Preferences, Confusion and Competition

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Abstract

Existing literature has argued that firms benefit from confusing consumers of homogeneous goods. This paper shows that this insight generally breaks down with differentiated goods and heterogeneous preferences: With polarized taste distributions, firms fully educate consumers. In cases where firms nevertheless confuse consumers, the welfare consequences are worse than for homogeneous goods, as consumers choose dominated options. Similar insights are also obtained for political contests, in which candidates compete for voters with heterogeneous preferences: Parties choose ambiguous platforms only when preferences are "indecisive", featuring a concentration of indifferent voters.

Keywords: obfuscation, consumer confusion, differentiated products, price competition, polarized/indecisive preferences, political competition.

JEL Classification: D43, L13, M30.

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1 Introduction

When purchasing goods such as smartphones, motor vehicles or insurance, consumers often make mistakes. To a certain extent, such mistakes reflect confusion. This has been documented across various sectors, including retailing, financial services, utilities, telecommunication, the hospitality industry or health insurance (Eppler and Mengis, 2004; Walsh et al., 2007; Loewenstein et al., 2013; Kasabov, 2015).[1]

Based on several decades of evidence, there is a broad consensus in marketing science that "consumers mix up, misidentify, or make wrong (i.e. illfounded) inferences about products and/or erroneous product selections" (Mitchell and Kearney, 2002, p. 357).

Firms can influence the degree of confusion through their own activities. On the one hand, firms can engage in measures to educate consumers: They can describe products transparently to facilitate comparison, or they can inform consumers about their true needs (e.g., by providing free trials). On the other hand, firms have means to confuse consumers. For instance, insurance companies may issue contracts with complicated premium-deductible schemes that impede comparisons with those of other firms. When advertising differentiated products, firms may emphasize irrelevant product details rather than those characteristics that really matter to consumers. Manufacturers of sophisticated products, such as smartphones, digital cameras, or laundry machines, may add attributes with unclear value to their products. Importantly, consumer confusion often results from the joint activities of the firms in the market. For instance, the use of product labels has often been associated with confusion (Langer et al., 2007).

Do firms seek to *educate* or *confuse* their consumers? The existing literature might suggest that the answer is clear-cut. Oligopolistic producers of homogeneous goods suffer from the temptation to undercut each others' prices, resulting in a zero-profit equilibrium under well-known conditions.

The literature on behavioral industrial organization has shown that obfuscation techniques often allow firms to escape the "Bertrand trap", allowing suppliers of homogeneous goods to secure positive profits in environments where this would otherwise be impossible.[2] This paper points to the limitations of such reasoning. We show that consumer

[1] See "Why the confusion of the cell phone market has caused millions to switch," *Forbes*, May 2017, for a recent report about consumer confusion in the cell phone industry.

[2] For instance, firms can benefit from hidden fees (Gabaix and Laibson, 2006; Ellison and Ellison, 2009; Heidhues et al., 2016), spurious differentiation resulting from the credulity of consumers (Spiegler, 2006), product complexity (Gabaix and Laibson, 2004) or combative marketing (Eliaz and Spiegler, 2011a,b), from coarse thinking (Mullainathan et al., 2008), incomparable price formats (Carlin, 2009;

confusion may hurt firms when choice options are *heterogeneous*. This is surprising, as the scope for confusion with heterogeneous products is larger. There can be many ways to present the differences between products, and the dimensions that firms emphasize are likely to influence the perceived valuations. Nevertheless, the incentives to confuse consumers are less clear-cut, because firms usually obtain positive profits in differentiated goods markets even without obfuscation. Accordingly, by blurring the perception of consumers, obfuscation may reduce rather than increase profits.

These issues are not specific to consumption. For example, political candidates facing heterogeneous voters can reduce confusion by stating their policies clearly, or they can "becloud their policies in a fog of ambiguity" (Downs, 1957). The literature suggests that "complexity, obfuscation, vagueness, and uncertainty are permanent features of American electoral politics" (Gill, 2005, p. 372) and that candidates deliberately obscure their positions (Downs, 1957; Franklin, 1991; Tomz and Van Houweling, 2009; Rovny, 2012; Jacobson and Carson, 2015).

We thus introduce a general framework to uncover under which conditions strategic contestants (firms, political candidates, etc.), who compete for heterogeneous agents (consumers, voters, etc.), communicate their choice options clearly or ambiguously, respectively. Agents' true preferences are characterized by a distribution of match values for two contestants, who can influence their payoffs by choosing their *communication strategies* and *efforts*. Communication strategies are activities affecting the precision with which the addressees understand the value of the choice options. By contrast, efforts are activities that unambiguously help to make the contestant appear more attractive.

The central feature of communication strategies is that they jointly determine the agents' perceptions of product valuations, potentially distorting comparisons. *Confusion* arises if the perceived and true valuations disagree. We first assume that communication strategies induce stochastic perturbations, which do not affect the *average* valuation *differences*. Agent confusion then results in unbiased decision mistakes, meaning that the agents cannot be systematically fooled. Different from communication strategies, efforts unambiguously improve the evaluation of a contestant by the agents. Examples are price reductions (a lower price increases the relative attractiveness for all consumers) or certain advertising measures (a more prominent option captures more attention).

Piccione and Spiegler, 2012; Chioveanu and Zhou, 2013; Spiegler, 2014), from increasing consumer search costs (Ellison and Wolitzky, 2012), or from consumers lacking (intertemporal) self-control (Heidhues and Kőszegi, 2017); for comprehensive surveys, see Grubb (2015) and Heidhues and Köszegi (2018).

We formalize the above notions in a two-stage complete information game, where the contestants first simultaneously choose their communication strategies and then their efforts. Finally, each agent selects the contestant (buys a product, casts a vote) which she perceives as offering the higher value to her. Our main finding is that the contestants may prefer to educate rather than confuse, which is in sharp contrast with the case of homogeneous agents. While contestants always favor minimal competition, whether they can achieve this by confusing or educating agents depends critically on the true dispersion of preferences. We identify intuitive properties of the preference distribution that determine whether contestants will engage in obfuscation or education. Specifically, we distinguish between *indecisive* preferences, for which, loosely speaking, indifferent agents are relatively common, and *polarized* preferences, for which strong opinions prevail. For instance, in a textbook Hotelling model with symmetric firms, consumer preferences are indecisive (polarized) if the density of the consumer distribution has a maximum (minimum) at the midpoint of the Hotelling interval.

In our benchmark model, education is an equilibrium outcome with polarized preferences. Moreover, if the communication profiles can be ordered in terms of the error dispersion they induce (e.g., by mean-preserving spreads), education is the *only* equilibrium outcome, which is in strong contrast to conventional wisdom. By contrast, there is no equilibrium without confusion when preferences are indecisive. Further, if the error dispersions are ordered, then the unique equilibrium features maximal confusion, and the equilibrium communication profile leads to a more extreme dispersion of perceived valuations compared to the true distribution. Crucially, these results reflect the interplay between the shape of the true preference distribution and the effects of communication. Competition forces contestants to fight for the marginal agents, that is, for the agents who perceive the two options as equally valuable. The larger the mass of such perceptually indifferent agents, the fiercer is competition, and the less profitable the market becomes. When true preferences are indecisive, so that undecided "moderates" are common and "extremists" with strong opinions are rare, confusion reduces the mass of perceptually indifferent agents, as it converts more indecisive agents into extremists than vice versa.

While we are not aware of any systematic analysis on the shape of the valuation difference distribution across different product or political markets, casual evidence indicates that both polarized and indecisive dispersions are relevant. For example, consider the hospitality industry. It is hard to imagine that many guests will be indifferent when faced with the choice between a "family" hotel and a "business" hotel. Instead, most consumers will clearly prefer one alternative over the other, resulting in a polarized preference distribution across hotel categories. By contrast, some food products such as different brands of cereal or soft-drinks, seem to feature a large share of consumers that are truly indifferent between the various choice options, suggesting indecisive preferences. A famous case in point is the indifference of many study participants between Coke and Pepsi in blind tests (see Van Doorn and Miloyan, 2018); similarly for yoghurts (Maison et al., 2004). Even if an outside observer does not know which case applies, it seems reasonable to expect that the marketing divisions of the firms know which case applies to the market under consideration, and can adjust communication strategies accordingly. To investigate the determinants of communication strategies, empirical research should therefore attempt to get a better understanding of preference dispersions. Obtaining such evidence may be challenging precisely because observed preferences may have been influenced by the communication activities of firms or politicians, as predicted by our framework.

Our results for indecisive consumers may help to understand why heterogeneous labels persist in many food markets despite their confusing effects, or why a positive correlation between consumer confusion and brand loyalty may emerge (Kurtulmuşoğlu and Atalay, 2020). Conversely, with polarization undecided agents are rare and those with strong opinions are common. Education decreases the mass of perceptually indifferent agents, and thus arises in equilibrium. This may explain why it is very easy for consumers to identify "family-style" or "business-style" hotels in online market environments.[3]

Going beyond the benchmark model, we show that the role of the preference distribution remains similar if contestants differ in their effort costs, or if the chosen communication strategies may result in biased valuation differences. By contrast, when we allow for "massive" obfuscation, meaning that even the agents who are most loyal to a contestant could be confused enough to choose the dominated option, confusion may arise in equilibrium even with polarized preferences.

Under price competition, the welfare analysis differs substantially from the well-known homogeneous goods case where, absent a binding outside option, confusion merely redistributes rents from consumers to firms. With differentiated goods, confused consumers

[3]In an education equilibrium, we would expect the market to provide the tools or assistance that consumers need to choose in accordance with their true tastes. Indeed, popular infomediaries, such as tripadvisor.com, kayak.com or trivago.com, provide highly user-friendly navigational tools, that allows consumers to easily (un-)select family hotels, business hotels or wellness hotels, meaning that hotels of different categories are not likely to be forced into a tough competition with each other.

may choose dominated options. Thus, though confusion arises less often than with homogeneous goods, if it does, its effects are more severe.[4] We also show that policy measures directed at fostering competition, e.g., by means of product standardization, can backfire by increasing incentives to confuse. Further, we illustrate that a binding outside option for consumers need not diminish incentives to obfuscate with indecisive preferences: On the contrary, firms may deliberately choose to engage in obfuscation even if this means that some consumers (inefficiently) abstain from purchasing any product. This may help to understand why confusion remains prevalent in many cases, despite the development of a large body of "confusion reduction strategies" (Mitchell and Papavassiliou, 1999).[5]

Our analysis extends beyond the market context to contest settings. Specifically, when applied to political competition for voters, it explains why parties may choose ambiguous platforms rather than specifying their intended policies clearly (e.g., Shepsle, 1972; Aragones and Neeman, 2000; Callander and Wilson, 2008; Kartik et al., 2017).

The paper is organized as follows. Section 2 introduces the general framework. Section 3 presents the main results. Section 4 deals with price competition, Section 5 with all-pay contests. Section 6 concludes. Appendices A and S contain further results and the proofs.

2 The Model

We introduce the formal framework in Section 2.1, postponing discussion and interpretation to Section 2.2.

2.1 Competition and Confusion

A unit measure of agents needs to decide between the choice options offered by two contestants $i = 1, 2$. Agent preferences are characterized by a distribution of match values $(v_1^k, v_2^k) \in \mathbb{R}^2$ for the contestants, where the *match advantage* of contestant $i = 2$ for agent $k \in [0, 1]$ is given by $v_\Delta^k \equiv v_2^k - v_1^k$. The match advantages v_Δ^k are dispersed over the agent population according to an exogenous, commonly known CDF G_0 with a zero-symmetric density function g_0 (i.e., $g_0(x) = g_0(-x)$ $\forall x \in \mathbb{R}$).

[4]Our results contrast with Spiegler (2020), who asks whether agents with mis-specified causal models can be systematically fooled (to have biased expectations): We show that even if there is no average perception bias, contestants may exploit the agents as confusion softens competition.

[5]See Chernev et al. (2015) for a survey of related issues. The marketing literature has conjectured that consumer confusion might raise revenues (Mitchell and Papavassiliou, 1999; Mitchell and Kearney, 2002; Haan and Berkey, 2002), but this issue has not been further explored yet (Kasabov, 2015).

Each contestant has two instruments to influence the agents' choices. First, each contestant chooses a *communication strategy* $a_i \in A$. The chosen communication profile $(a_1, a_2) = \mathbf{a} \in \mathcal{A} \equiv A^2$ influences the agents' perceptions of match advantages. For any $\mathbf{a} \in \mathcal{A}$, agent k's *perceived match advantage* $\tilde{v}^k_\Delta(\mathbf{a})$ of contestant $i = 2$ is

$$\tilde{v}^k_\Delta(\mathbf{a}) = v^k_\Delta + \varepsilon_{\mathbf{a}}, \tag{1}$$

where $\varepsilon_{\mathbf{a}}$ is a (possibly degenerate) random variable with CDF $\Gamma_{\mathbf{a}}$ independent of v^k_Δ.[6]

Second, each contestant can exert an "*effort*" $s_i \in \mathcal{S} \subset \mathbb{R}$ to persuade the agents to choose in his favor, given a dispersion of perceived match advantages $\tilde{v}^k_\Delta$. A higher s_i unambiguously increases every agent's evaluation of contestant i relative to his competitor, where agent k chooses $i = 1$ if and only if $\tilde{v}^k_\Delta(\mathbf{a}) \leq s_1 - s_2$, i.e., the perceived match advantage of contestant 2 is smaller than the effort advantage of contestant 1.

Let $G_{\mathbf{a}}$ denote the distribution of the perceived match advantages $\tilde{v}^k_\Delta(\mathbf{a})$ of contestant $i = 2$. For any given communication-effort profile $(\mathbf{a}, (s_1, s_2))$, the *market shares*, i.e., the fraction of agents who choose contestant $i = 1$ and 2, are $G_{\mathbf{a}}(s_1 - s_2)$ and $1 - G_{\mathbf{a}}(s_1 - s_2)$, respectively. We consider the following general form of the contestants' expected payoffs

$$\begin{aligned} \Pi^{\mathbf{a}}_1(s_1, s_2) &= R(s_1, s_2) G_{\mathbf{a}}(s_1 - s_2) - C(s_1), \\ \Pi^{\mathbf{a}}_2(s_1, s_2) &= R(s_2, s_1)\left(1 - G_{\mathbf{a}}(s_1 - s_2)\right) - C(s_2), \end{aligned} \tag{2}$$

where both $R : \mathbb{R}^2 \to \mathbb{R}_+$ and $C : \mathbb{R} \to \mathbb{R}_+$ are continuously differentiable. The idea behind (2) is that, besides affecting market shares, the efforts could be costly and influence the revenues earned per unit of market share.

We study the competition between the contestants as a two-stage complete information game. In the *communication stage*, both contestants simultaneously choose their communication strategies $a_i \in A$. In the subsequent *effort stage*, they decide how much effort s_i to exert, upon observing the chosen communication profile.[7] We do not consider mixed strategy equilibria in either stage. Our main interest is whether competition leads to a communication profile which educates consumers about true valuations, or whether

[6]Our main insights do not hinge on independence of $\varepsilon_{\mathbf{a}}$ and v^k_Δ; see Appendix S.7 for an example.

[7]We view communication strategies as the long-term decision variable, relative to efforts, which motivates the timing of the game. For example, in the case of price competition adjusting the design of a marketing campaign or the strategy of brand-image building is more laborious and less instantaneous for firms compared to adjusting prices (the "effort" variable). This also is a notable difference to the literature on hidden fees or add-on costs, which typically studies a simultaneous-move game.

competition results in confusion instead. The following definition clarifies these notions.[8]

Definition 1 *A communication profile* $\mathbf{a} \in \mathcal{A}$ *induces agent confusion (is obfuscating) if* $\tilde{v}^k_\Delta(\mathbf{a})$ *and* v^k_Δ *are not equal in distribution, i.e.,* $G_\mathbf{a} \neq G_0$. *A communication profile* $\mathbf{a} \in \mathcal{A}$ *induces agent education (is educating) if* $G_\mathbf{a} = G_0$.

In the main analysis, we suppose $\varepsilon_\mathbf{a}$ in (1) is zero-symmetrically dispersed $\forall \mathbf{a} \in \mathcal{A}$, so that communication has an *unbiased* effect on perceived match advantages.[9] Hence, for $\mathbf{a} \in \mathcal{A}$, either (i) $\varepsilon_\mathbf{a} = O$, where O is any random variable that is (almost surely) equal to zero, or (ii) $\Gamma_\mathbf{a}(x) = 1 - \Gamma_\mathbf{a}(-x)\ \forall x \in \mathbb{R}$. If $\varepsilon_\mathbf{a} \neq O$, we assume $\Gamma_\mathbf{a}$ has a density $\gamma_\mathbf{a}$. The symmetry of G_0 and unbiasedness of $\varepsilon_\mathbf{a}$ imply that the perceived match advantage distribution, given by $G_\mathbf{a}(v) = \int G_0(v-e)d\Gamma_\mathbf{a}(e)$ with density $g_\mathbf{a}(v) = \int g_0(v-e)d\Gamma_\mathbf{a}(e)$, is zero-symmetric. Moreover, these expressions accentuate the interaction between true preferences (G_0) and the perception errors ($\Gamma_\mathbf{a}$) induced by different communication profiles, which will be decisive for whether confusion or education are equilibrium phenomena.

2.2 Interpretation and Discussion of the Framework

We first motivate our notion of agent confusion and then offer several interpretations for second-stage effort competition.

2.2.1 Agent Confusion

In our main analysis, we assume that agents are confused in their comparisons *in an unsystematic way.* From an analytical perspective, the focus on unbiased communication is helpful to isolate whether the contestants desire a precise, unequivocal or an ambiguous, unclear type of communication. Thus, in a sense the unbiasedness assumption plays a similar disciplining role as the "conformity with the prior" assumption in Bayesian models of persuasion (Kamenica and Gentzkow, 2011), costly information acquisition (Caplin and Dean, 2015), or information design (Armstrong and Zhou, 2019).[10] Note that unbiasedness does not rule out that communication may bias the *levels* of the perceived match

[8] Given our interest in the strategic incentives to confuse or educate, we define these concepts in terms of final effects that the various communication profiles have on perceived match advantages. As such, we remain agnostic about details of the information or signal structure that firms may use to achieve their goals or about the update process consumer may use; studying such aspects is of interest in its own right but beyond the scope of this paper.

[9] Gabaix and Laibson (2004),Kalaycı and Potters (2011), and Gaudeul and Sugden (2012) consider a similar structure of "decision utility" in the homogeneous goods case.

[10] Unlike in Bayesian models, the agents in our model can be swayed by the face value of communication. Therefore, the standard logic with rational agents, stating that complete information provision

values $(\tilde{v}_1^k, \tilde{v}_2^k)$; we consider such a possibility in Section S.2. Moreover, unbiasedness does not require v_Δ and $\varepsilon_{\mathbf{a}}$ to be independent; see Appendix S.7 for an example. Finally, communication that leads to unsystematic perception mistakes is hard to interpret by an agent population. It is therefore not immediately obvious whether a standard unravelling logic would apply even with fully rational agents; analyzing this issue is beyond the scope of this paper.

Another reason to focus on unbiased communication is the extensive evidence in marketing and more broadly in perceptual psychology and neuroscience, documenting that consumer confusion is manifested through *unsystematic* and *heterogeneous* perception mistakes; see Walsh et al., 2007; Kasabov, 2015 for surveys. As we now argue intuitively, such confusion can reflect product complexity, attribute uncertainty or limited comparability, e.g., due to framing; for a formalization see Appendix S.3.

Complexity The marketing literature documents that confusion occurs once consumers are "confronted with more product information and alternatives than they can process in order to get to know, to compare and to comprehend alternatives" (Walsh et al., 2007), particularly if the choice alternatives are complex, e.g., due to a large number of attributes or marketing messages (Jacoby, 1977; Malhotra, 1982). Such complexity confusion was found for computers, mobile phones, automobiles, digital cameras, buildings or insurance policies (see Walsh et al., 2007; Kasabov, 2015; Mützel and Kilian, 2016), and has been associated with product packaging (Mitchell and Papavassiliou, 1999), or lengthy and complicated contracts involving "fine print" (Turnbull et al., 2000). It has been linked to heterogeneous perceptions (Solomon et al., 2014), a lack of critical evaluation and reduced decision accuracy (Eppler and Mengis, 2004), or a "failure to develop correct interpretations of various facets of a product or service" (Turnbull et al., 2000).

We can capture complexity confusion by assuming that each firm can choose "features" (attributes, advertising slogans,...), each of which has an i.i.d. effect on how consumers evaluate his product. Then, the number $a_i \in A \subset \mathbb{N}$ of features corresponds to the communication strategy of firm i, and quantifies i's contribution to agent confusion. This model presumes that adding more features increases confusion in the comparison of the products, because the perception of individual products becomes more diffuse. However, as we now illustrate, (1) captures more general notions of confusion.

softens competition as it imparts horizontal differentiation through more dispersed posteriors (see, e.g., Anderson and Renault, 2009), does not transfer to our setting.

Product Labels A central aspect of (1) is that communication strategies may not only affect the perceived valuation for the respective contestant, but also for the competitor. This is in line with a general insight that confusion in product comparisons is a synthetic phenomenon of all marketing messages interacting with each other (Mützel and Kilian, 2016). For instance, comparisons may be impeded if one food brand uses the label "original" while another one uses "authentic" (Langer et al., 2007; Leek et al., 2015). Likewise, a study by the British Food Advisory Committee concluded that food labels like "fresh", "natural", "original" or "pure" lead to confusion because they seem similar but still can mean different things for different consumers. To quote the principal policy advisor at the British Consumer Association: "Labels are all too often more of a marketing gimmick than a way of providing meaningful information to help consumers make choices about the food they eat."[11] As we detail in Appendix S.3, we can accommodate confusion due to heterogeneous labeling in (1) by assuming that the communication strategies aggregate in a non-independent manner to unsystematic perception errors $\varepsilon_{\mathbf{a}}$.

Frames The possibility that a communication strategy of a contestant could affect the evaluation of the competitor is consistent with the approaches of Carlin (2009), Piccione and Spiegler (2012), Chioveanu and Zhou (2013) or Spiegler (2014). These authors study homogeneous goods models with price competition, where the mutual choice of a "frame", i.e., a way to present the product price, determines if a consumer can compare products. The notion of framing can be incorporated in our setting by assuming that communication profiles induce a probability distribution over the different frames that a consumer could adopt to compare the products.

Hiding Product Features The behavioral IO literature has argued that firms may hide disadvantageous features from consumers (Ellison, 2005; Gabaix and Laibson, 2006; Dahremöller, 2013; Heidhues et al., 2016). On a related note, we can interpret the communication strategy as determining the precision with which the firms inform about hidden features. Imprecise communication then leads to heterogeneity in consumer opinions about these features which influences the ensuing price competition. If a firm chooses not to advertise a certain attribute as part of its communication strat-

[11] See "Report reveals food label confusion," *The DailyMail*, July 25, 2001. Also see the report by GlobalVision emphasizing that "unless you work in the industry, it is very difficult to decipher food labels accurately." ("The nutrition facts: Food label confusion", July 2016.)

egy, this forces consumers to guess the relevant valuation of the product. We think of such guessing as idiosyncratically sampling the valuations from a given interval, where the range of the possible valuations correlates with the amount of information that is hidden from consumers by firms (see Appendix S.3 for a formalization). This means that some consumers may become overly optimistic about a certain firm, while others become equally optimistic about the other firm. In this context, we ask whether rational contestants would ever seek to implement features with such double-edged effects.

Among the attributes that firms might want to hide, the literature has focused on add-on fees, meaning that consumers misperceive the effective product price (see Grubb, 2015 for a survey). While formulation (1) pertains to misperceptions in valuations, it could also capture that communication strategies can amount to choosing the presentation of the final price, rather than of gross match advantages. What ultimately matters for our analysis only is that the agents may be confused about *net* match advantages, which are composed of perceived gross match advantages and perceived price differences.[12]

Agent Naiveté Behavioral IO models often partition the agent population into "naives" and "sophisticates", where the latter do not show any behavioral bias. Likewise, marketing and psychometric research has shown that consumers may differ in how susceptible they are to obfuscation, captured by their individual "confusion proneness" (Walsh et al. (2007) and Murray (1993)). Generalizing this idea, we show in Appendix S.3 that formulation (1) and the subsequent equilibrium analysis apply if naiveté is an exogenous trait that is dispersed by some random variable over the population.

2.2.2 Second-Stage Competition

The unbiasedness assumption notwithstanding, reality suggests that contestants may have powerful means to bias the audience in their favor. For example, a ceteris paribus increase of advertising or campaigning intensities by a politician may bias the evaluation by the agents towards that politician by capturing their attention. A similar argument applies to unilateral price cuts in price competition. As we will exemplify next, our model reflects such activities in the second-stage effort variable.[13]

As a leading example, price competition between differentiated firms can be expressed

[12] However, one might argue that the size of the consumers' errors could depend on the realized price vector, which would then not be consistent with (1).

[13] In Section 5.2, we explore the implications when communication has *asymmetric* effects.

by defining $p_i \equiv -s_i$ as the price set by firm i and letting $\mathcal{S} \equiv (-\infty, 0]$ (hence $G_{\mathbf{a}}(s_1 - s_2) = G_{\mathbf{a}}(p_2 - p_1)$), as well as $C(s_i) = 0$ and $R(s_i, s_j) \equiv -s_i - c \ \forall s_i, s_j$, where $c \geq 0$ is the constant unit cost. Likewise, the model of *retail bank competition* for customer deposits (see Freixas and Rochet, 2008) can be integrated by assuming that efforts correspond to the interest rates granted to depositors.

The general payoff functions (2) allow us to show that our results on price competition similarly apply to other important forms of competition. As an example, suppose firms are limited in their pricing flexibility by regulation, but may resort to quality competition instead. We can capture this by interpreting $R(s_i, s_j)$ as quality-dependent revenues at regulated prices and $C(s_i)$ as quality costs. Similarly, firms could compete in advertising efforts s_i to persuade consumers to choose their products. Here, the formulation $R(s_i, s_j)$ allows that the efforts of both firms could jointly determine willingness-to-pay, e.g., as in Von der Fehr and Stevik (1998), and $C(s_i)$ are advertising costs. In a *political economy* interpretation, we think of parties as competing for voters. Then the communication strategies a_i correspond to the ambiguity of the political platforms (see Section 5.3), whereas efforts s_i reflect campaigning intensity. We interpret the "market share" $G_{\mathbf{a}}$ as the share of favorable voters, $C(s_i)$ as the campaigning costs, and $R(s_i, s_j)$ as the value of recruiting an additional voter.

