Challenging the incumbent: Entry in markets with captive consumers and taste heterogeneity

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09 Mar 2022

Abstract

We analyze entry of a firm with a new and differentiated product into a market with two properties: An existing incumbent has a captive consumer base, and all consumers have heterogeneous tastes. The interaction between the share of captive consumers and the degree of taste heterogeneity leads to non-monotone effects of both parameters on entry: The captive share can have an inverse-U relation with entry profits, and higher taste heterogeneity (i.e., less product substitutability) can impede entry in the presence of captive consumers. Considering these effects together leads to new insights on entry, horizontal product innovation, and price discrimination.

Keywords: entry, captive consumers, asymmetric competition, product innovation

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1 Introduction

The provision of new products is a core function of markets. A larger variety of goods enables consumers to satisfy their wants better. Knowing which factors influence entry of new products into markets is thus important from a welfare perspective. In this paper, we analyze two determinants of entry: incumbency advantage and intensity of competition. We will show that their interaction generates non-monotone effects on entry.

The *incumbency advantage* we consider manifests itself in exclusive consumer awareness. An incumbent firm has been present and perceived by consumers for a longer time, so that some of them are unable or unwilling to consider emerging alternatives. They are captive to the incumbent. This generates an asymmetry that can impede entry. If the share of captive consumers is high, then potential demand for the entrant is low and so are her expected profits. The *intensity of competition* is determined by consumer taste heterogeneity. If this heterogeneity is large, consumers have stronger preferences regarding their optimal product and are thus less willing to substitute one good for the other. Less consumer substitution means less intense competition and therefore more potential for a firm to generate profits, making entry more feasible in the absence of captive consumers.[1]

On their own, incumbency advantage and the intensity of competition thus have clear effects on entry prospects: Entry is difficult when the incumbent has many captive consumers and when products are close substitutes. We show how the interaction between the share of captive consumers and the degree of substitution leads to more complex, non-monotone effects: An increase in the share of captive consumers may favor entry for some parameters, but impede it for others. Likewise, a decline in product substitutability may impede entry for some parameter constellations, and make it easier for others. Crucially, while an increase in the share of captive consumers reduces the market size for the entrant, and thereby her demand potential, it may also lead the incumbent to price less aggressively to exploit his

[1] For variants of this argument see, for instance, Sutton (1991), Boone (2000), Raith (2003), Vives (2008).

captive base. Thus, an increase in the share of captive consumers may foster entry, despite its negative effect on potential demand. This leads to a non-monotonicity, where entry is profitable only for intermediate shares of captive consumers.

In turn, while greater heterogeneity increases prices without captive consumers, it may reduce prices when they are present. This happens because the incumbent considers the impact of his pricing on the captive monopoly segment as well, in particular, when the share of captive consumers is large.[2] It is well-known that, contrary to the duopoly case, increasing heterogeneity *may* have negative price effects in monopolies. In this case, with captive consumers, the interaction between the price-increasing effect of heterogeneity in a duopoly without captive consumers and the price-reducing effect in monopoly generates a non-monotonicity where entry is feasible only for intermediate levels of heterogeneity.

This non-monotonicity result relates to the large literature on the effects of competition on process innovation (cost reductions) and vertical product innovations (product improvements), which argues that these effects need not be monotone and may depend on firm or market characteristics.[3] As argued above, for *horizontal* product innovations (the introduction of differentiated products), without captive consumers, the increasing intensity of competition associated with greater substitutability makes innovative entry less attractive. We show that this insight should be taken with a grain of salt with captive consumers, as the relation between substitutability and horizontal innovation is non-monotone.

We identify the main effects in a model with general demand. Then, we illustrate the results in a standard discrete-choice model, showing that the positive effect of the share of captive consumers on entry is more likely to arise when consumers are not too heterogeneous.[4] Moreover, we find that a large share of captive consumers fosters the negative effect of

[2] We restrict the incumbent to set a uniform price for all consumers. If he could set separate prices, the presence of captive consumers would have no effect on the entrant beyond the reduction in market size.

[3] See Aghion et al. (2005), Gilbert (2006), Vives (2008), Schmutzler (2010, 2013). Empirically, Aghion et al. (2005) find an inverse-U relation between competition and innovation, whereas Aghion et al. (2009) show that competition differentially affects stronger and weaker firms.

[4] This model is isomorphic to the Salop and Hotelling duopolies with linear transportation cost.

heterogeneity arising from the monopolistic segment of the market.

Our analysis might suggest that the well-known concerns of competition authorities against incumbents who take measures to generate captive consumers are misguided.[5] This conclusion is not justified, however, because, it does not take the endogeneity of these measures into account. Indeed, the incumbent tends to shy away from increasing the share of captive consumers where this would induce entry.

The analysis of competition with captive consumers has produced a vast literature that goes back to Varian (1980) who analyzed price dispersion in a symmetric model.[6] Sinitsyn (2008, 2009) shows that mixed-strategy price equilibria emerge for differentiated goods if heterogeneity between non-captive consumers is low. Mixed-stratategy equilibria also arise in our model for low heterogeneity, but for wide parameter regions, we obtain simple pure-strategy equilibria, on which we focus.[7] Entry when an incumbent has the advantage of a separate monopolistic market has been analyzed first by Armstrong and Vickers (1993). They focus on the effects of price discrimination on entry prospects and highlight the competition-softening effect of a larger captive segment.[8] In our model, captive consumers and switchers are part of the same market, and we add heterogeneity.

The idea that consumers fail to consider alternative products is supported by a broad empirical literature. Barroso and Llobet (2012) document that consumer awareness for new car models takes time to build. An incumbency advantage emerges because a product which

[5]Such concerns arise most frequently in the context of loyalty rebates, see for instance the *Michelin* case in the E.U.(https://eur-lex.europa.eu/legal-content/EN/TXT/HTML/?uri=CELEX:62001TJ0203) and several follow-up cases, also in other jurisdiction. For instance, the Swiss Competition commission sanctioned the *Swiss Post* for abuse of dominance in 2017. Loyalty rebates were at center stage in the case (https://www.tagesanzeiger.ch/post-akzeptiert-millionen-busse-183825630839).

