

Networks in Conflict: Theory and Evidence from the Great War of Africa*

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Abstract

Many wars involve complicated webs of alliances and rivalries between multiple actors. Examples include the recent civil wars in Somalia, Uganda, and the Democratic Republic of Congo. We study from a theoretical and empirical perspective how the network of military alliances and rivalries affects the overall conflict intensity, destruction and death toll. The theoretical analysis combines insights from network theory and from the politico-economic theory of conflict. We construct a non-cooperative model of tactical fighting featuring two novel externalities: each group's strength is augmented by the fighting effort of its allied, and weakened by the fighting effort of its rivals. We achieve a closed form characterization of the Nash equilibrium of the fighting game, and of how the network structure affects individual and total fighting efforts. We then perform an empirical analysis using data for the Second Congo (DRC) War, a conflict involving many groups and a complex network of alliances and rivalries. We obtain structural estimates of the fighting externalities, and use them to infer the extent to which the removal of each group involved in the conflict would reduce the conflict intensity.

1 Introduction

Alliances and enmities among armed actors play a key role in warfare. In many instances, especially in civil conflicts, these are not even sanctioned by formal treaties or war declarations, but remain informal and loose relationships. The commands of allied forces are often decentralized and only engage in a limited extent of coordination. More generally, allied groups typically pursue separate goals and compete one with another for the same resource pool over which they fight common enemies. It is not rare that even open fights between belligerent groups that are supposed to be on the same side erupt.

An example of a loose alliance in the context of international wars is the alliance between the Soviet Union and the Anglo-Americans to bring down Nazi Germany during World War II. While sanctioned by international treaties, this was little more than a tactical alliance to defeat a common

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enemy. Well before the war was over, the Soviet Union and the Anglo-Americans were fighting strategically for conflicting objectives, each trying to secure the best political and military outcome at the end of the conflict. This goal informed their choice of military targets (e.g., the Red Army did not intervene in support of the Warsaw insurrection regarding it as a hostile attempt orchestrated by London to gain control over Poland; Stalin geared its military campaign to reach Berlin before the allied forces; etc.) and the investments of human and physical resources in the conflict. Similar considerations apply to earlier wars, from the Peloponnesian War in the ancient Greece, to the Napoleonic Wars (cf. for example Ke et al, 2013), or to the alliances between warlords in China after the proclamation of the Republic in 1912. The recent civil conflicts in Afghanistan, first at the time of the Soviet occupation, and after the NATO intervention in 2001, were instances in which informal alliances and enmities played an important role (see Bloch, 2012). The same is true for many civil wars in Africa, including Somalia, Uganda, Sudan or the Democratic Republic of Congo.

In this paper, we construct a theory of conflict focusing explicitly on the role of the network of alliances and enmities. The maintained assumption is that each group involved in the conflict maximizes its individual pay-off given by the share of resources controlled at the end of the conflict (the "prize") net of the resources sank in the battlefield. The equilibrium is determined as the standard Nash equilibrium of a non-cooperative game. The benchmark is a *contest success function*, henceforth CSF (see, e.g., Hirshleifer 1989; Skaperdas, 1992; Grossman and Kim, 1995). In a standard CSF, the share of the prize accruing to each group is determined by the relative amount of resources (*fighting effort*) that each of them commits to the conflict. In our model, the network of alliances and enmities introduces additional externalities. More precisely, the share of the prize accruing to each group hinges on its relative strength, that we label as *operational performance*. In turn, this is determined by its own fighting effort augmented by the fighting effort of all allied groups (weighted by an allied externality parameter) and diminished by the fighting effort of all enemy groups (weighted by an enemy externality parameter). Thus, when a group increases its fighting effort, it also affects, positively, the operational performance of its allied groups, and negatively that of its enemy groups.¹ The complex externality web affects the optimal fighting effort of all groups. Enemy relationships induce strategic complementarity in fighting efforts, whereas alliances induce strategic substitution.

We provide a full analytical solution for the Nash equilibrium of the game. Absent other sources of heterogeneity, the fighting effort of each agent is pinned down by its centrality in the network. Our novel centrality measure is related to the Bonacich centrality (see Ballester, Calvo-Armengol and Zenou, 2006). More precisely, it is approximately equal to the sum of the Bonacich centrality related to the network of hostilities, and the (negative-parameter) Bonacich centrality related to the network of alliances. The share of the prize accruing to each player has a particularly simple solution: it can be expressed as the ratio between a simple function of the first-degree links (i.e., the number of direct allied and enemies a group has) – a measure that we label *local hostility level* – and the sum of the total hostility levels of all agents involved in the conflict. Interestingly, the more allies a group has, the smaller the share of the prize it appropriates – although the same result need not apply to the set of allies taken together. In contrast, welfare (i.e., the share of the prize net of the fighting effort) is increasing in the number of allies and decreasing in the number of enemies. Intuitively, alliances suffer from a free riding problem. Consider, for instance, a conflict involving three groups, two of them being allied against the third group. When each member of the alliance exerts costly effort, part of its effects spills over to the allied group. Hence, the marginal benefit of fighting is smaller for each of the two allies than for the isolated group.

Together with determining the fighting effort and success of each individual player, the network

¹These spillovers compounds with those already present in ordinary CSFs, as discussed in more detail below.

of alliances and hostilities determines the size of the conflicts, or the total rent dissipation, which is the inverse measure of aggregate welfare. In general, the abundance of hostility links lead to more rent dissipation, while the opposite is true for alliances. This is well illustrated in regular graphs, in which the number of alliances and enmities is invariant across groups. The conflict escalation (and rent dissipation) is maximum when all groups are enemy of each other (as in Hobbes' *homo homini lupus*), whereas it is minimized in networks where all groups are in friendly terms (as in Rousseau's *well-order society*). The well-order society is a somewhat surprising outcome, as it results from a selfish behavior in a non-cooperative contest. The reason is that the marginal product of fighting effort becomes small when all agents are allied and the alliance externality is sufficiently small. Free riding is socially desirable, since war effort has no social value. This peaceful outcome can be viewed as a paradigmatic representation of societies in which the system of institutional check and balances reduces the incentive for opportunistic behavior.

In the second part of the paper, we perform an empirical analysis based on the structural equations of the model. We focus on the Second Congo War, which is also sometimes referred to as the "Great African War" and whose estimated death toll ranges between 3 and 5 million lives (Olsson and Fors, 2004; Autesserre, 2008). More details about the historical context of the conflict are provided in Section 3.1. This conflict involves a large number of groups, and a rich network of alliances and enmities. We use information from Armed Conflict Location Event Database (ACLED) and the Stockholm International Peace Research Institute (SIPRI) to identify the network of alliances and enmities. We assume that the network is constant over time (an assumption that conforms with the data, at least for the period we consider) and proxy fighting effort by the annual observations for the number of fighting events in which each group is involved. The estimation uses the panel of annual observations controlling for group fixed effects and time-varying observed heterogeneity. In particular, we focus on weather shocks, that have been shown in the previous literature to have important effects on fighting intensity (cf. e.g. Miguel et al. 2004, Vanden Eynde 2011, and Rogall 2013). Weather shocks are especially important in our analysis, as they provide the exogenous source of variation that allows us to achieve identification, following the methodology proposed by Bramoullé *et al.* 2009 (see also Liu *et al.* 2011). In particular, our identification exploits the exogenous variation in the average weather conditions facing, respectively, the set of allies and of enemies of each group. Without imposing any restriction on the estimation procedure, we find that the two estimated externalities have the signs predicted by theory.

After estimating the model, and in particular gauging the size of the network externalities, we can calculate how important each player is in the determining the total intensity of the conflicts. More formally, we perform a key player analysis, i.e., we ask how the total rent dissipation would change if each of the player were individually removed, and all other players were to readjust their fighting effort. We view this as a policy-relevant exercise, as it allows international organizations (e.g., the UN) to identify the actors whose decommissioning would be most effective to scale down conflict. Interestingly, while on average large groups are more crucial than small ones, the relationship is not one-to-one. The hypothetical removal of some relatively small players such as the Lord Resistance Army turns out to have large effects on the containment of the DRC conflict. Intuitively, the war activity of the LRA increases the military operations of its traditional enemy, the armed forces of Uganda. This, in turns, spills over to the activity of the DRC army and its allied, that are enemies of the Ugandan army for different reasons.

Our contribution is related to the various strands of the existing literature. First, our paper is linked to the growing literature on the economics of networks (cf. e.g. Jackson and Zenou 2014; Acemoglu and Ozdaglar 2011; Jackson 2008). There exist only very few papers in the literature studying strategic interaction of multiple agents in networks of conflict. Franke and Öztürk (2009) study agents being embedded in a network of bilateral conflicts, where agents can

choose their fighting efforts to attack their neighbors. However, differently to the current paper, they do not allow for alliances. Moreover, they can only characterize equilibrium efforts for very specific networks (regular, star-shaped, and complete bipartite graphs). In contrast, we provide an equilibrium characterization for any network structure. This allows us to apply our model to the data, and to perform a key player analysis. Hiller (2012) studies the formation of networks where agents can form alliances to coerce payoffs from enemies with fewer friends. However, the payoff structure is rather specific, and, more importantly, it does not allow for endogenous choice of fighting efforts in conflict. Moreover, none of the above papers is applied to data, and neither do they provide a structural estimation of the model's parameters which is necessary for our key player analysis. A notable exception is the recent paper by Acemoglu, Garcia-Jimeno and Robinson (2014) that estimates a structural model of political economy of public goods provision using a network of Colombian municipalities. However their object of inquiry is very different as they are interested in estimating the spillover effects of local state capacity across municipalities.

The problem of identifying key players in strategic games on networks is pioneered by Ballester et al. (2006). The authors determine equilibrium effort choices in a game of strategic complements between neighboring nodes, and identify key players, i.e. the agents whose removal reduces equilibrium aggregate effort the most. However, their payoff structure is substantially different from ours, and does not incorporate an environment in which agents are competing in a contest over common resources. More recently, the key player policy has also been tested empirically. Liu et al. (2011) test the key player policy for juvenile crime in the United States, while Lindquist and Zenou (2013) identify key players for co-offending networks in Sweden. Differently to these works we analyze key players in armed conflicts, and provide an application to the war in Congo.

Further, our study is also related to the growing politico-economic conflict literature.² Papers in this literature typically focus on settings with two large groups facing each other, and they do not consider network relationships. A few papers consider multiple groups comprising each a large number of players, and study collective action problems. Esteban and Ray (2001) show that the Olson paradox does not generally hold and that sometimes large groups can be more effective than small groups. To this purpose they build a model with n different groups composed each of a varying number of individual players. Individual players select costly fighting effort and the winning chances of each collective group depends on its total effort as a share of the sum of all group efforts in society. Their model is different and complementary to ours, since it does not consider general network structures, but focuses on a setting comprising n different cliques, where there are no links between cliques.³

There is also a small numbers of papers that study explicitly the role of alliances in settings with either three players, or identical players (cf. Konrad, 2009, 2011, and Bloch, 2012 for surveys). Some papers note, as we do, that alliances may be socially desirable since they reduce rent dissipation in wars (cf. Olson and Zeckhauser 1966, Wärneryd 1998, and Garfinkel 2004). In this context, a few papers regard the mere existence of alliances as a "puzzle" given the problem they generate (cf. Nitzan 1991, Skaperdas 1998, Esteban and Sakovics 2004, Sanchez-Pagés 2007, Konrad and Kovenock 2009). These papers emphasize that (i) there is a collective action problem within the alliance, leading to free-riding, and hence lower aggregate effort; (ii) after being successful there

²Recent work in this field links conflict to state capacity (Besley and Persson, 2011), trust and social capital (Rohner, Thoenig and Zilibotti, 2013, Acemoglu and Wolitzky, 2014), trade (Martin et al. 2008; Dal Bo and Dal Bo, 2011), political bias and institutions (Jackson and Morelli, 2007; Conconi, Sahuguet, and Zanardi, 2014), natural resource abundance and inequality (Caselli et al., 2013; Morelli and Rohner, 2014), and the effects of ethnic diversity (Esteban and Ray, 2008; Caselli and Coleman, 2013).

³Another conflict model allowing for n aggregate ethnic groups composed of a given number of individual players each is Rohner (2011), but his setting does not contain contest success functions.

may be a second stage with conflict within the victorious alliance, which further reduces incentives for fighting effort in the first stage of the game. To address the puzzle, Konrad and Kovenock (2009) argue that capacity constraints can explain the establishment of alliance, while Skaperdas (1998) argues that alliances can only form when the CSF has increasing returns characteristics. This literature focuses either on settings with only three players, or alternatively on frameworks with n identical and symmetric players. In contrast, our analysis focuses on complex networks where the different centrality of different players play a decisive role. We take the web of alliances as given and do not try to rationalize them.

Our paper is also embedded in the empirical literature on civil war (cf. e.g. Fearon and Laitin, 2003; Collier and Hoeffler, 2004), and in particular in the recent literature that studies conflict using very disaggregated micro-data on geo-localised fighting events, such as for example Dube and Vargas (2013), Cassar, Grosjean, and Whitt (2013), Michalopoulos and Papaioannou (2013), Rohner, Thoenig, and Zilibotti (2013b), and La Ferrara and Harari (2012).

The paper is organized as follows: Section 2 presents the theoretical model, the equilibrium and the welfare analysis; Section 3 discusses the application to the Second Congo War; Section 4 concludes. A technical appendix (not included in this version, and available upon request) contains the proofs of the Lemmas, Propositions and Corollaries.

2 The Model

2.1 Environment

The model economy is populated by a network of n agents (groups), $G \in \mathcal{G}^n$, where \mathcal{G}^n denotes the class of graphs on n nodes. Each pair of agents can be in one of three bilateral states: alliance, hostility, or neutrality. We represent the set of bilateral states by the matrix $\mathbf{A} = [a_{ij}]_{1 \leq i, j \leq n}$ where $a_{ij} \in \{-1, 0, 1\}$. More formally, \mathbf{A} is the signed adjacency matrix associated with the network G (cf. Zaslavsky 1982), where:

$$a_{ij} = \begin{cases} 1, & \text{if } i \text{ and } j \text{ are allied,} \\ -1, & \text{if } i \text{ and } j \text{ are enemies,} \\ 0, & \text{if } i \text{ and } j \text{ are in a neutral relationship.} \end{cases}$$

Note that a neutral relationship is modelled as the absence of links. If \mathbf{A} does not contain any zero entries, we say that we have complete signed network.

Let $a_{ij}^+ \equiv \max\{a_{ij}, 0\}$ and $a_{ij}^- \equiv -\min\{a_{ij}, 0\}$ denote the positive and negative parts of a_{ij} , respectively. Then, $a_{ij} = a_{ij}^+ - a_{ij}^-$, respectively, for all $1 \leq i, j \leq n$. Similarly, $\mathbf{A} = \mathbf{A}^+ + \mathbf{A}^-$ where $\mathbf{A}^+ = [a_{ij}^+]_{1 \leq i, j \leq n}$ and $\mathbf{A}^- = [a_{ij}^-]_{1 \leq i, j \leq n}$. We denote the corresponding subgraphs as G^+ and G^- , respectively, so that G can be written as the graph join $G = G^+ \oplus G^-$. Finally, we define by $d_i^+ \equiv \sum_{j=1}^n a_{ij}^+$ agent i 's number of alliances, and by $d_i^- \equiv \sum_{j=1}^n a_{ij}^-$ his number of enmities.

