic demands.

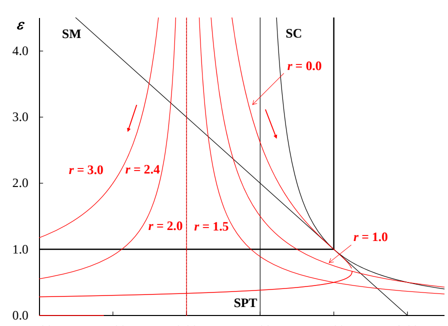


Figure 15: Demand Manifolds for QMOR Demand Functions

G.4 Properties of PIGL Demands

With $\nu = 1$, the elasticity of demand becomes $\varepsilon = \frac{\gamma p^{-1}+\sigma\delta p^{-\sigma}}{\gamma p^{-1}+\delta p^{-\sigma}}$. Subtracting one gives: $\varepsilon - 1 = \frac{(\sigma-1)\delta p^{-\sigma}}{\gamma p^{-1}+\delta p^{-\sigma}} = (\sigma - 1)\frac{px-\gamma}{px}$. It follows that $\sigma - 1$, δ and $px - \gamma$ must have the same sign. In addition, the demand manifold is $\rho = \frac{(\sigma+2)\varepsilon-\sigma}{\varepsilon^2}$, so convexity is increasing in σ. Combining these results with Proposition 6, there are three possible cases of this demand function. For σ less than one, the demand function is less convex than the translog (i.e., PIGLOG) case, δ is negative and γ is positive. For σ greater than one, δ is positive, the demand function is more convex than the translog case, and it is subconvex if γ is negative, otherwise it is superconvex. These properties are dual to those of the inverse PIGL demand functions in Appendix F.3, and, like the latter, they can be related to whether the elasticity of marginal revenue with respect to *price* is greater or less than one (the value of one corresponding to the PIGLOG case). Note finally that the limiting case of PIGL demand function when σ approaches zero is the LES, the only demand function which is a subset of both PIGL and Pollak. The LES case is special in another respect: as can be seen in Figure 9(a), it is the only member of the PIGL family for which ε is monotonic in ρ along the manifold. In all other cases the manifold is vertical at $\{\varepsilon, \rho\} = \{\frac{2\sigma}{\sigma+2}, \frac{(\sigma+2)^2}{4\sigma}\}$. For $\sigma < 0$ it is not defined for $\rho < \frac{(\sigma+2)^2}{4\sigma}$, while for $\sigma > 0$ it is not defined for $\rho > \frac{(\sigma+2)^2}{4\sigma}$.

G.5 Proof of Lemma 3: Uniqueness of the Translog

It is obvious by inspection that no member of the inverse bipower family can be always both strictly subconvex and strictly supermodular. Consider next the direct bipower family. The slope of its demand manifold is:

$$\frac{d\varepsilon}{d\rho} = -\frac{\varepsilon^3}{(\nu+\sigma+1)\varepsilon - 2\nu\sigma} \tag{56}$$

This cannot be zero, since ε is greater than one in the admissible region. Consider the behavior of the demand manifold as ε approaches one from above. If it lies between the superconvexity and supermodularity loci, it must approach the Cobb-Douglas point $(\varepsilon, \rho) = (1, 2)$. In addition, since both loci have slopes equal to -1 at that point, equation (56) must also equal -1 at that point. Evaluating the demand manifold and its slope at $(1, 2)$ gives two restrictions which the parameters must exhibit:

$$1 = \nu + \sigma - \nu\sigma \quad \text{and} \quad \frac{1}{\nu + \sigma + 1 - 2\nu\sigma} = 1 \tag{57}$$

Both of these can hold only if $\nu\sigma = 1$. This in turn implies from (57) that $\nu + \sigma = 2$. Combining these two conditions implies that $(\nu - 1)^2 = 0$, so both ν and σ must equal one. Only the translog is consistent with this, which proves the result.