[6]More recently, Armstrong and Vickers (2019) have analyzed asymmetric mixed-strategy equilibria where consumers differ with respect to the firms they take into consideration when deciding on their purchases. Their analysis allows for rich types of competitive interactions, but focuses on homogeneous-goods oligopolies.

[7]In Sinitsyn (2009), firms have symmetric captive bases. Like our paper, Sinitsyn (2008), analyzes an asymmetric setup where only one of two firms has captive consumers. He focuses on price dispersion rather than entry. Further, he assumes that the captive consumers all have the same valuations for the good. By contrast, in our model all consumers, captive or not, have heterogeneous tastes.

[8]Anton, Vander Weide, and Vettas (2002) analyze the competition-softening effect in the context of auctions for the right to serve a new market.

has been in the market for longer has had more chances to build awareness.[9] Hortaçsu, Madanizadeh, and Puller (2017) identify inattentiveness as a significant factor preventing consumers from switching to a newly available electricity provider. Ho, Hogan, and Scott Morton (2017) find similar consumer inattention with insurance plans.

Chen and Schwartz (2013) compare the incentives of an incumbent for innovation of horizontally differentiated products with those of an entrant. They neither investigate incumbency advantages nor the effects of horizontal product substitutability.[10]

Section 2 provides the analysis with general demand functions. Section 3 provides sharper results for a simple discrete-choice model. In Section 4, we discuss extensions and policy implications. Section 5 concludes.

2 The general framework

We consider a market in which an incumbent faces potential competition from an entrant. Consumers have heterogeneous tastes. Moreover, some of them (the *captive consumers*) only consider the incumbent's product, while the others (the *switchers*) choose between both products.

2.1 Assumptions

An incumbent $i = 0$, and a potential entrant $i = 1$ can both produce a single product. In the first stage, firm 1 decides whether to incur a fixed cost $F \geq 0$ to enter the market with a differentiated product. In the second stage, if entry has taken place, firms compete as duopolists; otherwise the incumbent is a monopolist. After observing the entry decision, the incumbent sets a price p_0 and, if firm 1 has entered, the latter simultaneously sets a

[9] In a theoretical model with two periods, Chioveanu (2008) has firms engaging in advertising in the first period to create captive consumers.

[10] More broadly related, Boone (2000) considers the interaction of horizontal product innovation incentives with firms' cost efficiency levels.

price p_1. Profits from sales to consumers are then realized.[11] Variable production costs are assumed to be constant and equal to 0 for both firms. If the incumbent is a monopolist, his market demand function is given as $D_M(p_0;\tau)$, where τ is a demand parameter, taken from a non-degenerate subinterval $\mathcal{T}$ of the real numbers. When firms compete for consumers, the demand function is $D_i(p_i, p_j; \tau)$, $i = 1, 2$. We assume that all demand functions are twice continuously differentiable almost everywhere where prices and demands are positive and τ is in the interior of $\mathcal{T}$. Further, we make the following more substantial assumptions:

Assumption 1.

(a) $D_i(0, 0; \tau) > 0 \forall \tau$; *$D_i$ is (weakly) decreasing in* p_i *and (weakly) increasing in* p_j $(j \neq i)$.

(b) $\frac{\partial D_i}{\partial p_j} + p_i \left(\frac{\partial^2 D_i}{\partial p_i \partial p_j} \right) \geq 0 \ \forall i, j \in \{0, 1\},\ j \neq i.$

(c) $D_0(p_0, p_1; \tau) \leq D_M(p_0, \tau)$ *for all* $(p_0, p_1; \tau) \in \mathbb{R}^+ \times \mathbb{R}^+ \times \mathcal{T}$.

(d) $p \frac{\partial D_i(p,p)}{\partial p_i} + D_i(p, p) \leq p D_M'(p) + D_M(p) \ \forall p \geq 0.$

These assumptions mean that (a) the goods are imperfect substitutes, (b) prices are strategic complements, (c) the incumbent's duopoly demand is weakly lower than his monopoly demand, and (d) the marginal effect of increasing prices is higher for a monopolist than for symmetric duopolists. Assumption 1 is compatible with standard models of price competition with horizontally differentiated goods, including the model of Section 3.

We assume that a fraction $\mu \in [0, 1]$ of consumers never buy from the entrant. Being captive is not correlated with preferences.[12] This is in line with research on consumers' consideration sets, which finds that these sets are formed separately from preferences over products.[13]

[11] We assume that entry takes place if the entrant is indifferent between entering and not entering.

[12] We allow for such correlation in Section A.3.

[13] Kardes et al. (1993) analyze the decomposition of brand choice into distinct steps, from awareness, to inspection, to eventual choice. They show that a brand that was present earlier in the market is more likely to be included in the early awareness step, producing an incumbency advantage. Campbell (2013) presents a model of a social network in which consumers have to be informed before they can decide whether or not to purchase a product. Their probability of being informed is independent of their valuation for the good.

More importantly, we assume that the incumbent cannot price discriminate between captive consumers and switchers. The total demand functions are thus given by

$$\begin{aligned}\widetilde{D}_0\left(p_0, p_1; \tau, \mu\right) &= \mu D_M\left(p_0; \tau\right) + \left(1-\mu\right) D_0\left(p_0, p_1; \tau\right) \\ \widetilde{D}_1\left(p_1, p_0; \tau, \mu\right) &= \left(1-\mu\right) D_1\left(p_1, p_0; \tau\right)\end{aligned}$$

We denote the resulting profit functions for $i = 0, 1$; $j \neq i$, as

$$\Pi_i\left(p_i, p_j; \tau, \mu\right) = p_i \widetilde{D}_i\left(p_i, p_j; \tau, \mu\right).$$

The polar cases $\mu = 1$ and $\mu = 0$ correspond to the monopoly with demand function D_M and the *pure duopoly* with demand functions D_0 and D_1, respectively. The next assumption sharpens the role of the heterogeneity parameter τ.

Assumption 2.

(a) $\frac{\partial D_i(p_i,p_j;\tau)}{\partial \tau} = 0$ *if* $p_i = p_j$ *and* $D_i(p_i, p_j; \tau) > 0$.

(b) $\frac{\partial^2 D_i}{\partial p_i \partial \tau} \geq 0$.