The n agents compete for a prize whose total value is denoted by $V > 0$. We assume agents' payoffs to be determined by a CSF. The CSF maps the relative fighting intensity each agent devotes to a conflict into the share of the prize he appropriates after the conflict. More formally, we postulate a payoff function $\pi_i : \mathcal{G}^n \times \mathbb{R}_+^n \rightarrow \mathbb{R}$ such that

$$\pi_i(G, \mathbf{x}) = \frac{\varphi_i(G, \mathbf{x})}{\sum_{j=1}^n \varphi_j(G, \mathbf{x})} V - x_i, \quad (1)$$

where \mathcal{G}^n is the class of graphs on n node, $\mathbf{x} \in \mathbb{R}_+^n$ is a vector describing the fighting effort of each player and φ_i is agent i 's *operational performance* (henceforth, OP) The latter is assumed to depend on agent i 's own fighting effort, as well as on his allied's and enemies's efforts. More formally, we assume that

$$\varphi_i(G, \mathbf{x}) \equiv x_i + \beta \sum_{j=1}^n a_{ij}^+ x_j - \gamma \sum_{j=1}^n a_{ij}^- x_j, \quad \beta, \gamma \in [0, 1]. \quad (2)$$

Note that the specification of Equation (2) assumes no heterogeneity across agents other than their position (i.e., the number of allies and enemies) in the network. We will introduce heterogeneity in Section 2.5 below.

Equation (2) is our main theoretical assumption. It postulates that each agent's OP increases in the total effort exerted by the allied and decreases in the total effort exerted by the enemies. These externalities compound with those embedded in the pay-off of the standard CSF, which equation (2) nests as the particular case in which $a_{ij}^+ = a_{ij}^- = 0$ for all i and j . In this case, $\pi_i = \left(x_i / \sum_{j=1}^n x_j\right) V - x_i$, and each agent's effort imposes a negative externality on all other agents in the contest only by increasing the denominator of the CSF. Consider, next, a case in which $a_{ij}^+ > 0$ and $\beta > 0$, for one and only one pair (i, j) (while $a_{ij}^- = 0$ for all i and j). Then, $\pi_i = \left((x_i + \beta x_j) / \sum_{j=1}^n x_j\right) V - x_i$. In this case, an increase in agent j 's effort entails both the standard negative externality through the denominator, and, in addition, the positive externality captured by an increase in the numerator. Thus, holding efforts constant, a newly established alliance between i and j increases the share of V accruing jointly to i and j , at the expenses of the remaining groups. To, the opposite hostility links strengthen the negative externality of the standard CSF. For instance, suppose that $n = 3$ and that all agents exert the same fighting effort, $x_1 = x_2 = x_3 = x$. Then, the standard CSF prescribes an equal division of the pie. However, if agents 1 and 2 are enemies, while agent 3 is in a neutral relationship with both, then agents 1 and 2 earn a smaller share of the pie each, while agent 3 earn a larger share of it. For instance, if $\gamma = 1/2$, agents 1 and 2 receive a quarter of the pie each, while agent 3 appropriates half of it.

2.2 Equilibrium Fighting Effort

In this section, we endogenize the fighting effort choice, and characterize the Nash equilibrium of the contest. More formally, each agent chooses effort (x_i) non-cooperatively in order to maximize $\pi_i(G, \mathbf{x})$, given all other agents' efforts, \mathbf{x}_{-i} . We impose no restrict to the effort choice, and allow, in principle, negative effort choices.⁴ The First Order Conditions yield, for all $i = 1, 2, \dots, n$:

$$\frac{\partial \pi_i(G, \mathbf{x})}{\partial x_i} = 0 \iff \varphi_i = \frac{1}{1 + \beta d_i^+ - \gamma d_i^-} \left(1 - \frac{1}{V} \sum_{j=1}^n \varphi_j\right) \sum_{j=1}^n \varphi_j,$$

where we assume that, for all i , $\beta d_i^+ - \gamma d_i^- > -1$.⁵ Solving the system of n equations yields the equilibrium OP levels:

$$\varphi_i^*(G) = \Lambda^{\beta, \gamma}(G) \left(1 - \Lambda^{\beta, \gamma}(G)\right) \Gamma_i^{\beta, \gamma}(G) \times V, \quad (3)$$

⁴Formally, the zero effort level is purely a matter of normalization. One could as well rewrite the model by replace x_i with $(\bar{x} - x_i)$, where $(\bar{x} - x_i)$ denotes the effort level. All results would be unchanged. More importantly, we rule out a participation decisions. Agents have no option to stay out of the conflict (e.g., because they would be subject to expropriation of their endowments).

⁵This condition is necessary and sufficient for the Second Order Conditions to hold for all players.

where

$$\Gamma_i^{\beta,\gamma}(G) \equiv \frac{1}{1 + \beta d_i^+ - \gamma d_i^-} \text{ and } \Lambda^{\beta,\gamma}(G) \equiv 1 - \frac{1}{\sum_{i=1}^n \Gamma_i^{\beta,\gamma}(G)}. \quad (4)$$

Summing over i yields the total economy-wide OP level,

$$\sum_{i=1}^n \varphi_i^* = \Lambda^{\beta,\gamma}(G) \times V, \quad (5)$$

which in turn implies that

$$\frac{\varphi_i^*}{\sum_{j=1}^n \varphi_j^*} = \frac{\Gamma_i^{\beta,\gamma}(G)}{\sum_{j=1}^n \Gamma_j^{\beta,\gamma}(G)} \quad (6)$$

$\Gamma_i^{\beta,\gamma}$ is a measure of *local hostility level* capturing the externalities associated with agent i 's first-degree alliance and hostility links. $\Lambda^{\beta,\gamma}$ is a measure of *total OP* in the network. Both $\Gamma_i^{\beta,\gamma}(G)$ and $\Lambda^{\beta,\gamma}(G)$ are decreasing with β , and increasing with γ . Equation (5) implies that the total OP is decreasing in the number of bilateral alliances and in β , and increasing in the number of hostility links and in γ . Moreover, equations (1) and (6) show that the share of the prize accruing to each agent in equilibrium increases in the number of direct alliances and decreases in the number of enmities.

Next, we characterize agents' equilibrium fighting efforts, and how these depend on the structure of the network. The following Proposition provides a complete characterization of the Nash equilibrium:

Proposition 1 *Assume that $\beta + \gamma < 1/\max\{\lambda_{\max}(G^+), d_{\max}^-\}$ and $\beta\lambda_{\max}(\mathbf{A}^+) < 1 - \gamma\lambda_{\max}(\mathbf{A}^-)$, where $\lambda_{\max}(\mathbf{A}^\pm)$ denotes the largest eigenvalue associated with the matrix \mathbf{A}^\pm . Let $\Gamma_i^{\beta,\gamma}(G)$ and $\Lambda^{\beta,\gamma}(G)$ be defined as in (4), and let*

$$\mathbf{c}^{\beta,\gamma}(G) \equiv (\mathbf{I}_n + \beta\mathbf{A}^+ - \gamma\mathbf{A}^-)^{-1} \mathbf{\Gamma}^{\beta,\gamma}(G) \quad (7)$$

be a centrality vector, whose generic element $c_i^{\beta,\gamma}(G)$ describes the centrality of agent i in the network. Then, there exists a unique Nash equilibrium with effort levels given by

$$x_i^*(G) = \Lambda^{\beta,\gamma}(G) \left(1 - \Lambda^{\beta,\gamma}(G)\right) c_i^{\beta,\gamma}(G) \times V, \quad (8)$$

for all $i = 1, \dots, n$. Moreover, the equilibrium OP levels are given by (5), and the payoffs are given by

$$\pi_i^*(G) = \pi_i(\mathbf{x}^*, G) = V(1 - \Lambda^{\beta,\gamma}(G)) \left(\Gamma_i^{\beta,\gamma}(G) - \Lambda^{\beta,\gamma}(G) c_i^{\beta,\gamma}(G)\right). \quad (9)$$

The centrality measure, $c_i^{\beta,\gamma}(G)$, plays a key role in Proposition 1. Note, in particular, that the relative fighting efforts of any two agents only depends on their relative centrality in the network:

$$\frac{x_i^*(G)}{x_j^*(G)} = \frac{c_i^{\beta,\gamma}(G)}{c_j^{\beta,\gamma}(G)}.$$

While $c_i^{\beta,\gamma}(G)$ depends in general on the links of all degrees, it is enlightening to focus on the case in which the spillover parameters β and γ are small. In this case, our centrality measure can be approximated by the the sum of (i) the Bonacich centrality related to the network of hostilities,

G^- , (ii) the (negative-parameter) Bonacich centrality related to the network of alliances, G^+ , and (iii) the local hostility vector, $\Gamma^{\beta,\gamma}(G)$ (cf. Ballester et al., 2006; Liu et al., 2011).⁶

Corollary 1 *As $\beta \rightarrow 0$ and $\gamma \rightarrow 0$, the centrality measure defined in Equation (7) can be written as*

$$\mathbf{c}^{\beta,\gamma}(G) = \mathbf{b}_{\Gamma^{\beta,\gamma}(G)}(\gamma, G^-) + \mathbf{b}_{\Gamma^{\beta,\gamma}(G)}(-\beta, G^+) + \Gamma^{\beta,\gamma}(G) + O(\beta\gamma),$$

where $O(\beta\gamma)$ involves second and higher order terms, and the (μ -weighted) Bonacich centrality with parameter α is defined as

$$\mathbf{b}_\mu(\alpha, G) \equiv \sum_{k=1}^{\infty} \alpha^k \mathbf{A}^k \mu, \quad (10)$$

as long as the invertibility condition $|\alpha| < 1/\lambda_{\max}(G)$ is satisfied.

Corollary 1 states that the centrality $\mathbf{c}^{\beta,\gamma}(G)$ can be expressed as a linear combination of the weighted Bonacich centralities $\mathbf{b}_{\Gamma^{\beta,\gamma}(G)}(\gamma, G^-)$ and $\mathbf{b}_{\Gamma^{\beta,\gamma}(G)}(-\beta, G^+)$ plus the vector $\Gamma^{\beta,\gamma}(G)$. Each Bonacich centrality gauges the network multiplier effect attached to the system of hostilities and alliances, respectively. In particular, $\mathbf{b}_{\Gamma^{\beta,\gamma}(G)}(\gamma, G^-)$ captures how a group i is influenced by all its (direct and indirect) enemies.⁷ In the case of small γ , $\mathbf{b}_{\Gamma^{\beta,\gamma}(G)}(\gamma, G^-)$ can be itself approximated as follows (ignoring terms of degree three or higher):

$$b_{\Gamma^{\beta,\gamma}(G),i}(\gamma, G^-) = \Gamma_i^{\beta,\gamma}(G) + \gamma \sum_{j=1}^n a_{ij}^- \Gamma_j^{\beta,\gamma}(G) + \gamma^2 \sum_{j=1}^n a_{ij}^- \sum_{k=1}^n a_{jk}^- \Gamma_k^{\beta,\gamma}(G) + O(\gamma^3).$$

Similarly,

$$b_{\Gamma^{\beta,\gamma}(G),i}(-\beta, G^+) = \Gamma_i^{\beta,\gamma}(G) + (-\beta) \sum_{j=1}^n a_{ij}^+ \Gamma_j^{\beta,\gamma}(G) + (-\beta)^2 \sum_{j=1}^n a_{ij}^+ \sum_{k=1}^n a_{jk}^+ \Gamma_k^{\beta,\gamma}(G) + O(\beta^3).$$

Thus, Corollary 1 suggests that, when higher degree terms can be neglected, our centrality measure is increasing in γ and in the number of degree-one and degree-two enmities, whereas it is decreasing in β and in the number of degree-one alliances. Degree-two alliances have instead a positive effect on the centrality measure.⁸

In the case of weak network externalities (i.e., $\beta \rightarrow 0$ and $\gamma \rightarrow 0$, as above) we can as well obtain simple approximate expression for the equilibrium efforts and the payoffs in Proposition 1.⁹

⁶See Appendix B for a more detailed discussion of the Bonacich centrality. A discussion of the Bonacich centrality with a negative parameter can be found in Bonacich (2007).

⁷The Bonacich centrality measure related to the network of hostilities, $b_{\Gamma^{\beta,\gamma}(G),i}(\gamma, G^-)$, measures as the local hostility levels along all walks reaching i using only hostility connections, where walks of length k are weighted by the geometrically decaying hostility externality γ^k . Due to the approximation, we only consider links up to degree two.

⁸The intuition for this last, perhaps surprising, property is related to the relationship of complementarity and substitution among fighting efforts, as will become clear below. To anticipate the argument: If i is allied with j and j is allied with k , an increase in the fighting effort of k reduces the fighting effort of j and this, in turn, increases the fighting effort of i . Consider, instead, the case in which i is enemy to j and j is enemy to k . Then, an increase in the fighting effort of k increases the fighting effort of j and this, in turn, increases the fighting effort of i .

⁹It is also useful to note that, when $\beta = \gamma = 0$, then $\Lambda^{\beta,\gamma} = 1 - \frac{1}{n}$, and $c_i^{\beta,\gamma} = \Gamma_i^{\beta,\gamma} = 1$. Then, the equilibrium expressions in Proposition 1 simplify to $x_i^* = V(n-1)/n^2$ and $\pi_i^* = V/n^2$ which are the standard solutions in the Tullock CSF.

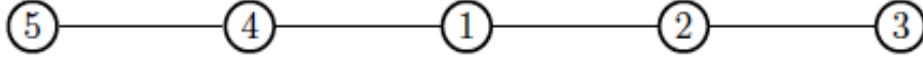


Figure 1: The figure shows an example of a line graph L_5 with five agents.

Corollary 2 *As $\beta \rightarrow 0$ and $\gamma \rightarrow 0$, the equilibrium effort and payoff of agent i in network G can be written as*

$$\begin{aligned} x_i^*(G) &= (X_o(G, \beta, \gamma, n) + X_1(G, \beta, \gamma, n) (d_i^- \gamma - d_i^+ \beta)) \times V + O(\beta\gamma) \\ \pi_i^*(G) &= (\Pi_o(G, \beta, \gamma, n) + \Pi_1(G, \beta, \gamma, n) (d_i^+ \beta - d_i^- \gamma)) \times V + O(\beta\gamma) \end{aligned}$$

where X_0, X_1, Π_0, Π_1 are unimportant positive constants (see the proof of Corollary 2 in Appendix A).

Corollary 2 shows that, when network externalities are small, an agent's fighting effort increases in the weighted difference between the number of enmities (weighted by γ) and of alliances (weighted by β). The opposite is true for the equilibrium pay-off, that is increasing in $d_i^+ \beta - d_i^- \gamma$. Thus, *ceteris paribus*, an increase in the spillover from alliances (enmities), parameterized by β (γ), as well as an increase in the number of allied (enemies) decreases (increases) agent i 's fighting effort and increases (reduces) its pay-off. Intuitively, an agent with many enemies tends to fight harder and to appropriate a smaller share of the prize, whereas an agent with many friends tends to fight less and to appropriate a large size of the pie. One must remember that this simple result may be reversed if higher-degree links have sizable effects.

2.3 Example: a line graph

In this section, we consider the illustrative example of a line graph L_5 with five agents, as depicted in Figure 1. This example highlights the role of the centrality of different players.