A common feature of these latter applications is that effort costs $C(s_i)$ arise independently of the equilibrium market shares $G_{\mathbf{a}}$, so that the game becomes an all-pay *contest.* By contrast, the "efforts" in strategic price competition amount to implicit costs due to price reduction that depend directly on the equilibrium market shares of each firm. While all-pay contests differ from price competition with respect to the cost structure, our main results show that the incentives to confuse or to educate are similar in both settings.

3 Equilibrium Analysis

We now derive the SPE of the game. Section 3.1 characterizes the symmetric second-stage effort equilibrium. Section 3.2 shows how preferences determine whether confusion arises in equilibrium. Section 3.3 sharpens the predictions. Section 3.4 discusses generalizations.

3.1 The Effort Stage

Together with the symmetry of G_0 and $\Gamma_{\mathbf{a}}$, payoff functions (2) imply that effort competition is a symmetric subgame for any given $\mathbf{a} \in \mathcal{A}$. We focus on interior symmetric equilibria ($s_1 = s_2 = s > 0$) in the effort stage which, if they exist, are characterized by the first-order condition

$$\frac{\partial \Pi_i^{\mathbf{a}}(s,s)}{\partial s_i} = \frac{1}{2} R_1(s,s) + R(s,s) g_{\mathbf{a}}(0) - C'(s) = 0, \tag{3}$$

where $G_{\mathbf{a}}(0) = 1/2$, R_ℓ denotes the partial derivative of R with respect to its ℓ-th argument, and $g_{\mathbf{a}}(0) \equiv \int g_0(-e) d\Gamma_{\mathbf{a}}(e) > 0$ is the measure of agents who *perceive* the two contestants as equally valuable after exposure to the communication profile $\mathbf{a}$.[14] To interpret condition (3), we rewrite it per unit $R(s,s)$ of revenue earned, i.e.,

$$g_{\mathbf{a}}(0) = \underbrace{\frac{1}{R(s,s)} \cdot \left(C'(s) - \frac{R_1(s,s)}{2} \right)}_{\equiv z(s)}. \tag{4}$$

Plainly, the LHS of (4) quantifies the marginal market share earned from increasing own efforts; it reflects the intuition that competition forces contestants to concentrate on perceptually indifferent agents. The RHS of (4) amounts to *net* marginal effort costs per unit of revenue.[15] We impose a technical assumption (which holds in our major applications) to assure that a *unique* symmetric effort equilibrium exists in every subgame, and it is indeed determined by equation (4).

Assumption 1 *The following conditions are satisfied:*

(A1.1) $\Pi_i^{\mathbf{a}}(s_i, s_j)$ *is strictly quasi-concave in* s_i, $\forall s_j \in \mathcal{S}$, $\mathbf{a} \in \mathcal{A}$ *and* $i, j = 1, 2$.

(A1.2) $\forall \mathbf{a} \in \mathcal{A}\ \exists\, s \in \mathcal{S}$ *such that* $g_{\mathbf{a}}(0) = z(s)$,

(A1.3) $z(s)$ *is strictly increasing, and* $R_1(s,s) + R_2(s,s) < 2C'(s)\ \forall s \in \mathcal{S}$.

Lemma 1 *Suppose that Assumption 1 holds. For any* $\mathbf{a} \in \mathcal{A}$ *a unique pure-strategy symmetric equilibrium exists, and both contestants choose* $s^*_{\mathbf{a}} = z^{-1}(g_{\mathbf{a}}(0))$. *The equilibrium payoff is strictly decreasing in* $g_{\mathbf{a}}(0)$ *for each contestant.*

[14] Symmetric equilibria are the generic equilibrium type in symmetric (stage) games as the one studied here (Hefti, 2017).

[15] As an illustration, the case of a diminishing revenue $R_1(s,s) < 0$ has the effect of increasing net costs $z(s)$ above explicit costs per unit of revenue $C'(s)/R(s,s)$.

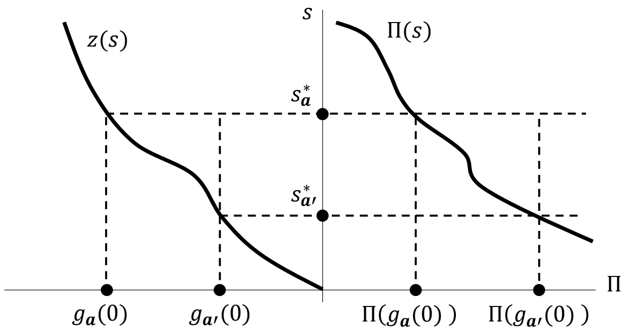
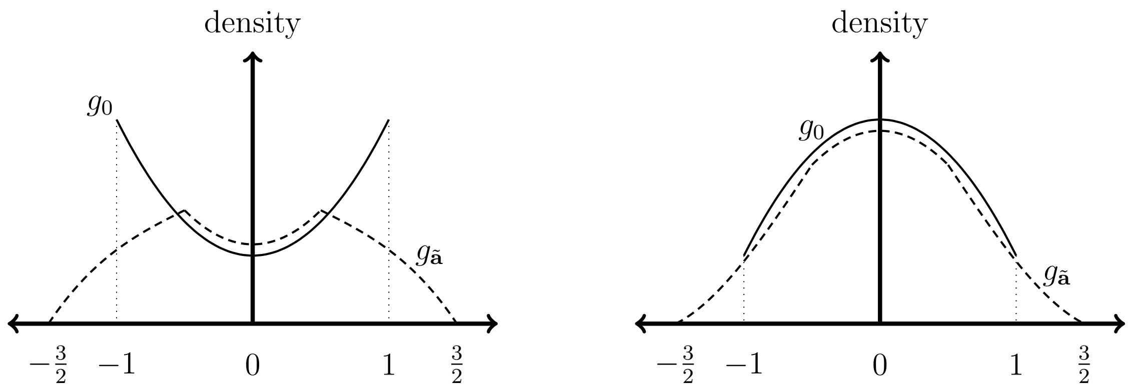


Figure 1: Equilibria in the effort stage for two different values $\mathbf{a}, \mathbf{a}'$ (left), and the corresponding level of equilibrium payoffs (right)

Figure 1 illustrates Lemma 1. (A1.1) and (A1.2) jointly assure equilibrium existence, where the first-order condition $g_{\mathbf{a}}(0) = z(s^*_{\mathbf{a}})$ determines equilibrium effort $s^*_{\mathbf{a}}$. Next, the standard regularity condition (A1.3) on $z(\cdot)$ states that marginal benefits must intersect marginal costs from above. It assures that the equilibrium effort $s^*_{\mathbf{a}}$ is uniquely determined for any given $\mathbf{a} \in \mathcal{A}$. This assumption also implies that an exogenous increase in the measure of perceptually indifferent agents $g_{\mathbf{a}}(0)$ intensifies competition, resulting in a higher equilibrium effort. Finally, the second part of (A1.3) means that, under optimal behavior, more effort increases revenue less than expenditures explaining the negative correlation between equilibrium payoffs and efforts (see the right panel of Figure 1).

3.2 Communication Behavior

Because of Lemma 1, whether contestants want to confuse or educate agents depends on how communication affects the measure of perceptually indifferent agents $g_{\mathbf{a}}(0)$. The contestants' interests are fully aligned *at this stage*: They both want to minimize $g_a(0)$ to soften competition. To see the implications of this common goal, first note that $g_{\mathbf{a}}$ is the convolution of two zero-symmetric functions g_0 and $\Gamma_{\mathbf{a}}$. This shows that the dispersion of true preferences g_0 has major implications on how communication affects $g_{\mathbf{a}}(0)$. To illustrate, suppose g_0 is convex around zero, and the chosen communication strategies add a small amount of noise to perception, captured by a non-degenerate, zero-symmetric CDF $\Gamma_{\mathbf{a}}$. This induces an upward shift of the dispersion of perceived valuations, $g_{\mathbf{a}}$, around zero; see Figure 2(a). By contrast, if g_0 is concave around zero, adding the same noise reduces $g_{\mathbf{a}}(0)$, thereby softening second-stage competition; see Figure 2(b). These arguments suggest that the local properties of g_0 around zero determine equilibrium



(a) With polarization, $g_0(0) < g_{\tilde{\mathbf{a}}}(0)$. (b) With indecisiveness, $g_0(0) > g_{\tilde{\mathbf{a}}}(0)$.

Figure 2: The heterogeneous effects of agent confusion, $\varepsilon_{\tilde{\mathbf{a}}} \sim U[-0.5, 0.5]$.

communication, motivating the following properties of true preferences.

Definition 2 *Let $\delta > 0$ be such that $[-\delta, \delta] \subset supp(g_0)$.*

(i) ***(Indecisiveness)*** *True match advantages are*

(a) weakly δ-indecisive if $g_0(0) > g_0(x)\ \forall x \in [-\delta, 0) \cup (0, \delta]$,

(b) δ-indecisive if g_0 is strictly quasi-concave on $[-\delta, \delta]$, and

(c) strongly δ-indecisive if g_0 is strictly concave on $[-\delta, \delta]$.

(ii) ***(Polarization)*** *True match advantages are*

(a) weakly δ-polarized if $g_0(0) < g_0(x)\ \forall x \in [-\delta, 0) \cup (0, \delta]$,

(b) δ-polarized if g_0 strictly quasi-convex on $[-\delta, \delta]$, and

(c) strongly δ-polarized if g_0 is strictly convex on $[-\delta, \delta]$.

Strong δ-indecisiveness implies δ-indecisiveness and thus weak δ-indecisiveness. While weakly δ-indecisive preferences only require that indifference ($v_\Delta = 0$) occurs more often than all other alternatives on $[-\delta, \delta]$, the two other concepts assure that less pronounced valuation differences occur more frequently than more pronounced ones. The relation between the different concepts of polarization is similar. Our most general result (Theorem 1) only requires the weakest notions of indecisiveness and polarization; the stronger concepts help to obtain equilibrium uniqueness and monotonicity (Theorem 2).

The analysis of SPE requires more structure on the communication technology $\mathcal{A}$. Let $\mathcal{A} \subset \mathbb{R}^2_+$, and assume $\mathbf{0}$ is an educating profile, i.e., $\varepsilon_{\mathbf{0}} = O$. For definiteness, we further suppose that (i) agent education is among the feasible options for the contestants, and (ii) each contestant can always force some confusion unilaterally. As we sketch in Section 3.4, it is straightforward to adjust the analysis to the cases where (i) or (ii) is violated.

Assumption 2 *The set of feasible communication profiles $\mathcal{A} \subset \mathbb{R}^2_+$ verifies:*

(A2.1) $\mathbf{0} \in \mathcal{A}$.

(A2.2) $\forall i = 1, 2$, $j \neq i$ *and* $\forall a_j \in A$, $\exists\, a_i \in A$, *such that* $\varepsilon_{(a_i,a_j)} \neq O$.

We are now ready to state our first main result.

Theorem 1 *Suppose that Assumptions 1 and 2 hold.*

(i) If there exists $\delta > 0$ with supp($\varepsilon_{\mathbf{a}}$) $\subset [-\delta, \delta]$ $\forall \mathbf{a} \in \mathcal{A}$ and the true match values are weakly δ-polarized, then an SPE without consumer confusion exists.

(ii) If there exists $\delta > 0$ with supp($\varepsilon_{\mathbf{a}}$) $\subset [-\delta, \delta]$ $\forall \mathbf{a} \in \mathcal{A}$ and the true match values are weakly δ-indecisive, then no SPE without consumer confusion exists.

Theorem 1 vindicates the decisive role of the preference dispersion for the communication equilibrium. The rationale is intuitive. Every obfuscating communication profile (i.e., $\varepsilon_{\mathbf{a}} \neq O$) distorts the perceived distribution of match advantages. Thus, some truly indifferent agents come to perceive one contestant as strictly superior, while other agents, who favor one contestant, may become indifferent. By Lemma 1, the contestants benefit from confusion if and only if the former effect dominates the latter. With *polarized* match advantages, confusion pushes more agents towards indifference than vice versa, which intensifies the competition for market shares in the effort stage, as illustrated in Figure 2(a). This shows that both contestants have a strict incentive to avoid obfuscation. Because full agent education is feasible by (A2.1), education must be part of an SPE.

By contrast, confusion reduces the measure of perceptually indifferent agents if true match values are indecisive, as Figure 2(b) shows. Thus, even though the contestants are fully aware that some agents switch to the competitor in an obfuscated market, confusion is an effective means to soften competition. As each contestant can unilaterally force some confusion on the market by (A2.2), education cannot be an equilibrium outcome.

3.3 How Much Confusion?

So far, we have not put any order structure on the noise distributions. Given unbiased consumer confusion, it seems natural to assume that the distribution functions $\{\Gamma_{\mathbf{a}}\}_{\mathbf{a}\in\mathcal{A}}$ are ordered by statistical principles, such as by *mean-preserving spreads* (MPS). A random variable X is an MPS of another random variable Y, if Y is distributed as $X+\eta$, where $\eta \neq O$ and $E[\eta|X]=0$. That is, Y is a noisy version of X. Assuming an order structure like MPS yields stronger equilibrium predictions.[16]

Assumption 3 (MPS ordering) *$A \subset \mathbb{R}_+$ is compact, and $\varepsilon_{\mathbf{a}} = O \Leftrightarrow \mathbf{a} = \mathbf{0}$. Moreover, $\forall \mathbf{a}, \mathbf{a}' \in \mathcal{A}$ with $\mathbf{a} \neq \mathbf{a}'$ and $\mathbf{a} \leq \mathbf{a}'$, $\Gamma_{\mathbf{a}'}$ is an MPS of $\Gamma_{\mathbf{a}}$ ($\Gamma_{\mathbf{a}'} \succeq_{MPS} \Gamma_{\mathbf{a}}$).*

With the MPS order, the communication profile where both contestants choose $\bar{a} \equiv \max A$ ($\underline{a} \equiv \min A$) induces maximal (minimal) agent confusion as measured by the *variance* of the perceived match advantages. Further, Assumption 3 implies Assumption 2 if $0 \in A$ and A contains more than one element. An example of an MPS is an increase in the number of i.i.d features implemented by a contestant (see Section 2.2.1). Similarly, the MPS applies when the members of the family $\{\Gamma_{\mathbf{a}}\}_{\mathbf{a}\in\mathcal{A}}$ are *truncations* of each other.

Replacing Assumption 2 with Assumption 3, we can strengthen Theorem 1:

Theorem 2 *Suppose that Assumptions 1 and 3 hold.*

(i) If there exists $\delta > 0$ such that supp$(\varepsilon_{\mathbf{a}}) \subset [-\delta, \delta], \forall \mathbf{a} \in \mathcal{A}$ and true match values are strongly δ-indecisive, then there exists a unique SPE, and confusion is maximal.

(ii) If there exists $\delta > 0$ such that supp$(\varepsilon_{\mathbf{a}}) \subset [-\delta, \delta], \forall \mathbf{a} \in \mathcal{A}$ and true match values are strongly δ-polarized, then there exists a unique SPE, and confusion is minimal.

Intuitively, by Assumption 3, a contestant i can unilaterally enforce more confusion as long as $a_i < \bar{a}$. This results in a smaller mass of perceptually indifferent consumers when preferences are strongly indecisive. Thus, each firm will choose a_i maximally for indecisive preferences and both contestants coordinate on the communication profile that induces maximal confusion. The opposite reasoning applies in the strongly polarized case.

3.4 Discussion and Generalizations

We first argue that our main insights hold more generally, and convey additional insights. Then we extend the analysis to the case where obfuscation opportunities become large.

[16] See Appendix S.4 for an alternative order structure.

Relaxing Assumption 2 Assumption 2 is not universally applicable. First, violating part (i), cognitive limitations of the consumers may make full education impossible. Appropriately modified, Theorem 2 applies even in this case: Both contestants would coordinate on the SPE with minimal agent confusion by playing $\underline{\mathbf{a}}$ in the communication stage under polarization. Second, Assumption 2(ii) can clearly be appropriate in some circumstances: For example, confusion may already arise if only one competitor uses a product label while the other does not. The case where all firms seek to hide a given disadvantageous feature may be equally relevant: Then, if one firm decides to disclose this feature, all consumers become educated about the market. Such a strong form of education, which seems particularly relevant with homogeneous choice options, was studied by Heidhues et al. (2016). If, contradicting 2(ii), education could be enforced unilaterally, then part Theorem 1(i) would be strengthened in that only SPE with education exist. More generally, there could be SPE with education, regardless of the true match value distribution. However, with indecisive preferences any SPE featuring confusion strictly dominates such an education equilibrium from the perspectives of the contestants.

The above discussion shows that assumptions on the communication technology $\mathcal{A}$ may affect results. For instance, $\mathcal{A}$ can express whether the environment is *education-favoring* or *confusion-favoring* (Heidhues and Köszegi, 2018). A flexible way of formalizing such notions is to assume that communication profiles map to parameter values $f(\mathbf{a}) = \omega_{\mathbf{a}} \in \mathbb{R}$, where the corresponding CDF's $\Gamma_{\mathbf{a}}$ are ordered by the MPS criterion (i.e., $\Gamma_{\mathbf{a}'} \succeq_{MPS} \Gamma_{\mathbf{a}}$ if $\omega_{\mathbf{a}'} \geq \omega_{\mathbf{a}}$). Then, the function $f(a_1, a_2) = \min\{a_1, a_2\}$ captures an education-favoring communication technology, where one firm can ensure low confusion on its own, while $f(a_1, a_2) = \max\{a_1, a_2\}$ is confusion-favoring; mixed cases are also conceivable.[17] Our findings do not rely on a specific functional structure. The details of the communication technology $\mathcal{A}$ can affect certain aspects of the equilibrium set, but our analysis shows that the general tenor about the role of preferences remains the same: Contestants desire education with polarized and confusion with indecisive preferences.

Relaxing Assumption 3 The requirement of an MPS ordering in Assumption 3 is not necessary for the existence of an equilibrium with maximal (or minimal) confusion: It suffices to assume that there exist $\bar{\mathbf{a}}, \underline{\mathbf{a}} \in \mathcal{A}$, such that $\Gamma_{\bar{\mathbf{a}}} \succeq_{MPS} \Gamma_{\mathbf{a}} \succeq_{MPS} \Gamma_{\underline{a}}\ \forall \mathbf{a} \in \mathcal{A}$, so that $\bar{\mathbf{a}}$ ($\underline{\mathbf{a}}$) generates more (less) confusion than any other communicate profile.

[17]For example, the technology $f(a_1, a_2) = \max\{a_1 + a_2, 0\}$, $a_i \in [-\bar{a}, \bar{a}]$, captures that firms may have means to "de-obfuscate" the communication of the other firm.

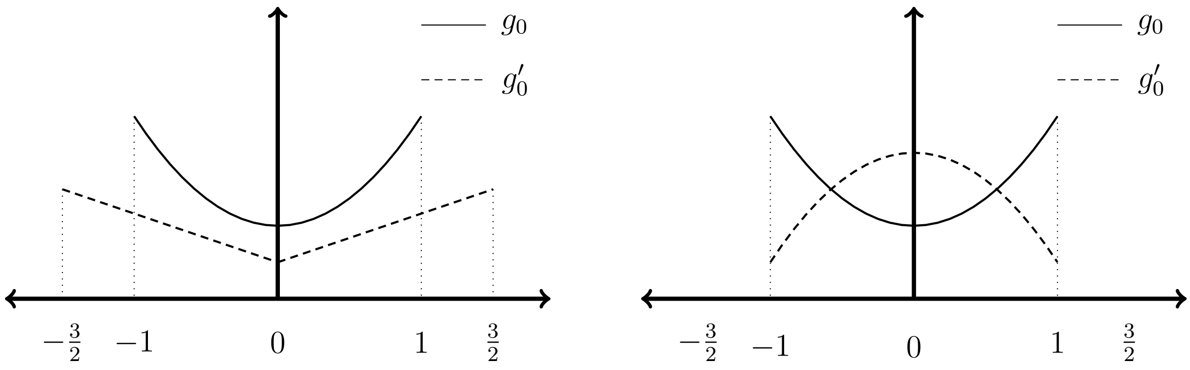
Under this weaker assumption, the set of feasible communication profiles $\mathcal{A}$ is no longer completely ordered, and multiple equilibria are possible. However, the contestants should prefer to coordinate on the equilibrium with maximal (minimal) confusion, because that would maximize each of their expected payoffs given that the true match values are strongly indecisive (strongly polarized). Further, in Appendix S.4, we provide a variant of Theorem 2 that invokes indecisiveness (polarization) rather than strong indecisiveness (strong polarization). In order to handle the more general preferences, we need to use a more restrictive ordering defined on $\mathcal{A}$ than MPS, which we call *two-sided single crossing.*

Massive Confusion Theorems 1 and 2 apply when true preferences are dispersed enough that the scope for confusion is constrained by the degree of taste differentiation, i.e., $supp(\varepsilon_{\mathbf{a}}) \subset supp(g_0)$ $\forall \mathbf{a} \in \mathcal{A}$. Thus, for equal second-stage efforts, obfuscation never induces agents with the strongest preferences for one contestant to switch to the other one. To allow for such reversals (where true tastes are narrowly dispersed), we say that confusion can be *massive* whenever $supp(g_0) \subsetneq supp(\varepsilon_{\mathbf{a}})$ for some $\mathbf{a} \in \mathcal{A}$. We use the tractable case of uniformly distributed perception errors $\varepsilon_{\mathbf{a}}$ to show that, even with polarized preferences, obfuscation may arise if massive confusion is possible.

Theorem 3 *Suppose that Assumptions 1 and 3 hold, and $\varepsilon_{\mathbf{a}}$ is uniformly distributed on $[-\omega_{\mathbf{a}}, \omega_{\mathbf{a}}]$, where $\omega_{\mathbf{a}} \geq 0$, $\forall \mathbf{a} \in \mathcal{A}$. Let $\bar{\omega} \equiv \max_{\mathbf{a} \in \mathcal{A}} \omega_{\mathbf{a}}$. Then, the unique SPE features maximal confusion ($\omega_{\mathbf{a}^*} = \bar{\omega}$) if either (i) true match values are indecisive on $supp(g_0)$, or (ii) true match values are polarized on $supp(g_0)$ and $\bar{\omega}$ is sufficiently large.*

For indecisive agents, the prediction from Theorem 2 that contestants want to obfuscate as much as possible immediately generalizes to the case of massive confusion. However, maximal confusion can now arise as the unique SPE, despite polarized preferences: If contestants can induce sufficiently large differences in the perceived match advantages, then the measure of indifferent agents must eventually decrease, regardless of g_0.[18] Finally, the key insight from the literature on competition with boundedly rational consumers and *homogeneous* products, according to which obfuscation arises in equilibrium, can be seen as the limiting case of Theorem 3: If true match advantages are arbitrarily close to zero, virtually any confusion is massive, so that contestants must benefit from introducing it.

[18]Note, however, that Theorem 3 relies on the assumption that agents do not choose an outside option. In Section 4.3, we allow for confusion that is so large that agents may opt for the outside option ex post.



(a) Unequal taste differentiation

(b) Equal taste differentiation

Figure 3: Taste differentiation and preference distribution

Taste Differentiation and Taste Dispersion It is instructive to link our central concept of taste dispersion to an alternative idea that is often used in locational models, namely that tastes are more differentiated the larger the (Lebesgue) measure $\lambda(supp(g_0))$ is. Theorem 1 relies only on the shape of g_0 rather than specifically on the extent of taste differentiation. For example, if preferences are δ-indecisive, confusion is the only possible equilibrium, even with arbitrarily large taste differentiation. To illustrate, suppose that $supp(\gamma_{\mathbf{a}}) \subset [-1, 1]\ \forall \mathbf{a} \in \mathcal{A}$. For g_0 and g_0' in Figure 3(a), taste differentiation differs, but the SPE involves education. By contrast, g_0 and g_0' in Figure 3(b) have the same degree of taste differentiation, but an education equilibrium only exists for g_0.

4 Price Competition

In this section, we apply our general framework to study price competition between firms. Section 4.1 provides the main results. Section 4.2 considers welfare, dealing in particular with pitfalls of competition-enhancing regulation. In Section 4.3, we study the case of a binding outside option, and relate our model to the empirical literature on choice overload.

4.1 Main Results with Price Competition

We now capture price competition by specifying $\mathcal{S} = (-\infty, 0]$ and $R(s_i, s_j) = -s_i$, $C(s_i) = 0$, and letting $p_i = -s_i$ be the price of firm i (see Section 2.2.2), so that the effort stage becomes the *pricing stage*. To apply the results from Section 3, we need to verify that the conditions in Assumption 1 are satisfied. (A1.3) always holds because

$$z(s) = -\frac{1}{2s}, \text{ and } R_1(s,s) + R_2(s,s) - 2C'(s) = -1 < 0, \forall s = -p \leq 0.$$

The following proposition provides a simple set of sufficient conditions on the distribution functions G_0 and $\Gamma_{\mathbf{a}}$ assuring that (A1.1) and (A1.2) hold, allowing us to identify the unique symmetric pure-strategy price equilibrium of the pricing stage.

Proposition 1 *Suppose that: (i) G_0 is log-concave on supp(g_0); (ii) g_0 is continuous at zero and $g_0(0) > 0$; (iii) If $\varepsilon_{\mathbf{a}} \neq O$, it has a density $\gamma_{\mathbf{a}}$ that is log-concave on supp($\gamma_{\mathbf{a}}$). Then Assumption 1 holds, and every pricing subgame has a unique symmetric pure-strategy equilibrium where both firms choose $p^*_{\mathbf{a}} = \frac{1}{2g_a(0)}$, $\forall \mathbf{a} \in \mathcal{A}$.*

Conditions (i) and (iii) assure the strict quasiconcavity of the payoff (A1.1). Jointly with the technical condition (ii), the requirement in (iii) that $\varepsilon_{\mathbf{a}}$ has a density function if it is not degenerate implies (A1.2), assuring that the equilibrium prices p^* are well-defined.