(c) For $\mu = 0$ *and any* $\tau \geq 0$*, the duopoly game has a unique price equilibrium* $p_0^* = p_0^*(\tau, \mu), p_1^* = p_1^*(\tau, \mu)$*, which is symmetric.*

(d) $\frac{\partial D_M}{\partial \tau} + p_0 \frac{\partial^2 D_M}{\partial p_0 \partial \tau} \leq 0$*, so that the monopoly profit* Π_M *satisfies* $\frac{\partial^2 \Pi_M}{\partial p_0 \partial \tau} \leq 0$.

Parts (a)-(c) hold in standard oligopoly models. (a) and (b) underline that τ parameterizes the taste heterogeneity of a population with fixed size, capturing the notion that increasing heterogeneity reduces the adverse demand effect of higher prices for a duopolist. We denote the equilibrium profits as $\Pi_0^*\left(\tau, \mu\right)$ and $\Pi_1^*\left(\tau, \mu\right)$, dropping $\left(\tau, \mu\right)$ wherever convenient. Condition (d) applies for instance in discrete choice models where the underlying population changes from being very homogeneous (with sufficiently high willingness-to-pay) to becoming more heterogeneous.[14] When it holds, it is often profitable for a monopolist to reduce prices so as to continue to keep all or most potential buyers on board (see Observation 1 below). Nonetheless, (d) is violated in some examples (see Section 4.1.2). In addition to Assumption 1 and 2, we assume that the functions Π_i are concave and the stability condition $\frac{\partial^2\Pi_0}{\partial p_0^2}\frac{\partial^2\Pi_1}{\partial p_1^2} > \frac{\partial^2\Pi_0}{\partial p_0 \partial p_1}\frac{\partial^2\Pi_1}{\partial p_1 \partial p_0}$ holds.

2.2 Results

In this section, we show how changes in μ and τ affect equilibrium prices and profits. Importantly, we focus exclusively on interior solutions where the effects are easiest to describe at a general level. We obtain the following straightforward observations:

Observation 1:

(i) *In a pure duopoly ($\mu = 0$), equilibrium prices and profits are increasing in τ.*

(ii) *In a monopoly ($\mu = 1$), prices are decreasing in τ.*

(iii) *Prices are higher in monopoly than in pure duopoly ($\mu = 0$).*

(i) holds because, for $\mu = 0$, Assumption 2(a) and (b) imply that τ increases $\frac{\partial \Pi_i}{\partial p_i}$, shifting reaction curves outwards symmetrically. As prices are strategic complements, these positive direct price effects are mutually reinforcing. (ii) and (iii) are directly implied by Assumption 2(d) and 1(d) and, respectively.

Our results concern the profit effects of a change in a parameter $\theta \in \{\tau, \mu\}$. We use the simple fact that the total effect of a marginal change of θ on the profit of firm i is

$$\frac{d\Pi_i^*}{d\theta} = \frac{\partial \Pi_i}{\partial \theta} + \frac{\partial \Pi_i}{\partial p_j}\frac{dp_j^*}{d\theta} \text{ for } j \neq i. \tag{1}$$

[14] Consider, e.g., a discrete-choice model with valuations distributed according to F with density f, so that the monopoly profit is $\pi_M(p_0) = p_0(1 - F(p_0))$. If $p_M(\tau)$ an interior solution, Assumption 1 requires that $\frac{\partial^2\Pi_0}{\partial p_0 \partial \tau} = -\frac{\partial F(p_0,\tau)}{\partial \tau} - p\frac{\partial f(p_0,\tau)}{\partial \tau} < 0$.

Thus, the total profit effect $\frac{d\Pi_i^*}{d\theta}$ of a parameter change consists of a direct effect (which ignores price adjustments), captured by the partial derivative $\frac{\partial \Pi_i}{\partial \theta}$, and an indirect effect induced by the adjustments in the rival's equilibrium price.[15] To understand the total profit effect of a parameter, it is thus essential to understand how it affects prices.

2.2.1 The Effect of Captive Consumers

We first characterize the price effect of the share μ of captive consumers.

Lemma 1. *p_0^* and p_1^* are both weakly increasing in μ if and only if*

$$D_M(p_0^*) - D_0(p_0^*, p_1^*) > p_0^* \left(\frac{\partial D_0}{\partial p_0}(p_0^*, p_1^*) - D_M'(p_0^*) \right), \tag{2}$$

or, equivalently,

$$D_M(p_0^*) - D_0(p_0^*, p_1^*) > \varepsilon_M(p_0^*) D_M(p_0^*) - \varepsilon_0(p_0^*, p_1^*) D_0(p_0^*, p_1^*), \tag{3}$$

where ε_M and ε_0 are the own-price elasticities of D_M and D_0, respectively (in absolute values).

Intuitively, μ scales the entrant's profit by a constant and thus does not affect her reaction function, but only the incumbent's. By strategic complements, if a higher μ shifts the incumbent's reaction curve out, both prices increase. As μ increases, the incumbent pays more attention to the captive market, raising prices as monopoly prices are higher than pure duopoly prices. Thus, the incumbent has more demand from captive consumers and less demand from switchers; the marginal effect is $D_M - D_0$, which is positive as he has to share the switchers with the competitor (Assumption 1(c)). However, the term $\frac{\partial D_0}{\partial p_0}(p_0^*, p_1^*) - D_M'(p_0^*)$ on the right-hand side of (2) is usually also positive in parametric examples. With competition,

[15] Using the logic of the envelope theorem, there is no indirect effect of θ on π_i induced by a change in p_i^*.

the demand losses from higher prices are lower than in monopoly.[16] If, as required by (2), this *elasticity effect* is smaller than the former *demand effect*, then prices are increasing in μ. (3) illustrates the elasticity effect. The incumbent price rises when the direct gains from inframarginal consumers outweigh the adverse effects of demand reductions by marginal consumers or, equivalently, when monopoly prices are higher than pure duopoly prices. Because of Assumption 1(d) and the implied Observation 1(iii), this always holds in the model described above. Nonetheless, we included conditions (2)) and (3) explicitly because, as argued by Chen and Riordan (2008), there are reasonable cases where duopoly prices could be higher than monopoly prices.[17]

The next result gives conditions for profits to increase with the share of captive consumers.

Lemma 2.