G.6 Properties of Pollak Demands

With $\nu = 0$, the elasticity of demand becomes $\varepsilon = \frac{\sigma\delta p^{-\sigma}}{\gamma + \delta p^{-\sigma}} = \sigma\frac{\sigma\delta p^{-\sigma}}{x} = \sigma\frac{x-\gamma}{x}$. It follows that σ, δ and $x - \gamma$ must have the same sign. The sign of σ also determines whether the inverse demand function is logconvex or not. The CARA demand function is the limiting case when $\sigma \to 0$: the direct demand function becomes $x = \gamma' + \delta' \log p$, $\delta' < 0$, which implies that the inverse demand function is log-linear: $\log p = \alpha + \beta x$, $\beta < 0$.[41] The CARA manifold is $\bar{\rho}(\varepsilon) = \frac{1}{\varepsilon}$, which is a rectangular hyperbola through the point $\{1.0, 1.0\}$. Hence the CARA function is the dividing line between two sub-groups of demand functions and their corresponding manifolds, with σ either negative or positive. For negative values of σ, γ is an upper bound to consumption: the best-known example of this class is the linear demand function, corresponding to $\sigma = -1$. By contrast, for strictly positive values of σ, γ is the lower bound to consumption and there is no upper bound. Especially in the LES case, it is common to interpret γ as a "subsistence" level of consumption, but this requires

[41] As noted by Pollak, this demand function was first proposed by Chipman (1965), who showed that it is implied by an additive exponential utility function. Later independent developments include Bertoletti (2006) and Behrens and Murata (2007). Differentiating the Arrow-Pratt coefficient of absolute risk aversion defined in footnote 27 gives $\frac{\partial A(x)}{\partial x} = -\frac{u'u''' - (u'')^2}{(u'')^2} = -\frac{pp'' - (p')^2}{(p')^2} = 1 - \varepsilon\rho$, so absolute risk aversion is constant if and only if $\varepsilon = \frac{1}{\rho}$.

that it be positive and not greater than x, which only holds if demand is superconvex. All members of the Pollak family with positive σ are translated-CES functions, and, as the arrows in Figure 9(b) indicate, they asymptote towards the corresponding "untranslated-CES" function as sales rise without bound; for example, the LES demand function, with σ equal to one, asymptotes towards the Cobb-Douglas. Summarizing, there are three possible cases of this demand function:

	$\gamma > 0$	$\gamma < 0$
$\sigma > 0$, $\delta > 0$	1. Superconvex; logconvex: $x > \gamma > 0$	2. Subconvex; logconvex
$\sigma < 0$, $\delta < 0$	3. Subconvex; logconcave: $\gamma > x > 0$	n/a

H Demand Functions that are not Manifold-Invariant

In this section we introduce two new demand systems whose demand manifolds can be written in closed form, though they depend on all the parameters, and so are not manifold invariant. We consider in turn: the "Doubly-Translated CES" super-family, which nests both the Pollak and Bulow-Pfleiderer families; and the "Translated Bipower-Inverse" super-family, which nests both the "Apt" (Adjustable pass-through) system of Fabinger and Weyl (2012) and a new family which we call the inverse "iso-temperance" system.[42] We also introduce a demand function, the inverse exponential, which is partly manifold-invariant, and which is an example of a demand function that can be both sub- and superconvex.

[42] A third super-family is the dual of the second, the "Translated Bipower-Direct" super-family. Reversing the roles of p and x in equation (60) below leads to a "dual" manifold giving the inverse elasticity e as a function of the direct convexity r with the same form as (62). Special cases of this include the dual of the Apt system and the direct "iso-temperance" system (i.e., the demand system necessary and sufficient for $-px'''/x''$ to be constant). It does not seem possible to express the manifold $\bar{\varepsilon}(\rho)$ in closed form for this family.

H.1 The "Doubly-Translated CES" Super-Family

We can nest the Pollak and Bulow-Pfleiderer families as follows: $p(x) = \alpha + \beta (x - \gamma)^{-\theta}$. The elasticity and convexity of this function are:

$$\varepsilon(x) = \frac{1}{\theta} \frac{p}{p-\alpha} \frac{x-\gamma}{x} \qquad \rho(x) = (\theta + 1) \frac{x}{x - \gamma} \tag{58}$$

When γ is zero this reduces to the Bulow-Pfleiderer case. Assuming $\gamma \neq 0$, we have $\rho \neq \theta + 1$, and so the expression for ρ in (58) can be solved for x: $x = \frac{\rho}{\rho - (\theta+1)} \gamma$. Substituting into the expression for ε yields:[43]

$$\bar{\varepsilon}(\rho) = \left[1 + a_1 \left(\frac{1}{\rho - a_2} \right)^{a_3} \right] \frac{a_4}{\rho} \tag{59}$$

where: $a_1 = \frac{\alpha}{\beta} \left\{ (\theta + 1) \gamma \right\}^{\theta}$, $a_2 = \theta + 1$, $a_3 = \theta$, and $a_4 = \frac{\theta+1}{\theta}$. This is a closed-form expression for the manifold but it depends on all four parameters, except in special cases such as the Pollak family, when, with $\alpha = 0$, it reduces to $\bar{\varepsilon}(\rho) = \frac{\theta+1}{\theta} \frac{1}{\rho}$. Even in this case, the demand manifold allows for some economy of information, since three of its four parameters depend only on the exponent θ in the demand function.