Suppose, first, that $\gamma > 0$, and all links in Figure 1 are enmities. If two agents are not linked, they are in neutral terms. Thus, agent 1 is an enemy of agents 2 and 4, while agent 3 is an enemy of agent 2 and agent 5 is an enemy of agent 4. Because of pair-wise symmetry, equilibrium efforts and payoffs for agents 5 and 3 as well as for 4 and 2 are identical. The ranking of the effort level, for any γ , features:¹⁰

$$x_1^* > x_2^* = x_4^* > x_3^* = x_5^*.$$

Agent 1 always exerts a higher effort than agent 2, because agent 1 is more central in the network and thus experiences a higher network multiplier effect. This consistent with the result that effort levels are proportional to the Bonacich centrality (cf. Corollary 1). Conversely, the equilibrium

¹⁰The analytical expressions yield:

$$\begin{aligned} x_1^* &= V \frac{2(\gamma(\gamma+2)-2)(\gamma(3\gamma^2+\gamma-1)-1)}{(5-7\gamma)^2(3\gamma^2-1)} \\ x_2^* = x_4^* &= V \frac{2((\gamma(3\gamma+5)-9)\gamma^2+2)}{(5-7\gamma)^2(1-3\gamma^2)} \\ x_3^* = x_5^* &= V \frac{2(\gamma(\gamma+2)-2)((\gamma-1)\gamma(3\gamma+1)+1)}{(5-7\gamma)^2(3\gamma^2-1)} \end{aligned}$$

payoff of agent 1 is smaller than the payoff of agent 2. Moreover, equilibrium effort of agent 3 is lowest, and the payoff is highest as this agent is the least central in the network and thus experiences the least network multiplier effect from conflicts. The upper left and upper right panel of Figure 2 show, respectively, effort levels and pay-off for varying values of γ . Fighting effort and payoffs are increasing and decreasing in γ , respectively.

Consider, next the polar opposite case in which $\beta > 0$, and all links in Figure 1 are alliances. If two agents are not linked, they are in neutral terms, as before. The following equilibrium fighting effort ranking results:¹¹

$$x_2^* = x_4^* < x_1^* < x_3^* = x_5^*.$$

The lower left and right panels of Figure 2 show, respectively, effort levels and payoffs for different values of β . We observe that, for a low range of β 's, the least central agents, 3 and 5, exert the highest fighting effort and earn the lowest payoff. However, for higher β 's, agents 3 and 5 earn a higher pay-off than agent 1. It is interesting to note, in addition, that it is agents 2 and 4 who exert lowest effort, in spite of agent 1 being the most central player. The reason is that agents 2 and 4 are connected, respectively, to agents 3 and 5 who lie at the periphery of the network, and have no other connections. Thus, agents 3 and 5 exercise very high effort, and this is exploited by agents 2 and 4, who can reduce their fighting effort and earn a higher payoff.¹² Finally, the non-monotonicity of efforts and payoffs for agents 1 and 3 (and 5) is related to the fact that, when β is high, agent 2 (and 4) reduces a lot its fighting effort, inducing substitution (i.e., higher effort) from the neighbor. Interestingly, for very high β , this effect is stronger for agent 1 than for agent 3, so the most central agent ends up earning the lowest pay-off among all agents in the contest.

Note that in both cases (upper and lower panels in Figure 2) the results are consistent with Corollary 2, which requires externalities to be small. In the upper panel, as $\gamma \rightarrow 0$ the effort is higher and pay-off is lower for the agents who have two enemies (i.e., agents 1, 2 and 4) than for the agent in the periphery (agents 3 and 5) who have only one enemy. In this case, this is true for any γ . In the lower panel, as $\beta \rightarrow 0$ the effort is lower and pay-off is higher for the agents who have two allies (i.e., agents 1, 2 and 4) than for the agents with only one ally (agents 3 and 5). In this case, however, the result changes as one takes larger β 's, as discussed above.

2.4 Welfare Analysis

In this section, we discuss welfare and policy implications of the theory. Our (negative) welfare measure is the extent of *rent dissipation*, which is equal to the total equilibrium fighting effort as a share of the prize of the context. This is given by:

$$\text{RD}^{\beta,\gamma}(G) \equiv \frac{1}{V} \sum_{i=1}^n x_i^*(G) = \Lambda^{\beta,\gamma}(G)(1 - \Lambda^{\beta,\gamma}(G)) \sum_{i=1}^n c_i^{\beta,\gamma}(G).$$

¹¹The analytical expressions are:

$$\begin{aligned} x_1^* &= V \frac{2(\beta^2(\beta+1)(\beta(3\beta-10)+5)-2)}{(7\beta+5)^2(3\beta^2-1)} \\ x_2^* &= V \frac{2(\beta^2(\beta(5-3\beta)+9)-2)}{(7\beta+5)^2(3\beta^2-1)} \\ x_3^* &= V \frac{2((\beta-2)\beta-2)(\beta(\beta+1)(3\beta-1)-1)}{(7\beta+5)^2(1-3\beta^2)}. \end{aligned}$$

¹²This is related to the interpretation of the Bonacich centrality with a negative parameter. In this case a node is more powerful to the degree its connections themselves have few alternative connections (see Sec. 1.1.1 in Bonacich, 2007).

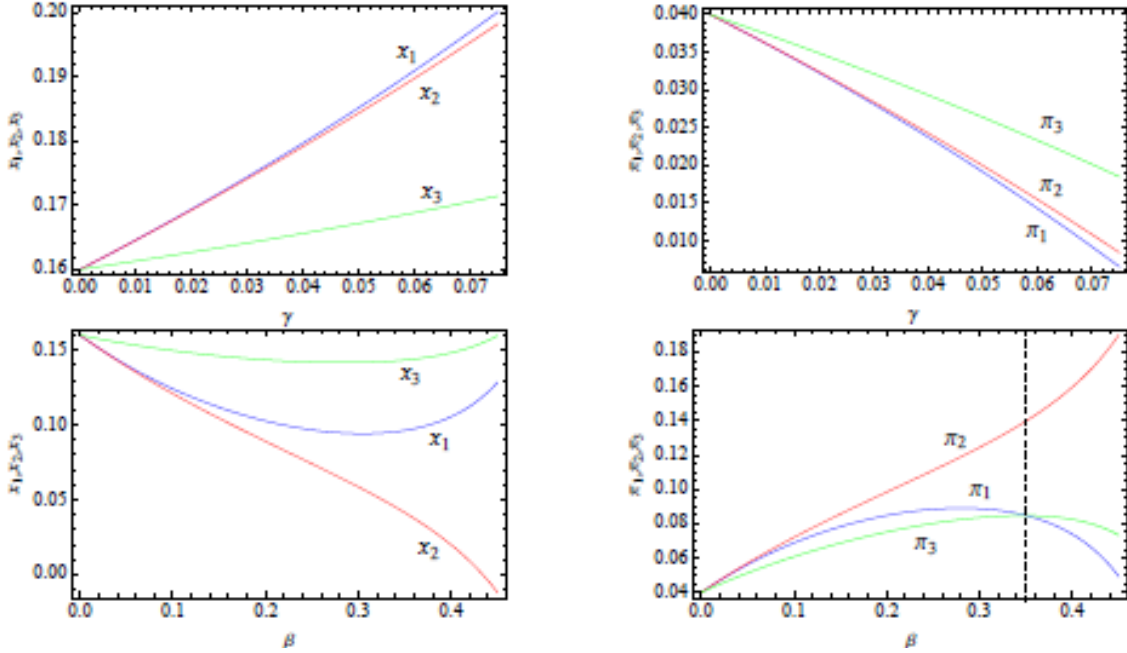


Figure 2: The figure shows the equilibrium efforts (left panels) and payoffs (right panels) as functions of γ and β for two line graphs in which there are only hostile relationships (upper panels) and only alliances (lower panels), respectively.

Since the aggregate welfare is $\sum_{i=1}^n \pi_i^*(G) = V(1 - \text{RD}^{\beta, \gamma}(G))$, then, minimizing rent dissipation is equivalent to maximizing welfare.

2.4.1 Efficient Networks

We start by analyzing how the network structure affects rent dissipation. Intuitively, one might expect that abundant alliances tend to reduce rent dissipation, by decreasing the marginal return of individual fighting effort, the opposite being true for enmities. While this becomes complicated in general networks, a simple proof can be provided for the particular case of regular graphs, i.e., networks in which every agent i has $d_i^+ = k^+$ alliances and $d_i^- = k^-$ enmities. An illustration is shown in Figure 3.

We denote a regular graph by G_{k^+, k^-} . The Nash equilibrium is in this case symmetric: all agents exercise the same effort, and $\varphi_i^* = \varphi^* = 1/n$, implying an equal division of the pie. Under the conditions of Proposition 1,¹³ the equilibrium effort and payoff are given by, respectively,

$$x_i^*(G_{k^+, k^-}) = x^*(k^+, k^-) = \left(\frac{1}{1 + \beta k^+ - \gamma k^-} - \frac{1}{n} \right) \times \frac{V}{n}, \quad (11)$$

$$\pi_i^*(G_{k^+, k^-}) = \pi^*(k^+, k^-) = \frac{1 + (1+n)(\beta k^+ - \gamma k^-)}{n(1 + \beta k^+ - \gamma k^-)} \times \frac{V}{n}. \quad (12)$$

¹³We require that $\beta + \gamma < 1/\max\{\lambda_{\max}(G^+), d_{\max}^-\}$ and $\beta \lambda_{\max}(\mathbf{A}^+) < 1 - \gamma \lambda_{\max}(\mathbf{A}^-)$ with $\lambda_{\max}(\mathbf{A}^+) = k^+$, $\lambda_{\max}(\mathbf{A}^-) = d_{\max}^- = k^-$.

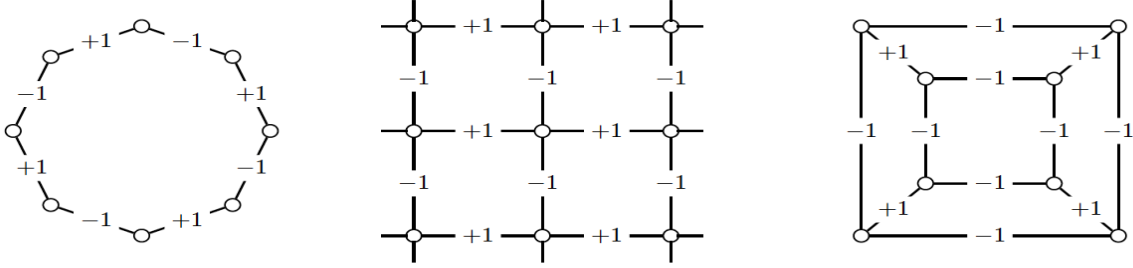


Figure 3: The figure shows three examples of regular graphs G_{k^+,k^-} : The left panel shows a regular graph with $k^+ = k^- = 1$, the middle panel shows a regular graph with $k^+ = k^- = 2$ (assuming periodic boundary conditions, i.e. a torus), and the right panel a Cayley graph with $k^+ = 1$ and $k^- = 2$.

Standard differentiation implies that

$$\frac{\partial x^*(k^+, k^-)}{\partial k^+} < 0, \quad \frac{\partial x^*(k^+, k^-)}{\partial k^-} > 0, \quad \frac{\partial \pi^*(k^+, k^-)}{\partial k^+} > 0, \quad \frac{\partial \pi^*(k^+, k^-)}{\partial k^-} < 0.$$

Intuitively, fighting effort and rent dissipation increase (decrease) in the number of enmities (alliances). The regular graph nests three interesting particular cases. First, if $\beta = \gamma = 0$, we obtain the standard equilibrium of the Tullock game, with $\text{RD}^{0,0}(G_{k^+,k^-}) = (n-1)/n$. Second, consider a complete network of alliances ($k^+ = n-1$), where, in addition, $\beta \rightarrow 1$. Then, $x^* \rightarrow 0$ and $\text{RD}^{1,\gamma}(G_{n-1,0}) \rightarrow 0$, i.e., there is no rent dissipation. Namely, the society attains peacefully the equal split of the surplus, as in Rousseau's harmonious society. The lack of conflict does not stem here from social preferences or cooperation, but from the equilibrium outcome of a non-cooperative game between selfish individuals. The crux is the strong fighting externality across allied agents, which takes the marginal product of individual fighting effort down to zero. Third, consider, conversely, a society in which all relationships are hostile, i.e., $k^- = n-1$. Then, $\text{RD}^{\beta,\gamma}(G_{0,n-1}) \rightarrow 1$ as $\gamma \rightarrow 1/(n-1)^2$: all rents are dissipated through fierce fighting, as in Hobbes pre-contractual society where *homo homini lupus est*.

Figure 4 shows how the equilibrium effort and payoffs change as a function of k^+ and β assuming no enmities (or $\gamma = 0$). Higher β and higher k^+ induce lower effort and higher payoffs. The reason is that the substitutability effect of allied agents increases with both β (strength of the alliances) and k (number of alliances), and this allows agents to reduce their fighting efforts, while increasing their payoffs as they can decrease their costs of fighting. The opposite is true for k^- and γ . Consider, next, general networks. While it is difficult to characterize in general which graph minimizes rent dissipation, we can provide an upper bound to the rent dissipation associated with the efficient graph. The bound follows from the observation that the efficient graph cannot induce more rent dissipation than the complete graph of alliances, $\text{RD}^{\beta,\gamma}(G_{n-1,0})$. This result is summarized by the following Proposition.

Proposition 2 *Let the efficient graph $G^* \in \mathcal{G}_n$ be the graph that minimizes the rent dissipation. Then we have that*

$$0 \leq \text{RD}^{\beta,\gamma}(G^*) \leq \text{RD}^{\beta,\gamma}(G_{n-1,0}) = \frac{1}{1 + \beta(n-1)} - \frac{1}{n},$$

where $G_{n-1,0}$ is the complete graph where all agents are allies. Consequently, when either $\beta \rightarrow 1$ or $n \rightarrow \infty$, then, $\text{RD}^{\beta,\gamma}(G^*) \rightarrow 0$, and the efficient graph is $G^* = G_{n-1,0}$.

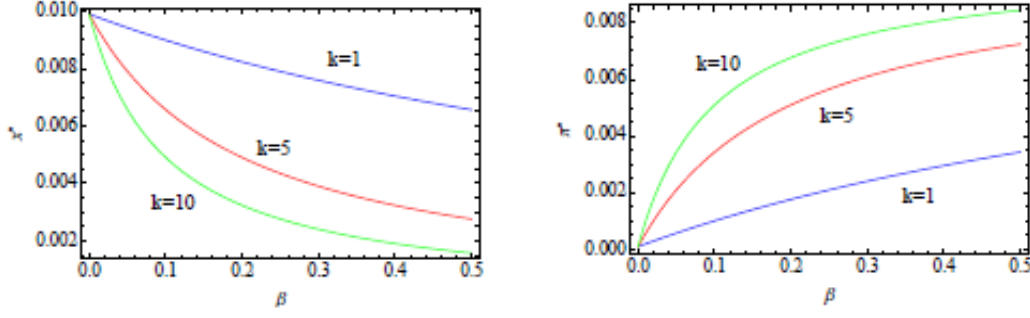


Figure 4: The figure shows the equilibrium efforts (left panel) and pay-off (right panel) in the case in which there are no enmities. Parameters: $k^+ \in \{1, 5, 10\}$, $V = 1$ and $n = 100$. Equilibrium efforts are decreasing in β and k .