Proposition 1 shows that the equilibrium price $p^*_{\mathbf{a}}$ is determined by and decreasing in $g_{\mathbf{a}}(0)$, the measure of perceptually indifferent consumers. Since higher prices imply higher profits, firms prefer communication profiles that reduce the measure of perceptually indifferent consumers, consistent with Lemma 1. By Theorems 1 – 3, SPE without consumer confusion exist for weak polarization, but not for weak indecisiveness. With stronger indecisiveness (polarization) conditions and suitable dispersion orders of the noise induced by communication, there is a unique SPE with maximal (minimal) confusion. Reflecting the logic of our general analysis, equilibrium forces push both firms to compete for the perceptually indifferent and therefore most price-sensitive consumers. If $g_{\mathbf{a}}(0)$ is low, there are only few such consumers. Thus, consistent with empirical observations (Ellison and Ellison, 2009), obfuscation is an effective means to lower price elasticities and increase markups if either tastes are indecisive or obfuscation can become massive (e.g., because products are near to homogeneous).

Competition on the line As in a well-known textbook example, suppose each consumer is characterized by a parameter $\theta \in \Theta = [-1, 1]$, drawn from a commonly known distribution H with zero-symmetric density $h(\theta) = \alpha\theta^2 + \frac{1}{2} - \frac{\alpha}{3}$ on Θ, for $\alpha \in \left[-\frac{3}{4}, \frac{6-3\sqrt{3}}{4}\right]$. The true match value of consumer θ for product $i \in \{1, 2\}$ is $v_i^\theta = \mu - (x_i - \theta)^2$, where $\mu > 0$ is sufficiently large, and $x_1 = -1$, $x_2 = 1$ are the locations of the firms. Thus,

θ captures the location on a (Hotelling) line, with true match values determined by a quadratic distance function. We obtain $G_0(x) = H(\frac{x}{4})\ \forall x \in \mathbb{R}$. If $\alpha > 0$ ($\alpha < 0$), G_0 is strongly polarized (indecisive) on its support $[-4, 4]$. If the error distribution is uniformly distributed on $[-\omega_{\mathbf{a}}, \omega_{\mathbf{a}}]$ where $\omega_{\mathbf{a}} < 4\ \forall \mathbf{a} \in \mathcal{A}$ (i.e., confusion cannot be massive), then, by Proposition 1, the symmetric price equilibrium is

$$p^*_{\mathbf{a}} = \frac{1}{2g_{\mathbf{a}}(0)} = \frac{288}{\alpha\omega^2_{\mathbf{a}} + 72 - 48\alpha} > 0. \tag{5}$$

Hence, the equilibrium price (and thus payoffs) increase in $\omega_{\mathbf{a}}$ if $\alpha < 0$ (indecisiveness), and decrease if $\alpha > 0$ (polarization), confirming the results from Theorems 1 and 2.[19]

The Logit model The Logit model is among the most frequently used models for the theoretical and empirical analysis of discrete choice. It is well known that whenever true tastes are described by a linear random utility model with an i.i.d. Gumbel distribution, a Logit demand system results (see Anderson et al., 1992), and the true match advantages, v_Δ, follow a zero-mean logistic distribution. As this type of distribution is zero-symmetric, log-concave and features (global) indecisiveness on $\mathbb{R}$, our main analysis directly implies that only SPE featuring agent confusion can exist in a Logit context.

Connection to the Advertising Literature We found that more precise information about the products increases prices and profits with indecisive preferences, but decreases them with polarized preferences. This contrasts with the literature on informative advertising, surveyed in Bagwell (2007), where more information provided by firms typically reduces prices by intensifying competition. Our results also relate to papers emphasizing the prisoners' dilemma nature of persuasive advertising: Firms engage in costly advertising races, which do not affect prices and gross profits (see, e.g., Dixit and Norman, 1978; Von der Fehr and Stevik, 1998; Bagwell, 2007). In our setting, obfuscating communication strategies can be interpreted as persuading some consumers at the cost of alienating others.[20] Our analysis shows that firms refrain from such advertising measures (with polarized consumers) or use the measures to soften competition (with indecisive consumers), which contrasts with the prisoners' dilemma situations. Finally, Johnson and Myatt (2006) show that a monopolist does not always benefit from advertising that

[19] In the knife-edge case of uniformly distributed consumers ($\alpha = 0$), confusion has no price effect (unless it may become massive).

[20] An example are the cold-calls of tele-marketing agents (Schumacher and Thysen, 2021).

increases the heterogeneity of consumer valuations. In their paper, the entire valuation distribution matters for the optimal price, while in our setting competitive forces imply that equilibrium prices depend only on the mass of perceptually indifferent consumers (absent a binding outside option). Our main result that the shape of the true match advantage distribution determines whether obfuscation fosters such heterogeneity has no counterpart in Johnson and Myatt (2006).

4.2 Welfare Implications

With homogeneous goods, confusion increases prices, and firms benefit at the expense of consumers. With differentiation, confusion could reduce prices in principle. However, in such a case (when preferences are polarized), firms avoid obfuscation according to Theorems 1 and 2. Moreover, confusion can lead to inefficiency with differentiated goods, because some consumers might acquire a dominated product. For any equilibrium communication profile $\mathbf{a}^*$, the expected welfare loss from mismatch is

$$L = 2\int_0^{+\infty} x\Gamma_{\mathbf{a}^*}(-x)g_0(x)dx \geq 0. \tag{6}$$

Intuitively, if a consumer chooses the dominated option, the welfare loss is $|v_2 - v_1|$. Let $x = v_2 - v_1 > 0$ w.l.o.g. Then, $g_0(x)$ is the likelihood of type x and $\Gamma_{\mathbf{a}^*}(-x)$ is the probability that x buys from the wrong firm. (6) shows that confusion is necessary and sufficient for a positive welfare loss ($L > 0$) to occur. In view of the results in Sections 3.2 – 3.4, indecisive preferences imply confusion and thus an inefficient equilibrium outcome, while no such inefficiency arises with polarization unless confusion is massive. In Appendix S.5, we establish that the welfare loss (6) with indecisive preferences is increasing in the size of confusion if confusion follows a uniform distribution as in Proposition 3.[21]

Our analysis further informs the evaluation of regulations aimed at increasing competition between incumbent firms. An example is a compulsory product standard or norm, which all products need to match, and which therefore increases the true similarity between different products. For instance, manufacturers of LED and LCD televisions follow

[21] In addition, we show in the example with competition on the line that an increase in the indecisiveness of preferences (captured by an increase in the parameter $|\alpha|$ of the density h) has an ambiguous effect on welfare. The ambiguity follows from two competing effects. As $|\alpha|$ increases, more almost indifferent consumers buy the wrong product, but at the same time, these welfare losses are rather low. The former effect dominates (and the welfare loss increases in $|\alpha|$) only if obfuscation possibilities are large enough relative to true differentiation.

standardization rules that ensure that all products have certain identical features, such as inputs (HDMI port, USB ports, etc.) or internet connectivity. In the example with competition on the line, such a regulation can be captured as a relocation of both firms towards the middle, resulting in a truncation of the true match advantage distribution G_0. More generally, let $supp\, g_0 = [-\lambda, \lambda]$, where $\lambda > 0$ captures the true extent of product differentiation. Consider a policy with the effect of reducing this differentiation to $supp\, g_0^r = [-r, r]$, $0 < r < \lambda$, where g_0^r is a truncation of g_0. Absent any consumer confusion, it is easy to see that such a regulation lowers equilibrium prices, as $g_0^r(0) > g_0(0)$, independent of the shape of g_0. We find that, depending on consumer preferences, such regulations may have unintended, adverse effects on welfare due to consumer confusion:

Proposition 2 *Suppose that Assumptions 1 and 3 are satisfied, and consider the regulation with* $0 < r < \lambda$ *outlined above. (i) If true preferences are strongly indecisive on* $supp\, g_0$ *and* $supp\, \gamma_{\mathbf{a}} \subset [-r, r]$, $\forall \mathbf{a} \in \mathcal{A}$, *the regulation strictly increases the welfare loss due to consumer confusion. (ii) If true preferences are polarized on* $supp\, g_0$*, the regulation has no adverse welfare effects, unless possibly if confusion becomes massive.*

Intuitively, regulation does not affect the qualitative shape of the true match advantage distribution and therefore by Theorem 2 it has no impact on confusion, at least as long as the scope of confusion is limited. However, when confusion does take place (i.e., with indecisive preferences), more consumers will be confused for any given valuation difference as the distribution of match advantages becomes more concentrated, while firms nevertheless obfuscate at maximal intensity. This implies that more consumers choose a dominated product following the regulation. By contrast, firms continue to educate consumers with polarized preferences, at least as long as massive confusion is infeasible.[22]

Similar reasoning applies to changes in the environment that are not policy-induced. For instance, pundits believed that the internet would lead to the "the death of distance" (Cairncross, 1997) in banking competition, as the possibility of online transactions was expected to dramatically reduce the importance of (geographical) distance between banks. By contrast, our model predicts that banks, competing in interest rates for depositors, are likely to counter such increasing competitive pressure by obfuscating the market.[23]

[22]See also Spiegler (2006) Piccione and Spiegler (2012), Hefti (2018) and Heidhues et al. (2021) for further vindication that regulating product features, e.g., by means of standardization or the facilitation of new product entry, may have adverse welfare effects.

[23]As argued in Section 2.2.2, it is easy to adopt our framework to capture such strategic competition in interest rates between banks.

This is consistent with the fact that, indeed, a pro-competitive effect of the Internet has not been observed in the data (Degryse and Ongena, 2005).

4.3 Binding Outside Options

The marketing literature has highlighted that confused consumers may inefficiently abstain from buying a product (Iyengar and Lepper, 2000; Iyengar et al., 2004; Eppler and Mengis, 2004; Lee and Lee, 2004; Scheibehenne et al., 2010; Bertrand et al., 2010; Chernev et al., 2015). But if consumer confusion leads to a loss in demand from consumers who would prefer this firm in terms of perceived match advantage, would this not impose an upper bound on the willingness of firms to confuse? We show that, in the presence of binding outside options, firms may choose a confusing communication strategy in equilibrium, even if this induces some consumers to exit the market. This provides an additional rationale for why the market does not eliminate confusion and its negative consequences.

Using a Hotelling approach, suppose that the perceived match values are given by $\tilde{v}_1 = 1 + \frac{v}{2} + \frac{\varepsilon}{2}$ and $\tilde{v}_2 = 1 - \frac{v}{2} - \frac{\varepsilon}{2}$, where $v \in [-1, 1]$ is governed by a zero-symmetric distribution G_0 with density g_0. Reservation values are normalized to zero. Thus, a consumer purchases the good with the higher perceived net value $\tilde{v}_j - p_j$ if this is non-negative, and does not purchase otherwise.[24] Compared to the previous analysis, each firm must take into account that a price increase may now mean losing some consumers to whom the firm actually is offering the better deal. Therefore, one might believe that the possibility of a binding outside option may discipline firms against obfuscating too much. We now illustrate why this need not be the case; Appendix S.6 gives a formal analysis. Figure 4 depicts second-stage prices, firm demand and payoff if G_0 follows a simple "tent" distribution on $[-1, 1]$ (so that true preferences are indecisive),[25] and $\{\Gamma_{\mathbf{a}}\}_{\mathbf{a}\in\mathcal{A}}$ is given by a family of uniform distributions that differ in their support $[-\omega, \omega]$, $\omega \geq 0$. Here ω captures the intensity of confusion in a market. The figure shows that despite an increasing share of exiting consumers the equilibrium price and profit are globally increasing in ω, and strictly so if ω is small ($\omega < 1$) or large ($\omega > 2$) enough.

Intuitively, as long as obfuscation cannot reduce the measure of perceptually indifferent consumers enough ($\omega \leq 1$), prices remain so low that the outside option does not

[24] In the above formulation, the true match values are (perfectly) negatively correlated, implying that the distribution of the true match advantages, $v_\Delta = -v$, also is given by g_0. This makes the formal analysis tractable. The normalization $supp(g_0) = [-1, 1]$ is not essential, but simplifies the presentation.

[25] The tent distribution has $g_0(x) = 1 + x$ for $x \in [-1, 0]$ and $g_0(x) = 1 - x$ for $x \in (0, 1]$.

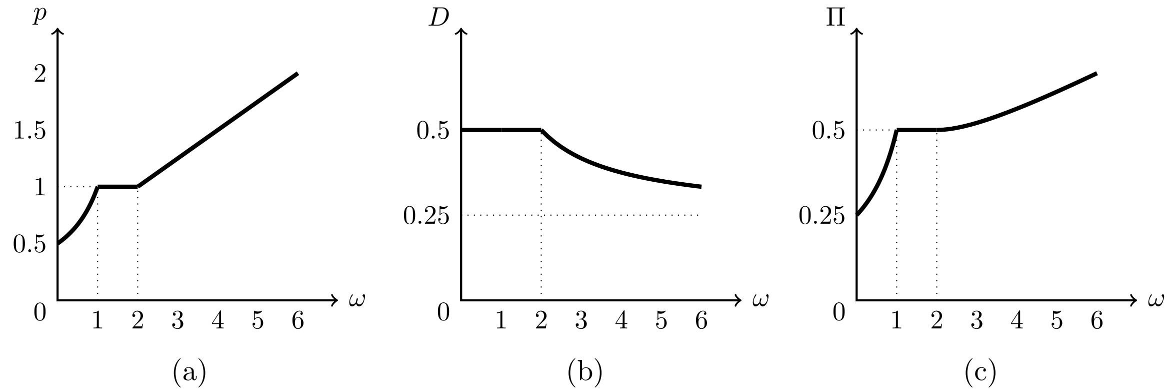


Figure 4: (a) Price, (b) demand, and (c) payoff as a function of ω

bind in equilibrium. In this case, the analysis is identical to the one of our main model: More confusion (higher ω) increases prices and profits without reducing demand. For $\omega > 1$, the outside option may become binding for some consumers. Thus, the firms need to strategically balance the corresponding loss in demand against the benefits of higher prices. For intermediate confusion ($1 < \omega < 2$), this leads firms to choose their prices exactly so that no consumer decides to leave the market. The key point is that firms benefit from confusion if the latter becomes strong enough ($\omega > 2$), because they can then concentrate on those consumers that have become highly enthusiastic about it due to confusion, and raise their prices so that the loss in demand from perceptually indifferent consumers is more than compensated by the prices paid by highly loyal consumers.[26]

Figure 4 implies that, whenever obfuscation possibilities are sufficiently large ($\omega > 2$ is feasible), a communication profile inducing maximal obfuscation must be part of an SPE.[27] Thus, our model rationalizes the possibility that consumers erroneously choose the outside option as an event that firms are willing to accept as part of their profit-maximizing strategies. Put differently, a binding outside option does not generally discipline firms against using confusing communication strategies; equilibrium payoffs are weakly increasing in ω, becoming flat only for intermediate values of ω. Such firm behavior creates an additional source of market inefficiency, besides the inefficiency originating from the mismatch between some consumers and firms.

According to our analysis, confusion can make some consumers overly enthusiastic about a product, manifested in a higher willingness-to-pay, which eventually dominates the fact that other consumers abstain from a purchase. More generally, the prediction

[26] Thus, the model predicts a positive association between brand loyalty and confusion – a pattern, which has been recently detected in survey data (Kurtulmuşoğlu and Atalay, 2020).

[27] This result holds more generally, as we show in Appendix S.6.

that overly optimistic consumers can coexist with consumers who abstain from purchases as a result of strategic communication behavior suggests a possible relation between the literature on choice overload and a literature emphasizing that marketing can increase the individual willingness-to-pay by making consumers more confident about the product they intend to choose (see, e.g., Andrews, 2016). These strands seem to have developed independently, while our results highlight a connection between them. Investigating the relation between them could therefore be a promising avenue for empirical research.

5 Confusion in All-Pay Contests

Our framework not only applies to price competition where efforts (lower prices) are conditional on success (sales). It also allows for contests with *non-contingent* efforts that arise independently of the players' success in attracting market share, as in the examples of voting, advertising and quality competition. In Section 5.1, we deal with equilibrium existence and characterization. Section 5.2 uses the model with unconditional effort to illustrate the robustness of our main insights to asymmetries between contestants. Section 5.3, discusses potentially counterintuitive implications for the political economy example.

5.1 Equilibrium

To capture unconditional efforts, we specify (2) by setting $R(s_1, s_2) = 1$ and $C(s_i) = ks_i^\eta$, $\forall s_1, s_2 \in \mathcal{S} = \mathbb{R}_+$, where $k > 0$ and $\eta > 1$. In Appendix S.1, we prove that payoffs are strictly quasi-concave if the curvature of the distribution of true preferences is not too strong, and the effort cost function is convex enough (high η). Lemma 1 thus shows that the first-order condition $g_\mathbf{a}(0) = C'(s^*_\mathbf{a})$ describes an equilibrium (see Proposition S1). It implies that $s^*_\mathbf{a}$ is strictly increasing and equilibrium payoffs are strictly decreasing in the mass of perceptually indifferent agents $g_\mathbf{a}(0)$, reflecting increasing equilibrium effort levels. Both contestants have an incentive to evade such intense competition, and our main insights about the relation between communication strategies and the true dispersion of preferences remain valid: If Assumptions 2 and 3 hold, confusion arises if preferences are indecisive (but not if they are polarized).

5.2 Asymmetric Contestants

While our analysis allows for ex post asymmetric contestants (after the choice of efforts), we have assumed that they are symmetric ex ante (before the choice of communication strategies) and ex interim (between the choices of communication and efforts). In Appendix S.2, we discuss the robustness of our main insights with respect to these symmetry assumptions in the model with unconditional efforts; similar results apply to price competition. There, we first consider contestants who are asymmetric ex ante (have heterogeneous effort costs), finding that a weak contestant's decision to confuse still depends on whether preferences are indecisive or polarized. By contrast, a strong contestant will tend to have more negative views about confusion.[28] In the case that contestants can use biased communication strategies resulting in ex-interim asymmetries, we find that the driving forces behind the confusion decisions are very similar to the symmetric case.

5.3 Competing for Voters

A large literature has asked why political candidates often choose ambiguous platforms, rather than describe their policies exactly. In the Hotelling-Downs voting model, ambiguous platforms are never optimal with risk-averse voters (Shepsle, 1972). Some authors have argued that candidates choose ambiguity to maintain the flexibility to adapt to future circumstances (Aragones and Neeman, 2000; Kartik et al., 2017), while others have provided behavioral explanations, relying on context-dependent preferences (Callander and Wilson, 2008) or projection bias (Jensen, 2009). Our paper contributes to this discussion by studying how the incentives of political candidates to confuse or educate potential voters about their platforms depend on the heterogeneity of voter preferences.

We interpret the market share as the share of votes, and we assume that the two political contestants (henceforth "parties"), value their votes identically. Parties are heterogeneous with respect to ideology, and voters have heterogeneous preferences, captured by the distribution G_0 of true match advantages. Voters evaluate a party according to perceived match advantages. Communication strategies (a_1, a_2) determine the distribution of perceived match advantages, $G_{\mathbf{a}}$.[29] A party can avoid being precise, leading to a

[28] See also Dahremöller (2013) who studies an extension of the add-on model by Gabaix and Laibson (2006) where firms differ in their ex ante costs of the add-ons. In this vertical differentiation setting he finds that, depending on the circumstances, the more or the less efficient firm seeks to educate consumers.

[29] Higher values of a_i capture greater ambiguity due to, for example, a larger set of policies (leading to a larger support of the perceived match advantages), as is common in the literature.

noisy perception, whereby some voters get a too positive impression of the party's value for them and others get a too negative impression. Parties not only influence election outcomes by their platforms. It is well known that media prominence has a strong persuasive effect on voting behavior, potentially influencing undecided voters (Gerber et al., 2011; Gallego and Schofield, 2017). We capture persuasion efforts in s_i, where the party with $s_i > s_j$ is more prominent and, consequently, obtains a larger share of voters.

The results of Section 5.1 show that parties want to confuse voters only when preferences are indecisive, not when they are polarized. They do this to soften competition in the effort stage, which maximizes their expected payoffs by lowering the costs of campaigning. This might be counterintuitive at first sight, as anecdotal evidence would suggest that polarization and *low* communication quality go hand in hand in the political realm. However, this observation is consistent with our model if one distinguishes carefully between ex-ante and ex-post (or true and perceived) voter preferences. In a nutshell, in a world where *true* preferences are indecisive, candidates rely on confusion for the reasons discussed in our model, resulting in a move of *perceived* preferences towards polarization – voters are pushed into having stronger opinions. A potential explanation for indecisive voter preferences may be the complexity of the problems at stake. As a result of such complexity, given the current state of knowledge, it may be hard to judge which of the candidates has the better approach to solving the problems even from the perspective of a voter who is very well-informed about the plans of a politician. Such lack of understanding should generate indecisiveness of true preferences – and thus is an ideal breeding ground for confusion.

We think of the political economy application as pointing to possible relations between voter preferences and communication strategies – that are in need of empirical verification. The model is incomplete in this regard. For instance, communication strategies might be multi-dimensional, with parties that have asymmetric expertise. One might then expect parties to be more precise on issues where they have more expertise.[30]

6 Conclusion

Our paper studies the connection between the preference distribution of heterogeneous agents and the strategic use of communication by contestants, such as firms or political

[30] We are grateful to a referee for this suggestion.

parties, who compete for the agents. Our main insight is that the distribution of true agent preferences plays a decisive role for whether contestants choose an educating or a confusing communication strategy. Education emerges if true preferences are polarized, meaning that tastes are concentrated near the contestants' positions. By contrast, confusion arises with indecisive preferences, characterized by a concentration of agents who are indifferent between the choice alternatives in terms of their true preferences. Further, we show that, other than with homogeneous goods, firms do not always confuse consumers, but if they do, the welfare effects are more detrimental. Consumers can be harmed by choosing dominated options, paying higher prices or inefficiently forgoing purchases.

Applied to political competition, our analysis shows that the distribution of voter preferences affects whether parties choose ambiguous platforms: With many undecided voters, such ambiguity softens political competition as it moves the perceived valuations for the candidates towards polarization. While political competition differs from price competition in that it features an "all-pay" cost scheme rather than costs that are contingent on obtaining successful transactions, our results show that the effects of agent preferences on the contestants obfuscation decisions are identical in both cases. Thus, our insights do not depend on the precise form of the competition between the contestants.

Our arguments are particularly transparent in the case of unbiased communication, which is suggested by many examples, and also seems like a natural environment to isolate the effects of noisy communication. However, even in its current form, our framework can incorporate the possibility that obfuscation affects the expected perceived valuations of both contestants, as long as it leaves the expected perceived valuation *differences* untouched. Moreover, as we detail in Appendix S.2, we can modify our approach to incorporate asymmetries where certain communication strategies may allow a contestant to bias the perceived valuation differences in its favor. While such modifications affect the details of the underlying equilibrium structure, our main insight that indecisiveness works in favor of obfuscation, whereas polarization works against it, remains valid.

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A Main Appendix

A.1 General Analysis: Proofs

Proof of Lemma 1 Let $\mathbf{a} \in \mathcal{A}$. By (A1.2) there exists $s^*_{\mathbf{a}} \in \mathcal{S}$ such that $z(s^*_{\mathbf{a}})$ is well-defined (i.e., $R(s^*_{\mathbf{a}}, s^*_{\mathbf{a}}) \neq 0$) and satisfies $z(s^*_{\mathbf{a}}) = g_{\mathbf{a}}(0)$, or, equivalently,

$$\frac{R_1(s^*_{\mathbf{a}}, s^*_{\mathbf{a}})}{2} + R(s^*_{\mathbf{a}}, s^*_{\mathbf{a}})g_{\mathbf{a}}(0) - C'(s^*_{\mathbf{a}}) = 0. \tag{A.1}$$

Moreover, $s^*_{\mathbf{a}}$ is uniquely determined because $z(s)$ is strictly increasing by (A1.3). Because $G_{\mathbf{a}}$ is zero-symmetric, (A.1) corresponds to the first-order condition for an interior symmetric equilibrium in the effort stage. By strict quasi-concavity (A1.1), we may conclude that the mutual choice of $s^*_{\mathbf{a}} = z^{-1}(g_{\mathbf{a}}(0))$ by both contestants constitutes the unique symmetric equilibrium in the effort stage. Because $z(s)$ is strictly increasing, the equilibrium effort $s^*_{\mathbf{a}} = z^{-1}(g_{\mathbf{a}}(0))$ is strictly increasing in $g_{\mathbf{a}}(0)$. Since by (A1.3) we have

$$\frac{d\Pi^{\mathbf{a}}_i(s, s)}{ds} = \frac{R_1(s, s) + R_2(s, s)}{2} - C'(s) < 0,$$

the equilibrium payoff must be strictly decreasing in $g_{\mathbf{a}}(0)$. ■

Proof of Theorem 1 Part (i): For any $\mathbf{a} \in \mathcal{A}$ such that $\varepsilon_{\mathbf{a}} \neq O$, we have

$$g_{\mathbf{a}}(0) = \int_{supp(\varepsilon_{\mathbf{a}})} g_0(e)d\Gamma_{\mathbf{a}}(e) > \int_{supp(\gamma_{\mathbf{a}})} g_0(0)d\Gamma_{\mathbf{a}}(e) = g_0(0), \tag{A.2}$$

where the inequality follows from $supp(\gamma_{\mathbf{a}}) \subset [-\delta, \delta]$, the zero-symmetry of $\Gamma_{\mathbf{a}}$, and the fact that the true preferences are weakly polarized. Hence, by Lemma 1 we can conclude that $\Pi^{\mathbf{a}}_i(s^*_{\mathbf{a}}, s^*_{\mathbf{a}}) < \Pi^{\mathbf{0}}_i(s^*_{\mathbf{0}}, s^*_{\mathbf{0}})$ for all $\mathbf{a} \in \mathcal{A}$ such that $\varepsilon_{\mathbf{a}} \neq O$. It then immediately follows that any choice of $\mathbf{a} \in \mathcal{A}$ for which $\varepsilon_{\mathbf{a}} = O$ must be an equilibrium of the communication stage, followed by $s^*_1 = s^*_2 = s^*_{\mathbf{0}}$ in the competition stage. In such SPE, communication strategies are thus chosen such that no agent confusion results, and by (A2.1) at least one such communication profile exists. For part (ii), the inequality in (A.2) is reversed by weak indecisiveness of the agents' preferences. Hence, by Lemma 1, any communication profile with $\varepsilon_{\mathbf{a}} \neq O$, followed by the symmetric equilibrium $s^*_{\mathbf{a}}$ would be strictly preferred by the contestants than the respective outcome under $\varepsilon_{\mathbf{a}} = O$. (A2.2) then assures that any possible SPE must involve agent confusion. ■

Proof of Theorem 2 Part (i): By Theorem 1, for this part of the proof it is without loss to assume that $\mathbf{0} \notin \mathcal{A}$, since even if it is available the communication profile $\mathbf{a} = \mathbf{0}$ will not be chosen in any SPE. Take any $\mathbf{a}, \mathbf{a}' \in \mathcal{A}$ such that $\mathbf{a} \neq \mathbf{a}'$ and $\mathbf{a} \leq \mathbf{a}'$. By Assumption 3, $\varepsilon_{\mathbf{a}'}$ has the same distribution as $\varepsilon_{\mathbf{a}} + \eta$, where $\eta \neq O$. Thus

$$g_{\mathbf{a}'}(0) = \mathbb{E}\left[g_0(\varepsilon_{\mathbf{a}'})\right] = \mathbb{E}\left[\mathbb{E}\left[g_0\left(\varepsilon_{\mathbf{a}} + \eta\right)|\varepsilon_{\mathbf{a}}\right]\right]$$
$$< \mathbb{E}\left[g_0\left(\mathbb{E}[\varepsilon_{\mathbf{a}} + \eta|\varepsilon_{\mathbf{a}}]\right)\right] = \mathbb{E}\left[g_0\left(\varepsilon_{\mathbf{a}}\right)\right] = g_{\mathbf{a}}(0). \tag{A.3}$$

The second equality follows from the law of iterated expectations and that $\varepsilon_{\mathbf{a}'}$ and $\varepsilon_{\mathbf{a}} + \eta$ are equal in distribution. The strict inequality follows from Jensen's inequality because g_0 is strictly concave on $[-\delta, \delta] \supset supp(\varepsilon_{\mathbf{a}})$. Hence, $g_{\mathbf{a}}(0)$ achieves its minimum on $\mathcal{A}$ if and only if $\mathbf{a} = (\bar{a}, \bar{a})$. By Lemma 1, this also maximizes the payoffs of the contestants in the competition stage. Hence, $(\bar{a}, \bar{a})$ must be part of an SPE. Moreover, $(\bar{a}, \bar{a})$ is the only possible equilibrium outcome, because for any alternative $(a_1, a_2) \in \mathcal{A}$, a forward-looking contestant with $a_i < \bar{a}$ would always want to deviate to $\bar{a}$.