H.2 The "Translated Bipower-Inverse" Super-Family

This demand function adds an intercept α_0 to the bipower-inverse family of Section 3.3:

$$p(x) = \alpha_0 + \alpha x^{-\eta} + \beta x^{-\theta} \tag{60}$$

Differentiating gives the elasticity and convexity:

$$\varepsilon(x) = \frac{\alpha_0 x^{\eta} + \alpha + \beta x^{\eta - \theta}}{\eta \alpha + \theta \beta x^{\eta - \theta}} \qquad \rho(x) = \frac{\eta (\eta + 1) \alpha + \theta (\theta + 1) \beta x^{\eta - \theta}}{\eta \alpha + \theta \beta x^{\eta - \theta}} \tag{61}$$

[43] Here and elsewhere, the parameters must be such that, when the exponent (here θ) is not an integer, the expression which is raised to the power of that exponent is positive.

Assuming as before that $\rho \neq \theta + 1$, and also that $\eta \neq \theta$, we can invert $\rho(x)$ to solve for x: $x(\rho) = \left[\frac{\eta\alpha}{\theta\beta}\frac{(\eta+1)-\rho}{\rho-(\theta+1)}\right]^{\frac{1}{\eta-\theta}}$. Substituting into $\varepsilon(x)$ gives a closed-form expression for the manifold:

$$\bar{\varepsilon}(\rho) = \frac{\rho - a_1}{a_2} + (a_3 - \rho)^{a_4}(\rho - a_5)^{a_6} a_7 \tag{62}$$

where: $a_1 = \eta + \theta + 1$, $a_2 = -\eta\theta$, $a_3 = \eta + 1$, $a_4 = \frac{\eta}{\eta-\theta}$, $a_5 = \theta + 1$, $a_6 = -\frac{\theta}{\eta-\theta}$, and $a_7 = \left(\frac{\eta}{\beta}\right)^{\frac{\eta}{\eta-\theta}}\left(\frac{\theta}{\alpha}\right)^{-\frac{\theta}{\eta-\theta}}\frac{\alpha_0}{\eta\theta(\eta-\theta)}$. In general, this depends on the same five parameters as the demand function (60), though once again all but a_7 depend only on the two exponents η and θ. It is best understood by considering some special cases:

(1) Bipower Inverse: The cost in additional complexity of the "translation" parameter α_0 is apparent. Setting this equal to zero, the expression simplifies to give the bipower inverse manifold as in Proposition 3: $\bar{\rho}(\varepsilon) = 1 + \eta + \theta - \eta\theta\varepsilon$.

(2) Apt Demands: Fabinger and Weyl (2012) show that the pass-through rate (in our notation, $\frac{dp}{dc} = \frac{1}{2-\rho}$) is quadratic in the square root of price if and only if the inverse demand function has the form of (60) with $\eta = 2\theta$. This reduces the number of parameters by one, so the demand manifold simplifies to: $\bar{\varepsilon}(\rho) = \frac{1+3\theta-\rho}{2\theta^2} - \frac{[(2\theta+1)-\rho]^2}{\rho-(\theta+1)}\frac{2\alpha}{\beta^2\theta^2}\alpha_0$.

(3) Iso-Temperance Demands: Setting $\eta = -1$ is sufficient to ensure that temperance, $\chi \equiv -\frac{xp'''}{p''}$, is constant, equal to $\theta + 2$. It is also necessary. To see this, write $xp''' = -\chi p''$, where χ is a constant, and integrate three times, which yields $p(x) = c_0 + c_1 x + \frac{c_2}{(1-\chi)(2-\chi)} x^{2-\chi}$, where c_0, c_1 and c_2 are constants of integration. This is identical to (60) with $\eta = -1$ and $\theta = \chi - 2$. Note that iso-convexity implies iso-temperance, but the converse does not hold; just as CES implies iso-convexity, but the converse does not hold.

It should be apparent that the demand manifold in (62) is not particularly convenient to work with. However, if we are mainly interested in pass-through, then we do not need to work with the demand manifold at all, since the key conditions in (7) and (44) do not depend on the elasticity of demand (a point stressed by Weyl and Fabinger (2013)). In such cases, our approach can be applied to the *slope* rather than the *level* of demand. By relating the elasticity and convexity of this slope to each other, we can construct a "Demand Slope

Manifold" corresponding to any given demand function, and the properties of this manifold are very informative about when pass-through is increasing or decreasing with sales. It can be shown that the demand slope manifolds of the Apt and iso-temperance demand functions are particularly convenient in this respect.