2.4.2 The Key Player

The *key player* is defined as the agent whose removal triggers the largest reduction in rent dissipation Ballester et al. (2006) Identifying the key player is important to determine which policy intervention can reduce fighting activity.

Definition 1 Let G^{-i} be the network obtained from G by removing agent i , and assume that the conditions of Proposition 1 hold. Then the key player $i^* \in \bar{\mathcal{N}} = \{1, \dots, n\} \cup \emptyset$ is defined by

$$i^* = \arg \max_{i \in \bar{\mathcal{N}}} \left\{ \text{RD}^{\beta, \gamma}(G) - \text{RD}^{\beta, \gamma}(G^{-i}) \right\}, \quad (13)$$

where $\text{RD}^{\beta, \gamma}(G) \equiv \sum_{i=1}^n x_i^*(G) = V \Lambda^{\beta, \gamma}(G) (1 - \Lambda^{\beta, \gamma}(G)) \sum_{i=1}^n c_i^{\beta, \gamma}(G)$, and $x_i^*(G)$ is the generic element of the vector $\mathbf{x}^*(G)$ defined in Equation (8).

Note that the welfare difference $\text{RD}^{\beta, \gamma}(G) - \text{RD}^{\beta, \gamma}(G^{-i})$ can be interpreted as the maximum cost a benevolent policy maker would be willing to pay to induce or force agent i not to participate in the contest. Note also that the key player in Definition 1 might be empty if none of the agents can be removed so that the rent dissipation is reduced.

The identity of the key player is related to the centrality measure defined in Equation (7). Consider, for instance, the case in which all agents have the same technology. Then, the definition of the key player simplifies to (see also Proposition 4 in Appendix A)

$$\begin{aligned} i^* = & \arg \max_{i \in \bar{\mathcal{N}}} \left\{ \sum_{j=1}^n c_j^{\beta, \gamma}(G) + \sum_{j \neq i} \frac{h_i^{\beta, \gamma}(G) (1 - (1 - \Lambda^{\beta, \gamma}(G)) h_i^{\beta, \gamma}(G))}{1 - \Lambda^{\beta, \gamma}(G)} \right. \\ & \times \sum_{k=1}^n \left[\left(m_{jk}^{\beta, \gamma}(G) - \frac{m_{ij}^{\beta, \gamma}(G) m_{ik}^{\beta, \gamma}(G)}{m_{ii}^{\beta, \gamma}(G)} \right) \Gamma_k^{\beta, \gamma}(G) \left(\frac{1 + \beta d_k^+ - \gamma d_k^-}{1 + \beta (d_k^+ - 1) - \gamma d_k^-} 1_{\{k \in \mathcal{N}_i^+\}} \right. \right. \\ & \left. \left. + \frac{1 + \beta d_k^+ - \gamma d_k^-}{1 + \beta d_k^+ - \gamma (d_k^- - 1)} 1_{\{k \in \mathcal{N}_i^-\}} + 1_{\{k \notin (\mathcal{N}_i^+ \cup \mathcal{N}_i^-\)}} \right) \right] \left. \right\}, \quad (14) \end{aligned}$$

where $\bar{\mathcal{N}} \equiv \{1, \dots, n\} \cup \emptyset$; $1_{\{k \in \mathcal{N}_i^+\}}$ and $1_{\{k \in \mathcal{N}_i^-\}}$ are indicator variables taking the unit value if, respectively, $k \in \mathcal{N}_i^+$ and $k \in \mathcal{N}_i^-$, and zero otherwise, we have defined by

$$h_i^{\beta, \gamma}(G) = \left(1 - \beta \sum_{j \in \mathcal{N}_i^+} \frac{\Gamma_j^{\beta, \gamma}(G) (1 - \Lambda^{\beta, \gamma}(G))}{1 + \beta (d_j^+ - 1) - \gamma d_j^-} - \gamma \sum_{j \in \mathcal{N}_i^-} \frac{\Gamma_j^{\beta, \gamma}(G) (1 - \Lambda^{\beta, \gamma}(G))}{1 + \beta d_j^+ - \gamma (d_j^- - 1)} \right)^{-1},$$

and $m_{ij}^{\beta, \gamma}(G)$ is the ij -th element of the matrix $\mathbf{M}^{\beta, \gamma}(G) = (\mathbf{I}_n + \beta \mathbf{A}^+ - \gamma \mathbf{A}^-)^{-1}$. It is important to note that the key player identified in Equation (14) differs significantly from the one introduced in Ballester et al. (2006), where the key player is defined as $i^* = \arg \max_{i \in \mathcal{N}} \frac{b_{\mathbf{u}, i}(G, \alpha)}{N_i(G, \alpha)}$ with $b_{\mathbf{u}, i}(G, \alpha)$ being the Bonacich centrality of agent i in G (see also Equation (10) and Appendix B), and $N_i(G, \alpha)$ counting the number of closed walks starting and ending at i where the length of the walks is discounted by powers of α . Our key player formula is more involved due to the non-linearity inherent in the contest success function in the agents' payoffs.

2.5 Heterogenous Fighting Technologies

So far, we have maintained that all agents have access to the same fighting technology to turn effort into fighting intensity. This has allowed us to focus sharply on the network as the only source of heterogeneity. In reality, military groups typically differ in size, wealth, access to weapons, etc. In this section we generalize our model by allowing heterogeneous fighting technologies. We study both additive and multiplicative effects

Suppose, first, that agent i 's fighting strength can be written as

$$\varphi_i = \tilde{\varphi}_i + x_i + \beta \sum_{j=1}^n a_{ij}^+ x_j - \gamma \sum_{j=1}^n a_{ij}^- x_j. \quad (15)$$

where $\tilde{\varphi}_i$ is an additive shock affecting group i 's OP (e.g., its military capability). The payoff function in Equation (1) can then be written as follows:

$$\pi_i(G, \mathbf{x}) = V \frac{\varphi_i}{\sum_{j=1}^n \varphi_j} - x_i = V \frac{x_i + \beta \sum_{j=1}^n a_{ij}^+ x_j - \gamma \sum_{j=1}^n a_{ij}^- x_j + \tilde{\varphi}_i}{\sum_{j=1}^n \left(x_j + \beta \sum_{k=1}^n a_{jk}^+ x_k - \gamma \sum_{k=1}^n a_{jk}^- x_k + \tilde{\varphi}_j \right)} - x_i. \quad (16)$$

One can show that the equilibrium OP is unchanged, and continues to be given by Equation (3). Likewise, Equation (6) continues to characterize the share of the prize appropriated by each agent. Somewhat surprising, $\frac{\varphi_i^*}{\sum_{j=1}^n \varphi_j^*}$ is independent of $\tilde{\varphi}_i$. In contrast, $\tilde{\varphi}_i$ affects the equilibrium effort exerted by each agent. In particular, the equilibrium fighting effort vector is now given by (see Proposition 5 in Appendix A)

$$\mathbf{x}^* = (\mathbf{I}_n + \beta \mathbf{A}^+ - \gamma \mathbf{A}^-)^{-1} (V \Lambda^{\beta, \gamma}(G) (1 - \Lambda^{\beta, \gamma}(G)) \mathbf{\Gamma}^{\beta, \gamma}(G) - \tilde{\boldsymbol{\varphi}}), \quad (17)$$

under the same assumptions and definitions of $\Lambda^{\beta, \gamma}$ and $\mathbf{\Gamma}^{\beta, \gamma}(G)$ given above, and under the additional assumption that $V \Lambda^{\beta, \gamma}(G) (1 - \Lambda^{\beta, \gamma}(G)) \mathbf{\Gamma}^{\beta, \gamma}(G) > \tilde{\boldsymbol{\varphi}}$. This extension is important, as it will allow us to introduce observable and unobservable sources of heterogeneity into the econometric model below.

Another generalization involves allowing heterogeneity in the productivity of fighting effort. Suppose for instance that agent i 's fighting strength can be written as

$$\varphi_i = \lambda_i x_i + \beta \sum_{j=1}^n a_{ij}^+ \lambda_j x_j - \gamma \sum_{j=1}^n a_{ij}^- \lambda_j x_j, \quad (18)$$

with $\lambda_i \in \mathbb{R}_+$ is a measure of the individual fighting technology. The payoff function of Equation (1) can then be written as follows

$$\pi_i(G, \mathbf{x}, \boldsymbol{\lambda}) = V \frac{\varphi_i}{\sum_{j=1}^n \varphi_j} - x_i = V \frac{\lambda_i x_i + \beta \sum_{j=1}^n a_{ij}^+ \lambda_j x_j - \gamma \sum_{j=1}^n a_{ij}^- \lambda_j x_j}{\sum_{j=1}^n \left(\lambda_j x_j + \beta \sum_{k=1}^n a_{jk}^+ \lambda_k x_k - \gamma \sum_{k=1}^n a_{jk}^- \lambda_k x_k \right)} - x_i, \quad (19)$$

where $\boldsymbol{\lambda} \equiv (\lambda_1, \dots, \lambda_n)^\top \in \mathbb{R}_+^n$. Proposition 1 can then be generalized to the heterogenous case. The Nash equilibrium is given by (see Proposition 6 in Appendix A)

$$\mathbf{x}^*(G, \boldsymbol{\lambda}) = V \tilde{\Lambda}^{\beta, \gamma}(G, \boldsymbol{\lambda}) \mathbf{D}(\boldsymbol{\lambda})^{-1} (\mathbf{I}_n + \beta \mathbf{A}^+ - \gamma \mathbf{A}^-)^{-1} (\mathbf{I}_n - \tilde{\Lambda}^{\beta, \gamma}(G, \boldsymbol{\lambda}) \mathbf{D}(\boldsymbol{\lambda})^{-1}) \boldsymbol{\Gamma}^{\beta, \gamma}(G), \quad (20)$$

where

$$\tilde{\Lambda}^{\beta, \gamma}(G, \boldsymbol{\lambda}) \equiv \frac{\mathbf{u}^\top \boldsymbol{\Gamma}^{\beta, \gamma}(G) - 1}{\mathbf{u}^\top \mathbf{D}(\boldsymbol{\lambda})^{-1} \boldsymbol{\Gamma}^{\beta, \gamma}(G)}$$

is a measure of "local fighting intensity", \mathbf{D} is a diagonal matrix, and \mathbf{u}^\top is a vector of ones. Intuitively, *ceteris paribus*, a high- λ agent can afford to exert low effort because each unit of his effort translates into a high fighting intensity. Consequently, his payoff tends to be high. The definition of rent dissipation is modified, accordingly: $\text{RD}^{\beta, \gamma}(G, \boldsymbol{\lambda}) \equiv \frac{1}{V} \sum_{i=1}^n (1 + \xi \lambda_i) x_i^*(G, \boldsymbol{\lambda})$, where $x_i^*(G, \boldsymbol{\lambda})$ is the generic element of the vector $\mathbf{x}^*(G, \boldsymbol{\lambda})$ defined in Equation (20) and $\xi > 0$ a positive constant.

3 Empirical Application - The Second Congo War

In this section, we apply the theoretical model constructed in Section 2 to the study of the recent civil conflict in the Democratic Republic of Congo (henceforth, DRC). Our goal is to estimate key externality parameters β and γ from a structural equation such as (15) characterizing the Nash equilibrium of the model. The estimates are used, on the one hand, to test some restrictions imposed by the theory, and, on the other hand, to perform some policy analysis. We start by presenting the historical context of the DRC conflict. Then, we discuss the data sources. Next, we discuss how we estimate the network structure from the data. We proceed then to the econometric model and the discussion of identification and estimation procedure. Finally, we discuss some policy analysis (key player analysis).

3.1 Historical Context

We study the Second Congo War, sometimes referred to as the "Great African War". Detailed accounts of this conflict can be found in Prunier, (2011) and Stearns (2011). The DRC is the largest Sub-Saharan African country in terms of area, and is populated by about 75 million inhabitants. After gaining independence from Belgium in 1960, it has gone through great political and military turbulences, and is an example of a failed state. Despite (or partly because of) its abundance of natural resources (including diamonds, copper, gold and cobalt), the DRC remains today one of the poorest countries in the world. It is also a heavily ethnically fragmented country with over 200 ethnic groups. The Congo conflict has been emblematic for the role of natural resource rents and for the involvement of a large number (64) of inter-connected domestic and foreign actors. In particular, the conflict has "involved three Congolese rebel movements, 14 foreign armed groups, and countless militias" (Autesserre, 2008). This abundance of fighting actors participating has resulted in a setting of particularly complex warfare where links of alliances and enmities have played a big role.

The combination of natural resource abundance, weak institutions, low productivity and a large ethnic diversity have been fertile conditions for the proliferation of rebel movements and militias (cf. Collier, Hoeffler and Rohner, 2009). The Congo Wars are intertwined with the ethnic conflicts in neighboring Rwanda and Uganda. The culminating event was the genocide of 1994, where the Hutu-dominated government of Rwanda supported by ethnic militias such as the Interahamwe persecuted and kill nearly a million of Tutsis and moderate Hutus within less than one hundred days. After losing power to the Tutsi rebels of the Rwandan Patriotic Front (RPF), over a million Hutus fled Rwanda and found refuge in the DRC, governed at that time by the dictator Mobutu Sese Seko. The refugee camps hosted, along with civilians, former militiamen responsible of the Rwandan genocide. These continued to harassed the Tutsis population living both in Rwanda and in the DRC, most notably in the Kivu region (cf. Seybolt, 2000).

As ethnic tensions escalated, a broad coalition comprising the Ugandan government, of the new Tutsi-dominated Rwandan government, and a heterogenous coalition of African states, supported the rebel group Alliance of Democratic Forces for the Liberation of Congo (ADFL) led by Laurent-Désiré Kabila in ousting Mobutu in what became known as first Congo War (1996-97). Kabila became the new president of the DRC. However, his relationship with the former Tutsi allies and their political sponsors (Rwanda and Uganda) deteriorated rapidly. A new war started then in 1998 where Kabila received the support of some old foreign allies (Angola, Chad, Namibia, Sudan and Zimbabwe) and of the same Hutu militias that had supported Mobutu in the First Congo War. The main enemy were Uganda, Rwanda and a network of rebel groups including the Uganda-sponsored Rallye for Congolese Democracy - Liberation Movement (RCD-ML) (also known as RCD - Kisangani) and Congolese Liberation Movement (MLC); and the Rallye for Congolese Democracy - Goma (RCD-G), closely tied with Rwanda (cf. Seybolt, 2000). Other actors took part in the conflict out of their hostility to specific actors. These include, among others, the anti-Ugandan rebel forces of the Allied Democratic Forces and the Lord's Resistance Army, or the anti-Angolan UNITA forces.

The Second Congo War started in 1998 and ended officially in 2003, although in reality the fighting has continued until today. This war corresponds to the deadliest conflict since World War II, with between 3 and 5 million lives lost (Olsson and Fors, 2004; Autesserre, 2008). Contrary to the shifts that occurred in 1997, the web of alliances and enmities remained relatively stable throughout the conflict. Laurent-Désiré Kabila was assassinated in 2001, being replaced by his son Joseph Kabila.

Many actors were only active in limited parts of the country. As we argue below, weather conditions play an important role in determining the intensity of the conflict in different regions. Figure5 displays the fighting intensity and average climate conditions for different ethnic homelands in the DRC. The data used for generating this figure is discussed in detail below.

3.2 Data

We build a panel dataset at the fighting group - year level, covering the period 1998-2010 that includes both the official years of the Second Congo War (until 2003) and its turbulent aftermath. To construct our main variables we draw on a variety of dataset.