(i) The entrant's profit Π_1^* *is locally increasing in* $\mu \in [0, 1)$ *if and only if*

$$\frac{\mathrm{d}p_0^*}{\mathrm{d}\mu} > \frac{D_1\left(p_1^*, p_0^*\right)}{(1-\mu)\frac{\partial D_1}{\partial p_0}(p_1^*, p_0^*)}. \tag{4}$$

(ii) The incumbent's profit Π_0^* *is locally increasing in* $\mu \in [0, 1)$ *if and only if*

$$\frac{\mathrm{d}p_1^*}{\mathrm{d}\mu} > \frac{D_0\left(p_0^*, p_1^*\right) - D_M\left(p_0^*\right)}{(1-\mu)\frac{\partial D_0}{\partial p_1}(p_0^*, p_1^*)}. \tag{5}$$

To understand (i), note that $D_1\left(p_1^*, p_0^*\right)$ captures the entrant's demand loss from a marginal increase in μ, the negative direct effect. As $(1-\mu)\frac{\partial D_1}{\partial p_0}$ is positive by Assumption 1(a), the entrant can only benefit from an increase in the share of captive consumers if the incumbent responds with a sufficiently high price increase. Similarly, Result (ii) provides a lower bound on the effect of μ on the opponent's price, guaranteeing a positive effect of μ on incumbent

[16] This is for instance true in the example in Section 3.

[17] In our specific model in Section 3, we will rule this out by limiting heterogeneity, except in Section 4.1.2.

profits. However, contrary to (4), the right-hand side of (5) is negative because the direct effect on the incumbent's profit, the demand increase $D_M(p_0^*) - D_0(p_0^*, p_1^*)$, is positive. Thus, as long as it does not lead to a substantial drop in the entrant's price, an increase in the incumbent's share of captive consumers increases the incumbent's profits. We now use Lemma 2 to derive a sufficient condition for global non-monotonicity.

Corollary 1. *For any fixed $\tau > 0$, the entrant's profit Π_1^* is non-monotone in μ on $[0,1]$ if*

$$\left.\frac{\mathrm{d}p_0^*}{d\mu}\right|_{\mu=0} > \frac{D_1(p_1^*, p_0^*; \tau, 0)}{\frac{\partial D_1}{\partial p_0}(p_1^*, p_0^*, \tau, 0)}. \tag{6}$$

Obviously, entry profits always decline to 0 as μ approaches 1. (6) makes sure that, by contrast, introducing a small share of captive consumers has a positive effects on entry profit, as the increase in incumbent prices suffices to make up for the reduction in the number of switchers. The condition becomes easier to fulfill if $\frac{\partial D_1}{\partial p_0}$ is high relative to D_1, as a competition-softening effect of captive consumer matters more for the entrant when competition is otherwise intense.

2.2.2 The Effect of Increasing Heterogeneity

We now analyze the effects of increasing heterogeneity τ.

Lemma 3. *The entrant's profit $\Pi_1^*(p_i, p_j; \tau)$ is locally weakly increasing in τ if and only if*

$$\frac{\mathrm{d}p_0^*}{\mathrm{d}\tau} > -\frac{\frac{\partial D_1}{\partial \tau}(p_1^*, p_0^*, \tau)}{\frac{\partial D_1}{\partial p_0}(p_1^*, p_0^*, \tau);}. \tag{7}$$

Similarly to Lemma 2, (ii) states that a positive effect of τ on the incumbent's price is conducive to higher entry profits. Under reasonable conditions, an increase in the incumbent price is in fact *necessary* for τ to increase entry profits: In applications, the entrant's equilibrium price is usually smaller than the incumbent's, because the incumbent gives some weight to the monopolistic market where prices are higher than on the pure duopoly market. This implies that $\frac{\partial D_1}{\partial \tau} < 0$ by Assumption 2(a) and (b), because the entrant's demand advantage translates less into a higher market share with higher τ.[18] Thus, the right-hand side of equation (7) is positive, so that an incumbent price increase is necessary for greater heterogeneity to increase entry profits. This shows why an increase in τ may reduce entry profits: By Observation 1(ii), monopoly prices fall as τ increases, capturing the idea that the incumbent responds to an increase in heterogeneity by lowering prices to keep captive consumers on board. When $\mu < 1$, but sufficiently large, the effect of increasing τ on the incumbent price is still negative and thus potentially also the effect on the entrant's profit.

3 A discrete-choice example

We now specify the demand functions D_M, D_0 and D_1 in a simple discrete choice example, so as to derive price and entry decisions explicitly.

3.1 Assumptions and Benchmark Results

The incumbent $i = 0$ and an entrant $i = 1$ face a unit mass of consumers, each of which buys one unit of one good or nothing at all. Each consumer has valuations v_0 and v_1, where the average valuation for the two goods is the same for each consumer, $\frac{v_0+v_1}{2} = 1$. The valuation difference $v_1 - v_0$ is uniformly distributed on an interval $[-2\tau, 2\tau]$, where $\tau \in (0, 1/3]$ captures taste heterogeneity. Marginal production costs are constant at $c = 0$. The fixed entry cost is $F > 0$. When there is no entry, the incumbent has monopoly demand $D_M(p_0, \tau) = \min\left\{\frac{2(1-p_0)+2\tau}{4\tau}, 1\right\}$.[19] We restrict prices to $p_i \in [0, 1+\tau]$. This is without loss of generality, as $1 + \tau$ is the highest possible consumer valuation.[20]

[18] For $p_0 = p_1$, $\frac{\partial D_1}{\partial \tau} = 0$ by Assumption 2(a). Reducing p_1 lowers $\frac{\partial D_1}{\partial \tau}$ below 0 by Assumption 2(b).

[19] This holds because $v_0 = p_0 \Leftrightarrow v_1 - v_0 = 2(1 - p_0)$ as $v_1 = 2 - v_0$.

[20] $v_i + v_j = 2$ implies that, for $v_i = 1 + \tau$, $v_i - v_j = (1 + \tau) - (1 - \tau) = 2\tau$.