Part (ii): If g_0 is strictly convex, the inequality in (A.3) is reversed. By Lemma 1, any $\varepsilon_{\mathbf{a}'}$ which is MPS of $\varepsilon_{\mathbf{a}}$ therefore is payoff-dominated by $\varepsilon_{\mathbf{a}}$. Further, any $\varepsilon_{\mathbf{a}} \neq O$ trivially is an MPS of O. Thus, regardless of whether $\mathbf{0} \in \mathcal{A}$ or not, $\mathbf{a}^* = (\underline{a}, \underline{a})$ is the only possible equilibrium outcome. Indeed, setting $a_i = \underline{a}$ is a dominant action for each contestant i, because $\forall a_j \in A$, with any alternative $a_i > \underline{a}$ the resulting $\varepsilon_{(a_i, a_j)}$ is a MPS of $\varepsilon_{(\underline{a}, a_j)}$, which can only lead to a lower equilibrium payoffs in the competition stage. ■

Proof of Theorem 3 For every $\omega \geq 0$, let

$$g_\omega(0) \equiv \int_{-\omega}^{\omega} g_0(\varepsilon) d\Gamma_\omega,$$

where Γ_ω is the uniform distribution on $[-\omega, \omega]$:

$$\Gamma_\omega(x) = \begin{cases} 1 & \text{if } x > \omega, \\ \frac{x+\omega}{2\omega} & \text{if } x \in [-\omega, \omega], \\ 0 & \text{otherwise.} \end{cases}$$

By construction and the assumptions of the theorem, $\Gamma_{\mathbf{a}}(x) = \Gamma_{\omega_{\mathbf{a}}}(x)\ \forall x \in \mathbb{R}, \mathbf{a} \in \mathcal{A}$.

Consider first case (i). We start by showing that $g_\omega(0)$ is strictly decreasing in ω on

$[0, +\infty)$. Since match values are indecisive on $supp(g_0)$ and $g_0(x) = 0\ \forall x \notin supp(g_0)$, we have $g_0(\varepsilon) < g_0(0)\ \forall \varepsilon \neq 0$. It then follows that, $\forall \omega > 0$,

$$g_\omega(0) = \int_{-\omega}^{\omega} g_0(\varepsilon)\Gamma_\omega < \int_{-\omega}^{\omega} g_0(0) d\Gamma_\omega = g_0(0).$$

Further, since $g_\omega(0)$ is differentiable with respect to ω for all $\omega > 0$, we have

$$\begin{aligned}
\frac{\partial g_\omega(0)}{\partial \omega} &= \frac{g_0(-\omega)}{2\omega} + \frac{g_0(\omega)}{2\omega} - \int_{-\omega}^{\omega} \frac{1}{2\omega^2}\, g_0(-\varepsilon) d\varepsilon \\
&= \frac{g_0(\omega)}{\omega} - \int_{-\omega}^{\omega} \frac{1}{2\omega^2}\, g_0(\varepsilon) d\varepsilon \\
&< \frac{g_0(\omega)}{\omega} - \int_{-\omega}^{\omega} \frac{1}{2\omega^2}\, g_0(\omega) d\varepsilon \\
&= \frac{g_0(\omega)}{\omega} - \frac{g_0(\omega)}{\omega} \\
&= 0,
\end{aligned}$$

where the inequality follows that match values are indecisive on $supp(g_0)$, g_0 is zero-symmetric, and $g_0(x) = 0\ \forall x \notin supp(g_0)$. By Lemma 1, there exists a unique symmetric equilibrium in which both contestants choose the effort $s^*_{\mathbf{a}} = z^{-1}(g_{\omega_{\mathbf{a}}}(0))$ following every $\mathbf{a} \in \mathcal{A}$ in the effort stage . Since $g_\omega(0)$ is strictly decreasing in ω, $g_{\omega_{\mathbf{a}}}(0)$ is minimized at $\omega_{\mathbf{a}} = \bar{\omega}$. Therefore, the equilibrium payoff is maximized at $\omega_{\mathbf{a}} = \bar{\omega}$ which, as a consequence of Assumption 3, implies that the unique SPE features maximal confusion.

Next, consider case (ii). We first prove the following two lemmas.

Lemma A1 *If $supp(g_0)$ is bounded, then* $\lim_{\omega \to +\infty} g_\omega(0) = 0$.

Proof. Since $supp(g_0)$ is bounded, we must have $supp(g_0) \subset [-\omega, \omega]$ for sufficiently large ω. As a result,

$$\lim_{\omega \to +\infty} g_\omega(0) = \lim_{\omega \to +\infty} \int_{-\omega}^{\omega} \frac{g_0(\varepsilon)}{2\omega} d\varepsilon = \lim_{\omega \to +\infty} \int_{supp(g_0)} \frac{g_0(\varepsilon)}{2\omega} d\varepsilon = 0. \qquad \square$$

Lemma A2 *If $supp(g_0)$ is bounded, then $g_\omega(0)$ is strictly decreasing in ω for all $\omega >$* $\sup supp(g_0)$

Proof. Since $supp(g_0)$ is bounded, we must have $\sup supp(g_0) < +\infty$, and $g_0(\omega) = 0$

for all $\omega > \sup supp(g_0)$. It then follows that, for every $\omega > \sup supp(g_0)$,

$$\frac{\partial g_\omega(0)}{\partial \omega} = \frac{g_0(\omega)}{\omega} - \int_{-\omega}^{\omega} \frac{g_0(\varepsilon)}{2\omega^2} d\varepsilon = - \int_{-\omega}^{\omega} \frac{g_0(\varepsilon)}{2\omega^2} d\varepsilon < 0. \qquad \square$$

Now consider any match value distribution that is polarized on its support $supp(g_0)$. By definition, g_0 is strictly decreasing on $(\inf supp(g_0), 0]$ and is strictly increasing on $[0, \sup supp(g_0))$. This implies that the support of g_0 must be bounded, as otherwise we would have

$$\int_{supp(g_0)} g_0(x)dx = \int_{-\infty}^{+\infty} g_0(x)dx \geq \int_{-\infty}^{+\infty} g_0(0)dx = +\infty,$$

contradicting the definition of g_0 as a density function. Applying Lemmas A1 and A2, we can conclude that $\lim_{\omega \to +\infty} g_\omega(0) = 0$ and that $g_\omega(0)$ is strictly decreasing in ω on $(supp(g_0), +\infty)$. Hence, given $g_0(0) > 0$ there must exist $\hat{\omega} > 0$, such that if $\omega \geq \hat{\omega}$, then $g_\omega(0) \leq g_0(0)$ and it is further decreasing as ω increases. Therefore, if $\bar{\omega}$ is sufficiently large, the subgame equilibrium payoff is maximized at $\omega_{\mathbf{a}} = \bar{\omega}$ which, by Assumption 3, implies that the unique SPE then features maximal confusion. ■

A.2 Price Competition: Proofs and Additional Results

Proof of Proposition 1 First, we argue that the log-concavity assumptions in (i) and (iii) imply that each firm i's expected payoff $\Pi_i^{\mathbf{a}}(p_i, p_j) = p_i G_{\mathbf{a}}(p_j - p_i)$ is strictly quasi-concave in p_i, $\forall \mathbf{a} \in \mathcal{A}$ and $p_j \geq 0$. To see this, first note that $\forall \mathbf{a} \in \mathcal{A}$, the distribution function $G_{\mathbf{a}}$, which is defined by

$$G_{\mathbf{a}}(x) = \int_{supp(\varepsilon_{\mathbf{a}})} G_0(x - \varepsilon) d\Gamma_{\mathbf{a}}(\varepsilon), \forall x \in \mathbb{R},$$

is log-concave on $supp(g_{\mathbf{a}})$. This is trivial if $\varepsilon_{\mathbf{a}} = O$, since in this case we have $G_{\mathbf{a}} = G_0$, and G_0 is log-concave on $supp(g_0)$ as condition (i) assumes. If $\varepsilon_{\mathbf{a}} \neq O$, then by (iii) it has a log-concave density $\gamma_{\mathbf{a}}$, and we thus have $G_{\mathbf{a}}(x) = \int_{supp(\varepsilon_{\mathbf{a}})} G_0(x - \varepsilon)\gamma_{\mathbf{a}}(\varepsilon)d\varepsilon, \forall x \in \mathbb{R}$. Then, we can again conclude that $G_{\mathbf{a}}$ is log-concave on $supp(g_{\mathbf{a}})$, because the convolution of log-concave functions is also log-concave.[31] Further, since the function $f(p) = p$ is strictly log-concave on $[0, +\infty)$, for all $p_j \geq 0$ the profit function $\Pi_i^{\mathbf{a}}(p_i, p_j)$ is strictly log-concave (and hence strictly quasi-concave) in p_i on $\left[\max\{\underline{p}(p_j), 0\}, \bar{p}(p_j)\right]$, where, for

[31]For an overview of the properties of log-concave distributions, see Bagnoli and Bergstrom (2005).

every $p_2 \geq 0$, we define

$$\underline{p}(p_2) \equiv \sup\{p_1 \in \mathbb{R} \,|\, G_{\mathbf{a}}(p_2 - p_1) = 1\}, \text{ and } \bar{p}(p_2) \equiv \inf\{p_1 \in \mathbb{R} \,|\, G_{\mathbf{a}}(p_2 - p_1) = 0\},$$

and similarly, for every $p_1 > 0$,

$$\underline{p}(p_1) \equiv \sup\{p_2 \in \mathbb{R} \,|\, G_{\mathbf{a}}(p_2 - p_1) = 0\}, \text{ and } \bar{p}(p_1) \equiv \inf\{p_2 \in \mathbb{R} \,|\, G_{\mathbf{a}}(p_2 - p_1) = 1\}.$$

Since $\Pi_i^{\mathbf{a}}(p_i, p_j) = p_i$ is strictly increasing if $p_i \leq \max\{\underline{p}(p_j), 0\}$, and $\Pi_i^{\mathbf{a}}(p_i, p_j) = 0$ $\forall p_i \geq \bar{p}(p_j)$, we can conclude that $\forall p_j \geq 0$, $\Pi_i^{\mathbf{a}}(p_i, p_j)$ is strictly quasic-concave in p_i on the entire domain $[0, +\infty)$.

Next, we prove that (A1.2) is satisfied. Since in the current setting we have

$$z(s) = -\frac{1}{2s} \; \forall s = -p \leq 0,$$

it suffices to show that $g_{\mathbf{a}}(0) > 0 \; \forall \mathbf{a} \in \mathcal{A}$. If $\varepsilon_{\mathbf{a}} = O$, then $g_{\mathbf{a}}(0) = g_0(0) > 0$ as condition (ii) directly implies. If $\varepsilon_{\mathbf{a}} \neq O$, then again by (iii) it has a density $\gamma_{\mathbf{a}}$ which is log-concave on its support. By definition, it follows that $supp(\gamma_{\mathbf{a}})$ must be a convex set, i.e., an interval on $\mathbb{R}$. It then follows that $0 \in supp(\gamma_{\mathbf{a}})$, for if $0 \notin supp(\gamma_{\mathbf{a}})$ then $supp(\gamma_{\mathbf{a}})$ must reside entirely either in $(-\infty, 0)$ or in $(0, \infty)$, contradicting the symmetry of $\Gamma_{\mathbf{a}}$ at zero. The zero-symmetry of $\Gamma_{\mathbf{a}}$ further assures that $supp(\gamma_{\mathbf{a}})$ is an interval symmetric around zero. By (ii), g_0 is continuous and strictly positive at the point $x = 0$. Hence, there must exist $\delta > 0$ such that $g_0(x) > 0 \; \forall x \in [-\delta, \delta]$. In particular, we can choose this $\delta > 0$ so small to assure that $[-\delta, \delta] \subset supp(\gamma_{\mathbf{a}})$. Accordingly, we have

$$g_{\mathbf{a}}(0) = \int_{-\infty}^{+\infty} g_0(-\varepsilon)\gamma_a(\varepsilon)d\varepsilon \geq \int_{-\delta}^{\delta} g_0(-\varepsilon)\gamma_a(\varepsilon)d\varepsilon > 0.$$

The existence of a unique symmetric pure-strategy equilibrium in every subgame then immediately follows from Lemma 1. ■

Proof of Proposition 2 (i) Because $g_0(\cdot, r)$ is strongly indecisive on its support, the only SPE involves maximal confusion by Theorem 2. Let $L(\mathbf{a}, r)$ denote the welfare loss (6) for a given communication profile $\mathbf{a}$ and regulation r. Then

$$L(\mathbf{a}, r) = \frac{2}{2G_0(r) - 1} \int_0^{\varepsilon} x\Gamma_{\mathbf{a}}(-x)g_0(x)dx > 2\int_0^{\varepsilon} x\Gamma_{\mathbf{a}}(-x)g_0(x)dx = L(\mathbf{a}, \lambda),$$

showing that the welfare loss increases for $r < \lambda$. For (ii), note that the unique SPE involves full education also under the regulation as long as $supp\, \gamma_{\mathbf{a}} \subset supp\, g_0(\cdot, r)$, $\forall \mathbf{a} \in \mathcal{A}$, by Theorem 2, meaning that $L(\mathbf{0}, r) = L(\mathbf{0}, \lambda) = 0$. If the regulation leads to a sufficiently tight support of $g_0(\cdot, r)$, then Theorem 3 shows that maximal confusion becomes the unique SPE, at least if $\{\Gamma_{\mathbf{a}}\}_{\mathbf{a}\in\mathcal{A}}$ is given by a family of uniform distributions. ■

Proposition A1 *Consider the model with competition on the line. Suppose that* $\alpha \leq \hat{\alpha} \equiv (6 - 3\sqrt{3})/4\lambda^3$*, (A1.3) holds, and supp*$(\gamma_{\mathbf{a}}) \subset [-4\lambda^2, 4\lambda^2]$ $\forall \mathbf{a} \in \mathcal{A}$*.*

(i) If Assumption 2 also holds, then there exists (does not exist) an SPE without consumer confusion if $\alpha > 0$ $(\alpha < 0)$*.*

(ii) If Assumption 3 also holds and $\alpha \neq 0$*, then there exists a unique SPE. This SPE features minimal (maximal) consumer confusion if* $\alpha > 0$ $(\alpha < 0)$*.*

Proof of Proposition A1 We start by arguing that if $\alpha \leq \hat{\alpha}$, then (A1.1) holds, that is, G_0 is log-concave on its support. To show this, we will make use of Lemma A3 below, which states that H is log-concave on its support if $\alpha \leq \hat{\alpha}$.

Lemma A3 *If* $\alpha \leq \hat{\alpha}$*, then* H *is log-concave on* $[-\lambda, \lambda]$*.*

Proof. If $\alpha \leq 0$, the statement of the lemma immediately follows because in these cases the density function h is log-concave, which is sufficient (but not necessary) for the distribution function H to be log-concave on $[-\lambda, \lambda]$.

Suppose now that $\alpha \in (0, \hat{\alpha}]$. We will show that H remains to be log-concave despite that the density function h is actually log-convex. By continuity, it suffices to show that H is log-concave on the open interval $(-\lambda, \lambda)$. Since h is differentiable on $(-\lambda, \lambda)$, H is log-concave on this interval if and only if for all $\theta \in (-\lambda, \lambda)$,

$$\begin{aligned}
& h'(\theta)H(\theta) - (h(\theta))^2 \leq 0 \\
\iff\ & 2\alpha\theta \cdot \left(\frac{1}{3}\alpha\theta^3 + \beta\theta + \frac{1}{2}\right) \leq \left(\alpha\theta^2 + \beta\right)^2 \\
\iff\ & \frac{2}{3}\alpha^2\theta^4 + 2\alpha\beta\theta^2 + \alpha\theta \leq \alpha^2\theta^4 + \beta^2 + 2\alpha\beta\theta^2 \\
\iff\ & -\frac{1}{3}\alpha^2\theta^4 + \alpha\theta \leq \left(\frac{1}{2\lambda} - \frac{\alpha\lambda^2}{3}\right)^2 .
\end{aligned} \tag{A.4}$$

Given that $\alpha > 0$, the inequality obviously holds when $\theta \leq 0$. But given that $\theta > 0$, the LHS of (A.4) is increasing in θ on $[0, \lambda]$, since

$$\left(-\frac{1}{3}\alpha^2\theta^4 + \alpha\theta\right)' = -\frac{4}{3}\alpha^2\theta^3 + \alpha \geq -\frac{4}{3}\alpha^2\lambda^3 + \alpha > 0,$$

where the last inequality holds as $\hat{\alpha} < 3/(4\lambda^3)$. Hence, inequality (A.4) holds for all $\theta \in (-\lambda, \lambda)$ if and only if

$$\begin{aligned} & -\frac{1}{3}\alpha^2\lambda^4 + \alpha\lambda \leq \frac{1}{4\lambda^2} + \frac{\alpha^2\lambda^4}{9} - \frac{\alpha\lambda}{3} \\ \iff & -\frac{4}{9}\alpha^2\lambda^4 + \frac{4}{3}\alpha\lambda \leq \frac{1}{4\lambda^2} \\ \iff & -\alpha^2\lambda^4 + 3\alpha\lambda \leq \frac{9}{16\lambda^2} \\ \iff & \left(\alpha\lambda^2 - \frac{3}{2\lambda}\right)^2 \geq \frac{27}{16\lambda^2}. \end{aligned} \tag{A.5}$$

Since $\lambda > 0$ and $\hat{\alpha} \leq 3/(2\lambda^3)$, (A.5) is further equivalent to $\alpha \leq \frac{6-3\sqrt{3}}{4\lambda^3} = \hat{\alpha}$. □

Since $G_0(x) = \Pr(4\lambda\theta \leq x) = \Pr\left(\theta \leq \frac{x}{4\lambda}\right) = H\left(\frac{x}{4\lambda}\right)$ $\forall x \in \mathbb{R}$, and the function $t(x) = x/(4\lambda)$ is increasing and concave in x, G_0 is log-concave on $[4\lambda a, 4\lambda b]$ if H is log-concave on $[a, b] \subset \mathbb{R}$. Hence, by Lemma A3, G_0 is log-concave on $[-4\lambda^2, 4\lambda^2]$ provided that $\alpha \leq \hat{\alpha}$.

It is straightforward to check that all other conditions in Assumption 1 are satisfied. Hence, by Lemma 1, we can conclude that there exists a unique pure-strategy equilibrium in every pricing subgame, where each firm chooses the same price $p^*_{\mathbf{a}} = \frac{1}{2g_{\mathbf{a}}(0)}$ $\forall \mathbf{a} \in \mathcal{A}$. The statements of the proposition then immediately follow from Theorems 1 and 2. ■

S Supplementary Material (for Online Publication)

This appendix contains a number of extensions of the main analysis. In Section S.1, we provide the formal details of the model with all-pay contests from Section 5, which we phrase as competition for voters. In Section S.2, we relax the unbiasedness assumption on communication in the context of this model. In Section S.3, we elaborate on the foundations for the perceived match advantages (1). In Section S.4, we present the two-sided single crossing ordering of the perception errors, and show that it implies similar results as the MPS ordering. Section S.5 elaborates on the size of the welfare loss due to confusion in the context of price competition (see Section 4.2), and Section S.6 formally analyzes the model with outside option from Section 4.3. In Section S.7, we study confusion about needs in case of a Salop model, which exemplifies a situation where $\varepsilon_{\mathbf{a}}$ and v_{Δ}^{k} are naturally correlated.

S.1 All-Pay Contests: Formal Analysis

The following proposition presents sufficient conditions for the existence of a symmetric second-stage equilibrium in efforts in the model of Section 5.

Proposition S1 *Consider the all-pay contests and suppose that the following conditions are satisfied: (i) $\exists\, M \geq 0$, such that for almost all $x \in supp(g_0)$, $g_0(x) > 0$, $g_0'(x)$ exists, and $\left|\frac{g_0'(x)}{g_0(x)}\right| \leq M$; (ii) If $\varepsilon_{\mathbf{a}} \neq O$, it has a density $\gamma_{\mathbf{a}}$; (iii) $\forall \mathbf{a} \in \mathcal{A}$, $C'^{-1}(g_{\mathbf{a}}(0)) <$ $\max\{k, 1/k\}$. Then there exists η^* such that if $\eta \geq \eta^*$, every subgame in the competition stage has a unique symmetric pure-strategy equilibrium, where both contestants choose the effort $s_{\mathbf{a}}^* = C'^{-1}(g_{\mathbf{a}}(0))\ \ \forall \mathbf{a} \in \mathcal{A}$.*

Proof of Proposition S1 First, note that since

$$\Pi_i^{\mathbf{a}}\left(\max\{k, 1/k\}, s_j\right) \leq 1 - k(\max\{k, 1/k\})^{\eta} \leq 0,$$

no candidate will ever choose an effort level higher than $\max\{k, 1/k\}$. We now argue that if η is sufficiently large, then $\Pi_i^{\mathbf{a}}(s_i, s_j)$ is strictly quasi-concave in $s_i \in [0, \max\{k, 1/k\}]$, $\forall s_j \in \mathbb{R}_+$, $\mathbf{a} \in \mathcal{A}$ and $i = 1, 2$. This will be proved by using the well-known fact that a twice differentiable real-valued function f, defined on some open interval $X \subset \mathbb{R}$, is strictly quasi-concave if $f'(x) = 0$ implies $f''(x) < 0$ for any $x \in X$.

Take any $\mathbf{a} \in \mathcal{A}$ and $s_2 \in \mathbb{R}_+$. Note that by conditions (i) and (ii),

$$g_{\mathbf{a}}(s_1 - s_2) = \int_{supp(\varepsilon_{\mathbf{a}})} g_0(s_1 - s_2 - \varepsilon) d\Gamma_{\mathbf{a}}(\varepsilon)$$

is differentiable. Now suppose that $\frac{\partial \Pi_1^{\mathbf{a}}(s_1,s_2)}{\partial s_1} = 0$, for some $s_1 \in (0, \max\{k, 1/k\})$, i.e.,

$$g_{\mathbf{a}}(s_1 - s_2) - k\eta s_1^{\eta-1} = 0. \tag{B.1}$$

Then, we have

$$\begin{aligned}\frac{\partial^2 \Pi_1^{\mathbf{a}}(s_1, s_2)}{\partial s_1^2} &= g'_{\mathbf{a}}(s_1 - s_2) - k\eta(\eta - 1)s_1^{\eta-2} \\ &= g'_{\mathbf{a}}(s_1 - s_2) - (\eta - 1)g_{\mathbf{a}}(s_1 - s_2)s_1^{-1}.\end{aligned}$$

Since (B.1) also implies that $g_{\mathbf{a}}(s_1 - s_2) > 0$, we further have $\frac{\partial^2 \Pi_1^{\mathbf{a}}(s_1,s_2)}{\partial s_1^2} < 0$ if and only if

$$s_1 \cdot \frac{g'_{\mathbf{a}}(s_1 - s_2)}{g_{\mathbf{a}}(s_1 - s_2)} < \eta - 1. \tag{B.2}$$

By condition (i) and $s < \max\{k, 1/k\}$, the LHS of (B.2) is bounded from above by $\max\{kM, M/k\}$. Thus the inequality (B.2) must hold whenever η is sufficiently large, implying that $\Pi_i^{\mathbf{a}}(s_i, s_j)$ is strictly quasi-concave in s_i on the open interval $(0, \max\{k, 1/k\})$. By continuity, it is also strictly quasi-concave on $[0, \max\{k, 1/k\}]$.