H.3 Exponential Inverse Demand

Our last example is a demand function which is sometimes subconvex and sometimes superconvex:

$$p(x) = \alpha + \beta \exp(-\gamma x^{\delta}) \tag{63}$$

where $\gamma > 0$ and $\delta > 0$. The elasticity and convexity of demand are found to be:

$$\varepsilon(x) = \frac{p}{p-\alpha}\frac{1}{\gamma\delta x^{\delta}} \quad \text{and} \quad \rho(x) = \gamma\delta x^{\delta} - \delta + 1 \tag{64}$$

Solving the latter for γx^{δ} as a function of ρ and substituting into the former yields a closed-form expression for the demand manifold:

$$\bar{\varepsilon}(\rho) = \frac{1}{\rho+\delta-1}\frac{\frac{\alpha}{\beta} + \exp\left(-\frac{\rho+\delta-1}{\delta}\right)}{\exp\left(-\frac{\rho+\delta-1}{\delta}\right)} \tag{65}$$

This is invariant with respect to γ and also depends only on the ratio of α and β, not on their levels. Differentiating with respect to ρ shows that, provided $\frac{\alpha}{\beta}$ is strictly positive, the demand function is subconcave for low values of ρ, which from (64) implies low values of x, but superconcave for high ρ and x:

$$\bar{\varepsilon}_{\rho} = \frac{-\delta + \frac{\alpha}{\beta}(\rho-1)\exp\left(\frac{\rho+\delta-1}{\delta}\right)}{\delta(\rho+\delta-1)^2} \tag{66}$$

Figure 16 illustrates some demand functions and the corresponding manifolds from this class for a range of values of α, assuming $\beta = \gamma = 1$ and $\delta = 2$. A superconvex range in the admissible region is possible only for parameter values such that the minimum point of the manifold lies above the Cobb-Douglas point, $\{\varepsilon, \rho\} = \{1, 2\}$, i.e., only for $\alpha > \beta\delta \exp\left(-\frac{\delta+1}{\delta}\right)$, which for the assumed values of β and δ is approximately $\alpha > 0.446$.

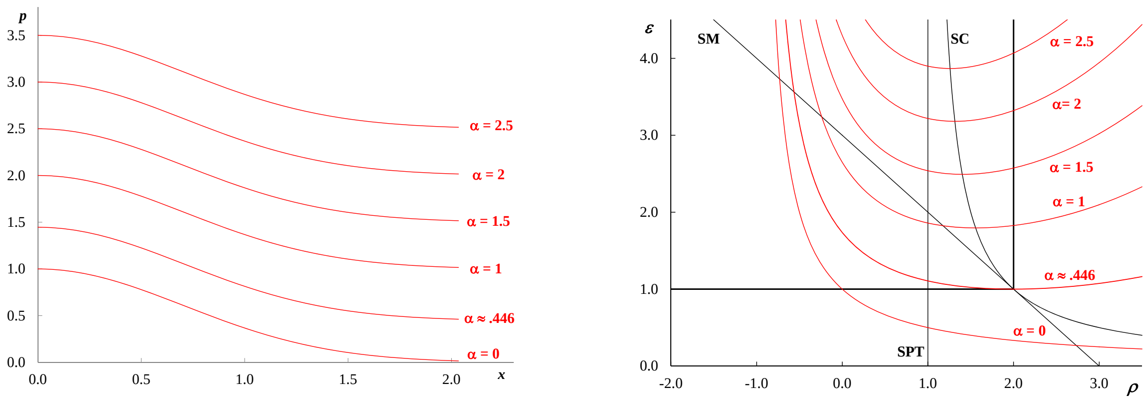


Figure 16: Inverse Exponential Demand Functions and Manifolds

I Calculating the Effects of Globalization

To solve for the results in (27), use (25) to eliminate $\hat{x}$ from (23) and then solve (23) and (24) for $\hat{p}$ and $\hat{y}$, with $\hat{n}$ determined residually by (26). The results in (28) are obtained by using $\hat{x} = \hat{y} - \hat{k}$ and $\hat{N} = \hat{k} + \hat{n}$.