Fighting efforts: We measure fighting efforts at the group-year level using data from the Armed Conflict Location and Events Dataset (ACLED, 2012), a well established data source in the literature.¹⁴ This dataset contains 4765 geolocalised violent events taking place in the DRC between 64 fighting groups over the 1998-2010 period. For each violent event, information is provided on

¹⁴Recent papers that use ACLED data for measuring fighting efforts include among others Cassar,Grosjean, and Whitt (2013), Michalopoulos and Papaioannou (2013), and Rohner, Thoenig, and Zilibotti (2013b).

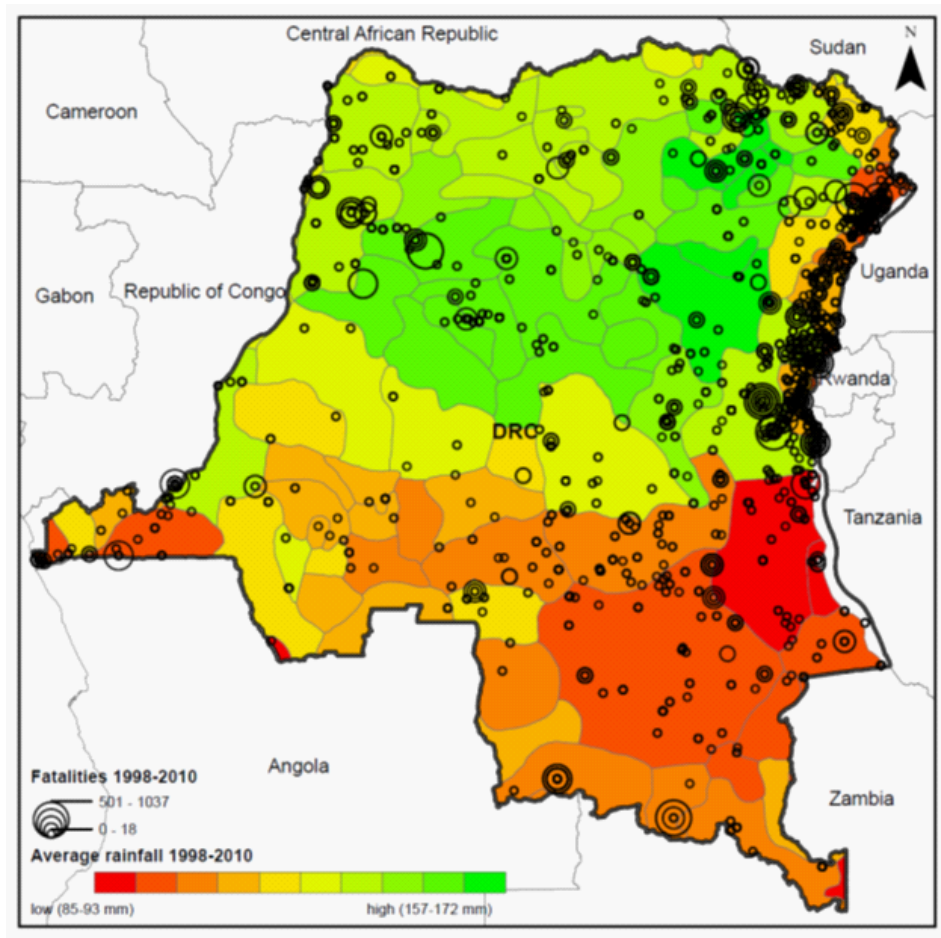


Figure 5: Map of DRC

the exact location, the date and the identities of the fighting groups involved in the event. To construct our main variable of *fighting effort* we take the sum over all fighting events against other armed groups in which a given fighting group is involved during a year. Our results are robust to restricting our attention to different types of events (e.g. only to battles) or to a variant of the variable focusing on the number of fatalities occurring in events involving a given armed group in a given year (e.g. drop all events with a below-median number of fatalities).

Alliances and enmities: Unique to the ACLED data is also the information on the composition of each opposing side involved in a given event. Consider the following three examples: On the 18th of May 1999 a battle took place between "RCD: Rally for Congolese Democracy (Goma)" and "Military Forces of Rwanda", on one side, and the "Military Forces of Democratic Republic of Congo" on the other side. On the 13th of January 2000 there was a battle between "Lendu Ethnic Militia" and "Military Forces of Democratic Republic of Congo", on the one side, and "Hema Ethnic Militia" and "RCD: Rally for Congolese Democracy", on the other side. On the 3rd of February 2000, the "MLC: Congolese Liberation Movement" together with "Military Forces of Uganda" confronted the allied forces of the "Military Forces of Democratic Republic of Congo" and "Interahamwe Hutu Ethnic Militia".

While the ACLED data has been widely used in recent years to measure geo-referenced fighting efforts, the dyadic information it contains about which groups fight together or against each other in given events has received limited attention so far. To the best of our knowledge, our study is the first that exploits this information to construct a network of alliances and enmities. Using ACLED to construct the network of alliance and enmity links has the major advantage of covering smaller groups and militias that most qualitative case studies of the Second Congo War miss. However, ACLED cannot recover links between groups that may take place without them physically meeting in the battlefield. Some such alliances may actually be very important, especially since some of the foreign actors only become involved sporadically in battlefields. In order not to miss such links, we supplement the ACLED data with the alliance relationships listed in the Yearbook of the Stockholm International Peace Research Institute (SIPRI) (Seybolt, 2000).

In a nutshell, the two main variables of this dataset provide us with direct measures of our main theoretical variables, namely \mathbf{x}_t , the vector of equilibrium fighting efforts in year t , and $\mathbf{A}^+ \cup \mathbf{A}^-$ the adjacency matrices of alliances and enmities observed on the battlefield. We measure x_{it} , the fighting effort of a group i in year t , as the total number of ACLED violent events the group participates to. From the matrix of alliances and enmities we are also able to retrieve two additional variables, " $d-(\#Enemies)$ ", which corresponds to the numbers of enmity links, as well as " $d+(\#Allies)$ ", which captures the number of alliance links.

Other variables:

The following variables are used to generate the set of standard control variables and of Instrumental variables (IVs). Below we shall describe in detail the exact specification of controls and IVs.

- *Government Organization:* This variable takes a value of 1 for fighting groups that are official government organizations of one of the countries involved, and 0 otherwise. In particular, are coded as one the Military Forces of Burundi, Chad, Namibia, Rwanda, South Africa, Sudan, Uganda, Zambia and Zimbabwe.

- *Foreign:* This dummy variable takes a value of 1 for all foreign actors, and a value of 0 of all domestic fighting groups that originate from the DRC.

- *Fighting Effort Outside the DRC:* For all groups we also compute the total number of fighting events in which they are involved outside the DRC. This proxies for the global scope of operation

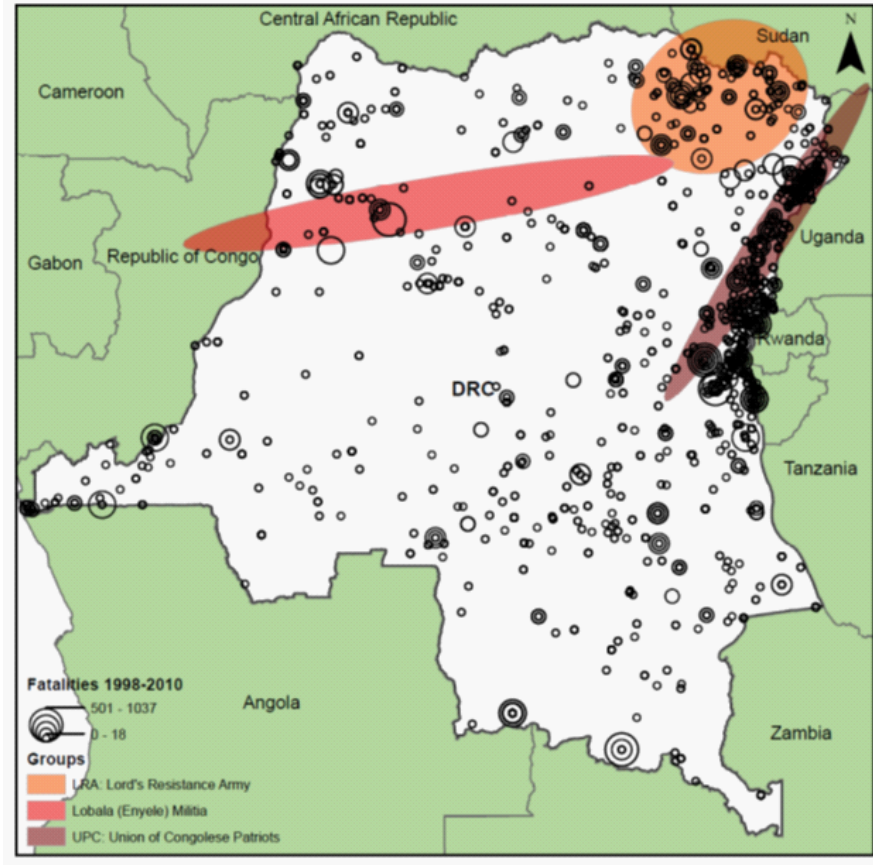


Figure 6: Construction of group-level rain fall measures

of a group.

- *Rain*: We first construct, for each fighting group, the geolocalised ellipses containing all the ACLED fighting events involving the group. This gives us a measure of the spatial zone of operation of the group. Then we use the rainfall data from Schneider et al. (2011), which has a resolution of $0.5^\circ \times 0.5^\circ$, to compute the average rainfall for each year in the zones of operation. We compute both the level of rainfall and its square, both for current and lagged rainfall. The Figure 6 below illustrates the construction of rainfall measures for three particular armed groups.

3.3 Network of Fighting Groups in DRC

In this section, we discuss how we construct the network of alliances and enmities among the belligerent groups. We code two groups i, j , as being *allied* (i.e. $a_{ij}^+ = a_{ji}^+ = 1$) if they have been fighting as brothers in arms in at least one event over the whole sample period, and, in addition, they have never fought on opposing sides in any event. Similarly, we code two groups as being *enemy* (i.e. $a_{ij}^- = a_{ji}^- = 1$) if they have fought on at least two occasions on opposing sides during the sample period, and have never been allied in any event.¹⁵ We code all other dyads as *neutral*

¹⁵Given that in our setting by definition all groups are competing for larger shares of the pie, we require at least two instances of fighting against each other to code two groups as *rivals*. Our results are robust to also coding as *rivals* group that fight on only one occasion against each other.

Variable	Obs	Mean	Std. Dev.
Alliances fighting as brothers in arms in a given year	101150	0.2174988	4.658625
Rivals fighting each other in a given year	101150	0.7355413	8.544813
Among which, rivals that have occasional co-fighting (9.1% of all rivals)	101150	0.0672269	2.591956
Alliances over the sample period	101150	2.242215	14.80527
Rivalries over the sample period	101150	6.15917	24.04137
Among which, rivals that have occasional co-fighting (15.7% of rivals)	101150	0.9688581	9.795309

Figure 7: Descriptive Statistics of the Network Links

(i.e. $a_{ij}^+ = a_{ij}^- = 0$). This neutral coding includes dyads whose groups have fought at some point on the same side and at some other point on opposing sides.

The links corresponding to these coding rules are described in Table 7. The upper part of the table reports statistics on an annual basis. In any given year roughly 1% of dyadic observations are reported as fighting on the same or on opposite sides. This small proportion reflects the fact that big battles are relatively rare and that most groups are not necessarily involved in fighting in every single year of the sample, though it is also possible that some events pass unrecorded. There are about 3.5 more enmities than alliances, reflecting the overall highly conflicted situation in the DRC during the war period. In a limited number of cases (less than a tenth of the observations) we see inconsistencies: groups that have fought on at least two occasions on opposing sides have also fought, in at least one occasion, as brothers in arms against a common enemy in a given year. As discussed above, we code such instances as neutral.¹⁶ The lower part of the table reports statistics over the entire sample (1998-2010). This is the information we use to construct the network. Over the whole period, about 8% of dyads are codes as either allied or enemies, enmities are about 3 times more frequent than alliances, and about 15% of enmities have some occasional co-fighting over the sample period. We view as reassuring the fact that the number of inconsistencies (i.e., the occasional fighting as brother in arms of enemy groups) does not rise sharply when we consider the whole sample period is reassuring. For this reason, we focus on a time-invariant network.¹⁷

Assuming a time-invariant nature is by necessity subject to some caveats. However we some clear advantages to our procedure: First, a time-varying network would raise rampant concerns of reversed causation, due to past fighting efforts affecting future link formation. Second, using an "automatic" coding rule reduces the risk of perception bias in manual coding of links by the researcher. The exhaustive data from ACLED makes sure that even links between small players are recorded, which would surely be missed when using exclusively manual coding of links based on background readings. Third, our coding rule is conservative: It is likely that we code as neutral

¹⁶As discussed below, our results are robust to alternative coding rules.

¹⁷Analyzing of network dynamics would be an interesting, but also a challenging task that goes beyond the scope of this study. In most cases, inconsistencies do not suggest that a group is changing camp altogether, but rather that some specific circumstance has made a case for a tactical temporary alliance.

Variable	Obs	Mean	Std. Dev.	Min	Max
Total Fighting	832	7.30529	26.7442	0	299
Total Fighting Enemies	832	73.0625	107.118	0	703
Total Fight Allies	832	44.3882	75.5271	0	471
d- (#Enemies)	832	3.5625	3.89494	1	20
d+ (#Allies)	832	2.28125	3.13172	0	15
Foreign	832	0.34375	0.475245	0	1
Government Organisation	832	0.171875	0.377499	0	1
Rain fall	832	128.877	25.8222	61.9577	197.346

Figure 8: Summary statistics

some dyads that are in fact allies or enemies but did not have the opportunity to participate into a common fighting event (e.g. due to spatial distance). This issue is partly alleviated by adding the alliance links described by the specialists of SIPRI (Seybolt, 2000), but it is likely that many links remain missing. Such missing links create measurement errors that lead to attenuation bias in the estimates of the fighting externalities (Chandrasekhar and Lewis, 2011). In our empirical analysis the 2SLS specification alleviates this issue (thus, we expect the 2SLS coefficients to larger than their OLS counterpart).

We document that our results are robust to other coding rules of alliances&enmities. First, our results go through if the coding thresholds are changed, e.g. when rivals with occasional co-fighting are coded as non-neutral (e.g. as enemy), or when groups are required to have had at least a n number of positive or negative interactions for being coded as having a positive or negative link. Second our results are robust to using of alternative data sources. In particular, in a robustness check we supplement our information with that provided in the "Non-State Actor Data" of Cunningham, Gleditsch and Salehyan (2013).

The summary statistics of the network is displayed in Table 8. We also display the network graphically with the aid of maps of the DRC: Figure 9 covers the whole of DRC, while Figure 10 focuses on the particularly on Eastern Congo and the disputed Kivu region. In the maps, the points indicate the centers of action of the various groups and the green and red lines show, respectively, alliance and enemy links. The polygons capture the homelands of different ethnic groups and the polygons painted in darker red identify areas characterized by higher fighting intensity.