Next, from the first-order conditions, we obtain a unique candidate for a symmetric pure-strategy equilibrium, where $s_1 = s_2 = s_{\mathbf{a}}^* = C'^{-1}(g_{\mathbf{a}}(0))$. By condition (iii), $s_{\mathbf{a}}^* < \max\{k, 1/k\}$. Hence, $(s_1, s_2) = (s_{\mathbf{a}}^*, s_{\mathbf{a}}^*)$ is indeed an equilibrium when $\Pi_i^{\mathbf{a}}(s_i, s_j)$ is strictly quasi-concave in s_i on $[0, \max\{k, 1/k\}]$. In particular, this is the case whenever η is sufficiently large. ■

S.2 Asymmetric Contestants

Ex-ante asymmetry We proceed by providing an intuitive discussion of the main insights. See Section S.2.1 for the formal analysis.

We first argue that, with ex-ante asymmetric contestants, the shape of preferences has a similar effect on the nature of equilibrium as in the symmetric case. Suppose that $i = 1$ is a *strong* and $i = 2$ a *weak* contestant in the sense that $C_1'(s) < C_2'(s)$ for

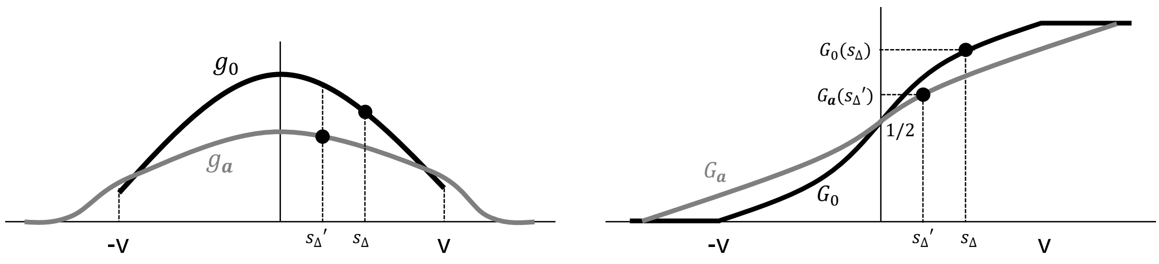


Figure S.1: The effects of confusion with ex-ante asymmetric contestants indecisiveness

any $s > 0$. For a given distribution of perceived match advantages $G_{\mathbf{a}}$, the first-order conditions in the effort stage yield $g_{\mathbf{a}}(s_1 - s_2) = C_1'(s_1) = C_2'(s_2)$, showing that marginal costs are equated by equilibrium forces. It follows that we must have $s_1 - s_2 \equiv s_\Delta > 0$ in equilibrium, reflecting the advantage of the strong contestant. Because then also $G_{\mathbf{a}}(s_\Delta) > 1/2$, it follows that, despite unbiased perception errors, communication can lead to a redistribution of market shares and thereby possibly induce a conflict of interests between the contestants. Figure S.1 illustrates the effects of agent confusion for indecisive preferences.[32]

As captured in the left part of Figure S.1, agent confusion spreads out the density of the perceived match values $g_{\mathbf{a}}$ below g_0, and accordingly rotates the distribution $G_{\mathbf{a}}$ at 0 clockwise (displayed in the right part of Figure S.1). On the one hand, both contestants benefit from confusion because the downward shift of $g_{\mathbf{a}}$ softens competition and allows them to choose lower equilibrium efforts. On the other hand, as we can see from the right part of Figure S.1, the market share of the strong (weak) contestant would be larger (smaller) without agent confusion. Therefore, the strong contestant may have a mixed view about the benefits of agent confusion, while the weak contestant can only benefit.[33] The opposite logic applies with polarized preferences. Then, agent confusion can only harm the weak contestant because it intensifies competition, and the weak contestant loses some otherwise favorable agents to the strong competitor. In sum, the weak competitor unambiguously strives to educate (to confuse) in the case of polarized (indecisive) preferences, while the incentive is less clear-cut for the strong contestant.[34]

[32]The suggestive insights portrayed by Figure S.1 are supported by formal results (see Section S.2.1).

[33]Kalaycı and Potters (2011) observe a similar result in a Hotelling model with quality-differentiated products; also see Gabaix and Laibson (2004).

[34]It is easy to see that a similar result emerges if $C_1(s) = C_2(s)$ but g_0 is shifted to the right, such that with equal effort, contestant $i = 1$ would obtain a strictly larger market share.

Ex-interim asymmetry Despite the many reasons suggesting an unbiased nature of the perception errors in the agents' comparison, the existence of communication profiles that lead to biased comparisons is conceivable. Are such biased communication profiles played in equilibrium provided that they are available? Do they overthrow the role of preferences that we identified in our previous results?[35] We formally study these questions in Section S.2 below in a simple extension with binary communication strategies $A = \{0, 1\}$, where $a_j = 0$ corresponds to accurate communication, and $a_j = 1$ to communication that leads to noisy but biased comparisons in j's favor given that $a_{-j} = 0$. Preferences still play a key role for whether education or confusion results with possibly biased communication profiles: Mutual obfuscation remains the unique equilibrium with indecisive preferences, while education remains the only equilibrium outcome with polarized preferences if the contestants can unilaterally educate the agents, e.g., as in Heidhues et al. (2016).[36]

S.2.1 Asymmetric contestants: Formal analysis

Ex-ante asymmetric contestants In this section, we consider the formal model on which the intuitive discussion about ex-ante asymmetric contestants in the previous section was based on. We suppose that every choice of communication profile $\mathbf{a}$ determines a parameter $\omega = \omega(\mathbf{a}) \in [0, \bar{\omega}]$ of the density function $\gamma(\cdot, \omega)$ of the perception errors ε, where $\bar{\omega}$ is exogenously given.

The following technical assumptions are imposed on $\gamma(\cdot, \omega)$. First, $supp\, \gamma(\cdot, \omega) \subset supp\, \gamma(\cdot, \omega')$ whenever $\omega < \omega'$. Second, $\gamma(\cdot, \omega)$ is a zero-symmetric and log-concave C^1-density function on its support for any given $\omega \in (0, \bar{\omega})$. Third, $\gamma(\cdot, \omega')$ is an MPS of $\gamma(\cdot, \omega)$ whenever $\omega' > \omega$. We also take $\gamma(x, \omega)$ to be continuously differentiable in ω at any $x \in supp\, \gamma(\cdot, \omega)$. We denote the distribution and density of the perceived match advantages for any given as $G(\cdot, \omega)$ and $g(\cdot, \omega)$, respectively. Further, we let $G(x, 0) \equiv G_0(x)$ and $g(x, 0) \equiv g_0(x)$.

The payoff functions are given by (2) with $R(s_1, s_2) = 1$, where we replace $G_{\mathbf{a}}(s_1 - s_2)$

[35] These issues are relevant for the price competition application as well: For instance, if Duracel's advertises its batteries as the "longest-living batteries", then all other batteries must be short-lived. If sufficiently many consumers are influenced by this logic, then this would be a clear case of biased communication.

[36] If education cannot be forced unilaterally, two possibilities arise. With very large biases, obfuscation becomes so attractive that contestants are trapped in a Prisoner's Dilemma, with mutual obfuscation as the sole equilibrium outcome. For biases of intermediate size, a coordination game results with two asymmetric equilibria, where exactly one firm obfuscates.

by $G(s_1 - s_2, \omega)$ given our notational convention, where $G(s_1 - s_2, \omega) = \int_{-\infty}^{s_1-s_2} g(s, \omega) ds$ is the market share of contestant 1 for effort profile (s_1, s_2). Further, we assume that C_1, C_2 are C^2-functions with $C_j'(s), C_j''(s) > 0$ for any $s > 0$ and $C_j(0) = 0$, $j = 1, 2$. We assume that payoffs are strictly quasi-concave in own strategies for any given $\omega \in [0, \bar{\omega})$, meaning that $g'(s_1 - s_2, \omega) - C_j''(s_j) < 0$ whenever $g(s_1 - s_2, \omega) = C_j'(s_j)$. For a given $\omega \in [0, \bar{\omega})$, an interior effort equilibrium $s_1(\omega), s_2(\omega)$ is then determined by the first-order system

$$g(s_1 - s_2, \omega) = {C_1}'(s_1), \qquad g(s_1 - s_2, \omega) = {C_2}'(s_2). \tag{B.3}$$

In the following we consider ex-ante asymmetry of the contestants in terms of ranked cost functions, where $C_1'(s) < C_2'(s)$ for any $s > 0$. We refer to $j = 1$ as the *strong*, and to $j = 2$ as the *weak* contestant, respectively. Our main result in this section shows that the incentive to obfuscate or educate is quite unambiguous for the weak contestant.

Proposition S2 *If, for some sufficiently large $\delta > 0$, g_0 is strongly indecisive on $[-\delta, \delta]$, the weak contestant unambiguously desires maximal agent confusion ($\omega = \bar{\omega}$). If g_0 is strongly polarized on $[-\delta, \delta]$, the weak contestant unambiguously desires minimal agent education ($\omega = 0$).*

Proof of Proposition S2 We prove the proposition in a series of lemmas. Note that in what follows, we take for granted the existence of a unique effort equilibrium $s_1(\omega), s_2(\omega) > 0$ for any given $\omega \in [0, \bar{\omega})$.[37]

Lemma S1 *For any given $\omega \in [0, \bar{\omega}]$, $s_1(\omega) > s_2(\omega)$ and $\Pi_1(\omega) > \Pi_2(\omega)$.*

Proof: By (B.3), $C_1'(s_1) = C_2'(s_2)$ in equilibrium, from which $s_1 > s_2$ follows. Equilibrium payoffs are $\Pi_1(\omega) = G(s_1 - s_2, \omega) - C_1(s_1)$ and $\Pi_2(\omega) = 1 - G(s_1 - s_2, \omega) - C_2(s_2)$. The fact that $s_1 > s_2$ implies $G(s_1 - s_2, \omega) > 1/2$. Then

$$\Pi_1(\omega) \geq \frac{1}{2} - C_1(s_2) \geq \frac{1}{2} - C_2(s_2) > \Pi_2(\omega),$$

where the second inequality follows that $C_1(0) = C_2(0)$ and $C_1'(s) < C_2'(s)$ $\forall s > 0$. □

Define $s_\Delta(\omega) \equiv s_1(\omega) - s_2(\omega)$. Note that for sufficiently large $\delta > 0$ (which we assumed), $s_\Delta(\omega) \in [0, \delta - \bar{\omega})$ $\forall \omega \in [0, \bar{\omega}]$.

[37] Our formal analysis below can be extended to show that, actually, the strong quasi-concavity assumption already assures that at most one equilibrium can exist in the effort game.

Lemma S2 *Suppose that g_0 is strongly indecisive (polarized) on $supp\, g_0$. For any given $x \in [0, \delta - \bar{\omega})$, $g(x, \omega)$ is strictly decreasing (increasing) in ω.*

Proof. The requirement $x \in [0, \delta - \bar{\omega})$ assures that $supp\, g_\omega(x) \subset (-\delta, \delta)$ for any $\omega \in (0, \bar{\omega}]$. The claims follow from the proof of Theorem 2 by replacing $g_{\mathbf{a}'}(0)$ with $g(x, \omega')$ and $g_{\mathbf{a}}(0)$ with $g(x, \omega)$ in (A.3). □

Lemma S3 *Suppose that g_0 is strongly indecisive (polarized) on $supp\, g_0$. For any given $x \in (0, \delta - \bar{\omega})$, $\omega, \omega' \in [0, \bar{\omega}]$ with $\omega < \omega'$, $G(x, \omega') < (>) G(x, \omega)$.*

Proof: The claim follows from Lemma S2 because

$$G(x, \omega) = \frac{1}{2} + \int_0^x g(s, \omega) ds > \frac{1}{2} + \int_0^x g(s, \omega') ds = G(x, \omega'). \qquad \square$$

Lemma S4 *If g_0 is strongly indecisive on $supp\, g_0$, then $s_1'(\omega), s_2'(\omega) < 0$. If g_0 is strongly polarized on $supp\, g_0$, then $s_1'(\omega), s_2'(\omega) > 0$*

Proof: The assumptions imposed in the current model assure that (B.3) is a system of C^1 functions, so by the Implicit Function Theorem, we have for each $j = 1, 2$,

$$s_j{}'(\omega) = \frac{C_{-j}{}''}{A} \frac{\partial g(s_\Delta, \omega)}{\partial \omega}, \qquad A \equiv C_1{}'' C_2{}'' + (C_1{}'' - C_2{}'') \frac{\partial g(s_\Delta, \omega)}{\partial s_\Delta},$$

where $A > 0$ is implied by strong quasi-concavity. Hence $sign\, s_j'(\omega) = sign\, \left(\frac{\partial g(s_\Delta, \omega)}{\partial \omega}\right)$, and the claim for the indecisive case follows because $g(s_\Delta, \omega)$ is strictly decreasing in ω by Lemma S2.[38] The claim for the polarized case holds because then $g(s_\Delta, \omega)$ is strictly increasing in ω. □

Finally, we apply the Envelope Theorem to see how equilibrium payoffs respond to marginal changes of ω:

$$\begin{aligned} \Pi_1{}'(\omega) &= -g(s_1 - s_2, \omega) s_1{}'(\omega) + \frac{\partial G(s_1 - s_2, \omega)}{\partial \omega}, \\ \Pi_2{}'(\omega) &= -g(s_1 - s_2, \omega) s_2{}'(\omega) - \frac{\partial G(s_1 - s_2, \omega)}{\partial \omega}. \end{aligned} \qquad \text{(B.4)}$$

[38] Strictly speaking, the strict monotonicity in Lemma S2 allows us only to conclude that $\frac{\partial g(s_\Delta, \omega)}{\partial \omega} \leq 0$. As $\frac{\partial g(s_\Delta, \omega)}{\partial \omega} = 0$ can never occur on any arbitrary small interval around ω, we ignore the knife-edge case where $\frac{\partial g(s_\Delta, \omega_0)}{\partial \omega_0} = 0$ for some ω_0.

Because $g(\cdot) > 0$, Lemmas S3 and S4 imply that $\Pi_2(\omega)$ is strictly increasing (decreasing) on $[0, \bar{\omega}]$ if g_0 is strongly indecisive (strongly polarized). ■

The comparative statics (B.4) shows that agent confusion increases equilibrium payoffs by a competition-softening effect. In the symmetric model, where $s_1 = s_2$ and, accordingly, $G(s_1 - s_2, \omega) = 1/2$, this is the only force, explaining why both firms unambiguously benefit from agent confusion. By contrast, the market share effect always works in opposite directions for the two contestants whenever $G(s_1 - s_2, \omega) \in (1/2, 1)$, showing a potential conflict of interest in the contestants' desire for confusion or education. Nevertheless, it turns out that the sensitivity and market share effects always work in the same direction for the weak, and always in opposite directions for the strong contestant. In particular, the weak (strong) contestant always gains (loses) market share if ω increases in case of indecisive preferences, and vice-versa in case of polarized preferences.

Ex-Interim Asymmetry We now consider a simple extension allowing for the possibility that biased perception errors $\varepsilon_{\mathbf{a}}$, leading to ex-interim asymmetries of the contestants, may result as a consequence of the chosen communication strategies. Suppose that $A = \{0, 1\}$, so that $\mathcal{A}$ consists of four ordered pairs, associated with four random variables $\varepsilon_{\mathbf{a}}$, each with $supp\,(g_{\mathbf{a}}) \subset supp\,(g_0)$. Then, $a_j = 0$ means that j communicates in a neutral way, while $a_j = 1$ means that j's communication is obfuscating with the possible effect of biasing perception towards the firm. We assume that $\varepsilon_{0,0} = O$, $\varepsilon_{1,1} \neq O$ is zero-symmetric, and $\varepsilon_{1,0} = -\varepsilon_{0,1}$. The symmetry of $\varepsilon_{1,1}$ captures that a communication profile where *both* contestants seek to bias valuations in their favor (e.g. both exaggerate the valuations of their offers) results in unbiased agent confusion.[39]

Suppose that for any such given $\varepsilon_{\mathbf{a}}$ the effort subgame following $\varepsilon_{\mathbf{a}}$ has a unique effort equilibrium, with corresponding equilibrium payoffs indicated by $\Pi_{\mathbf{a}}$, as depicted in the game matrix of Figure S.2.

In any SPE, the choice of communication profile must induce a Nash equilibrium in that game. If $\varepsilon_{1,0} = \varepsilon_{0,1}$ is zero-symmetric and $\varepsilon_{1,0} \neq O$, this is just a special case of our main model and, by Theorem 1, preferences alone are decisive for the type of SPE that results. It is easily observed that the same equilibrium pattern holds if the bias induced

[39]For example, $\varepsilon_{1,1}$ always is zero-symmetric if $\varepsilon_{1,1} = \varepsilon^2 - \varepsilon^1$ and $\varepsilon^1, \varepsilon^2$ are iid. In particular, let $\varepsilon_{\mathbf{a}} \equiv \varepsilon^2_{a_2} - \varepsilon^1_{a_1}$, where $\varepsilon^1_{a_1}, \varepsilon^2_{a_2}$ are independent. Let $\varepsilon^j_0 = O$, ε^2_1 be uniform on $[0, \omega]$, and ε^1_1 be uniform on $[-\omega, 0]$, $\omega > 0$. Note that then $\varepsilon_{1,1}$ has a zero-symmetric density (a "tent" distribution) on $[-\omega, \omega]$.

		Contestant 2 $a=0$	Contestant 2 $a=1$
Contestant 1	$a=0$	Π_{00} , Π_{00}	Π_{01} , Π_{10}
	$a=1$	Π_{10} , Π_{01}	Π_{11} , Π_{11}

Figure S.2: Equilibrium payoffs in effort stage game

by $a_j = 1$ is "weak" in the sense that $\varepsilon_{1,0}$ remains close to zero-symmetrically distributed. Then, the unilateral advantage of a biased communication profile is dominated by the perception noise it induces. As long as $\Pi_{00} < (>)\Pi_{10}$ continues to hold, education can never be (always is) an SPE in case of indecisive (polarized) preferences.

If perception errors are strongly biased in favor of $j = 1$ if $a = (1, 0)$ is chosen (and $j = 2$ is equally favored for $a' = (0, 1)$), it becomes conceivable that $\Pi_{10} > \Pi_{00} > \Pi_{01}$. This reflects a redistribution of the perceived match advantages in favor of $j = 1$. Consider first the case of indecisive preferences. Because a potential bias in favor of the competitor can be annihilated by choosing $a = 1$, and pure perception noise is beneficial with indecisive preferences, it follows that $\Pi_{11} > \Pi_{01}$. Thus, the only equilibrium involves mutual obfuscation ($a_1 = a_2 = 1$), similar to the case with unbiased perception errors.

Now consider the case of polarized preferences, meaning that $\Pi_{00} > \Pi_{11}$. The type of SPE now depends crucially on the obfuscation technology. In particular, full agent education remains the unique equilibrium prediction if consumer education can be unilaterally enforced. Then, $\varepsilon_{\mathbf{a}} = O$ whenever $\mathbf{a} \neq (1, 1)$, meaning that both contestants earn Π_{00}, whenever at least one contestant chooses $a = 0$. As a result, any $\mathbf{a} \neq (1, 1)$ constitutes a Nash equilibrium in the first stage, and agents are fully educated. If however, education cannot be unilaterally enforced, $\Pi_{10} > \Pi_{00}$ implies that agent education cannot be sustained as an SPE outcome. The type of equilibrium then depends on whether $\Pi_{01} > \Pi_{11}$ or $\Pi_{01} < \Pi_{11}$. If $\Pi_{01} < \Pi_{11}$, correcting the bias is more valuable than the loss in payoffs incurred from mutual obfuscation due to polarized tastes. Then, the contestants are trapped in a Prisoner's Dilemma, with mutual obfuscation as the sole equilibrium outcome, which is both inefficient and harmful for the contestants. If instead $\Pi_{01} > \Pi_{11}$, then the contestants end up in a coordination game with two (pure-strategy) equilibria, where one contestant earns the rents from a biased communication profile. In sum, agent preferences play a decisive role for the equilibrium outcome also with potentially biased communication strategies. In particular, neither a Prisoner's Dilemma nor a coordination game can arise with indecisive tastes.

S.3 Decision Noise: Examples and Foundations

In this section, we provide a formalization of the leading examples for confusion we discussed in Section 2.2.1. Suppose that the perceived match values of the agents for a contestant $i \in \{1, 2\}$ are distributed according to $\tilde{v}_i = v_i + \varepsilon_i(\mathbf{a})$, where v_i is the distribution of true valuations for contestant i over the agent population, $\varepsilon_i(\mathbf{a})$ is a random variable, and $\mathbf{a} \in \mathcal{A}$ is a communication profile. Then, the distribution of the perceived *match advantages* of contestant $i = 2$ is given by (1) with $v_\Delta \equiv v_2 - v_1$ and $\varepsilon_{\mathbf{a}} \equiv \varepsilon_2(\mathbf{a}) - \varepsilon_1(\mathbf{a})$.[40] For easy reference, we let $\hat{\mathcal{H}}$ denote the set of all random variables with a 0-symmetric and log-concave density function. As in the main text, O denotes a constant random variable that yields $x = 0$ with probability one, and $\mathcal{H} \equiv \hat{\mathcal{H}} \cup O$.

Product Complexity and Information Overload The intuitive arguments from Section 2.2.1 about *overload confusion*, e.g., due to product complexity in terms of lengthy contracts, numerous marketing messages or product attributes can be easily expressed with a random utility model. Suppose that each contestant can choose a number of "features" to implement in the marketing process. Each feature involves imperfect mental information processing due to cognitive capacity limitations, which results in a noisy attribution of the product's valuation, where some agents overestimate the value of a given feature to them, whereas others underestimate it.[41] Then, confusion due to complexity can be captured by assuming that each implemented feature has an i.i.d. effect on an agent's evaluation of the product, determined by a random variable $Z_s \in \hat{\mathcal{H}}$. If contestant j implements n_j such features, the perception noise is determined as

$$\varepsilon_j(n_j) = \sum\nolimits_{s=0}^{n_j} Z_s, \tag{B.5}$$

where $\varepsilon_j(n_j) \in \mathcal{H}$ if $n_j > 0$, and $\varepsilon_j(0)$ is degenerate. Then, the number of features implemented ($n_j \geq 0$) corresponds to the communication strategy of contestant j. The

[40] This decomposition of $\varepsilon_{\mathbf{a}}$ is suitable for some but not all examples we develop below. In this respect, it is helpful to note that expressing the effects of communication strategies directly in terms of $\varepsilon_{\mathbf{a}}$, rather than via $\varepsilon_2(\mathbf{a}) - \varepsilon_1(\mathbf{a})$, is without loss of generality in the sense that any given zero-symmetric random variable can always be decomposed as a sum of two zero-symmetric random variables, and vice-versa (Rubin and Sellke, 1986).

[41] Unsystematic evaluation errors resulting from overwhelming information stimuli due to cognitive limitations are a well established fact for human behavior at least since Miller (1956).

broad meaning of (B.5) is that more features result in more unsystematic perception errors across agents. If $\varepsilon_1, \varepsilon_2$ are independent and determined by (B.5), then $\varepsilon(n_1, n_2) \equiv \varepsilon_2(n_2) - \varepsilon_1(n_1) \in \mathcal{H}$, and $\varepsilon(n_1, n_2) \in \hat{\mathcal{H}}$ iff $n_1 + n_2 > 0$. Because $\varepsilon(n'_1, n'_2)$ is a mean-preserving spread of $\varepsilon(n_1, n_2)$ whenever $n'_1 + n'_2 > n_1 + n_2$, the type of obfuscation process captured by (B.5) has the additional feature that the resulting distributions ε can be ordered in the sense of MPS.[42] Note that model (B.5) can capture that confusion occurs only once a sufficient number of features have been implemented (i.e., information is rich enough). This is achieved, e.g., by introducing a threshold value $\bar{n} \in \mathbb{N}$ such that $\varepsilon_j(n_j) = \sum_{s=0}^{n_j} Z_s$ if $n_j > \bar{n}$ and $\varepsilon_j(n_j) = O$ otherwise.

A possible limitation of the above model is that in reality different features may affect consumer perception differentially, possibly with dependencies across features. We can adapt the model to encompass such dependencies. Formally, let $Z = \{Z_1, ...Z_K\}$ denote a set of random variables, where the random vector $(Z_1, ..., Z_K)$ has a joint density function $f(z_1, ..., z_K)$ that is coordinate-wise symmetric, i.e.

$$f(z_1, ..., z_k, ..., z_K) = f(z_1, ..., -z_k, ..., z_K), \quad \forall (z_1, ..., z_K) \in supp(f), \forall k = 1, ..., K.$$

A communication strategy corresponds to a selection $M_j \subset Z$ of features implemented by firm j, which affects the product valuation according to

$$\varepsilon_j(M_j) = \sum_{k \in M_j} Z_k. \tag{B.6}$$

The set of marketing strategies A now corresponds to all possible selections, i.e., any marketing activity a_j belongs to the power set of Z, where $a_j = \emptyset$ means that no feature is chosen and ε_j is degenerate. If the density of Z is log-concave and coordinate-wise symmetric, so is the density of any non-empty selection M_j, meaning that $\varepsilon_j(M_j)$ is symmetric and log-concave as well. Assuming independence between $\varepsilon_1(M_1)$ and $\varepsilon_2(M_2)$ implies that $\varepsilon(M_1, M_2) = \varepsilon_2(M_2) - \varepsilon_1(M_1) \in \mathcal{H}$, and $\varepsilon(M_1, M_2) \in \hat{\mathcal{H}}$ if and only if $M_j \neq \emptyset$ for some $j \in \{1, 2\}$. Finally, the current set of marketing technology can be partially ordered according to MPS. In particular, $\varepsilon(M'_1, M'_2)$ is a mean-preserving spread of $\varepsilon(M_1, M_2)$ whenever $M_j \subsetneq M'_j$, $j = 1, 2$.

[42] Note that (B.5) also allows for a non-cognitive explanation, where the "features" are perceived without error but of heterogeneous valuations to the agents, but some agents like certain features that others dislike in a way that the effects cancel out across the agent population.