J Change in Real Income: Details

With symmetric preferences and identical prices for all goods, the budget constraint becomes: $I = \int_0^N p(\omega)x(\omega)\,\mathrm{d}\omega = Npx$. So consumption of each good is: $x = \frac{I}{Np}$. Substituting into the direct utility function yields its indirect counterpart:

$$V(N, p, I) = F\left[Nu\left(\frac{I}{Np}\right)\right] \tag{67}$$

We can now define equivalent income $Y(N, p)$ as the income that preserves the initial level of utility U_0 following a shock:

$$V\left(N, p, \frac{I}{Y}\right) = U_0 \tag{68}$$

For small changes (so equivalent and compensating variations coincide), we logarithmically differentiate, with I fixed (since it equals exogenous labor income), to obtain: $\hat{N} - \xi(\hat{N} + \hat{p} + \hat{Y}) = 0$. Rearranging gives the change in real income in (29).

K Welfare with Bipower Inverse Preferences

K.1 Proof of Proposition 10

The proof of Proposition 10 follows by adapting that of Proposition 3. Consider sufficiency first. Just as the bipower inverse demand function in (17) implies equation (49), so the bipower utility function in (34) leads with appropriate relabeling to:

$$x^2u'' + [(\eta - 1) + (\theta - 1) + 1]\, xu' + (\eta - 1)(\theta - 1)u = 0. \tag{69}$$

Dividing by xu' and replacing $\frac{xu''}{u'}$ by $-\frac{1}{\varepsilon}$ and $\frac{u}{xu'}$ by $\frac{1}{\xi}$ yields equation (35) as required. The proof of necessity also proceeds as in Proposition 3, *mutatis mutandis*.

K.2 Proof of Proposition 11

From the proof of Proposition 4 in Section F.2, we have already seen that superconvexity of demand for the bipower inverse family is equivalent to $(\eta\varepsilon - 1)(\theta\varepsilon - 1) \leq 0$. Similarly, superconcavity of utility is equivalent from (31) to $\xi \geq \frac{\varepsilon - 1}{\varepsilon}$. Substituting for ξ from the bipower utility manifold (35) implies that superconcavity of utility holds for this family if and only if: $\frac{(\eta\varepsilon-1)(\theta\varepsilon-1)}{(1-\eta-\theta)\varepsilon+1} \geq 0$.

K.3 Welfare with Bulow-Pfleiderer Preferences

In the special case of Bulow-Pfleiderer preferences as in Corollary 6, the sufficient condition for gains from globalization from Corollary 5 simplifies to $(1-\theta)\,\varepsilon+1>0$. Since $\theta=\rho-1$, this is equivalent to $(2-\rho)\,\varepsilon+1>0$, which must hold from the firm's second-order condition. Hence the conditions for subconcavity of utility are the same as those for superconvexity of demand derived in Section F.4 above: $\theta>0$ and $p>\alpha>0$. Finally, the utility manifold (35) simplifies to: $\bar{\xi}(\varepsilon)=\frac{(1-\theta)\varepsilon}{(1-\theta)\varepsilon+1}$. Expressing this as a function of ε and ρ: $\xi(\varepsilon,\rho)=\frac{(2-\rho)\varepsilon}{(2-\rho)\varepsilon+1}$. Hence ξ always lies between zero and one.

L Welfare with Pollak Preferences

L.1 From Demands to Preferences

Recall from Section 3.4 that the Pollak demand function is $x(p)=\gamma+\delta p^{-\sigma}$, where δ, σ and $x-\gamma$ have the same sign, and γ has the same sign as δ and σ if and only if demand is superconvex. To derive the sub-utility function we must first invert to obtain the inverse demand function. This yields: $p(x)=\left(\frac{x-\gamma}{\delta}\right)^{-\frac{1}{\sigma}}$. It is convenient to redefine the constants as $\zeta\equiv-\gamma\sigma$ and $\beta\equiv(\delta/\sigma)^{1/\sigma}$ (i.e., we replace γ by $-\zeta/\sigma$, and δ by β^{δ}/σ), which yields: $p(x)=\beta(\sigma x+\zeta)^{-\frac{1}{\sigma}}$. Both β and $\sigma x+\zeta$ are positive. Integrating and setting the constant of integration equal to zero yields the sub-utility function (36).

L.2 Proof of Proposition 12

We have already seen in Proposition 6 that all bipower direct demand functions are superconvex if and only if both γ and δ are positive, and in Section G.6 that with Pollak demands σ and δ must have the same sign. Hence a negative value of $\zeta\equiv-\gamma\sigma$ is necessary and sufficient for demand to be superconvex. Exactly the same condition arises when we substitute from the Pollak utility manifold $\bar{\xi}(\varepsilon)=\frac{\sigma-1}{\varepsilon}$ into the condition for superconcavity of utility, $\xi \geq \frac{\varepsilon-1}{\varepsilon}$, which implies that superconcavity requires $\sigma \geq \varepsilon$. Since the elasticity ε equals $\sigma\frac{x-\gamma}{x} = \sigma + \frac{\zeta}{x}$, it follows that $\zeta \leq 0$ is also necessary and sufficient for utility to be superconcave.