Benchmarks: Straightforward arguments show that, for the pure duopoly without captive consumers ($\mu = 0$), a symmetric equilibrium with prices $p_i = 2\tau$ and profits $\Pi_i = \tau$ emerges. Entry therefore takes place if and only if $\tau \geq F$. In the monopoly (μ=1), the incumbent charges $p_0 = 1 - \tau$, thus serving the entire market. These results are in line with Observation 1. Crucially, while consumer heterogeneity softens competition, it reduces the monopoly price: Prices are decreasing in τ because increasing heterogeneity makes it harder to keep all consumers on board.[21]

3.2 Results

We now analyze the equilibria in the post-entry pricing stage. The following result gives existence conditions for different types of PSE and provides equilibrium prices. Moreover, it gives conditions under which only mixed-strategy equilibria (MSE) exist.

Proposition 1. *Suppose $\mu < 1$ and $\tau \leq \frac{1}{3}$. If one of the conditions (i)-(iii) below holds, then a pure-strategy equilibrium (PSE) exists where the market is fully covered.*

(i) For $\frac{1}{7} < \tau$ and $\tau \geq \frac{3(1-\mu)}{9-\mu}$, a low-surplus equilibrium (LSE) emerges: Both firms serve switchers; the incumbent sets the maximal price where all captive consumers buy. Prices are

$$p_0^* = 1 - \tau, \; p_1^* = \frac{\tau + 1}{2}.$$

(ii) For $\frac{9\mu(1-\mu)}{15\mu - 8\mu^2 + 9} \leq \tau < \frac{3(1-\mu)}{9-\mu}$, a consumer-friendly equilibrium (CFE) arises: Both firms serve switchers; all consumers obtain a positive surplus. Prices are

$$p_0^* = \frac{2\tau\,(3+\mu)}{3(1-\mu)}, \; p_1^* = \frac{2\tau\,(3-\mu)}{3(1-\mu)}.$$

[21] To repeat, Observation 1(iii) may seem obvious, but is not always true in differentiated product duopolies (Chen and Riordan (2008)). In fact, without the restriction $\tau < 1/3$, pure duopoly prices can be higher than monopoly prices (see Section 4.1.2).

(iii) For $\frac{1-\mu}{1+3\mu} \leq \tau \leq \frac{1}{7}$*, a market-partition equilibrium (MPE) arises: All captive consumers buy from the incumbent, and all switchers buy from the entrant. Prices are*

$$p_0^* = 1 - \tau,\ p_1^* = 1 - 3\tau.$$

If none of the conditions in (i)-(iii) holds, so that $\tau < \frac{1-\mu}{1+3\mu}$ *and* $\tau \leq \frac{9\mu(1-\mu)}{15\mu-8\mu^2+9}$*, there exists no PSE. Instead a mixed-strategy equilibrium (MSE) exists.*

The shaded areas in Figure 1 correspond to the equilibrium regions. Because of our restriction to $\tau \in (0, 1/3)$, the incumbent serves all captive consumers in all equilibria. For each τ, if μ is small, there is a CFE with low prices. Beyond a threshold number of captive consumers, the incumbent focuses on them and sets the monopoly price. If heterogeneity is low, the entrant finds it optimal to set a price just low enough to attract all the remaining switchers; an MPE emerges. If heterogeneity is not too low, the entrant sets a higher price, and some switchers buy from the incumbent; an LSE emerges.

The PSE regions in parts (i)-(iii) of Proposition 1 do not cover the entire parameter space. For low τ and intermediate μ as in (iv), there is no PSE. In the following discussion of entry, we focus on the PSE regions, but we briefly discuss the MSE in Section 4.1.1.

Given the equilibrium prices determined in Proposition 1, it is straightforward to identify the conditions under which entry takes part for the different PSE (see Proposition 2 in Appendix A.2.3). Figure 1 depicts the entry regions for different values of F. Entry profit is maximal ($\Pi_1 = 1/3$) when $(\tau, \mu) = (1/3, 0)$, that is, when τ is very large and μ is very low. For $F = 0.3$, the entry region consists of parts of the LSE and CFE regions in the lower right corner. For lower entry cost ($F = 0.25$ and $F = 0.15$), the entry region extends further to the upper left, containing parts of the MPE region. However, the shape of the entry regions shows that neither μ nor τ has a monotone effect on entry.

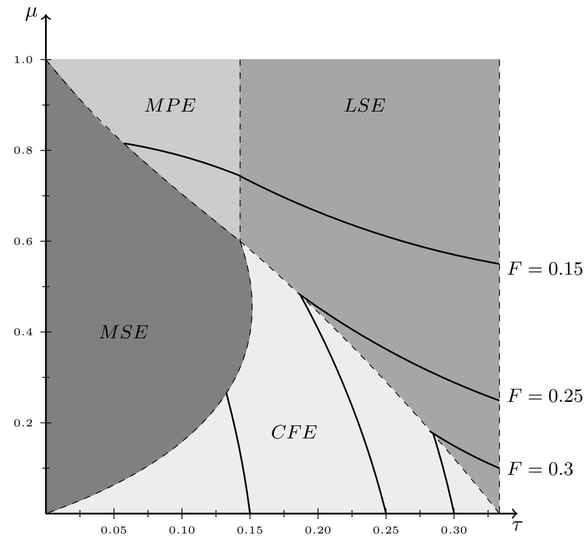

Figure 1: Entry Regions

To understand why an increase in the share of captive consumers does not generally lead to less entry, the following immediate implication of Proposition 1 is helpful.

Lemma 4. *(i) Both prices are increasing in μ in the CFE region. μ has no effect on prices in the remaining two regions. (ii) Entry profits are increasing in μ in the CFE region, decreasing in the LSE and MPE regions.*

Intuitively, (i) when μ is low, the incumbent focuses on the (large) switcher market, resulting in intense competition. As μ increases, he shifts attention to captive consumers, increasing prices in the CFE region. In the remaining regions, prices are independent of μ, determined by the requirement to keep all captive consumers on board. To understand the positive effect of higher μ on entry profits in CFE (ii), recall from Lemma 2 that such positive effects require a sufficient increase of the incumbent price.[22] Using (i), the price only increases in the CFE region; (ii) indeed shows that this incumbent price effect on the CFE dominates the lower switcher demand, and entry profits increase. This explains the observation from Figure 1 that, when fixed costs are so high that there is no entry in pure duopoly, it may

[22]The analysis of Section 2 directly applies to the CFE because equilibrium prices are interior.

nevertheless take place above a threshold level of μ near the upper boundary of the CFE region. In the remaining regions, the incumbent does not change the price, so that the loss of potential demand translates directly into lower entry profits.