Labels and Frames The examples of labels from the main text illustrate the common finding in marketing that product complexity should be viewed as a synthetic phenomenon of *all* marketing messages interacting with each other. This notion can be formalized in our framework by assuming that the chosen marketing activities (labels, ads, design aspects,...) by both contestants *jointly* affect the individual evaluations of each product in the sense that $\varepsilon_j = \varepsilon_j(a_1, a_2)$, $j = 1, 2$. For instance, if one food brand uses the label "original" (a_1) while another brand uses "authentic" (a_2), the comparison of the two labels by consumers may cause confusion ($\varepsilon_j(a_1, a_2) \neq O$), which could have been avoided if both firms had coordinated on the same label (say, a_1).

A related point is made by Piccione and Spiegler (2012) and Chioveanu and Zhou (2013) in a homogeneous goods model, where the mutual choices of "frames", specifically ways to present the price of a product, determines whether or not a consumer can *compare* the prices of both products. If a comparison can be made, the consumer chooses the cheaper product; otherwise the consumer picks at random. Our framework can accommodate a notion of limited comparability by supposing that the chosen communication profile determines the frame by which a consumer compares the products, and as such the extent to which an adequate comparison can be made.[43] To formalize this idea, suppose that each communication profile $(a_1, a_2) \in \mathcal{A}$ induces a frame, i.e., a random variable $\varepsilon(a_1, a_2) \in \mathcal{H}$. The frame captures the range and distribution of the possible errors an individual consumer can make in her product comparison. For example, a frame that facilitates a comparison is such that $\varepsilon(a_1, a_2)$ has much of its probability mass around zero. Accordingly, with such a frame the consumers are more likely to make only small comparative mistakes.

Spiegler (2014) also considers a homogeneous-goods model, where two firms simultaneously choose their marketing messages, which jointly determine the *distribution* of the frames a single consumer could adopt. The adopted frame, in turn, determines the choice probabilities of each firm. It is possible to implement this idea in our framework as well. Let $F \subset \mathbb{R}$ be the set of possible frames, where each $f \in F$ corresponds to a deterministic way how a consumer draws a product comparison. The actual frame adopted by the consumer after being exposed to a communication profile $\mathbf{a}$ is unknown to the firms, while they know the probability distribution $\varepsilon_{\mathbf{a}}$ over the possible frames induced by $\mathbf{a}$. It is easy to see that under the respective assumptions on F and $\varepsilon_{\mathbf{a}}$, $\mathbf{a} \in \mathcal{A}$, an error

[43] Given the additive structure of (1) we can interpret the chosen frames as affecting net valuations or the price percepts.

structure that is consistent with our framework results.[44]

Product and Preference Uncertainty Another source of confusion in the evaluation of products is the case of *inadequate* product information. A lack of communication by the firm may imply that consumers are forced to form conjectures about the value of a product and its attributes. Suppose that each firm can decide how much qualified information to display to consumer regarding their product. The less information is provided, the more consumers need to guess the relevant valuation of the product for them. To illustrate, let $A = [0, 1]$ and suppose that consumer guessing for the value of firm j's product is depicted by a random variable $\varepsilon_j(a_j)$ with a uniform distribution on $[-a_j, a_j]$. Then, $a_j = 0$ corresponds to the case that firm j provides all relevant information, and less information (higher a_j) leads to more noisy product perception.

This example highlights a connection to the literature on informative advertising (Bagwell, 2007). In that literature, information typically is of a binary nature, where a consumer can either be perfectly informed about a product with all its relevant attributes, or entirely uninformed about its existence. In our case, consumers always know that both products exist, but require information to judge the extent to which the product matches their needs. Our model then asks when firms may deliberately abstain from providing consumers with sufficient information to annihilate product uncertainty from a market.

Further, if consumers have a general understanding of the market structure, this could restrict the type of inference they draw during their product evaluations. As an illustration, suppose that consumers understand that they are located on a Hotelling line with firms sitting at the opposed edges, so they are aware of the negative correlation between true valuations. The communication profiles therefore can only affect perception in a way that preserves this correlation. Thus, the random variables $\varepsilon_1, \varepsilon_2$ are perfectly negatively correlated, such that $\varepsilon_1 = -\varepsilon_2$, where $\varepsilon_1 \in \mathcal{H}$. This is essentially a model where marketing has the effect of randomly moving each consumer around her true location on the Hotelling line. The interpretation is that marketing may either aid or obstruct consumers from properly orienting themselves in a market whose structure they principally are capable of understanding.

[44]As a more technical remark, a property called Weighted Regularity (WR) plays a critical role for the equilibrium analysis in the homogeneous-good model of Piccione and Spiegler (2012) and Spiegler (2014). It is quite easy to see that WR may or may not be satisfied under the assumptions we imposed on our model, meaning that WR is not critical for our analysis.

Spurious Correlations In the last example, the negative correlation of valuation shocks originated from a basic understanding of consumers about the market structure. However, it is also conceivable that consumers form spurious correlations between their evaluations of the product, independent of the true market structure. As an illustration, suppose that a firm chooses to present its product in a simplistic way, while its competitor emphasizes, in detail, how many features its product has. Some consumers may come to believe that the second product is better as it seems to offer many functionalities, while other consumers value positively the apparent simplicity of the first product. However, in such a situation consumers with a better impression about the second product may also tend to have a worse view of the first product as being too simple (and vice-versa). For example, digital cameras for amateurs sometimes even feature more pixels, typically heavily advertised, than professional cameras, to give the impression that the camera is able to shoot particularly sharp photos. Likewise, if a firm advertises in superlatives, such as Duracel claiming to have the "longest-living batteries", this may lead some consumers to believe that other batteries must be worse. These examples suggest a spurious correlation of the following type: If product A is so good, then B must be really bad.

Any such spurious correlation can be captured by the joint distribution of the random vector $(\varepsilon_1, \varepsilon_2)$. An interesting question to ask is how much correlation the firms desire, if they can influence it through the communication profiles. For example, if for any $(a_1, a_2) \in A$, the random vector $(\varepsilon_1(a_1, a_2), \varepsilon_2(a_1, a_2))$ is jointly normal,

$$(\varepsilon_1, \varepsilon_2) \sim N\left(0, \begin{pmatrix} \sigma_{11} & \sigma_{12}(a_1, a_2) \\ \sigma_{12}(a_1, a_2) & \sigma_{22} \end{pmatrix}\right),$$

or a 0-symmetric truncation thereof, then $\varepsilon = \varepsilon_2 - \varepsilon_1$ also is normal with a variance that depends on the correlation between $\varepsilon_1, \varepsilon_2$. An interesting insight is that a more negative correlation leads to a greater dispersion of opinions as measured by the variance.[45] In this sense, the firms' desire for obfuscation leads to an increasingly polarized evaluation culture, where consumers judge the products in a way that a better impression of product A also implies a worse impression of product B.

Consumer Sophistication In research with behavioral agents, the population is frequently partitioned into two types: the "naive" and the "sophisticated", where the latter

[45] This type of reasoning generalizes beyond the normal case if the family of densities resulted by the marketing strategies in $\mathcal{A}$ can be ordered alone by their variance.

do not exhibit any behavioral bias. We now demonstrate that our main results also apply in the presence of agents with different degrees of sophistication. Let naiveté be an exogenous trait that is dispersed over the consumer population according to the random variable ρ, such that $\tilde{v}_\Delta(\mathbf{a}) = v_\Delta + \rho\varepsilon_\mathbf{a}$, where $supp\,\rho \subset [0,1]$, and ρ, v_Δ and $\varepsilon_\mathbf{a}$ are independent for any given $\mathbf{a} \in \mathcal{A}$. A consumer with $\rho = 0$ is fully sophisticated, meaning that her perceptions are unaffected by the chosen communication strategies: $\tilde{v}_\Delta(\mathbf{a}) = v_\Delta$ $\forall \mathbf{a} \in \mathcal{A}$. By contrast, a larger value of ρ means less sophistication in that the possible distortions induced by $\varepsilon_\mathbf{a}$ are amplified.[46] Alternatively, ρ can be seen as a measure for the level of "confusion proneness" (Walsh et al., 2007), capturing that different consumers may be differ in how susceptible they are to obfuscation techniques. It is straightforward to use first-order conditions to verify that $p^*_\mathbf{a} = \frac{1}{2\hat{g}_\mathbf{a}(0)}$ results in the pricing stage, where $\hat{g}_\mathbf{a}(0) = \int\int g_0(\tilde{\rho}\tilde{\varepsilon})d\Gamma_\mathbf{a}(\tilde{\varepsilon})d\Gamma_\rho(\tilde{\rho})$ and Γ_ρ denotes the distribution function of ρ. [47] It is evident from this expression that the main firm-side incentives to obfuscate or educate still depend exclusively on true preferences, especially the shape of g_0.[48]

S.4 Two-Sided Single Crossing Ordering

In this section, we discuss the two-sided single crossing (TSC) ordering of distributions. Formally, let Γ, Γ' be two zero-symmetric distribution functions with supports $[-\omega, \omega]$ and $[-\omega', \omega']$, respectively. We say that Γ' is *more dispersed* than Γ in the sense of TSC, denoted by $\Gamma' \succ_{TSC} \Gamma$, if either (i) Γ' has a density function γ' while Γ is degenerate at zero, or (ii) Γ also has a density function γ, $\omega' \geq \omega$ and $\forall e, e' \in [0, \omega')$ with $e' > e$,

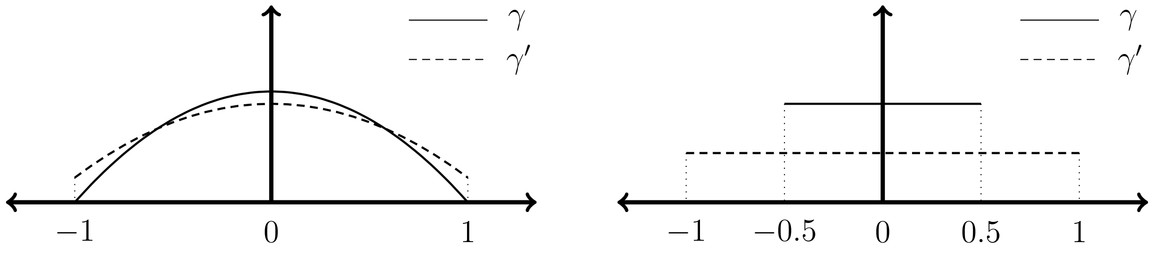
$$\gamma'(e) - \gamma(e) \geq 0 \Longrightarrow \gamma'(e') - \gamma(e') > 0. \tag{B.7}$$

In words, (B.7) requires that the two densities intersect at most once in $(-\omega', 0]$ and $[0, \omega')$, respectively; see Figure S.3.

[46]The information may be so complex that it cannot be fully assessed even by sophisticated agents (Eichenberger and Serna, 1996), in which case $\rho > 0$.

[47]The typical case of binary types occurs if all probability mass of Γ_ρ rests on $\rho = 0$ and $\rho = 1$, which yields $\hat{g}_\mathbf{a}(0) = \Gamma_\rho(0)g_\mathbf{a}(0) + (1 - \Gamma_\rho(0))g_0(0)$.

[48]In our framework, naive consumers impose a negative externality on sophisticated ones, independent of the shape of the preference distribution. This is because, in any case, the equilibrium measures taken by firms are such as to either confuse or educate the naive consumers, which increases the equilibrium price for sophisticated consumers as well. In fact, the desire for consumer education of firms and sophisticated consumers are exactly antipodal to each other.



(a) $\Gamma' \succ_{TSC} \Gamma$, $supp(\gamma') = supp(\gamma)$. (b) $\Gamma' \succ_{TSC} \Gamma$, $supp(\gamma') \supsetneq supp(\gamma)$.

Figure S.3: Examples of TSC orderings

Assumption S1 (TSC ordering) *$A \subset \mathbb{R}_+$ is compact, and $\varepsilon_{\mathbf{a}} = O \Leftrightarrow \mathbf{a} = \mathbf{0}$. Moreover, $\forall \mathbf{a}, \mathbf{a}' \in \mathcal{A}$ with $\mathbf{a} \neq \mathbf{a}'$ and $\mathbf{a} \leq \mathbf{a}'$, $\Gamma_{\mathbf{a}'} \succ_{TSC} \Gamma_{\mathbf{a}}$.*

The following theorem shows that we obtain the same type of result as with the MPS ordering (Theorem 2).

Theorem S1 *Suppose that Assumptions 1 and S1 hold.*

(i) If there exists $\delta > 0$ such that $supp(\varepsilon_{\mathbf{a}}) \subset [-\delta, \delta], \forall \mathbf{a} \in \mathcal{A}$ and true match values are δ-indecisive, then there exists a unique SPE, and confusion is maximal.

(ii) If there exists $\delta > 0$ such that $supp(\varepsilon_{\mathbf{a}}) \subset [-\delta, \delta], \forall \mathbf{a} \in \mathcal{A}$ and true match values are δ-polarized, then there exists a unique SPE, and confusion is minimal.

Proof: Consider part (i). Similar to the proof of Theorem 2, for this part of the proof it is without loss to assume that $\mathbf{0} \notin \mathcal{A}$. Take any $\mathbf{a}, \mathbf{a}'$ such that $\mathbf{a} \neq \mathbf{a}'$ and $\mathbf{a} \leq \mathbf{a}'$. Let $supp(\varepsilon_{\mathbf{a}}) = [-\omega_{\mathbf{a}}, \omega_{\mathbf{a}}]$ and $supp(\varepsilon_{\mathbf{a}'}) = [-\omega_{\mathbf{a}'}, \omega_{\mathbf{a}'}]$. Assumption S1 implies that there exists a unique $\hat{e} \in (0, \omega_{\mathbf{a}'})$, such that $\gamma_{\mathbf{a}'}(e) - \gamma_{\mathbf{a}}(e) < 0 \ \forall e \in [0, \hat{e})$, and $\gamma_{\mathbf{a}'}(e) - \gamma_{\mathbf{a}}(e) > 0$ $\forall e \in (\hat{e}, \omega_{\mathbf{a}'}]$. Since g_0 is strictly decreasing on $[0, \omega_{\mathbf{a}'}] \subset [0, \delta]$, we further have

$$\begin{aligned} \int_{\hat{e}}^{\omega_{\mathbf{a}'}} g_0(e) \left(\gamma_{a'}(e) - \gamma_a(e)\right) de &< \int_{\hat{e}}^{\omega_{\mathbf{a}'}} g_0(\hat{e}) \left(\gamma_{a'}(e) - \gamma_a(e)\right) de \\ &= \int_0^{\hat{e}} g_0(\hat{e})(\gamma_a(e) - \gamma_{a'}(e)) de \\ &< \int_0^{\hat{e}} g_0(e)(\gamma_a(e) - \gamma_{a'}(e)) de, \end{aligned} \tag{B.8}$$

where the equality makes use of the fact that, by symmetry and $\omega_{\mathbf{a}} \leq \omega_{\mathbf{a}'}$, we have

$$\frac{1}{2} = \int_0^{\omega_{\mathbf{a}'}} \gamma_{\mathbf{a}'}(e) de = \int_0^{\omega_{\mathbf{a}}} \gamma_{\mathbf{a}}(e) de = \int_0^{\omega_{\mathbf{a}'}} \gamma_{\mathbf{a}}(e) de.$$

Exploiting again the symmetry of g_0, $\gamma_{\mathbf{a}}$ and $\gamma_{\mathbf{a}'}$, we further have

$$\begin{aligned}
&\int_{-\omega_{\mathbf{a}'}}^{\omega_{\mathbf{a}'}} g_0(-e)\gamma_{\mathbf{a}'}(e)de - \int_{-\omega_{\mathbf{a}}}^{\omega_{\mathbf{a}}} g_0(-e)\gamma_{\mathbf{a}}(e)de \\
=& \int_{-\omega_{\mathbf{a}'}}^{\omega_{\mathbf{a}'}} g_0(-e)\left(\gamma_{\mathbf{a}'}(e) - \gamma_{\mathbf{a}}(e)\right) de \\
=& 2\int_{0}^{\omega_{\mathbf{a}'}} g_0(-e)\left(\gamma_{\mathbf{a}'}(e) - \gamma_{\mathbf{a}}(e)\right) de \\
=& 2\left[\int_{0}^{\hat{e}} g_0(-e)\left(\gamma_{\mathbf{a}'}(e) - \gamma_{\mathbf{a}}(e)\right) de + \int_{\hat{e}}^{\omega_{\mathbf{a}'}} g_0(-e)\left(\gamma_{\mathbf{a}'}(e) - \gamma_{\mathbf{a}}(e)\right) de\right] \\
=& 2\left[\int_{0}^{\hat{e}} g_0(e)\left(\gamma_{\mathbf{a}'}(e) - \gamma_{\mathbf{a}}(e)\right) de + \int_{\hat{e}}^{\omega_{\mathbf{a}'}} g_0(e)\left(\gamma_{\mathbf{a}'}(e) - \gamma_{\mathbf{a}}(e)\right) de\right] < 0,
\end{aligned}$$

where the last inequality follows from (B.8). We have thus shown that $g_{\mathbf{a}'}(0) < g_{\mathbf{a}}(0)$ for any feasible $\mathbf{a} \neq \mathbf{a}'$ with $\mathbf{a} \leq \mathbf{a}'$. Hence, if preferences are δ-indecisive, $g_{\mathbf{a}}(0)$ must be uniquely minimized at $\mathbf{a}^* = (\bar{a}, \bar{a})$. By arguments analogous to the case with MPS ordering (Theorem 2), we can conclude that there exists a unique SPE, and $\mathbf{a}^* = (\bar{a}, \bar{a})$ is the unique equilibrium outcome in the first stage. The proof for part (ii) is analogous, and thus omitted. ■

The MPS Theorem 2 and Theorem S1 cannot be ranked according to their generality. First, if the distributions $\{\Gamma_{\mathbf{a}}\}_{\mathbf{a}\in\mathcal{A}}$ are ordered by the TSC criterion, they are also ordered by the MPS criterion, while the converse generally is false. Second, we only need to impose indecisive or polarized match values with the TSC ordering in Theorem S1, while we need their strong counterparts with the MPS ordering in Theorem 2.

S.5 Welfare Loss: Additional Results

The following two propositions formalize the claims made in Section 4.2 about the size of the welfare loss in the case of indecisive preferences.

Proposition S3 *Consider the price competition application, and suppose that for any* $\mathbf{a} \in \mathcal{A}$, $\varepsilon_{\mathbf{a}}$ *is uniformly distributed on* $[-\omega_{\mathbf{a}}, \omega_{\mathbf{a}}]$, $\omega_{\mathbf{a}} > 0$, *whenever* $\varepsilon_{\mathbf{a}}$ *is non-degenerate. Then, the expected welfare loss* (6) *is strictly increasing in* $\omega_{\mathbf{a}}$.

PROOF: Let $\kappa \equiv \sup supp\,(g_0)$. We can write the expected welfare loss from mismatch as a function of the degree of confusion:

$$L(\omega) = 2\int_0^{\min\{\omega,\kappa\}} \left[x \cdot \frac{-x+\omega}{2\omega} \cdot g_0(x)\right] dx = \int_0^{\min\{\omega,\kappa\}} \left[x\left(1-\frac{x}{\omega}\right) g_0(x)\right] dx.$$

Taking the first derivative, we obtain

$$L'(\omega) = \int_0^{\kappa} \left[\frac{x^2}{\omega^2}\right] dG_0(x)$$

if $\omega \geq \kappa$, and

$$L'(\omega) = \int_0^{\omega} \left[\frac{x^2}{\omega^2}\right] dG_0(x) + \omega\left(1-\frac{\omega}{\omega}\right) g_0(\omega) = \int_0^{\omega} \left[\frac{x^2}{\omega^2}\right] dG_0(x)$$

if $\omega < \kappa$. Since by assumption G_0 is a non-degenerate distribution, $L'(\omega) > 0$ $\forall \omega > 0$. Hence, the expected welfare loss is strictly increasing in ω. ■

Proposition S4 *Consider the model with competition on the line. Suppose that $\varepsilon_{\mathbf{a}}$ is as in Proposition S3. If $\omega_{\mathbf{a}} < \hat{\omega} \equiv 64/15$, then the expected welfare loss (6) is strictly decreasing in α. If $\omega_{\mathbf{a}} > \hat{\omega}$, then the expected welfare loss is strictly increasing in α.*

PROOF: Since $G_0(x) = H\left(\frac{x}{4\lambda}\right)$ $\forall x \in \mathbb{R}$, the density function of G_0, which we denote as g_0, is given by

$$g_0(x) = \frac{1}{4\lambda} h\left(\frac{x}{4\lambda}\right) = \begin{cases} \frac{1}{4\lambda} \cdot \left(\alpha\left(\frac{x}{4\lambda}\right)^2 + \frac{1}{2\lambda} - \frac{\alpha\lambda^2}{3}\right) & \text{if } x \in [-4\lambda^2, 4\lambda^2], \\ 0 & \text{otherwise.} \end{cases}$$

The welfare loss can now be written as a function of α:

$$\begin{aligned} L(\alpha) &= \int_0^{\min\{\omega,4\lambda^2\}} \left[x\left(1-\frac{x}{\omega}\right) \cdot \frac{1}{4\lambda} \cdot h\left(\frac{x}{4\lambda}\right)\right] dx \\ &= \frac{1}{4\lambda}\int_0^{\min\{\omega,4\lambda^2\}} \left[x\left(1-\frac{x}{\omega}\right)\left(\alpha\left(\frac{x}{4\lambda}\right)^2 + \frac{1}{2\lambda} - \frac{\alpha\lambda^2}{3}\right)\right] dx. \end{aligned}$$

Taking derivative with respect to α, we have

$$L'(\alpha) = \frac{1}{4\lambda}\int_0^{\min\{\omega,4\lambda^2\}} \left[x\left(1-\frac{x}{\omega}\right)\left(\left(\frac{x}{4\lambda}\right)^2 - \frac{\lambda^2}{3}\right)\right] dx.$$

First, suppose that $\omega \leq 4\lambda^2$. In this case, we obtain

$$\begin{aligned}
L'(\alpha) &= \frac{1}{4\lambda}\int_0^{\omega}\left[x\left(1-\frac{x}{\omega}\right)\left(\left(\frac{x}{4\lambda}\right)^2-\frac{\lambda^2}{3}\right)\right]dx \\
&= \frac{1}{4\lambda}\left[\int_0^{\omega}\left(\frac{x^3}{16\lambda^2}-\frac{\lambda^2 x}{3}\right)dx - \int_0^{\omega}\left(\frac{x^4}{16\lambda^2\omega}-\frac{\lambda^2x^2}{3\omega}\right)dx\right] \\
&= \frac{1}{4\lambda}\left[\left(\frac{x^4}{64\lambda^2}-\frac{\lambda^2x^2}{6}\right)\Big|_0^{\omega} - \left(\frac{x^5}{80\lambda^2\omega}-\frac{\lambda^2x^3}{9\omega}\right)\Big|_0^{\omega}\right] \\
&= \frac{1}{4\lambda}\left[\frac{\omega^4}{64\lambda^2}-\frac{\omega^4}{80\lambda^2}-\frac{\lambda^2\omega^2}{6}+\frac{\lambda^2\omega^2}{9}\right].
\end{aligned}$$

Hence, provided that $\omega \in (0, 4\lambda^2]$, we further have

$$L'(\alpha) < 0 \iff \left(\frac{1}{64\lambda^2}-\frac{1}{80\lambda^2}\right)\omega^2 < \frac{\lambda^2}{6}-\frac{\lambda^2}{9} \iff \omega < \frac{4\sqrt{10}}{3}\lambda^2.$$

Since $4\sqrt{10}/3 \approx 4.22 > 4$, it follows that $L'(\alpha) < 0$ whenever $\omega \leq 4\lambda^2$.

Next, consider the case where $\omega > 4\lambda^2$. Expanding the equation $L'(\alpha)$ again, we have

$$\begin{aligned}
L'(\alpha) &= \frac{1}{4\lambda}\int_0^{4\lambda^2}\left[x\left(1-\frac{x}{\omega}\right)\left(\left(\frac{x}{4\lambda}\right)^2-\frac{\lambda^2}{3}\right)\right]dx \\
&= \frac{1}{4\lambda}\left[\left(\frac{x^4}{64\lambda^2}-\frac{\lambda^2x^2}{6}\right)\Big|_0^{4\lambda^2} - \left(\frac{x^5}{80\lambda^2\omega}-\frac{\lambda^2x^3}{9\omega}\right)\Big|_0^{4\lambda^2}\right] \\
&= \frac{1}{4\lambda}\left[\left(\frac{4^4\lambda^8}{64\lambda^2}-\frac{4^2\lambda^6}{6}\right)-\frac{1}{\omega}\left(\frac{4^5\lambda^{10}}{80\lambda^2}-\frac{4^3\lambda^8}{9}\right)\right] \\
&= 4\lambda^5\left[\left(\frac{16}{64}-\frac{1}{6}\right)-\frac{\lambda^2}{\omega}\left(\frac{64}{80}-\frac{4}{9}\right)\right].
\end{aligned}$$

Hence, provided that $\omega > 4\lambda^2$, we further have

$$L'(\alpha) > 0 \iff \frac{\lambda^2}{\omega}\left(\frac{4}{5}-\frac{4}{9}\right) < \frac{1}{4}-\frac{1}{6} \iff \omega > \frac{64}{15}\lambda^2.$$

Note that $64/15 \approx 4.27 > 4$. We can now conclude that $L'(\alpha) < 0$ whenever $\omega < \hat{\omega} \equiv 64\lambda^2/15$, and $L'(\alpha) > 0$ whenever $\omega > \hat{\omega}$. ■

S.6 The Role of Outside Options

In this section we analyze the SPE in the model with outside options from Section 4.3, using the more general formulation where perceived match values of a consumer k are

$\tilde{v}_1^k = \frac{m}{2} + \frac{v^k}{2} + \frac{\varepsilon^k}{2}$, $\tilde{v}_2 = \frac{m}{2} - \frac{v^k}{2} - \frac{\varepsilon^k}{2}$, where $m > 0$ is an exogenous constant, and v^k is symmetrically distributed over $[-1, 1]$ with distribution G_0 and density g_0. The parameter $m > 0$ is relevant only for the decision whether to buy any good, where a larger value of m means that, ceteris paribus, a consumer is more likely to purchase a good. The example in the main text corresponds to the special case $m = 2$.