3.4 Structural Estimation and Exclusion Restriction

In this Section, we present the econometric model. We consider a conflict with a given network that repeats itself over several years. We abstract from reputation and repeated game effects, and assume that each period is a one-shot game. Although the network is stable, there is variation in the outcome, driven by different realization of group-specific shocks. The inclusion of shocks has the dual purpose of matching more credibly real data, and of providing econometric identification. More precisely, we elaborate on the model of section 2.5 where OP, φ_{it} , is impacted by group-specific shocks, $\tilde{\varphi}_{it}$ (equation (15)) that are now assumed to capture observable and unobservable (for the

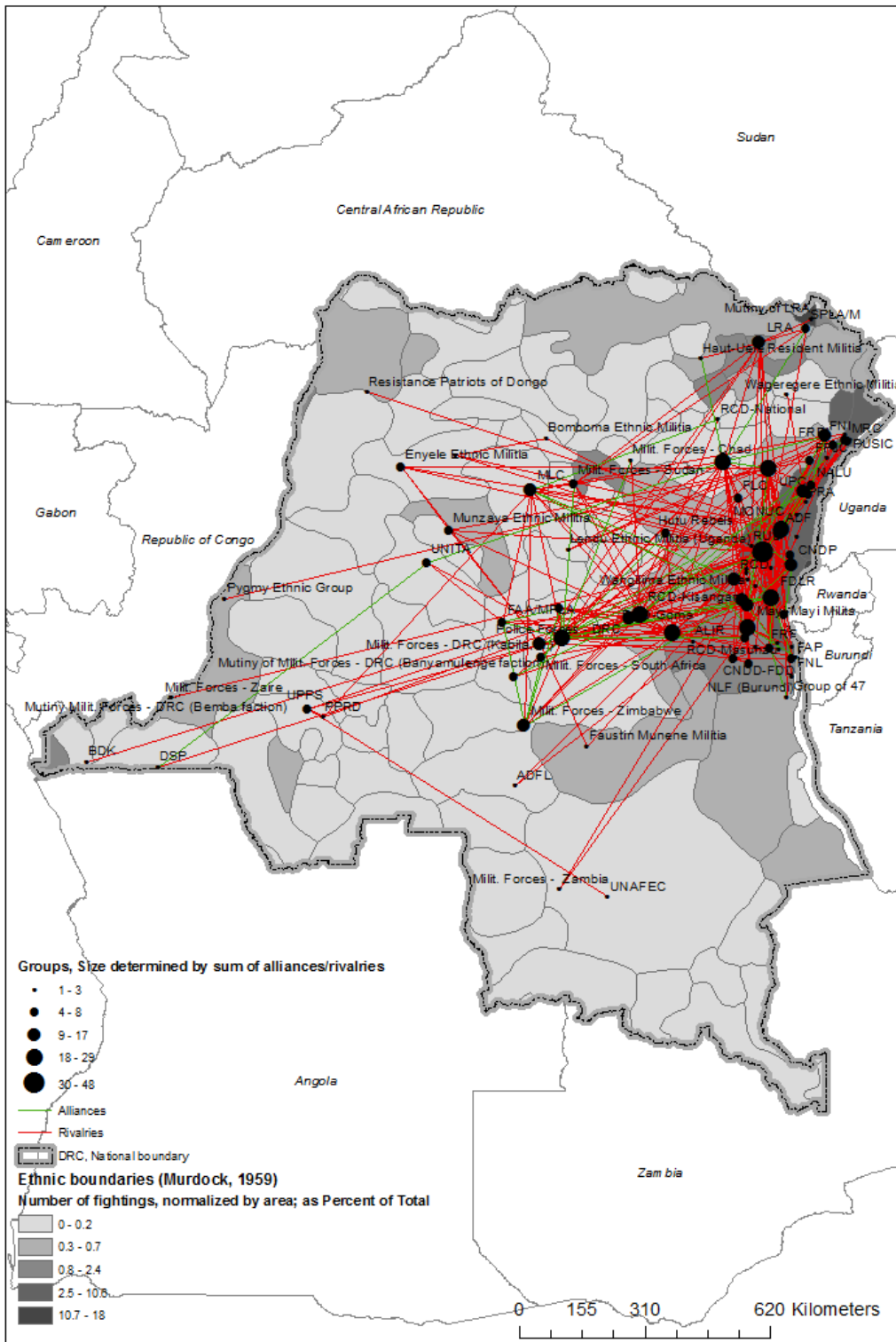


Figure 9: Alliance and rivalry networks and fighting in the DRC

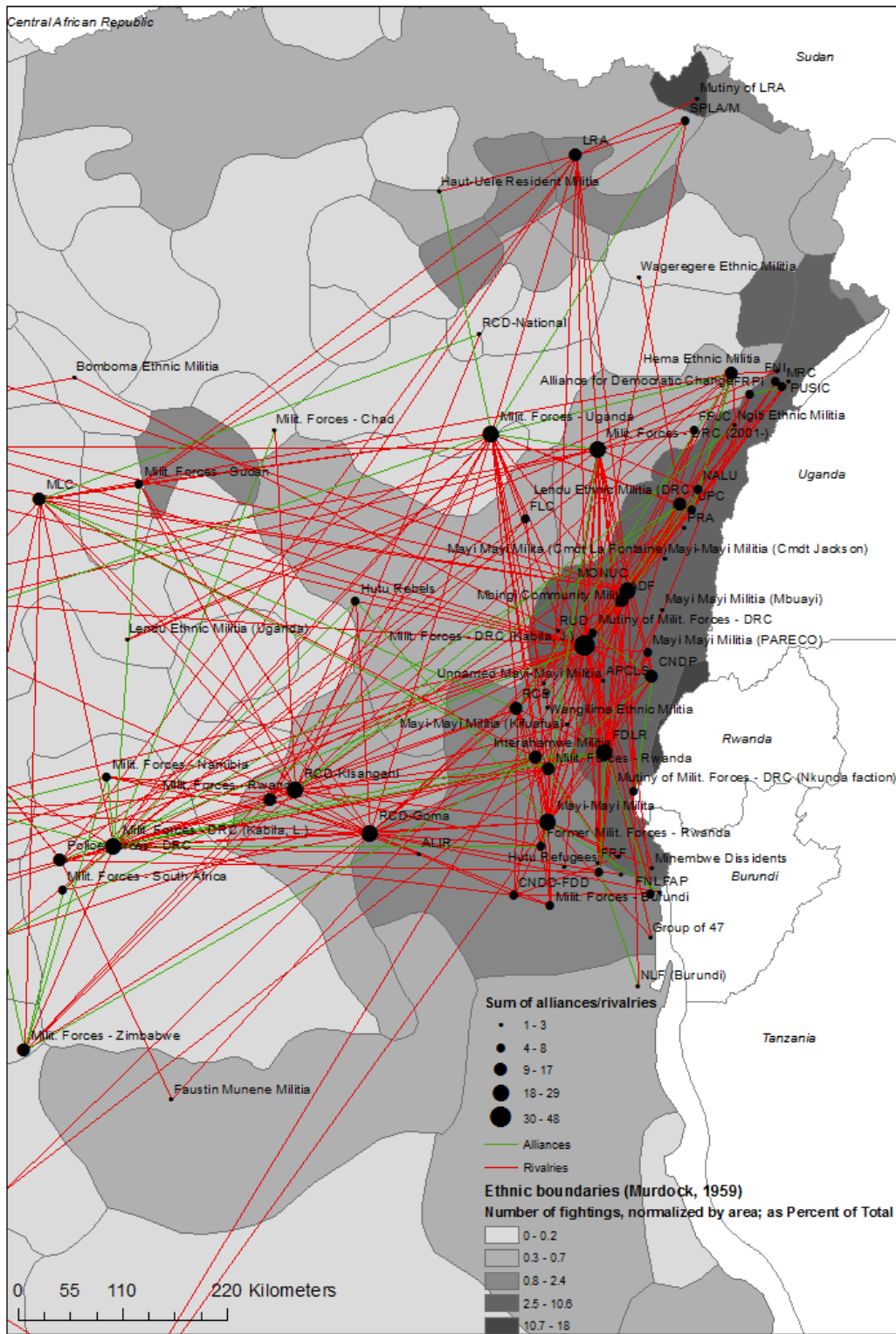


Figure 10: Close-up on alliance and rivalry networks and fighting in Eastern Congo

econometrician) heterogeneity. More formally, we let $\tilde{\varphi}_{it} = \mathbf{z}'_{it}\boldsymbol{\alpha} + e_i + \epsilon_{it}$, where \mathbf{z}_{it} is a vector of observable shifters with coefficients $\boldsymbol{\alpha}$ and e_i is a time-invariant group-specific unobservable shifter and ϵ_{it} is a iid, zero-mean unobservable shifter. Weather shocks are examples of observable shifters \mathbf{z}_{it} that will be key in our instrumental variable strategy; leadership or the moral of troops are examples of unobservable shifters, $e_i + \epsilon_{it}$.

Equation (15) yields, then, the following expression for the OP of group i in year t :

$$\varphi_{it} = x_{it} + \left[\beta \sum_{j=1}^n a_{ij}^+ x_{jt} - \gamma \sum_{j=1}^n a_{ij}^- x_{jt} \right] + \mathbf{z}'_{it}\boldsymbol{\alpha} + e_i + \epsilon_{it}. \quad (21)$$

The equilibrium characterization of the equilibrium follows the discussion in Section 2.5). Recall, in particular that the share of the prize appropriated by each group is independent of $\tilde{\varphi}_{it}$. In particular,

$$\sum_{i=1}^n \varphi_{it}^* = \left[1 - \frac{1}{\sum_{i=1}^n \frac{1}{1 + \beta d_i^+ - \gamma d_i^-}} \right] \times V, \quad (22)$$

which is identical to Equation (6) in the model without heterogeneity. Hence, the total economy-wide OP is fully characterized by the time-invariant network structure, $(\beta, \gamma, d_i^-, d_i^+)$, being independent of the realizations of individual shocks $(\mathbf{z}_{it}, e_i, \epsilon_{it})$. This result simplifies substantially the estimation procedure because it implies that there is no macro-feedback of unobserved heterogeneity on the equilibrium individual efforts.

Combining individual best-response (17), definition (21), and the equilibrium aggregate condition (22) we obtain the equilibrium effort for each agent i

$$x_{it}^* = -\beta \sum_{j=1}^n a_{ij}^+ x_{jt}^* + \gamma \sum_{j=1}^n a_{ij}^- x_{jt}^* - \mathbf{z}'_{it}\boldsymbol{\alpha} + u_i - \epsilon_{it}. \quad (23)$$

where the time-invariant term of unobserved individual heterogeneity is defined as

$$u_i \equiv e_i + \Lambda(G) (1 - \Lambda(G)) \Gamma_i(G) \times V \quad (24)$$

We estimate the structural equation (23). Given its linear form, this calls for a standard OLS or 2SLS specifications. We now discuss our three main identification issues:

1. *Correlated Effects* – in the structural equation (23) the term of time-invariant unobserved heterogeneity u_i correlates potentially with neighbor fighting efforts x_{jt}^* through the deep parameter $\Lambda(G)$ (an issue called "correlated effects" in the literature). Contrary to most existing papers, the panel structure of our data makes possible the inclusion of group specific fixed effects that absorb u_i .
2. *Reflection Problem* – The estimation of the fighting externalities β, γ requires exogenous sources of variations in allies/enemies fighting efforts distinct from shifters of own fighting effort. The structural equation (23) makes clear that the neighbors' observable shifters \mathbf{z}_{jt} are excluded from x_{it}^* : They do not affect directly the fighting effort x_{it}^* but only indirectly through the observable fighting effort of the allies/enemies x_{jt}^* . Hence, our identification strategy can exploit exogenous shifters of z_{jt} as instruments of neighbor outcomes $\sum_{j=1}^n a_{ij}^+ x_{jt}^*$, controlling for the shifters of z_{it} (in order to filter out spatial correlation between shifters). The existing

literature (e.g. Bramoullé et al., 2009) usually exploits covariates of neighbors of neighbors because the neighbors covariates cannot be excluded from the structural equation; an issue we do not face here. However in robustness checks we also use observable shifters of neighbors of neighbors fighting efforts as additional instruments.¹⁸

3. *Instrumental Variables* – Finding strong enough instruments that do not violate the exclusion restriction is not easy: While time-invariant potential candidate IVs would be multicollinear to the group fixed effects, time-varying shocks occurring at the country level would violate the exclusion restriction. We consequently use time-varying climatic shocks (rainfall) impacting fighting groups "homelands". As shown below, rainfall correlates indeed strongly with the allies' and enemies' fighting efforts, and the F-statistic of the first stage of the 2SLS regression is above the conventional thresholds. In line with the empirical literature and historical case studies, we expect groups affected by large positive rainfall shocks to fight *less*, for two reasons. First, local rainfalls are associated with larger agricultural surplus that increases the reservation wages of productive labor and hence leads to a greater opportunity cost of fighting. This channel linking rainfall to conflict has been documented by Miguel et al. (2004) and Vanden Eynde (2011), among others. Second, heavy rain imposes technical constraints on fighting, e.g. by making troop transportation more difficult (cf. for example, Rogall, 2013).

The exclusion restriction requires that rainfall taking place in the allies' and enemies' homelands does not have a direct impact on fighting efforts of a given group i other than through the fighting efforts' spillovers once we control for i 's own rainfall. Potential violation of the exclusion restriction could arise if there were intense within-country trade. For instance, a negative shock hitting crops in Western Congo could translate into strong price increases of agricultural products throughout the entire DRC, thereby affecting fighting in the Eastern part of the country. Such a channel may be somewhat plausible in a highly integrated country with large domestic trade. In a very poor country like the DRC a disintegrating government, very lacunary transport infrastructure and the disastrous security situation leads to very large transport costs reducing inter-regional trade for most goods, and leading to a very localized economy dominated by subsistence farming. Hence, in this context large-scale inter-regional economic spillovers are less plausible.

3.5 Estimates of the Fighting Externalities

We now present the estimates of the structural equation (23) based on a panel dataset containing 64 armed groups between 1998 and 2010. The observational unit is a given armed group in a given year; in all specifications, the standard errors are clustered at the group level.

Table 1 reports the results from the baseline specifications. In Column (1) we start with an OLS specification including group fixed effects and year dummies. The *total fighting of enemies* increase a group's fighting effort, whereas the *total fighting of allies* decreases its fighting effort. These results conform with the prediction of the theory, although the coefficient of *total fighting of allies* is not statistically significant. In column (2) we control for the current and lagged rainfall in the center of a given group's zone of activity, allowing for both linear and quadratic effects. The results are unchanged.

Column (3) aims at controlling other time-varying shocks affecting fighting at the group-level. Unfortunately, there are no data on such shocks (other than on weather conditions). Therefore,

¹⁸As discussed in the literature, this approach is valid because in our data we do not observe strong structural balance leading to systematic triadic closure of all triads. The fact that the network is not composed of a collection of cliques makes possible the use of degree 2 neighbors attributes as valid instruments (see Bramoullé et al. 2009).

we postulate that common shocks unobserved to the econometrician have heterogeneous effects across groups that differ by some observable characteristics. More specifically, we build four broad time-invariant group characteristics and interact each with year dummies. For instance, the intensity of international interventions to limit the influence of foreign armies may have an especially noticeable effect on foreign actors. Thus, construct a binary variable coding for foreign groups operating in DRC, and interact it with a time dummies. Another binary variable (also interacted with time dummies) captures groups affiliated to the DRC government - the activity of these groups may be affected by financial aid or external political pressure exercised on Kinshasa government. The other two variables proxy for group size. Note that we have no data on troop size.¹⁹ Therefore, we resort to two binary variables coding, respectively, large groups with at least 10 enemies (roughly the top decile of the sample), and groups fighting at least 20 violent events per year *outside* the DRC (again, this corresponds to the top decile of the sample). In column (3) we include this battery of time-varying controls. The estimated coefficients are stable, and more precisely estimated. The coefficient of *total fighting of allies* has now a p-value just above 5 percent.