For taste heterogeneity, Observation 1(i) implies that more intense competition (lower τ) reduces entry profits if $\mu = 0$.[23] Figure 1 shows that, for $\mu > 0$, entry profits may increase after a reduction in τ. The following implication of Proposition 1 explains this result.

Lemma 5. *(i) Increasing heterogeneity increases CFE prices and decreases MPE prices. Heterogeneity reduces the incumbent LSE price, but increases the entrant LSE price.*
(ii) Increasing heterogeneity increases CFE entry profits and decreases MPE and LSE entry profits. Thus, focusing on the PSE regions, for $\mu > 0.6$ *the relation between* τ *and entry profits is negative, whereas for* $0 < \mu < 0.6$*, it is inverse U-shaped.*[24]

Two countervailing forces are at work: Consumer heterogeneity softens competition for switchers, but may induce the incumbent to set lower prices to serve all captive consumers. In a pure duopoly, only the former positive effect is present, and, in the CFE region, it still dominates the negative effect from the monopoly market by (i). The negative effects on MPE and LSE prices explain why entry profits depend negatively on τ in those regions: Even though greater heterogeneity would increase pure duopoly profits, the negative effect on the prices of an incumbent who cares much about captive consumers dominates.

Differentiated entry is an instance of horizontal product innovation, and taste heterogeneity is inversely related to product substitutability – a standard measure of competition. We find that, unlike without captive consumers, increased competition from higher product substitutability can lead to more innovation. This relates to studies of the effects of competition on cost-reducing or demand-enhancing innovations, which show that even the qualitative

[23]This is a variant of the standard insight that more intense competition increases industry concentration (e.g., Sutton 1991).

[24]This statement holds because the conditions for a CFE in Proposition 1(i) can only hold for $\mu < 0.6$.

nature of the relation depends on market-specific details and that different types of non-monotonicities are conceivable (Gilbert, 2006; Vives, 2008; Schmutzler, 2010, 2013).[25]

4 Model Discussion and Policy Implications

In the following, we discuss the model, and we briefly sketch policy implications.

4.1 Model Discussion

We now discuss mixed-strategy equilibria. Then we relax the assumptions that (i) increases in heterogeneity reduce monopoly profits, and (ii) consumers' tastes are independent of whether they are captive or not.

4.1.1 Mixed-Strategy Equilibria

The non-existence of PSE in models with captive consumers and perfect competition is well known since Varian (1980). As Proposition 1 shows, it extends to imperfect competition as long as the degree of substitution is high enough. Intuitively, competition for switchers is intense when consumer heterogeneity is low. The incumbent wants to focus on exploiting the captive consumers by setting a high price, to which the entrant would respond by also setting a high price. As τ is low, switchers are sensitive to price differences, and the incumbent would be tempted to react with a low price to the entrant's high price. For intermediate values of μ, there is a large group of both consumer types, and the described reasoning has most bite: The incumbent's conflict between attracting switchers and exploiting captive consumers is particularly pronounced, and the region without PSE is large. Even for τ for which MSE exist for some values of μ, the basic conclusion about the effects of increasing μ holds: Introducing some captive consumers (below the MSE region) increases entry profits, but beyond the MSE region increasing μ further lowers these profits (see Figure 1).

[25] Aghion et al. (2005) provide evidence for an inverse-U relation between competition and innovation.

4.1.2 Large Heterogeneity

Assumption 2(d) (and $\tau < 1/3$ in the model of Section 3) imply that monopoly profits are decreasing in taste heterogeneity. While this is a perfectly reasonable possibility, it clearly is a restriction, which we made to avoid having to distinguish between too many different cases, thus making the intuition behind our results particularly transparent. Proposition 3 in Appendix A.2.5 provides the equilibrium for our parametric example when $1/3 < \tau \leq \frac{2\mu+3}{\mu+6}$. Monopoly profits are increasing in τ in this case, violating Assumption 2(d). Moreover, monopoly prices are now below pure duopoly prices. Hence, with such high heterogeneity, (1) shows that entry cannot become more likely when the share of captive consumers increases. Intuitively, the incumbent price effect now reinforces the adverse effect of the declining market size on entry profits. Similarly, because the negative effect of heterogeneity on prices is absent in this region, so is the adverse effect on entry profits. Therefore, if one extends the analysis to include $\tau > 1/3$, the non-monotonicity in τ becomes even more pronounced, with an N-shaped relation becoming possible for $\mu < 0.6$ and a U-shaped relation for $\mu > 0.6$.

4.1.3 Taste-Dependent Share of Captive Consumers

One might argue that consumers with stronger preferences for the incumbent's product are more likely to be captive. In Appendix A.3, we adjust the general model of Section 2 by allowing for such correlation. The main insights of our analysis are robust. As long as, for a fixed value of τ, introducing a small amount of captive consumers increases incumbent prices sufficiently, entry profits will be non-monotone in μ.[26] Similarly, increasing heterogeneity will continue to have negative effects on entry profits for large shares of captive consumers.

As in Section 3, we focus on a discrete choice setting. However, we allow general distributions of valuation differences $\nu = v_1 - v_0$, with support $G(v)$ on $(-\infty, \infty)$. On its (not necessarily full) support, G has a density function $g(\nu; \tau)$, which is differentiable in both arguments.

[26]What could change, however, is how τ affects the possibility that entry arises at intermediate values.

We assume that Assumption 2 holds. We allow the share of captive consumers to vary with preferences, denoting it as $L(\nu;\mu)$; where $\mu \in [0,1]$, $L(\nu;\mu)$ is increasing in μ, with $L(\nu;0) = 0$ and $L(\nu;1) = 1$ for all ν. $L(\nu;\mu)$ is differentiable in both arguments. In Appendix A.3, we formulate the resulting demand functions and we provide first-order conditions for an interior equilibrium. Using the logic of Section 2, we also provide conditions under which entry profits are locally increasing (Proposition 4) and globally non-monotone in μ (Corollary 4). As entry profits are zero when there are no switchers, non-monotonicity arises if, starting from a pure duopoly equilibrium without captive consumers, the price-increasing effect of introducing some captive consumers is sufficiently large. The argument resembles Corollary 1. Moreover, the effect of greater heterogeneity on entry profits still becomes negative for a large share of captive consumers. Again, increasing heterogeneity leads to increasing pure duopoly prices (and thus increasing entry profits) without captive consumers, but to decreasing monopoly prices and thus to decreasing entry profits for sufficiently high μ.