Assumption S2 *G_0 is log-concave on supp(g_0), g_0 is continuous on supp(g_0) with $g_0(0) > 0$, and $\forall \mathbf{a} \in \mathcal{A}$ either $\varepsilon_{\mathbf{a}} = O$ or $\varepsilon_{\mathbf{a}}$ has a density $\gamma_{\mathbf{a}}$ that is log-concave on supp$(\gamma_{\mathbf{a}})$.*

In the current setting, the distribution of v^k coincides with the distribution of the true match advantages. As a result, Assumption S2 implies conditions (i) – (iii) of Proposition 1. These conditions assured the existence of a unique symmetric equilibrium in the pricing stage of the game without outside option for any given communication profile $\mathbf{a} \in \mathcal{A}$, and play a similar role here. We begin our analysis with a characterization of the symmetric equilibria in the pricing stage.

Proposition S5 *Suppose that Assumption S2 holds. In the above game with outside option, there exists a unique symmetric pure-strategy equilibrium in the pricing stage, and $\forall \mathbf{a} \in \mathcal{A}$ both firms choose the price*

$$p^*_{\mathbf{a}} = \begin{cases} \frac{1}{2g_{\mathbf{a}}(0)} & \text{if } g_{\mathbf{a}}(0) > \frac{1}{m}, \\ \frac{m}{2} & \text{if } g_{\mathbf{a}}(0) \in \left[\frac{1}{2m}, \frac{1}{m}\right], \\ p^M_{\mathbf{a}} > \frac{m}{2} & \text{otherwise}, \end{cases} \tag{B.9}$$

where $p^M_{\mathbf{a}}$ solves the monopoly problem $\max_{p\geq 0} \Pi^M_{\mathbf{a}}(p) \equiv p\,(1 - G_{\mathbf{a}}(2p - m))$. The equilibrium demand of each firm is strictly less than $1/2$ if and only if $g_{\mathbf{a}}(0) < \frac{1}{2m}$.

All proofs are at the end of this section. The competition resulting from the presence of sufficiently many perceptually indifferent consumers ($g_{\mathbf{a}}(0) > 1/m$) disciplines both firms to choose an equilibrium price for which the outside option is non-binding for every consumer.[49] For lower values of $g_{\mathbf{a}}(0)$, competition is less intense, yielding a strong temptation to increase prices. As long as $g_{\mathbf{a}}(0) \in [\frac{1}{2m}, \frac{1}{m}]$, both firms settle exactly at the price $p = m/2$ that just keeps every consumer in the market.[50] By contrast, for $g_{\mathbf{a}}(0) < \frac{1}{2m}$ firms increase prices even though some consumers exit the market.

[49] See Armstrong and Zhou (2019) for a similar result in a different setup.

[50] Absent a binding outside option, both firms would increase their price above $m/2$.

In essence, the pricing pattern identified by Proposition S5 determines what type of communication strategies the firms choose in the first stage. Specifically, if $g_{\mathbf{a}}(0) \geq \frac{1}{m}$ $\forall \mathbf{a} \in \mathcal{A}$, the SPE identified by Theorems 1 and 2 apply given the respective assumptions on g_0. Further, if true preferences are strongly indecisive on $supp\, g_0 = [-1, 1]$, $g_{\mathbf{a}}(0) \geq \frac{1}{2m}$ $\forall \mathbf{a} \in \mathcal{A}$ and $\{\Gamma_{\mathbf{a}}\}$ verify an MPS ordering, maximal obfuscation is always an SPE outcome.[51] Most importantly, Proposition S5 suggests that the firms may desire to confuse on a massive scale so that they can then exploit some local monopoly power, even though many consumers choose to exit.

To confirm this idea, we now analyze the tractable case where $\Gamma_{\mathbf{a}}$ follows a uniform distribution with support $[-\omega_{\mathbf{a}}, \omega_{\mathbf{a}}]$, $\forall \mathbf{a} \in \mathcal{A}$, and $g_0(0) > \frac{1}{m}$. The latter assumption implies that without confusion the equilibrium price $p_0^* = \frac{1}{2g_0(0)}$ is such that the outside option is non-binding for all consumers. To simplify the presentation of results, we also assume $m \geq 2$. We first apply Proposition S5 to derive the equilibrium price, demand and payoff as a function of consumer confusion ω given that confusion is massive ($\omega \geq 1$).

Corollary S1 *Let $m \geq 2$ and suppose that Assumption S2 is satisfied. For any given $\omega > 0$, let Γ_ω follow a uniform distribution with support $[-\omega, \omega]$, and let p_ω denote the symmetric equilibrium price in the pricing game given consumer confusion Γ_ω. Likewise, let d_ω and Π_ω denote the corresponding demand and payoff of each firm. Then $p_\omega, d_\omega, \Pi_\omega : \mathbb{R}_{++} \to \mathbb{R}$ are continuous functions of ω, and if confusion is massive ($\omega \geq 1$), we have*

$$p_\omega = \begin{cases} \omega & \text{if } \omega \in \left[1, \frac{m}{2}\right) \\ \frac{m}{2} & \text{if } \omega \in \left[\frac{m}{2}, m\right] \\ \frac{\omega+m}{4} & \text{if } \omega > m \end{cases}, \quad d_\omega = \begin{cases} \frac{1}{2} & \text{if } \omega \in [1, m] \\ \frac{\omega+m}{4\omega} & \text{if } \omega > m \end{cases}, \text{ and } \Pi_\omega = p_\omega d_\omega.$$

If $1 \leq \omega \leq m$ all consumers buy a product; if $\omega > m$ the fraction of consumers leaving the market is $L(\omega) = \frac{\omega - m}{2\omega} > 0$, which is strictly increasing in ω with $\lim_{\omega \to \infty} L(\omega) = 1/2$.

Corollary S1 shows that equilibrium prices and payoffs are increasing and unbounded in the range of confusion ω, despite an increasing fraction of consumers who abstain from acquiring any product.

We now ask how the intensity of confusion $\omega = \omega_{\mathbf{a}}$ is determined by strategically behaving firms that fully anticipate the profit schedule Π_ω resulting from the various

[51]The only difference to the case without outside option is that uniqueness of equilibrium may fail, despite an MPS ordering. In particular, any $\mathbf{a} \in \mathcal{A}$ inducing a value $g_{\mathbf{a}}(0) \in [\frac{1}{2m}, \frac{1}{m}]$ is an SPE with second-stage price $p_{\mathbf{a}}^* = m/2$.

feasible confusion intensities. We impose a structure on the mapping $\mathbf{a} \mapsto \omega_{\mathbf{a}}$ that is consistent with the MPS order Assumption 3: $A \subset \mathbb{R}_+$ is compact, $0 \in A$, and $\omega : \mathcal{A} \to \mathbb{R}_+$ is such that $\omega_{\mathbf{a}'} > \omega_{\mathbf{a}}$ iff $\mathbf{a}' \geq \mathbf{a}$ and $\mathbf{a}' \neq \mathbf{a}$.[52] Further, define $\bar{a} \equiv \max A > 0$ and $\bar{\omega} \equiv \omega_{\bar{\mathbf{a}}}$ as the maximally feasible confusion. Given this structure, we now show that the SPE follow the same pattern as identified by Theorems 1 – 3 except that the chosen communication strategies may lead to consumer exit from the market.

Consider first the case where $\bar{\omega} \leq 1$, such that massive confusion is not feasible. If preferences are indecisive on $[-1, 1]$, then $g_{\mathbf{a}}(0) = \frac{1}{2\omega_{\mathbf{a}}} \int_{-\omega_{\mathbf{a}}}^{\omega_{\mathbf{a}}} g_0(e)de$ is strictly decreasing in the intensity of confusion $\omega_{\mathbf{a}}$, and $g_{\mathbf{a}}(0) \geq \frac{1}{2\bar{\omega}}$ $\forall \mathbf{a} \in \mathcal{A}$. Because $m \geq 2$ also assures $\frac{1}{2\bar{\omega}} > \frac{1}{m}$, it follows that $g_{\mathbf{a}}(0) > \frac{1}{m}$ $\forall \mathbf{a} \in \mathcal{A}$, and Proposition S5 implies that $p^*_{\mathbf{a}} = \frac{1}{2g_{\mathbf{a}}(0)}$, $\forall \mathbf{a} \in \mathcal{A}$. This shows that with indecisive preferences and $\bar{\omega} \leq 1$ (i) any SPE is such that no consumer leaves the market, (ii) there cannot be an SPE without confusion (as in Theorem 1), and (iii) maximal confusion is the unique SPE outcome in case of strongly indecisive preferences (as in Theorem 2). If preferences are polarized on $[-1, 1]$, then $g_{\mathbf{a}}(0)$ must be strictly increasing in the intensity of confusion $\omega_{\mathbf{a}}$. Together with $g_0(0) > \frac{1}{m}$ it follows from Proposition S5 that $p^*_{\mathbf{a}} = \frac{1}{2g_{\mathbf{a}}(0)}$, $\forall \mathbf{a} \in \mathcal{A}$. Thus, (i) any SPE is such that no consumer leaves the market, (ii) education always is an SPE outcome (as in Theorem 1), and education is the unique SPE outcome with strongly polarized preferences (as in Theorem 2).

The following result allows for the possibility that massive confusion may arise. The main point is that maximal confusion becomes the unique SPE outcome if confusion can become massive enough, even though a substantial portion of consumers chooses not to buy at all.

Proposition S6 *Consider the above example where $\Gamma_{\mathbf{a}}$ follows a uniform distribution for each $\mathbf{a} \in \mathcal{A}$. If G_0 is indecisive on supp g_0, then a unique SPE with maximal confusion always exists, and a fraction* $\max\{\frac{\bar{\omega}-m}{2\bar{\omega}}, 0\}$ *of consumers leaves the market. If G_0 is polarized on supp g_0, then maximal confusion is an SPE outcome whenever $\bar{\omega} \geq \frac{m}{2}$, and the unique SPE whenever $\bar{\omega} > m$, in which case a fraction $\frac{\bar{\omega}-m}{2\bar{\omega}} \in (0, \frac{1}{2})$ of consumers leaves the market.*

[52]It can be checked that the uniform distribution verifies the TSC order criterion (see Section S.4). A simple example is given by $\omega(\mathbf{a}) = z(a_1 + a_2)$ where z is any strictly increasing function with $z(0) = 0$.

S.6.1 Proofs

Proof of Proposition S5 For any given $\Gamma_{\mathbf{a}}$, the demand function of firm 1 is

$$
\begin{aligned}
D_{\mathbf{a}}(p_1,p_2) &= \int \Pr\left(\tilde{v}_1^k - p_1 \geq \max\{\tilde{v}_2^k - p_2, 0\}\right) d\Gamma_{\mathbf{a}} \\
&= \int \Pr\left(v \geq p_1 - p_2 - e,\ v \geq 2p_1 - m - e\right) d\Gamma_{\mathbf{a}}(e) \\
&= \int \min\left\{\Pr\left(v \geq p_1 - p_2 - e\right), \Pr\left(v \geq 2p_1 - m - e\right)\right\} d\Gamma_{\mathbf{a}}(e) \\
&= 1 - \int \max\left\{G_0\left(p_1 - p_2 - e\right), G_0\left(2p_1 - m - e\right)\right\} d\Gamma_{\mathbf{a}}(e).
\end{aligned}
$$

Recall that, for all $x \in \mathbb{R}$,

$$
G_{\mathbf{a}}(x) = \int G_0(x-e)d\Gamma_{\mathbf{a}}(e), \text{ and } g_{\mathbf{a}}(x) = \int g_0(x-e)d\Gamma_{\mathbf{a}}(e).
$$

Thus, for all $p \geq 0$, we have

$$
D_{\mathbf{a}}(p,p) = \begin{cases} \frac{1}{2} & \text{if } p \leq \frac{m}{2}, \\ 1 - G_{\mathbf{a}}(2p-m) & \text{if } p > \frac{m}{2}. \end{cases} \tag{B.10}
$$

Let $\Pi_1^{\mathbf{a}}(p_1,p_2) = p_1 D_{\mathbf{a}}(p_1,p_2)$. For every $p_2 > 0$, $\Pi_1^{\mathbf{a}}$ is differentiable in p_1 almost everywhere. In particular, if $p_1 < m - p_2$, we have $D_{\mathbf{a}}(p_1,p_2) = 1 - G_{\mathbf{a}}(p_1 - p_2)$, and

$$
\frac{\partial \Pi_1^{\mathbf{a}}(p_1,p_2)}{\partial p_1} = 1 - G_{\mathbf{a}}(p_1 - p_2) - p_1 g_{\mathbf{a}}(p_1 - p_2),
$$

which is also the left derivative of $\Pi_1^{\mathbf{a}}(p_1,p_2)$ at $p_1 = m - p_2$. Similarly, if $p_1 > m - p_2$, such that $D_{\mathbf{a}}(p_1,p_2) = 1 - G_{\mathbf{a}}(2p_1 - m)$, we have

$$
\frac{\partial \Pi_1^{\mathbf{a}}(p_1,p_2)}{\partial p_1} = 1 - G_{\mathbf{a}}(2p_1 - m) - 2p_1 g_{\mathbf{a}}(2p_1 - m),
$$

which is also the right derivative of $\Pi_1^{\mathbf{a}}(p_1,p_2)$ at $p_1 = m - p_2$.

Since log-concavity is preserved under convolution, the function $G_{\mathbf{a}}$ is log-concave on its support $supp(g_{\mathbf{a}})$. In addition, since $G_{\mathbf{a}}$ is a distribution function, its log-concavity also holds on $[0,+\infty)$. Hence, for all $p_2 > 0$ and $\mathbf{a} \in \mathcal{A}$, the demand function $D_{\mathbf{a}}(p_1,p_2)$ must be log-concave in p_1 on both $[0, m-p_2]$ and $[m-p_2, +\infty)$. Note that we are not claiming that $D_{\mathbf{a}}(p_1, p_2)$ is log-concave in p_1 on the entire interval $[0, +\infty)$. In what follows, we will show that although the global log-concavity of the demand function is not assured, Assumption S2 is still sufficient to guarantee the existence of a unique symmetric equilibrium in every subgame of the pricing stage.

First, suppose that $g_{\mathbf{a}}(0) > \frac{1}{m}$. Suppose also that firm 2 is choosing $p_2 = \frac{1}{2g_{\mathbf{a}}(0)} < \frac{m}{2}$. Then, for any $p_1 \leq \frac{m}{2}$ the whole market is guaranteed to be covered (i.e., every consumer will buy from one of the firms). In addition, since

$$\left.\frac{\partial \Pi_1^{\mathbf{a}}(p_1, p_2)}{\partial p_1}\right|_{p_1 = p_2 = \frac{1}{2g_{\mathbf{a}}(0)} < \frac{m}{2}} = 1 - G_{\mathbf{a}}(0) - \frac{1}{2g_{\mathbf{a}}(0)} \cdot g_{\mathbf{a}}(0) = 0,$$

and the function $\Pi_1^{\mathbf{a}}\left(p_1, \frac{1}{2g_{\mathbf{a}}(0)}\right)$ is strictly quasi-concave in p_1 on $\left[0, m - \frac{1}{2g_{\mathbf{a}}(0)}\right]$, and $g_{\mathbf{a}}(0) > \frac{1}{m}$ implies that $p_1 = \frac{1}{2g_{\mathbf{a}}(0)}$ maximizes the function $\Pi_1^{\mathbf{a}}\left(p_1, \frac{1}{2g_{\mathbf{a}}(0)}\right)$ over the range $\left[0, m - \frac{1}{2g_{\mathbf{a}}(0)}\right]$. We now argue that, in addition,

$$\Pi_1^{\mathbf{a}}\left(p_1, \frac{1}{2g_{\mathbf{a}}(0)}\right) < \Pi_1^{\mathbf{a}}\left(\frac{1}{2g_{\mathbf{a}}(0)}, \frac{1}{2g_{\mathbf{a}}(0)}\right) \ \forall p_1 > m - \frac{1}{2g_{\mathbf{a}}(0)}.$$

To see this, note that if $p_1 > m - \frac{1}{2g_{\mathbf{a}}(0)}$, then some consumers choose their outside options even though they would prefer firm 1 over firm 2. Therefore, a deviation to $p_1 > m - \frac{1}{2g_{\mathbf{a}}(0)}$ cannot be more profitable than it would have been in the case without outside option. But then, as we have shown in Lemma 1 and Proposition 1, in the absence of the outside option, choosing $p_1 = \frac{1}{2g_{\mathbf{a}}(0)}$ actually uniquely maximizes firm 1's expected profits over $[0, +\infty)$ given that its competitor plays $p_2 = \frac{1}{2g_{\mathbf{a}}(0)}$. This implies that deviating to $p_1 > m - \frac{1}{2g_{\mathbf{a}}(0)}$ cannot be profitable either in the presence of the outside option. Therefore, $p_1 = \frac{1}{2g_{\mathbf{a}}(0)}$ must be a global maximum of the function $\Pi_1^{\mathbf{a}}\left(p_1, \frac{1}{2g_{\mathbf{a}}(0)}\right)$, and $(p_1, p_2) = \left(\frac{1}{2g_{\mathbf{a}}(0)}, \frac{1}{2g_{\mathbf{a}}(0)}\right)$ indeed constitutes an equilibrium in the pricing subagme. It is easy to see that this is the only symmetric equilibrium with a price strictly less than $\frac{m}{2}$. In addition, since

$$\left.\frac{\partial \Pi_{\mathbf{a}}^M(p)}{\partial p}\right|_{p = \frac{m}{2}} = \frac{1}{2} - m \cdot g_{\mathbf{a}}(0) < \frac{1}{2} - 1 < 0$$

and $\Pi_{\mathbf{a}}^M(p)$ is strictly quasi-concave, even a monopoly firm would not choose a price $p \geq \frac{m}{2}$. Hence, when $g_{\mathbf{a}}(0) > \frac{1}{m}$, there cannot be any symmetric equilibrium in which both firms choose a price larger than $\frac{m}{2}$. As a result, $(p_1, p_2) = \left(\frac{1}{2g_{\mathbf{a}}(0)}, \frac{1}{2g_{\mathbf{a}}(0)}\right)$ is the unique symmetric pure-strategy equilibrium when $g_{\mathbf{a}}(0) > \frac{1}{m}$.

Next, consider the case $g_{\mathbf{a}}(0) \in [\frac{1}{2m}, \frac{1}{m}]$. Taking $p_2 = \frac{m}{2}$ as given, we will show that $p_1 = \frac{m}{2}$ is a best response for firm 1. As mentioned, the profit function $\Pi_1(p_1, p_2)$ is differentiable in p_1 on $\mathbb{R}_{++} \backslash \{m - p_2\}$ and semi-differentiable at the point $p_1 = m - p_2$. In particular, we have

$$\left.\frac{\partial^- \Pi_1^{\mathbf{a}}(p_1, p_2)}{\partial p_1}\right|_{p_1 = p_2 = \frac{m}{2}} = 1 - G_{\mathbf{a}}(0) - \frac{m}{2} \cdot g_{\mathbf{a}}(0) = \frac{1}{2} - \frac{m}{2} \cdot g_{\mathbf{a}}(0) \geq 0,$$

and

$$\left.\frac{\partial^+ \Pi_1^{\mathbf{a}}(p_1, p_2)}{\partial p_1}\right|_{p_1 = p_2 = \frac{m}{2}} = 1 - G_{\mathbf{a}}(0) - m g_{\mathbf{a}}(0) = \frac{1}{2} - m \cdot g_{\mathbf{a}}(0) \leq 0.$$

Since $\Pi_1^{\mathbf{a}}\left(p_1, \frac{m}{2}\right)$ is strictly quasi-concave on both $\left[0, \frac{m}{2}\right]$ and $[\frac{m}{2}, +\infty)$, the above inequalities imply that $p_1 = \frac{m}{2}$ is a maximum of $\Pi_1^{\mathbf{a}}\left(p_1, \frac{1}{2}\right)$ on each of these two intervals. This shows that $p_1 = \frac{m}{2}$ is a global maximum of $\Pi_1^{\mathbf{a}}\left(p_1, \frac{m}{2}\right)$ on $[0, +\infty)$. Hence, if $g_{\mathbf{a}}(0) \in [\frac{1}{2m}, \frac{1}{m}]$, the game in the pricing stage admits a symmetric equilibrium with $p_1 = p_2 = \frac{m}{2}$. Further, as $g_{\mathbf{a}}(0) \leq \frac{1}{m}$ a symmetric equilibrium with $p_1 = p_2 < \frac{m}{2}$ cannot exist. To see this, note that a symmetric equilibrium with $p_1 = p_2 < \frac{m}{2}$ involves full market coverage (as $p_1 < m - p_2$), and must be a solution to the first order condition

$$1 - G_{\mathbf{a}}(0) - p_1 g_{\mathbf{a}}(0) = 0.$$

Thus $p_1 = \frac{1}{2g_{\mathbf{a}}(0)}$, where the condition $p_1 < \frac{m}{2}$ therefore is equivalent to $g_{\mathbf{a}}(0) > \frac{1}{m}$, contradiction.

In addition, because

$$\left.\frac{\partial \Pi_{\mathbf{a}}^M(p)}{\partial p}\right|_{p = \frac{m}{2}} = \frac{1}{2} - m \cdot g_{\mathbf{a}}(0) \leq \frac{1}{2} - \frac{1}{2} = 0$$

and $\Pi_{\mathbf{a}}^M(p)$ is strictly quasi-concave, even a monopoly firm would not choose a price strictly higher than $\frac{m}{2}$. Hence, no symmetric equilibrium with $p_1 = p_2 > \frac{m}{2}$ can exist either. In sum, this shows that $(p_1, p_2) = \left(\frac{m}{2}, \frac{m}{2}\right)$ is the unique symmetric pure-strategy equilibrium when $g_{\mathbf{a}}(0) \in \left[\frac{1}{2m}, \frac{1}{m}\right]$.

Finally, suppose that $g_{\mathbf{a}}(0) < \frac{1}{2m}$. Observe that in this case, we have $p_{\mathbf{a}}^M > \frac{m}{2}$, because

$$\left.\frac{\partial \Pi_{\mathbf{a}}^M(p)}{\partial p}\right|_{p=\frac{m}{2}} = \frac{1}{2} - m \cdot g_{\mathbf{a}}(0) > 0$$

and $\Pi_{\mathbf{a}}^M(p)$ is strictly quasi-concave on $[0, +\infty)$. Now suppose that firm 2 plays $p_2 = p_{\mathbf{a}}^M$, and consider firm 1's profit function $\Pi_1(p_1, p_{\mathbf{a}}^M)$. Given the formula of the demand function and $p_{\mathbf{a}}^M > m - p_{\mathbf{a}}^M$, we have

$$\Pi_1^{\mathbf{a}}(p_{\mathbf{a}}^M, p_{\mathbf{a}}^M) = \Pi_{\mathbf{a}}^M(p_{\mathbf{a}}^M) > \Pi_{\mathbf{a}}^M(p_1) \geq \Pi_1^{\mathbf{a}}(p_1, p_{\mathbf{a}}^M) \quad \forall p_1 \in [0, +\infty)\backslash\{p_{\mathbf{a}}^M\},$$

which further implies that $p_1 = p_{\mathbf{a}}^M$ is the unique best response for firm 1. Hence, $(p_1, p_2) = \left(p_{\mathbf{a}}^M, p_{\mathbf{a}}^M\right)$ indeed constitutes an equilibrium in the pricing subgame where $g_{\mathbf{a}}(0) < \frac{1}{2m}$. Moreover, given $\frac{1}{2g_{\mathbf{a}}(0)} > \frac{m}{2}$, there cannot exist a symmetric equilibrium with $p_1 = p_2 < \frac{m}{2}$. Since

$$\left.\frac{\partial^- \Pi_1^{\mathbf{a}}(p_1, p_2)}{\partial p_1}\right|_{p_1=p_2=\frac{m}{2}} > \left.\frac{\partial^+ \Pi_1^{\mathbf{a}}(p_1, p_2)}{\partial p_1}\right|_{p_1=p_2=\frac{m}{2}} = \frac{1}{2} - m \cdot g_{\mathbf{a}}(0) > 0,$$

$p_1 = p_2 = \frac{m}{2}$ does not constitute an equilibrium either. In conclusion, $(p_1, p_2) = \left(p_{\mathbf{a}}^M, p_{\mathbf{a}}^M\right)$ is the unique symmetric pure-strategy equilibrium when $g_{\mathbf{a}}(0) < \frac{1}{2m}$.