L.3 Gains from Globalization with Pollak Preferences

Substituting for $\xi = \frac{\varepsilon\rho-2}{\varepsilon}$ into the general expression for welfare change in equation (33) gives in this case:

$$\hat{Y} = \frac{\varepsilon}{\varepsilon\rho - 2}\left[1 - \frac{(\varepsilon-1)^2}{\varepsilon^2(2-\rho)}\right]\hat{k} \tag{70}$$

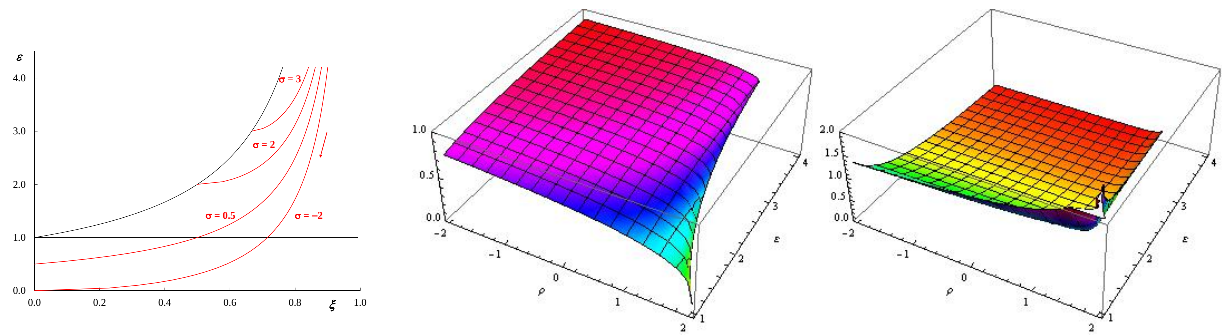
This is negative when $\rho > \rho^Y \equiv \frac{\varepsilon^2+2\varepsilon-1}{\varepsilon^2}$. To confirm that this lies in the admissible range $\rho \in \left[\underline{\rho}, \overline{\rho}\right] \equiv \left[\frac{2}{\varepsilon}, 1 + \frac{2}{\varepsilon}\right]$, note that $\rho^Y - \underline{\rho} = \frac{\varepsilon^2-1}{\varepsilon^2} > 0$ and $\overline{\rho} - \rho^Y = \frac{1}{\varepsilon^2} > 0$.

L.4 Alternative Normalizations of the Sub-Utility Function

In the text we follow Dixit and Stiglitz (1977) and set the constant of integration in the sub-utility function equal to zero. As Dixit and Stiglitz (1979) point out, this need not imply that $u(0)$ is strictly positive: we can define $u(x) = \max\left\{0, \frac{\beta}{\sigma-1}(\sigma x+\zeta)^{\frac{\sigma-1}{\sigma}}\right\}$, which is discontinuous at $x = 0$, but in all respects is a valid utility index. Nevertheless, it is unsatisfactory that new goods provide a finite level of utility, even when they are consumed in infinitesimal (though strictly positive) amounts. An alternative approach, due to Pettengill (1979), is to choose the constant of integration itself to ensure that $u(0) = 0$. This implies that the sub-utility function takes the following form:

$$u(x) = \frac{\beta}{\sigma-1}\left[(\sigma x+\zeta)^{\frac{\sigma-1}{\sigma}} - \zeta^{\frac{\sigma-1}{\sigma}}\right] \tag{71}$$

Note, however, that ζ must be positive, which implies from Proposition 12 that demand is always subconvex: a zero level of consumption is not in the consumer's feasible set if ζ is negative. Hence this normalization of the Pollak utility function implies a different restriction on the feasible region from that in the text, with the whole of the superconvex region now inadmissible.