4.2 Policy Discussion

The analysis of captive consumers raises several policy issues. First, policy measures can serve to reduce the share of captive consumers. Second, the literature has discussed the effects of price discrimination between captive consumers and switchers on entry. Our analysis sheds new light on this discussion in the context of differentiated goods.

4.2.1 Policy towards Captive Consumers

Governments can take measures to reduce the share of captive consumers themselves, e.g., by supporting the dissemination of information about new products to increase market transparency.[27] Less directly, they can restrict activities of incumbents to create captive consumers or foster activities of entrants to reduce the number of captive consumers.

[27] As an illustration, the European Competition Authorities recommended the adoption of transparent and objective price comparison sites for retail banking (OECD (2006)).

When the government can directly influence the share of captive consumers, increases in μ affect consumer surplus through several channels. First, keeping prices fixed, there is a direct weakly negative effect on those consumers who change from being switchers to being captive, as their choice set shrinks. Second, there are indirect effects for the consumers who keep their status as captive or flexible, because of induced changes on duopoly prices or market structure. The following result describes the total result.

Corollary 2. *(i) If it does not affect entry, an exogenous increase in μ reduces total consumer surplus. (ii) Suppose a marginal increase in μ changes the entry decision. Total consumer surplus falls in regions LSE and MPE; it increases in region CFE.*

The proof in Appendix A.2.4 uses Lemma 4, which gives explicit expressions for consumer surplus. Part (i) is intuitive given Proposition 1: Not only does the share of consumers who do not have access to the entrant rise; in addition, if prices change with μ, they increase. Result (ii) holds because an increase in μ impedes entry except in the CFE region (Lemma 4), and total consumer surplus is higher in duopoly than in monopoly.

Instead of the government, the incumbent may influence μ himself, for instance, by using persuasive advertising. If an incumbent engages in activities to create captive consumers, should one be concerned about adverse effects on consumer surplus? This is not obvious as consumers benefit from a marginal increase in captive consumers in the CFE region if this induces entry. However, the next result shows that the incumbent would not choose an entry-inducing strategy as this would reduce profits.

Corollary 3. *If a marginal increase in the share of captive consumers leaves market structure unaffected, it increases incumbent profits. If it does affect market structure, then it increases incumbent profits, except in the CFE region, where it decreases them.*

The result implies that any increase in μ that the incumbent would want to bring about himself is detrimental for consumers as it never implies entry.[28] Even though the effect of an exogeneous change in μ on consumer surplus is ambiguous, it therefore seems warranted for policy makers to keep a careful eye on strategies that create captive consumers.

Finally, the entrant can try to reduce the share of captive consumers by engaging in informative advertising. Usually, if the entrant benefits from reductions in the share of captive consumers, so do the consumers.[29] The only case where interests are not aligned arises in the CFE region, because, to keep competition soft, the entrant abstains from reducing the captive share, even though consumers would benefit from such a reduction.

4.2.2 Price discrimination

Up to now, we have considered the situation in which the incumbent is restricted to charging a uniform price to all his buyers. An important and active debate discusses the impact of price discrimination on entry, going back to Armstrong and Vickers (1993) who show that banning an incumbent's possibility to discriminate between captive and switching consumers fosters entry by softening competition in the switcher market.[30]

We have shown that, without price discrimination, a larger captive market share can help entry for low values of consumer heterogeneity. When the incumbent can set different prices for captive and switching consumers, this effect vanishes. Both prices are then independent of μ, and the incumbent now prices more aggressively in the switcher market, thereby reducing the entrant's profit. Thus, under the assumptions of Section 3, we confirm the standard insight of the literature that price discrimination impedes entry. However, if, as sketched in Section 4.1.2, we would allow for higher consumer heterogeneity, the result would be

[28] On a related note, Chioveanu (2008) who has ex-ante symmetric firms investing in advertising to create captive consumers. She finds that firms will chose asymmetric advertising intensities in equilibrium.

[29] An exception can arise in the large heterogeneity case of Section 4.1.2, where there is a small subregion where the entrant would want a reduction in μ, but the consumers would not.

[30] Anton, Vander Weide, and Vettas (2002) and Bouckaert, Degryse, and van Dijk (2013) obtain similar results.

reversed, because the incumbent would need to set a low price in order to attract his *captive* consumers.[31] At the same time, the price in the duopolistic segment would be relatively high because consumers substitute the products less willingly.

5 Conclusion

This paper asks under which circumstances market entry arises in spite of incumbency advantages. We analyze incentives for innovative entry with differentiated goods into a market previously dominated by a monopolist. For low initial levels of captive consumers and intermediate heterogeneity, an increase in the share of captive consumers is conducive to entry, reflecting its competition-softening effect. Beyond a threshold, further increases impede entry as a result of the reduction in the entrant's potential demand. Our analysis also contributes to the literature on competition and innovation. For positive shares of captive consumers, there exists an interval on which increasing taste heterogeneity (decreasing product substitutability) reduces entry profits. This is in marked contrast to the case without captive consumers, where greater consumer heterogeneity always fosters entry. Thus, once one introduces captive consumers, the relation between consumer heterogeneity and horizontal product innovation becomes similarly complex as the relation between consumer heterogeneity and "vertical" (cost-reducing or quality-enhancing) innovation.

[31] Recall that, for $\tau > 1/3$, monopoly prices are lower than pure duopoly prices.