In column (4) we report the result from the second stage of a 2SLS specification including (lagged) weather shocks but no other time-varying control variables. In this specification, *total fighting of enemies* and *total fighting of allies* are instrumented, respectively, by the one-year lag of average rainfall in enemies and allies homelands (where both a linear and square terms are included in the regression). The coefficients of the variables of interest have the expected sign and are significant. The coefficients are larger than in the OLS regressions, suggesting that the non-instrumented specification suffers from a severe estimation bias due to network externalities. Note that the direction of the bias is unclear because the two fighting externalities have opposite signs, and the network externalities compound them in a complicated way. Columns (1) and (2) in Table 2 display the first stage regressions for the *total fighting of enemies* and *total fighting of allies*. We see that the instruments have statistically significant coefficients, with the expected sign, but the overall statistical power is slightly too weak (Kleibergen-Paap F-stat equal to 8.5). The null hypothesis of the Hansen J test is not rejected indicating that the overidentification restrictions are valid. In column (5) of Table 1, we include a larger set of instruments comprising current-year rainfall (linear and quadratic terms) as well as current and lagged rainfalls of degree 2 neighbors (i.e. enemies of enemies and of enemies of allies).²⁰ The second-stage coefficients are stable and significant. As expected, the statistical power of the first stage (Columns (3)-(4) in Table 2) is now larger with a F-statistic above the conventional threshold of 10.

Equipped with this expanded set of instruments we can include again, in column (6) of Table 1, the battery of time-varying controls. This is our preferred specification. Our two variables of interest have the expected signs and are statistically significant at the 5% level. Due to the time-varying controls, the statistical power of the instruments in the first stage (Columns (5)-(6) in Table 2) is now reduced but with a F-statistic still above 10. The estimates of the fighting externalities are quantitatively large. A one standard deviation increase in *total fighting of enemies* (107 violent events) translates into a 0.36 s.d. increase in the fighting effort of the group (+9.6 violent events). A one standard-deviation increase in *total fighting of allies* (75 violent events) translates into a 0.51 s.d. decrease in the fighting effort of the group (-13.6 fighting events).

We performed a variety of robustness checks [VERY PRELIMINARY]. Table 3 reports the

¹⁹Such data only exist for a small subset of the 64 groups operating in DRC over the 1998-2010 period, see the International Institute of Strategic Studies (IISS) or the Small Arms Survey (SAS).

²⁰When we use the current and past average rain fall in enemies and allies homelands as instruments, we also control for the current and past average rain fall in the own group homeland in the second stage regression. This is important, since the rain fall in enemies and allies homelands is correlated with the rain fall in the own group homeland. Omitting the latter would lead to a violation of the exclusion restriction.

	Dependent variable: Total Fighting					
	(1)	(2)	(3)	(4)	(5)	(6)
Tot. Fight. Enemies	0.08*** (0.02)	0.08*** (0.02)	0.06*** (0.02)	0.08* (0.05)	0.13** (0.05)	0.09** (0.04)
Tot. Fight Allies	-0.03 (0.03)	-0.03 (0.03)	-0.04* (0.02)	-0.24* (0.12)	-0.18* (0.11)	-0.18** (0.08)
Group FE, annual time dummies	Yes	Yes	Yes	Yes	Yes	Yes
Control for rainfall variables	No	Yes	Yes	Yes	Yes	Yes
Additional controls	No	No	Yes	No	No	Yes
Estimator	OLS	OLS	OLS	IV	IV	IV
Set of Instrument Variables	n.a.	n.a.	n.a.	Restricted	Full	Full
Observations	832	832	832	832	832	832
R-squared	0.339	0.343	0.420	0.196	0.254	0.367

Notes: An observation is a given armed group in a given year. The panel contains 64 armed groups between 1998 and 2010. Robust standard errors allowed to be clustered at the group level in parentheses. Significance levels are indicated by * p<0.1, ** p<0.05, *** p<0.01.

Table 1: Baseline Regressions (Second Stage)

results. In column 1, we use as excluded instruments only the rainfalls in the homeland of degree 2 neighbors (i.e., the rain of enemies of enemies and of enemies of allies). We face a severe weak instrument problem (the F-stat is 1.3) due to the fact that the degree one rainfalls are now used as direct controls both in the first and in the second stage. Nevertheless, the coefficients estimated in the second stage regression are remarkably similar to our benchmark results.

In Column 2 we follow a conservative identification strategy using as excluded instruments the degree one and two rainfalls in the historical ethnic homelands of the fighting groups. By construction these homelands do not overlap spatially and this guarantees that the rainfall measures used for instrumenting fighting efforts of two different groups relate to different areas of the territory. We proceed as follows: As a first step we link as many armed groups as possible to a corresponding underlying main ethnic group. This is typically the ethnic affiliation of most fighters or at least of their leadership circle. A typical example would be the Lord's Resistance Army that is linked to the Acholi ethnic group. We find a clear match for 94% of all armed groups, and drop the remaining 6%. As a second step, we compute the rainfall averages on the polygons of all ethnic groups, using the digitalized version by Nunn and Wantchekon (2011) of the map of historical ethnic group homelands from Murdock (1959). Using these "ethnic homeland"-based rainfall IVs yields results that are similar than in the benchmark of Table 1, with the variables of interest having the expected sign and being statistically significant.

Restricting the set of IVs to only degree two neighbors rainfall is particularly appropriate when the zones used for computing the rainfall measures do not overlap spatially, as is the case for the groups' ethnic homelands. For this reason, in Column 3 we replicate Column 1 but use as IVs the rainfalls in ethnic homelands of degree two neighbors. This is a very demanding specification. The results are similar to Column 2, with the fighting effort of the allies being now borderline insignificant. However, the F-stat is still small. Consequently, in column 4 we still consider degree two IVs but we also use their two-year lagged values: The F-stat in the first-stage regression is now substantially larger (7.6). The coefficient of *total fighting of enemies* in the second stage is highly significant and the one of *total fighting of allies* is just below the margin of the 10% significance.

In Column 5 we restrict the sample to only groups with at least one enemy and at least one ally, while in Column 6 we restrict the sample of before 2006, where most of the fighting took place. In both columns our variables of interest still have the expected sign and are statistically

Dep. Variable:	IV regression of column (4)		IV regression of column (5)		IV regression of column (6)	
	Tot.Fi. Enemy	Tot.Fi.Allied	Tot.Fi. Enemy	Tot.Fi.Allied	Tot.Fi. Enemy	Tot.Fi.Allied
	(1)	(2)	(3)	(4)	(5)	(6)
Rain (t-1) Enemies	-1.71*** (0.29)	0.08 (0.12)	-1.55*** (0.23)	0.13 (0.10)	-1.34*** (0.29)	0.10 (0.09)
Sq. Rain (t-1) Ene.	0.00*** (0.00)	0.00 (0.00)	0.00*** (0.00)	-0.00 (0.00)	0.00*** (0.00)	0.00 (0.00)
Rain (t-1) Allies	-0.17 (0.29)	-1.21*** (0.18)	-0.24 (0.31)	-1.08*** (0.21)	-0.25 (0.32)	-1.01*** (0.19)
Sq. Rain (t-1) Alli.	0.00 (0.00)	0.00*** (0.00)	0.00 (0.00)	0.00*** (0.00)	0.00 (0.00)	0.00*** (0.00)
Current Rain Enemies			-1.04*** (0.25)	0.08 (0.12)	-1.02*** (0.27)	0.11 (0.11)
Sq. Curr. Rain Ene.			0.00*** (0.00)	-0.00 (0.00)	0.00 (0.00)	-0.00 (0.00)
Current Rain Allies			-0.05 (0.29)	-0.73*** (0.14)	-0.05 (0.33)	-0.64*** (0.15)
Sq. Curr. Rain Alli.			0.00 (0.00)	0.00*** (0.00)	0.00 (0.00)	0.00*** (0.00)
Current rain enemies of enemies			-0.20*** (0.05)	-0.06** (0.03)	-0.18*** (0.05)	-0.03 (0.03)
Sq. current rain enemies of enemies			0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)
Current rain enemies of allies			-0.18* (0.10)	-0.04 (0.05)	-0.13 (0.10)	-0.07 (0.05)
Sq. current rain enemies of allies			0.00** (0.00)	-0.00 (0.00)	0.00* (0.00)	0.00 (0.00)
Rain enemies of enemies (t-1)			-0.26*** (0.06)	-0.05 (0.03)	-0.28*** (0.06)	-0.04 (0.04)
Sq. rain enemies of enemies (t-1)			0.00*** (0.00)	0.00 (0.00)	0.00*** (0.00)	0.00 (0.00)
Rain enemies of allies (t-1)			-0.06 (0.14)	-0.05 (0.05)	-0.03 (0.14)	-0.03 (0.05)
Sq. rain enemies of allies (t-1)			0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)
F-Stat (Kleibergen-Papp)	8.5	8.5	31.5	31.5	12.8	12.8
Hansen J (p-value)	0.68	0.68	0.30	0.30	0.19	0.19
Observations	832	832	832	832	832	832
R-squared	0.457	0.649	0.560	0.682	0.630	0.720

Notes: An observation is a given armed group in a given year. The panel contains 64 armed groups between 1998 and 2010. Robust standard errors allowed to be clustered at the group level in parentheses. Significance levels are indicated by * p<0.1, ** p<0.05, *** p<0.01.

Table 2: Baseline Regressions (First Stage)

	Dependent variable: Total Fighting							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Degree 2 only	Ethnic IV benchmark	Ethnic IV degree 2	Ethnic IV extended	With at least 1 ally	Before 2006	Excluding bilat. evts.	With neutrals
Tot. Fight. Enemies	0.11* (0.07)	0.09*** (0.03)	0.08* (0.04)	0.12*** (0.03)	0.10* (0.05)	0.05* (0.03)	0.06 (0.04)	0.11** (0.05)
Tot. Fight Allies	-0.10 (0.23)	-0.23* (0.13)	-0.21 (0.14)	-0.13 (0.08)	-0.27** (0.12)	-0.13* (0.07)	-0.19** (0.09)	-0.20** (0.10)
Tot. Fight Neutrals								-0.01 (0.01)
Std controls, TE, FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Estimator	IV	IV	IV	IV	IV	IV	IV	IV
Zones used for IV construction	Group pr.	Ethnic home	Ethnic home	Ethnic home	Group pr.	Group pr.	Group pr.	Group pr.
Excluded instruments used	Deg 2, Lag 0&1	Deg 1&2, Lag 0&1	Deg 2, Lag 0&1	Deg 2, Lag 0&1&2	Deg 1&2, Lag 0&1	Deg 1&2, Lag 0&1	Deg 1&2, Lag 0&1	Deg 1&2, Lag 0&1
Observations	832	780	780	780	559	512	832	832
R-squared	0.425	0.338	0.364	0.404	0.294	0.489	0.331	0.332
F-Stat (Kleibergen-Papp)	1.3	4.3	4.2	7.6	10.7	11.7	11.5	10.1
Hansen J (p-value)	0.17	0.30	0.39	0.55	0.49	0.53	0.23	0.16

Notes: An observation is a given armed group in a given year. The panel contains 64 armed groups between 1998 and 2010. Robust standard errors allowed to be clustered at the group level in parentheses. Significance levels are indicated by * p<0.1, ** p<0.05, *** p<0.01.

Table 3: Main Robustness Checks

Dependent variable: Total Fighting						
	(1)	(2)	(3)	(4)	(5)	(6)
	Only violent events (above median fat.)	Only battles	Code rivals with co- fighting as enemies	Code 1 clash already as enemy link	Code links only when at least 3 joint evts	Add links from Cunningham et al
Tot. Fight. Enemies	0.31*** (0.07)	0.09** (0.05)	0.09*** (0.03)	0.12*** (0.04)	0.08** (0.04)	0.10* (0.05)
Tot. Fight Allies	-0.20* (0.11)	-0.19** (0.10)	-0.19** (0.09)	-0.18*** (0.07)	-0.10* (0.05)	-0.22* (0.12)
Observations	520	832	858	1105	832	819
R-squared	0.394	0.346	0.389	0.265	0.398	0.315
F-Stat (Kleibergen-Papp)	66.1	10.4	21.7	5.8	19.7	11.4
Hansen J (p-value)	0.64	0.21	0.34	0.85	0.34	0.19

Notes: An observation is a given armed group in a given year. The panel contains 64 armed groups between 1998 and 2010. Robust standard errors allowed to be clustered at the group level in parentheses. Significance levels are indicated by * p<0.1, ** p<0.05, *** p<0.01.

Table 4: Additional robustness checks on the network construction

significant. In Column 7 we exclude bilateral fighting events when computing the fighting effort variables. The coefficient of *total fighting of allies* is now significant at the 10% level and the *total fighting of enemies* is still significant at the 5% level. In Column 8 we control for the fighting effort of groups that are neutral with respect to the group of reference. If our network captures most of the connections, we expect the coefficient of *total fighting of neutrals* to be close to zero. This is indeed the case.

Another set of robustness checks are displayed in Table 4. In column 1 we consider only events with an above-median level of fatalities, i.e. with 4 or more casualties, for the construction of the network and of our key fighting effort variables. In column 2 we only use events that are classified in ACLED as battles, ignoring lower intensity events. In column 3 we code enemies with occasional co-fighting on the same sides not as neutral, but as enemies. In column 4 we do not require two fighting events between two groups to code them as enemy, but already code a dyad of armed groups as enemies if they fight each other at least once and never fight as brothers in arms on the same side. In column 5 we follow an opposite more restrictive rule and require three events against each other to be coded as enemies, and analogously require three events of fighting on the same side to count a dyad of groups as allied. Finally, in column 6 we add also links between groups listed in Cunningham et al. (2013). Our results are robust with both coefficients of interest being significant and of the expected sign for all six of these robustness checks.

3.6 Key Player Analysis

[VERY PRELIMINARY] In this section we assess the contribution of each group to the conflict, following the discussion of the key player analysis in Section 2.4.2. Knowing this has important policy implications. For instance, an international organization aiming to scale down conflict may be interesting in knowing which among the groups in conflict contribute mostly to the war. Such contribution cannot be simply measured by the fighting activity of each group. Removing a group would affect the incentives of the remaining groups to fight. For instance, removing a group with many allies may not be very useful, since the former allies would increase their efforts. Instead, removing a group with many strong enemies can have important effects on the containment of the conflict. In reality, the calculation is even more complicated, as it must take into account the response of all players through higher order links. For this reason, the information about the structure of the network and the size of the externalities is essential.

To perform the key player analysis, we must compute the set of counterfactual Nash equilibria corresponding to the sequential removal of each fighting group. In the following we denote with the

subscripts b and $c(k)$ variables that relate, respectively, to the benchmark equilibrium (i.e. when all groups fight) and the counterfactual equilibrium (i.e. removal of a given group k). Since our data have a panel structure, we apply the analysis to the "average scenario", i.e. we compare an average year of conflict in the benchmark model to its corresponding counterfactual. Namely, we assume that all shifters take on a value equal to the sample average for the corresponding group.