(a) Utility Manifold (b) Elasticity of Utility (c) Change in Real Income

Figure 17: Globalization and Welfare: Normalized Pollak Preferences

It goes without saying that the elasticity and convexity of demand are unaffected by this re-normalization of the sub-utility function. However, the elasticity of utility is very different. It now behaves more like the Bulow-Pfleiderer case, except that it is not consistent with superconvex demands:

$$\xi^N = H\xi, \quad H(\varepsilon,\sigma) = \frac{1}{1-\left(\frac{\varepsilon-\sigma}{\varepsilon}\right)^{\frac{\sigma-1}{\sigma}}}, \quad H(\varepsilon,\rho) = \frac{1}{1-\left(\frac{\varepsilon-\varepsilon\rho+1}{\varepsilon}\right)^{\frac{\varepsilon\rho-2}{\varepsilon\rho-1}}} \tag{72}$$

where H is a correction factor applied to the unnormalized elasticity of utility given by equations (37), $\xi(\varepsilon,\sigma) = \frac{\sigma-1}{\varepsilon}$, and (38), $\xi(\varepsilon,\rho) = \frac{\varepsilon\rho-2}{\varepsilon}$. The results are shown in Figure 17. Compared with Figure 13 in the text, the main differences are that utility is always superconcave and that the elasticity of utility now lies between zero and one for all admissible values of ε and ρ, i.e., throughout the subconvex region. Both the elasticity of utility and the change in real income behave qualitatively with respect to ε and ρ in a similar fashion to the case of Bulow-Pfleiderer preferences in Figure 12. All this confirms that the elasticity and convexity of demand are not sufficient statistics for the welfare effects of globalization, and that small changes in the parameterization of utility can have major implications for the quantitative effects of changes in the size of the world economy.

References

Aguirre, I., S. Cowan, and J. Vickers (2010): "Monopoly Price Discrimination and Demand Curvature," *American Economic Review*, 100(4), 1601–1615.

Antràs, P., and E. Helpman (2004): "Global Sourcing," *Journal of Political Economy*, 112(3), 552–580.

Arkolakis, C., A. Costinot, D. Donaldson, and A. Rodríguez-Clare (2012): "The Elusive Pro-Competitive Effects of Trade," Discussion paper, Yale University.

Arkolakis, C., A. Costinot, and A. Rodríguez-Clare (2012): "New Trade Models, Same Old Gains?," *American Economic Review*, 102(1), 94–130.

Atkin, D., and D. Donaldson (2012): "Who's Getting Globalized? The Size and Nature of Intranational Trade Costs," Discussion paper, Yale University.

Baldwin, R., and J. Harrigan (2011): "Zeros, Quality, and Space: Trade Theory and Trade Evidence," *American Economic Journal: Microeconomics*, 3(2), 60–88.

Behrens, K., and Y. Murata (2007): "General Equilibrium Models of Monopolistic Competition: A New Approach," *Journal of Economic Theory*, 136(1), 776–787.

Bergstrom, T., and M. Bagnoli (2005): "Log-Concave Probability and its Applications," *Economic Theory*, 26, 445–469.

Bertoletti, P. (2006): "Logarithmic Quasi-Homothetic Preferences," *Economics Letters*, 90(3), 433–439.

Bertoletti, P., and P. Epifani (2012): "Monopolistic Competition: CES Redux," Working paper, University of Bocconi.

Bulow, J., and P. Klemperer (2012): "Regulated Prices, Rent Seeking, and Consumer Surplus," *Journal of Political Economy*, 120(1), pp. 160–186.

Bulow, J. I., J. D. Geanakoplos, and P. D. Klemperer (1985): "Multimarket Oligopoly: Strategic Substitutes and Complements," *Journal of Political Economy*, 93(3), pp. 488–511.

Bulow, J. I., and P. Pfleiderer (1983): "A Note on the Effect of Cost Changes on Prices," *Journal of Political Economy*, 91(1), pp. 182–185.

Bustos, P. (2011): "Trade Liberalization, Exports and Technology Upgrading: Evidence on the Impact of MERCOSUR on Argentinian Firms," *American Economic Review*, 101(1), 304–340.

Chamberlin, E. H. (1933): *The Theory of Monopolistic Competition: A Re-Orientation of the Theory of Value*. Cambridge, Mass: Harvard University Press.

Chipman, J. S. (1965): "A Survey of the Theory of International Trade: Part 2, The Neo-Classical Theory," *Econometrica*, 33(4), pp. 685–760.

Cowan, S. (2012): "Third-Degree Price Discrimination and Consumer Surplus," *Journal of Industrial Economics*, 60(2), 333–345.

Deaton, A., and J. Muellbauer (1980): "An Almost Ideal Demand System," *American Economic Review*, 70(3), 312–326.

Dhingra, S., and J. Morrow (2011): "The Impact of Integration on Productivity and Welfare Distortions Under Monopolistic Competition," Discussion paper, Princeton University.

Diewert, W. E. (1976): "Exact and Superlative Index Numbers," *Journal of Econometrics*, 4(2), 115–145.

Dixit, A. K., and J. E. Stiglitz (1977): "Monopolistic Competition and Optimum Product Diversity," *American Economic Review*, 67(3), 297–308.