Taken together, the above derivations prove that the equilibrium price $p_{\mathbf{a}}^*$ is determined by (B.9). Further, (B.10) implies that market shares are below 1/2 iff $g_{\mathbf{a}}(0) < \frac{1}{2m}$, completing the proof. ■

Proof of Corollary S1 Let g_ω denote the density of the perceived match advantages given that the perception errors ε follow the uniform distribution Γ_ω. It is easily checked that for any $\omega > 0$, the density γ_ω is log-concave on its support. Thus, Assumption S2 is satisfied, and Proposition S5 applies to the case of uniformly distributed perception errors. Specifically, the equilibrium price is determined by (B.9), where we replace $p_{\mathbf{a}}^*$ by p_ω and $g_{\mathbf{a}}(0)$ by $g_\omega(0)$. For any given $\omega > 0$, we have $g_\omega(0) = \frac{1}{2\omega}\int_{-\omega}^{\omega} g_0(e)de$. Next, note that $g_\omega(0) = \frac{1}{2\omega}\int_{-1}^{1} g_0(e)de = \frac{1}{2\omega}$ whenever $\omega \geq 1$, which we assume in the following. Given that $\omega \geq 1$, the condition $g_\omega(0) > \frac{1}{m}$ is equivalent to $\omega < \frac{m}{2}$. Thus, for $\omega \in [1, \frac{m}{2})$, we must have $g_\omega(0) = \frac{1}{2\omega} > \frac{1}{m}$, and hence $p_\omega = \frac{1}{2g_\omega(0)} = \omega$ by (B.9). Next, given that $\omega \geq 1$, the condition $g_\omega(0) \geq \frac{1}{2m}$ is equivalent to $\omega \leq m$. Thus, for $\omega \in [\frac{m}{2}, m]$ also

$g_\omega(0) \in [\frac{1}{2m}, \frac{1}{m}]$, and thus $p_\omega = \frac{m}{2}$ for $\omega \in [\frac{m}{2}, m]$ by (B.9). Finally, $g_\omega(0) < \frac{1}{2m}$ iff $\omega > m$, in which case (B.9) implies that p_ω is given by the monopoly price p^M_ω. We now show that if $\omega > m$, then $p_\omega = p^M_\omega = \frac{\omega+m}{4}$, independent of the shape of g_0. If p_ω denotes the solution of this monopoly problem, then p_ω must be determined by the first order condition $\frac{\partial \Pi^M_\omega(p_\omega)}{\partial p} = 0$, which can be simplified to

$$\omega - \frac{1}{2}\int_{2p_\omega - m - \omega}^{2p_\omega - m + \omega} G_0(s)ds - p\int_{2p_\omega - m - \omega}^{2p_\omega - m + \omega} g_0(s)ds = 0. \tag{B.11}$$

To calculate the value of p_ω we need to evaluate the two integrals in the previous expressions. By Proposition S5, we know that $p_\omega > \frac{m}{2}$, which implies that $2p_\omega - m + \omega > 1$ for the upper bounds of the two integrals (recalling that $\omega \geq 1$). We now conjecture (and ex post verify) that $2p_\omega - m - \omega < -1$. Recalling that $supp\, g_0 = [-1, 1]$ and $\int_{-1}^{1} G_0(s)ds = 1$ as a consequence of symmetry, (B.11) evaluates to $p_\omega = \frac{\omega+m}{4}$ given the presumption that $2p_\omega - m - \omega < -1$. It now is easily verified that the last inequality indeed is satisfied for $p_\omega = \frac{\omega+m}{4}$, confirming that $p^M_\omega = \frac{\omega+m}{4}$ must be the monopoly price. In sum, these steps show that p_ω must be as stated by Corollary S1. Continuity of p_ω in ω then is obvious for $\omega \in [1, \infty)$. As the equilibrium price p_ω is determined by (B.9), it follows from (B.9) and the previous result that p_ω is continuous on the entire range $\omega \in (0, \infty)$ whenever $g_\omega(0)$ is continuous in ω on this range. As $g_\omega(0) = \frac{1}{2\omega}\int_{-\omega}^{\omega} g_0(e)de$ for any $\omega > 0$, the last property is obviously verified.

Turning to equilibrium demand, the proof of Proposition S5 shows that as long as $1 \leq \omega \leq m$ (i.e., $g_\omega(0) \geq \frac{1}{2m}$) the equilibrium price is such that the outside option is not binding for (almost) all consumers, meaning that $d_\omega = 1/2$ for $\omega \in [1, m]$. If $\omega > m$, the equilibrium price is given by $p^M_\omega = \frac{\omega+m}{4}$, and $d_\omega = 1 - G_\omega(2p^M_\omega - m)$, which evaluates to $d_\omega = \frac{\omega+m}{4\omega}$. The expression for equilibrium profits $\Pi_\omega = p_\omega d_\omega$ then follows immediately. Continuity of d_ω and of $\Pi_\omega = p_\omega d_\omega$ in ω follow from the continuity of p_ω. Finally, the claims about $L(\omega)$ follows from $L(\omega) = 1 - 2d_\omega$. ■

Proof of Proposition S6 Suppose that preferences are indecisive on $[-1, 1]$. If $\bar{\omega} \leq m$, the existence of a unique SPE with maximal confusion immediately follows from Theorem S1. If $\bar{\omega} > m$, the same result holds because it is shown in Corollary S1 that Π_ω is strictly increasing in ω for $\omega \geq m$. It is then clear that no consumer leaves the market in equilibrium when $\bar{\omega} \leq m$, while a fraction $L(\bar{\omega}) = \frac{\bar{\omega}-m}{2\bar{\omega}} \in (0, \frac{1}{2})$ of consumers leaves the market when $\bar{\omega} > m$.

Next, consider the case of polarized preferences. Corollary S1 assures that Π_ω is (weakly) increasing in ω whenever $\omega > \frac{m}{2}$ in this case. The firms benefit from confusion relative to an educated market if $\Pi_0 = \frac{1}{4g_0(0)} < \Pi_\omega$, $\omega > 0$. Because $g_0(0) > 1/m$ and thus $\Pi_0 < \frac{m}{4}$, Corollary S1 shows that a sufficient condition for $\Pi_0 < \Pi_{\bar{\omega}}$ is that $\bar{\omega} \geq \frac{m}{2}$. Thus, if $\bar{\omega} \geq \frac{m}{2}$, then maximal confusion is an SPE outcome (while education is not). As Π_ω is strictly increasing in ω for $\omega \geq m$, it follows that maximal confusion is the unique SPE outcome whenever $\bar{\omega} > m$. ■

S.7 Confusion About Needs on a Salop Circle

In the following application, we use a Salop circle to further pursue the idea that the communication profiles influence how precisely the agents can learn their true needs. This also exemplifies a situation where the perception errors and the true match advantages are interdependent. As the Salop model (Salop, 1979) has become a textbook-style workhorse model in IO and related fields, the subsequent analysis of strategic confusion or education upon a Salop circle is also interesting in itself.

Two firms are located at antipodal locations on a Salop circle, as illustrated in Figure S.4. Consumers are continuously and symmetrically distributed between the firms, where we indicate consumer locations in the clockwise direction with $\theta \in [0, 1)$. By symmetry, it suffices to specify the model only for the half-circle on the right-hand side. On this half-circle, consumers are dispersed over the $[0, 1/2]$-line according to a bounded function $h : \mathbb{R} \to \mathbb{R}_+$ with the following properties

$$\begin{aligned}
&(1) \quad h(\theta) > 0 \Leftrightarrow \theta \in [0, 1/2]\,, \\
&(2) \quad h \text{ is symmetric at } \tfrac{1}{4}. \\
&(3) \quad \textstyle\int_0^{1/2} h(x)dx = 1/2, \\
&(4) \quad h \text{ is differentiable on } (0, 1/4) \cup (1/4, 1/2),
\end{aligned} \tag{B.12}$$

The corresponding (half)-distribution function $H : \mathbb{R} \to [0, 1/2]$ is $H(\theta) = \int_0^\theta h(x)dx$, $\forall \theta \in \mathbb{R}$. For a consumer located at $\theta \in [0, 1/2]$, the true match values are

$$v_1^\theta = \mu - t\theta \quad \text{and} \quad v_2^\theta = \mu - t(1/2 - \theta),$$

where $\mu, t > 0$ are parameters. The match advantage of firm 2 then is $v_\Delta^\theta = v_2^\theta - v_1^\theta = 2t(\theta - 1/4)$. The consumer would truly prefer firm 1 if and only if $p_2 - p_1 \geq v_\Delta^\theta$. We

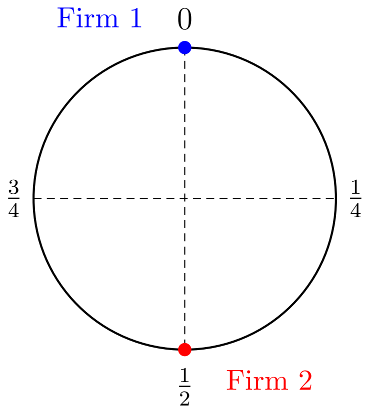


Figure S.4: The Salop circle

assume that μ is sufficiently large, such that every consumer will find it worthwhile to purchase one product in equilibrium. In accordance with the logic behind Definition 2, we say that the consumer preference distribution features indecisiveness (polarization) on $[0, 1/2]$ if $h(\cdot)$ is strictly increasing (decreasing) on $(0, 1/4)$ and thus strictly decreasing (increasing) on $(1/4, 1/2)$. As a simple example, suppose that $h(\cdot)$ is piecewise linear,

$$h(\theta) = \begin{cases} 1 + b(\theta - 1/8) & \text{if } \theta \in [0, 1/4] \,, \\ 1 - b(\theta - 3/8) & \text{if } \theta \in (1/4, 1/2] \,, \\ 0 & \text{otherwise,} \end{cases} \tag{B.13}$$

where $|b| < 8$ to assure that $h(\theta) > 0$ on $[0, 1/2]$. Then, $b > 0$ corresponds to indecisive and $b < 0$ to polarized preferences, while $b = 0$ gives the standard textbook Salop model with uniformly distributed consumers. Figure S.5 (a) depicts the preference distribution for the cases $b = 4, -2$ and 0, respectively.

Locational Confusion As in Section 4, firms first choose their communication strategies $\mathbf{a} \in \mathcal{A}$, and then compete in prices. In the current setting, consumer confusion enters the model in form of i.i.d. shocks $\varepsilon_{\mathbf{a}}$ to true consumer locations θ. Specifically, confusion means that each consumer's perceived location is distorted around the true location θ by a 0-symmetric, uniformly distributed shock $e \in [-\omega_{\mathbf{a}}, \omega_{\mathbf{a}}]$, where $\omega_{\mathbf{a}} \geq 0$ measures the size of confusion. The interpretation of this model is that communication strategies influence how well a consumer learns his true needs. If $\omega_{\mathbf{a}} = 0$ then $\hat{\theta} = \theta$ for each consumer $\theta \in [0, 1]$, meaning that communication allows each consumer to correctly learn her location. By contrast, if $\omega_{\mathbf{a}} = 1/2$, then $\hat{\theta} \in [0, 1)\ \forall \theta$, meaning that each consumer could find herself anywhere on the circle, independent of her true location. Note that obfuscation becomes *massive* in the sense of Section 3.4 whenever $\omega_{\mathbf{a}} > 1/4$, as then a consumer sitting exactly on a firm's location may, in principle, be so confused that she chooses the competitor's product. Nevertheless, $\omega_{\mathbf{a}} = 1/2$ corresponds to the natural upper bound of such massive obfuscation in the present model.

While the distribution of the perceived match advantages remains unbiased, here a notable difference to our main setting is that the true match advantages and the perception errors implied by locational confusion are not independent. Unbiasedness follows directly from the 0-symmetry of the locational shocks. The violation of independence occurs, in essence, because locational confusion confines the perceptions $\hat{\theta}$ to the circle, meaning that the range of perceived match advantages must always coincide with the range of true match advantages.[53]

Equilibrium Analysis For a given $\omega_{\mathbf{a}} \in [0, 1/2]$ and given prices p_1, p_2, a firm's demand consists of those consumers who perceive the firm as offering the better deal. We now derive a formal expression for the expected demand of firm $j = 1$. As $\omega_{\mathbf{a}} \in [0, 1/2]$, $e \in [-\omega_{\mathbf{a}}, \omega_{\mathbf{a}}]$ and $\theta \in [0, 1/2]$, it follows that $\theta + e \in [-1/2, 1]$. Thus, for each consumer $\theta \in [0, 1/2]$, the perceived distance to firm 1 is

$$\hat{d}_1 = \begin{cases} |\theta + e| & \text{if } \theta + e \leq 1/2, \\ 1 - (\theta + e) & \text{if } \theta + e > 1/2. \end{cases} \tag{B.14}$$

Fix prices $p_1, p_2 \geq 0$ and define $\Delta \equiv \frac{p_2 - p_1}{2t}$. If $|\Delta| \leq 1/4$, the market segment S_1 of firm 1 is $S_1 = \{\theta \in [0, 1] : \hat{d}_1 \leq \Delta + 1/4\}$. Further, $S_1 = \emptyset$ if $\Delta < -1/4$, and $S_1 = [0, 1]$ if $\Delta > 1/4$. A consumer transacts with firm 1 if her perceived location belongs to S_1. Hence, for $\Delta < 1/4$ the expected market demand of firm 1 from consumers on the right half-circle, $D_1 \in [0, 1/2]$, corresponds to the expected fraction of consumers for whom the true location is on $[0, 1/2]$ and the perceived locations is in S_1:

$$D_1(\Delta, \omega_{\mathbf{a}}) = \Pr\left(-1/4 - \Delta \leq \theta + \varepsilon_{\mathbf{a}} \leq 1/4 + \Delta\right) + \Pr\left(\theta + \varepsilon_{\mathbf{a}} \geq 3/4 - \Delta\right), \tag{B.15}$$

[53]To illustrate the violation of independence, let $t = 1$, $\omega_{\mathbf{a}} = 1/4$. Then, the perception errors, in terms of match advantages, implied by locational confusion for the consumer at $\theta = 0$ (hence $v_{\Delta}^{0} = 0$) have $supp\, \varepsilon_{\mathbf{a}}^{0} = [0, 1/2]$. By contrast, a consumer with location $\theta = 1/4$ (hence $v_{\Delta}^{1/4} = 0$) experiencing the same type of locational shock has $supp\, \varepsilon_{\mathbf{a}}^{1/4} = [-1/2, 1/2]$. Thus, $\varepsilon_{\mathbf{a}}$ and v_{Δ} cannot be independent.

where the probabilities in (B.15) should be interpreted as conditional on $\theta \in [0, 1/2]$. The second term in (B.15) captures that some consumers with location in the segment $(1/4 + \Delta, 1/2]$, who actually would be better-off by choosing firm 2, may obtain perceived locations in the segment $(3/4, 1)$ for sufficiently large obfuscation $\omega_{\mathbf{a}}$, and then (erroneously) choose firm 1.

We now turn to the equilibrium analysis, relying on the standard first-order approach as is common in applications of the Salop model (see, e.g., Grossman and Shapiro (1984)).[54] For $|\Delta(p_1, p_2)| < 1/4$ and $\omega_{\mathbf{a}} \in [0, 1/2]$, the expected profit of firm $j = 1$ in the pricing stage is

$$\Pi_1(\Delta(p_1, p_2), \omega_{\mathbf{a}}) = 2p_1 D_1(\Delta(p_1, p_2), \omega_{\mathbf{a}}) = 2p_1 D_1\left(\frac{p_2 - p_1}{2t}, \omega_{\mathbf{a}}\right), \tag{B.16}$$

where $D_1(\Delta(p_1, p_2), \omega_{\mathbf{a}})$ is given by (B.15). A symmetric pricing equilibrium $p_{\mathbf{a}}$ in the pricing stage corresponds to a solution of $\frac{\partial}{\partial p_1}\Pi_1(p_{\mathbf{a}}, p_{\mathbf{a}}) = 0$. As shown in the proof of the next proposition, such a unique solution $p_{\mathbf{a}}$ exists for every $\omega_{\mathbf{a}}$, and we assume that $p_{\mathbf{a}}$ then also corresponds to the equilibrium price in the pricing stage. In the following, we show how firms' profit $\Pi_i = \frac{p_{\mathbf{a}}}{2}$ in the symmetric pricing equilibrium of the pricing stage depends on the confusion parameter $\omega \in [0, 1/2]$ (we suppress the **a**-index for simplicity).

Proposition S7 *In the Salop model with locational confusion, the following cases can be distinguished:*

(i) (Indecisiveness) If the preference distribution is indecisive, there exists a unique $\omega_0 \in (1/4, 1/2)$ such that prices and profits increase strictly in ω up to ω_0, and decrease strictly thereafter. Moreover, prices and profits are minimized at $\omega = 0$.

(ii) (Polarization) If the preference distribution is polarized, there exists a unique $\omega_0 \in (1/4, 1/2)$ such that prices and profits decrease strictly in ω up to ω_0, and increase strictly thereafter. Moreover, prices and profits are maximized at $\omega = 0$.

(iii) (Uniform Dispersion) If the preference distribution is uniform, i.e., $h(\theta) = 1$ on $[0, 1/2]$ then the prices and profits are $p_\omega = t/2$ and $\Pi(\omega) = t/4$, $\forall \omega \in [0, 1/2]$.

The proof is at the end of this section. Proposition S7 shows that as long as confusion cannot become massive ($\omega_{\mathbf{a}} \leq 1/4 \ \forall \mathbf{a} \in \mathcal{A}$), the dispersion of preferences on the Salop

[54] This approach essentially takes the existence of a symmetric price equilibrium as given, in that sufficiency of the first-order condition for profit maximization at a symmetric solution of $\frac{\partial \Pi_1}{\partial p_1} = 0$ in the pricing stage is presumed.

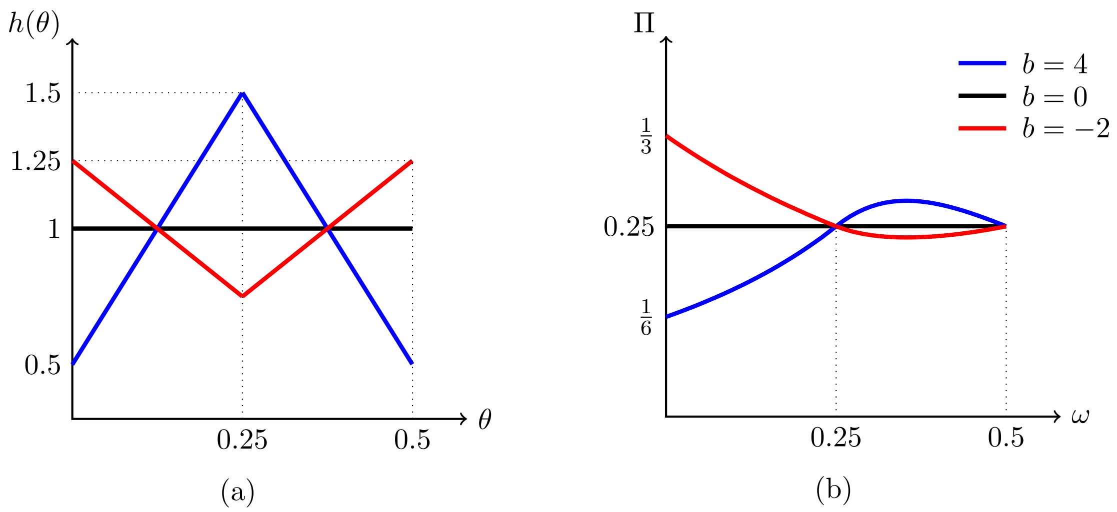


Figure S.5: (a) Preferences and (b) profits in the Salop model, $t = 1$.

circle has the same implication for the effects of confusion as in the baseline model. In particular, firms are only harmed (can only benefit) from such confusion if preferences are polarized (indecisive). Intuitively, this is the case because competition on each half-circle of the Salop model is akin to competition on the line for small enough confusion.

If confusion becomes massive, however, the Salop model offers new insights. Specifically, firms cannot benefit from maximally confused consumers independent of whether preferences are indecisive or polarized: In the latter case they prefer education ($\omega = 0$) and in the former case the intermediate level of confusion given by ω_0. In the knife-edge case where consumers are uniformly distributed, confusion has no effects on prices and profits at all. These different cases are illustrated in Figure S.5 (b). The intuition is that massive confusion has two effects. First, some right-hand side consumers actually favoring firm $j = 1$ may become indifferent on the right-hand circle ($\hat{\theta} = 1/4$). Second, some consumers who are located on the left-hand circle and who favor firm $j = 1$ may be so confused as to become indifferent on the right-hand circle. The second effect is absent in a Hotelling model. With indecisive preferences, the share of consumers that become perceptually indifferent in the sense of the second effect increases in confusion, which explains why firms are eventually harmed by confusion once it becomes large enough. While this effects is partly reversed with polarized preferences, it is not possible to soften competition more with confusion as given by the case of educated consumers. Finally, confusion has no impact on competition with uniform preferences, because the average inflow and outflow of perceptually indifferent consumers exactly compensate each other in this case.

Proof of Proposition S7 Let $|\Delta| < 1/4$. For $\omega = 0$, (B.15) then yields

$$D_1(\Delta, 0) = \Pr(\theta \leq 1/4 + \Delta) + \Pr(\theta \geq 3/4 - \Delta) = H(1/4 + \Delta).$$

For $\omega \in (0, 1/2]$ we obtain

$$\begin{aligned} & D_1(\Delta, \omega) \\ = & \int_{-\omega}^{\omega} \tfrac{1}{2\omega} \left(H(1/4 + \Delta - e) - H(-1/4 - \Delta - e)\right) de + \int_{-\omega}^{\omega} \tfrac{1}{2\omega} \left(1/2 - H(3/4 - \Delta - e)\right) de \\ = & \tfrac{1}{2\omega} \left(\int_{1/4+\Delta-\omega}^{1/4+\Delta+\omega} H(\theta) d\theta - \int_{-1/4-\Delta-\omega}^{-1/4-\Delta+\omega} H(\theta) d\theta - \int_{3/4-\Delta-\omega}^{3/4-\Delta+\omega} H(\theta) d\theta \right) + 1/2. \end{aligned} \tag{B.17}$$

Thenrefore

$$\begin{aligned} \frac{\partial D_1(\Delta, \omega)}{\partial \Delta} = & \frac{H(1/4 + \Delta + \omega) - H(1/4 + \Delta - \omega)}{2\omega} + \frac{H(-1/4 - \Delta + \omega) - H(-1/4 - \Delta - \omega)}{2\omega} \\ & + \frac{H(3/4 - \Delta + \omega) - H(3/4 - \Delta - \omega)}{2\omega}. \end{aligned} \tag{B.18}$$

Using (B.18) and $p_2 = p_1 = p_\omega$ in the first-order condition then yields

$$p_\omega = \frac{t/2}{\frac{\partial}{\partial \Delta} D_1(0, \omega)} \tag{B.19}$$

as the unique solution. Then, the corresponding equilibrium profit $\Pi(\omega) \equiv p_\omega/2$ satisfies

$$sign\, \Pi'(\omega) = sign\, \frac{\partial p_\omega}{\partial \omega} = -sign\, Z(\omega), \qquad Z(\omega) \equiv \frac{\partial^2 D_1(0, \omega)}{\partial \Delta \partial \omega}. \tag{B.20}$$

Let $\omega \in (0, 1/4)$. Noting that $h(\theta) = 0$ whenever $\theta \notin [0, 1/2]$, we obtain from (B.18)

$$Z(\omega) = \frac{2\omega h(1/4 + \omega) - (H(1/4 + \omega) - H(1/4 - \omega))}{2\omega^2}.$$

The symmetry of h at $\theta = 1/4$ implies

$$H(1/4 + \omega) - H(1/4 - \omega) = 2H(1/4 + \omega) - 1/2. \tag{B.21}$$

Using this and $H(1/4) = 1/4$ in (B.21), we have

$$Z(\omega) \geq 0 \quad \Leftrightarrow \quad h(1/4 + \omega)\, \omega \geq H(1/4 + \omega) - 1/4 = \int_{1/4}^{1/4+\omega} h(\theta) d\theta. \tag{B.22}$$

Therefore we can establish the claims in (i), (ii) and (iii) for the case where $\omega \in (0, 1/4)$.

(i) $\int_{1/4}^{1/4+\omega} h(\theta)d\theta > \int_{1/4}^{1/4+\omega} h(1/4+\omega)d\theta = h(1/4+\omega)\omega$, thus $Z(\omega) < 0$ by (B.22). Hence prices and profits increase in obfuscation by (B.20) for $\omega \in (0, 1/4)$.

(ii) $\int_{1/4}^{1/4+\omega} h(\theta)d\theta < \int_{1/4}^{1/4+\omega} h(1/4+\omega)d\theta = h(1/4+\omega)\omega$, thus $Z(\omega) > 0$ by (B.22). Hence prices and profits decrease in obfuscation by (B.20) for $\omega \in (0, 1/4)$.

(iii) $\int_{1/4}^{1/4+\omega} h(\theta)d\theta = 1/4+\omega-1/4 = \omega$, and $h\left(\frac{1}{4}+\omega\right)\omega = \omega$, thus $Z(\omega) = 0$ by (B.22). Hence obfuscation has no effects on prices and profits by (B.20) for $\omega \in (0, 1/4)$.

Next, suppose that $\omega \in (1/4, 1/2]$.[55] Then, (B.18) gives

$$Z(\omega) = \frac{1}{2\omega^2}\left[\omega\left(h(\omega - 1/4) + h(3/4 - \omega)\right) - \left(1 + H(\omega - 1/4) - H(3/4 - \omega)\right)\right].$$

Using $h(\omega - 1/4) = h(3/4 - \omega)$ and $H(\omega - 1/4) = 1/2 - H(3/4 - \omega)$, the nominator of $Z(\omega)$ becomes $z(\omega) \equiv 2\omega h(3/4 - \omega) - 3/2 + 2H(3/4 - \omega)$. Suppose now that h features indecisiveness (i). Then $h(1/2) < 1$, $h(1/4) > 1$, and

$$\lim_{\omega \downarrow 1/4} z(\omega) = \frac{h(1/2)}{2} - \frac{1}{2} < 0, \quad \text{and} \quad \lim_{\omega \uparrow 1/2} z(\omega) = h(1/4) - \frac{1}{2} > 0.$$

Further, for $\omega \in (\frac{1}{4}, \frac{1}{2})$ we have $z'(\omega) = -2\omega h'(3/4-\omega) > 0$. Together with the continuity of $z(\omega)$, these arguments assure that there exists a unique $\omega_0 \in (1/4, 1/2)$ such that

$$z(\omega), Z(\omega) \begin{cases} < 0 & \text{if } \omega < \omega_0, \\ = 0 & \text{if } \omega = \omega_0, \\ > 0 & \text{if } \omega > \omega_0. \end{cases} \text{ for } \omega \in [1/4, 1/2].$$

It then follows from (B.20) that $\Pi(\omega)$ and p_ω must have a global maximum at $\omega_0 \in [1/4, 1/2]$. Note from (B.18) that $\frac{\partial D_1(0,1/2)}{\partial \Delta} = 1$, which by (B.19) implies that $p_{1/2} = t/2$, and $\Pi(1/2) = t/4$. If $\omega = 0$, then $\frac{\partial D_1(0,0)}{\partial \Delta} = h(1/4)$, and thus $p_0 = t/(2h(1/4))$, and $\Pi(0) = t/(4h(1/4))$. Because $h(1/4) > 1$ with indecisive preferences, prices and profits must be minimal at $\omega = 0$, which completes the proof for (i). Case (ii) can be proved similarly. For (iii), note that if $h(\theta) = 1$ on $(1/4, 1/2]$, then $z(\omega) = 0$ on $(1/4, 1/2]$. ■

[55] The case $\omega = 1/4$ is not problematic, because $\partial D_1(0, \omega)/\partial \Delta$ is continuous at $\omega = 1/4$.