The first step of the analysis consists in retrieving from our estimates the appropriate set of deep parameters and the unobserved heterogeneity that are necessary for computing equilibria. We use the parsimonious specification of column 4 in Table 1, which yields point estimates of the fighting externalities equal $\hat{\beta} = 0.2436$ and $\hat{\gamma} = 0.0842$. Without loss of generality we set $V = 1$ as a numeraire (this corresponds to re-scaling the a-dimensional payoff and welfare). From the structural equation (23) we get $\hat{u}_i = \text{FE}_i$ where u_i corresponds to time-invariant heterogeneity and FE_i is the coefficient of the group i fixed effect in our estimates. From the definitions (4) and (24) we obtain an estimate of the time-invariant component of the unobserved shifter

$$\hat{e}_i = \text{FE}_i - \hat{\Lambda} (1 - \hat{\Lambda}) \hat{\Gamma}_i \quad (25)$$

where $\hat{\Gamma}_i = 1/(1 + \hat{\beta}d_i^+ - \hat{\gamma}d_i^-)$ and $\hat{\Lambda} = 1 - 1/(\sum_{j=1}^n \hat{\Gamma}_j)$. Finally the structural equation implies that $\hat{\epsilon}_{it} = -\text{RESID}_{it}$ where ϵ_{it} is the time-varying component of the unobserved shifter and RESID_{it} is the residual of our econometric specification.

Next, we compute the Nash equilibria. For the benchmark scenario we consider the average fighting equilibrium across the 1998-2010 period: Given the linearity of the structural equation the average equilibrium is also characterized by condition (23) where all RHS variables are now averaged across-time. By construction, $\hat{\Lambda}^b, \hat{\Gamma}_i^b$ and \hat{e}_i are constant through time while the time-average of $\hat{\epsilon}_{it}$ is zero; $\bar{\mathbf{Z}}$ denotes the matrix of time-averaged observed shifters. As in the baseline model, the vector of benchmark equilibrium fighting efforts is obtained by inverting the system of equilibrium conditions. In matrix form, this yields

$$\mathbf{x}^b = (\mathbf{I} + \hat{\beta}\mathbf{A}^+ - \hat{\gamma}\mathbf{A}^-)^{-1} \left[\hat{\Lambda}^b(1 - \hat{\Lambda}^b)\widehat{\mathbf{\Gamma}}^b - (\bar{\mathbf{Z}}\widehat{\boldsymbol{\alpha}} + \widehat{\mathbf{e}}) \right] \quad (26)$$

The procedure is similar for each counterfactual equilibrium $c(k)$ once we take into account that the structure of the network is affected by the removal of player k . The vector of equilibrium fighting efforts is characterized by an equation similar to the previous one except that the dimension of the system is reduced by 1, the adjacency matrix is now $\mathbf{A}^{c(k)}$ and the parameters attached to the network structure must be replaced by $(\Lambda^{c(k)}, \mathbf{\Gamma}^{c(k)})$.

For each agent k we then compute the change in rent dissipation before and after the removal of this agent (see Section 2.4.2). By definition it is equal to $\Delta\text{RD}^k \equiv \sum_{i=1}^n x_i^b - \sum_{i \neq k} x_i^{c(k)}$. The resulting key player ranking is displayed in Table 4. Moreover, Figure 11 shows the rent dissipation and its relative change before and after removing an agent.

The results are interesting: While the special role of the DRC government troops is not surprising, it is interesting to see which rebel groups are found to be particularly important players. The RCD-Goma and the RCD-Kisangani have been widely discussed in the qualitative political science literature as important players, and are the main armed groups supported by Rwanda and Uganda, respectively. What is surprising is the relatively big detrimental impact of relatively small foreign and domestic rebel groups such as the LRA, the CNDP, the FDLR, the UPC and the ADF, which all of them only had a few thousand men each (some even less than 1000).

This contrasts sharply with the very large groups like the MLC and the Mayi Mayi which had over ten thousand troops each and still only ranked relatively low in the key player ranking. One of the reasons may be that they have relatively many allies (i.e. almost as many allies as enemies)

Table Key player ranking for the first 25 actors in the Democratic Republic of the Congo (DRC).^a The total number of groups is $n = 64$. The total number of alliances is $m^+(G) = 73$ while the total number of conflicts is $m^-(G) = 114$. Moreover, we have that $\sum_j \varphi_j = 0.9912$.

Table 1: *

Actor	d_i^-	Δm^- (%) ^b	d_i^+	Δm^+ (%)	$\varphi_i / \sum_j \varphi_j$	x_i^*	$\Delta RD^{\beta,\gamma}$ (%) ^c	Rank
Military Forces of DRC (Joseph Kabila fraction)	20	17.5439	13	17.8082	0.0052	75.61	44.9881	1
RCD: Rally for Congolese Democracy (Goma)	15	13.1579	2	2.7397	0.0573	40.98	12.2813	2
RCD: Rally for Congolese Democracy (Kisangani)	15	13.1579	2	2.7397	0.0573	28.21	9.4684	3
Military Forces of DRC (Laurent Kabila fraction)	13	11.4035	15	20.5479	0.0036	49.08	9.4178	4
LRA: Lord's Resistance Army	7	6.1404	0	0.0000	0.0313	19.53	5.8750	5
CNDP: National Congress for the Defense of the People	4	3.5088	2	2.7397	0.0112	20.61	3.2267	6
FDLR: Democratic Forces for the Liberation of Rwanda	5	4.3860	4	5.4795	0.0083	31.38	2.6602	7
Military Forces of DRC	7	6.1404	13	17.8082	0.0036	11.85	2.6031	8
UPC: Union of Congolese Patriots	5	4.3860	1	1.3699	0.0156	10.23	2.0910	9
MLC: Congolese Liberation Movement	6	5.2632	4	5.4795	0.0088	14.07	1.8456	10
Mutiny of Military Forces of DRC	3	2.6316	2	2.7397	0.0104	7.77	1.3052	11
ADF: Allied Democratic Forces	3	2.6316	1	1.3699	0.0130	6.23	1.2800	12
RCD: Rally for Congolese Democracy	7	6.1404	2	2.7397	0.0143	8.31	1.1209	13
Mayi-Mayi Militia	6	5.2632	6	8.2192	0.0066	26.31	0.8845	14
FRPI: Front for Patriotic Resistance of Ituri	2	1.7544	1	1.3699	0.0120	4.07	0.8545	15
CNDD-FDD: Nat. Council for the Def. of Dem.	4	3.5088	0	0.0000	0.0194	3.30	0.8539	16
SPLA/M: Sudanese People's Liberation Army/Movement	1	0.8772	1	1.3699	0.0111	3.46	0.8365	17
FNI: Nationalist and Integrationist Front	1	0.8772	1	1.3699	0.0111	4.00	0.7841	18
FPJC: Popular Front for Justice in Congo	2	1.7544	0	0.0000	0.0155	3.07	0.7810	19
Hema Ethnic Militia	5	4.3860	3	4.1096	0.0098	4.30	0.6552	20
Hutu Rebels	2	1.7544	2	2.7397	0.0098	2.46	0.4572	21
ALIR: Army for the Liberation of Rwanda	2	1.7544	0	0.0000	0.0155	1.69	0.3882	22
Interahamwe Hutu Ethnic Militia	6	5.2632	6	8.2192	0.0066	6.61	0.3823	23
Military Forces of Burundi	1	0.8772	0	0.0000	0.0140	1.38	0.3212	24
PUSIC: Party for the Unity of Congo's Integrity	2	1.7544	0	0.0000	0.0155	1.23	0.2860	25

^a We have used the parameter estimates $\beta = 0.2436$ and $\gamma = 0.0842$ while setting $V = 1$.

^b The relative change in the number of rivalry/alliance links is computed as $m^\pm = \frac{m^\pm(G) - m^\pm(G^{-i})}{m^\pm(G)}$, where $m^\pm(G)$ denotes the total number of rivalry/alliance links in G and $m^\pm(G^{-i})$ the corresponding number in the network G^{-i} obtained after removing agent i from G .

^c The relative change in the rent dissipation is computed as $\frac{RD^{\beta,\gamma}(G) - RD^{\beta,\gamma}(G^{-i})}{RD^{\beta,\gamma}(G)}$ while the rent dissipation in the original network is $RD^{\beta,\gamma}(G) = 467.52$ (with $V = 1$).

which would make a removal of the MLC and the Mayi Mayi less effective, as after they are gone their numerous allies would not be able to free ride on their efforts anymore, and would start to fight harder themselves, which would dampen the pacifying effects of removing the MLC and Mayi Mayi.

4 Conclusions

[VERY PRELIMINARY] In this paper, we present the theoretical model of a conflict in which different groups compete for the appropriation of a fixed amount of resources. We introduce a network of alliances and enmities which generates externalities affecting the incentive to fight of each participant and the outcome predicted by a contest success function. Using the structural equations of the model characterizing the Nash equilibrium, we estimate the deep parameters capturing these network externalities. We apply the theory to the analysis of the Great War of Africa, following the estimation procedure suggested by Bramoullé *et al.* (2009). The signs of the estimated coefficients conform with the prediction of the theory: each group's fighting effort is increasing in the total fighting of its enemies and decreasing in the total fighting of its allies. The OLS estimates are inconsistent, due to a "reflection" problem. To correct the bias, we exploit an instrumental variable method exploiting the exogenous variation over space and time in weather conditions. The IV estimates have the same opposite sign pattern as the OLS ones, but are larger in absolute value. We use the estimates to perform a key player analysis, i.e., to identify which groups contribute the most to the escalation of the conflict, either directly or indirectly, via their

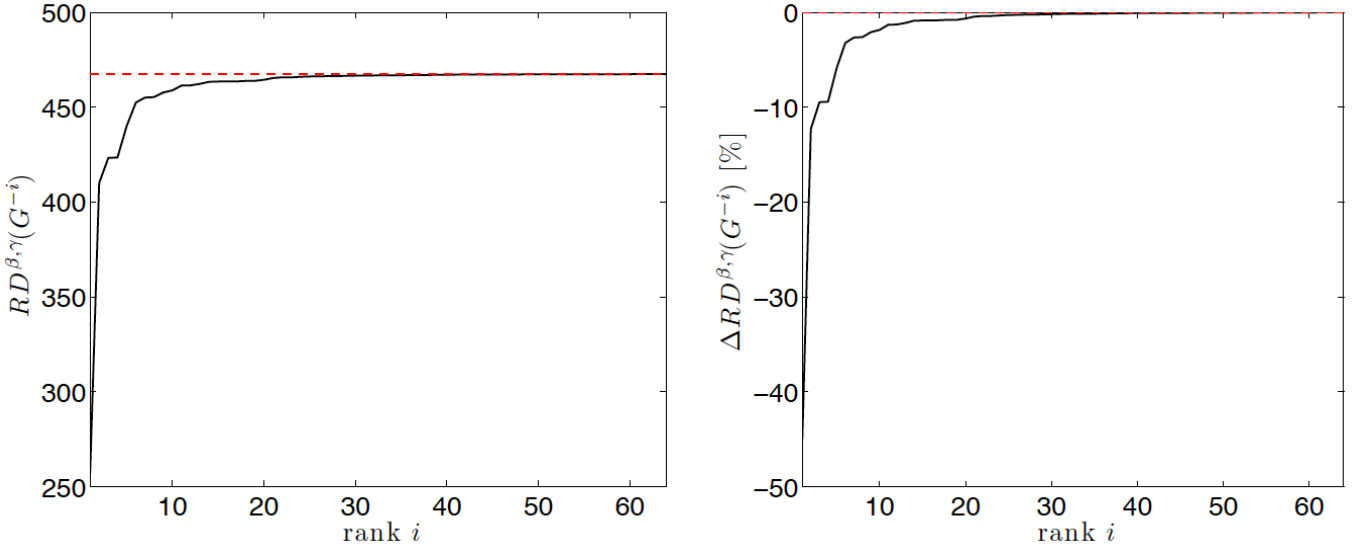


Figure 11: The rent dissipation (left panel) and the relative change in the rent dissipation (right panel) in the interior equilibria found after removing agent i . The (red) horizontal dashed line corresponds to the rent dissipation in the initial network G before the removal of an agent. The agents are ranked according to the reduction in the rent dissipation.

externalities they exercise on the other groups' fighting effort.

Our analysis represents the first step towards understanding how the web of alliances and enmities in a conflict can explain its escalation or containment. An important limitation is that we take the network as exogenous, and do not try to model its formation or dynamic evolution. This is an important *caveat* since adding or removing some players might affect the structure of alliances and rivalries. We leave this important extension to future research.

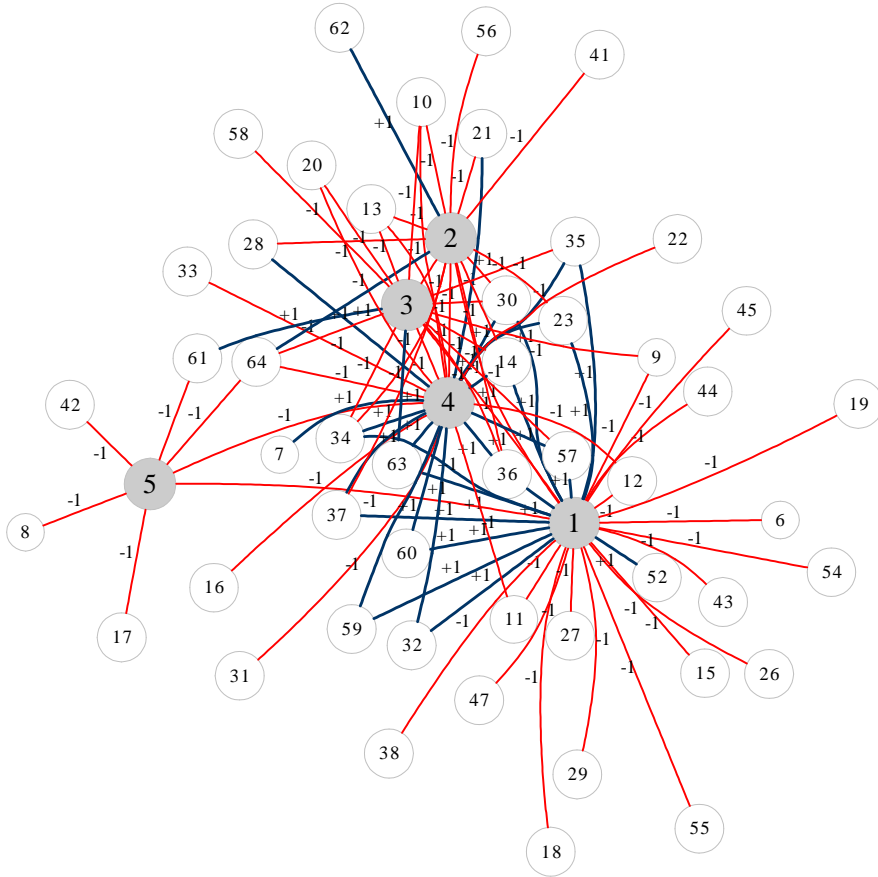


Figure 12: Network of the first 5 key players from Table 4 and their direct neighbors of alliances and conflicts in the Democratic Republic of the Congo (DRC). Alliance relationships are indicated in blue (+1) while conflict relationships are indicated in red (-1). The node numbers correspond to the ranking in Table 4.

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