——— (1979): "Monopolistic Competition and Optimum Product Diversity: Reply [to Pettengill]," *American Economic Review*, 69(5), 961–963.

Eeckhoudt, L., C. Gollier, and T. Schneider (1995): "Risk-Aversion, Prudence and Temperance: A Unified Approach," *Economics Letters*, 48(3), 331–336.

Fabinger, M., and E. G. Weyl (2012): "Pass-Through and Demand Forms," mimeo.

Feenstra, R. C. (2003): "A Homothetic Utility Function for Monopolistic Competition Models, without Constant Price Elasticity," *Economics Letters*, 78(1), 79–86.

——— (2014): "Restoring the Product Variety and Pro-Competitive Gains from Trade with Heterogeneous Firms and Bounded Productivity," NBER Working Paper No. 16796.

Gopinath, G., and O. Itskhoki (2010): "Frequency of price adjustment and pass-through," *Quarterly Journal of Economics*, 125(2), 675–727.

Helpman, E., M. J. Melitz, and S. R. Yeaple (2004): "Export Versus FDI with Heterogeneous Firms," *American Economic Review*, 94(1), 300–316.

Kimball, M. S. (1992): "Precautionary Motives for Holding Assets," in P. Newman, M. Milgate and J. Eatwell (eds.): *The New Palgrave Dictionary of Money and Finance*, New York: Stockton Press.

——— (1995): "The Quantitative Analytics of the Basic Neomonetarist Model," *Journal of Money, Credit and Banking*, 27(4), pp. 1241–1277.

Klenow, P. J., and J. L. Willis (2006): "Real Rigidities and Nominal Price Changes," Discussion paper, Research Division, Federal Reserve Bank of Kansas City.

Krugman, P. R. (1979): "Increasing Returns, Monopolistic Competition, and International Trade," *Journal of International Economics*, 9(4), 469–479.

Mathiesen, L. (2014): "On the Uniqueness of the Cournot Equilibrium: A Pedagogical Note," Discussion Paper, NHH, Bergen.

Melitz, M. J. (2003): "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica*, 71(6), 1695–1725.

Merton, R. C. (1971): "Optimum Consumption and Portfolio Rules in a Continuous-Time Model," *Journal of Economic Theory*, 3(4), 373–413.

Mrázová, M., and J. P. Neary (2011): "Selection Effects with Heterogeneous Firms," Discussion Paper No. 588, Department of Economics, University of Oxford.

Mrázová, M., and J. P. Neary (2014): "Together at Last: Trade Costs, Demand Structure, and Welfare," forthcoming in *American Economic Review, Papers and Proceedings.*

Muellbauer, J. (1975): "Aggregation, Income Distribution and Consumer Demand," *Review of Economic Studies*, 42(4), pp. 525–543.

Neary, J. P. (2009): "Putting the 'New' into New Trade Theory: Paul Krugman's Nobel Memorial Prize in Economics," *Scandinavian Journal of Economics*, 111(2), 217–250.

Novy, D. (2013): "International Trade without CES: Estimating Translog Gravity," *Journal of International Economics*, 89(2), 271–282.

Pettengill, J. S. (1979): "Monopolistic Competition and Optimum Product Diversity: Comment," *American Economic Review*, 69(5), 957–960.

Pollak, R. A. (1971): "Additive Utility Functions and Linear Engel Curves," *Review of Economic Studies*, 38(4), 401–414.

Pollak, R. A., R. C. Sickles, and T. J. Wales (1984): "The CES-Translog: Specification and Estimation of a New Cost Function," *Review of Economics and Statistics*, 66(4), pp. 602–607.

Schmalensee, R. (1981): "Output and Welfare Implications of Monopolistic Third-Degree Price Discrimination," *American Economic Review*, 71(1), 242–247.

Seade, J. (1980): "On the Effects of Entry," *Econometrica*, 48(2), 479–489.

Shapiro, C. (1989): "Theories of Oligopoly Behavior," in R. Schmalensee and R. Willig (eds.): *Handbook of Industrial Organization*, Elsevier, pp. 329-414.

Vives, X. (1999): *Oligopoly Pricing: Old Ideas and New Tools*. Cambridge, Mass.: MIT Press.

Weyl, E. G., and M. Fabinger (2013): "Pass-Through as an Economic Tool: Principles of Incidence under Imperfect Competition," *Journal of Political Economy*, 121(3), 528–583.

Zhelobodko, E., S. Kokovin, M. Parenti, and J.-F. Thisse (2012): "Monopolistic Competition: Beyond the Constant Elasticity of Substitution," *Econometrica*, 80(6), 2765–2784.