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A Appendix

A.1 General model

A.1.1 Proof of Lemma 1

Total differentiation of the first-order conditions implies that, for $i = 0, 1$ and $j \neq i$,

$$\frac{dp_i}{d\mu} = \frac{\frac{\partial^2 \Pi_i}{\partial p_i \partial p_j} \frac{\partial^2 \Pi_j}{\partial p_j \partial \mu} - \frac{\partial^2 \Pi_i}{\partial p_i \partial \mu} \frac{\partial^2 \Pi_j}{\partial p_j^2}}{\frac{\partial^2 \Pi_i}{\partial p_i^2} \frac{\partial^2 \Pi_j}{\partial p_j^2} - \frac{\partial^2 \Pi_i}{\partial p_i \partial p_j} \frac{\partial^2 \Pi_j}{\partial p_j \partial p_i}},$$

evaluated at the equilibrium. By the stability condition, the denominator is positive. Using the entrant's F.O.C., $\frac{\partial^2 \Pi_1}{\partial p_1 \partial \mu} = 0$. Hence, $dp_0/d\mu \geq 0$ if and only if $\frac{\partial^2 \Pi_0}{\partial p_0 \partial \mu} \frac{\partial^2 \Pi_1}{\partial p_1^2} \leq 0$, that is, using concavity of Π_1, $\frac{\partial^2 \Pi_0}{\partial p_0 \partial \mu} \geq 0$. Similarly, $dp_1/d\mu \geq 0$ if and only if $\frac{\partial^2 \Pi_1}{\partial p_0 \partial p_1} \frac{\partial^2 \Pi_0}{\partial p_0 \partial \mu} \geq 0$, that is, using strategic complements (Assumption 1(b)), $\frac{\partial^2 \Pi_0}{\partial p_0 \partial \mu} \geq 0$. Finally, $\frac{\partial^2 \Pi_0}{\partial p_0 \partial \mu} \geq 0$ if and only if

$$D_M - D_0 + p_0 \left(\frac{\partial D_M}{\partial p_0} - \frac{\partial D_0}{\partial p_0} \right) \geq 0.$$

This condition is equivalent to conditions (2) and (3) in the proposition.

A.1.2 Proof of Lemma 2

(i) Using (1), it suffices to show that (4) is equivalent to $\frac{\partial \Pi_1}{\partial \mu} + \frac{\partial \Pi_1}{\partial p_0} \frac{\partial p_0^*}{\partial \mu} > 0$. This follows from $\frac{\partial \Pi_1}{\partial \mu} = -p_1 D_1$ and $\frac{\partial \Pi_1}{\partial p_0} = p_1 (1 - \mu) \frac{\partial D_1}{\partial p_0}$, using $\frac{\partial D_1}{\partial p_0} > 0$ (by Assumption 1(a)).

(ii) Using (1), it suffices to show that (5) is equivalent to $\frac{\partial \Pi_0}{\partial \mu} + \frac{\partial \Pi_0}{\partial p_1} \frac{\partial p_1^*}{\partial \mu} > 0$. This follows from $\frac{\partial \Pi_0}{\partial \mu} = p_0 (D_M - D_0)$ and $\frac{\partial \Pi_0}{\partial p_1} = p_0 (1 - \mu) \frac{\partial D_0}{\partial p_1}$, using $\frac{\partial D_0}{\partial p_1} > 0$ (implied by Assumption 1(a)).

A.1.3 Proof of Corollary 1

For $\mu = 0$ and $\tau > 0$, equilibrium profits are positive, because $p_i = 0$ implies that $\frac{\partial \Pi_i(0,0)}{\partial p_i} = D_i(0,0) + p_i \frac{D_i(0,0)}{\partial p_i} = D_i(0,0) > 0$. For $\mu = 0$, the right-hand side of (6) is the right-hand side of (4). Thus, by Lemma 2(i), Π_1 is locally increasing in μ for $\mu = 0$. As $\Pi_1 = 0$ for $\mu = 1$, Π_1 is thus neither everywhere increasing nor everywhere decreasing.

A.1.4 Proof of Lemma 3

(i) Arguing as in the proof of Lemma 1, an increase in τ implies $\frac{dp_i}{d\tau} > 0$ if and only if

$$\frac{\partial^2 \Pi_i}{\partial p_i \partial p_j} \frac{\partial^2 \Pi_j}{\partial p_j \partial \tau} - \frac{\partial^2 \Pi_i}{\partial p_i \partial \tau} \frac{\partial^2 \Pi_j}{\partial p_j^2} > 0.$$

For $\mu = 0$, firms are symmetric, so that $\frac{\partial^2 \Pi_i}{\partial p_i^2} = \frac{\partial^2 \Pi_j}{\partial p_j^2}$ and $\frac{\partial^2 \Pi_i}{\partial p_i \partial \tau} = \frac{\partial^2 \Pi_j}{\partial p_j \partial \tau}$. Thus, using stability, $\frac{dp_i}{d\tau} > 0$ if and only if $\frac{\partial^2 \Pi_i}{\partial p_i \partial \tau} > 0$. This holds for $\mu = 0$, because $\frac{\partial \Pi_i}{\partial p_i} = p_i \frac{\partial D_i}{\partial p_i} + D_i$ is increasing in τ by Assumption 2(a) and (b), noting that $p_0^*(\tau, 0) = p_1^*(\tau, 0)$.

(ii) Equation (1) for $\theta = \tau$ gives $\frac{d\Pi_1^*}{d\tau} = \frac{\partial \Pi_1}{\partial \tau} + \frac{\partial \Pi_1}{\partial p_0} \frac{dp_0^*}{d\tau}$. Solving for $\frac{dp_0^*}{d\tau}$, using $\frac{\partial \Pi_1}{\partial p_0} = (1-\mu) \frac{\partial D_1}{\partial p_0} > 0$ and $\frac{\partial \Pi_1}{\partial \tau} = (1-\mu) \frac{\partial D_1}{\partial \tau}$ gives the result.

A.2 Discrete choice model

A.2.1 Auxiliary Results

We collect some results for convenience. First, note that $v_0 + v_1 = 2$ and $-2\tau \leq v_1 - v_0 \leq 2\tau$ imply that $1 - \tau \leq v_i \leq 1 + \tau$ for $i = 1, 2$. Thus, prices $p_i > 1 + \tau$ are dominated, and we can exclude them w.l.o.g. The monopoly demand function is:

$$D_M(p_0) = \begin{cases} \frac{2(1-p_0)+2\tau}{4\tau} & \text{if} \quad 1 - \tau < p_0 \\ 1 & \text{if} \quad p_0 \leq 1 - \tau \end{cases} \tag{8}$$

Demand functions under pure duopoly are (for $i = 1, 2; j \neq i$):

$$D_i(p_i, p_j) = \begin{cases} 0 & \text{if} \quad p_i \geq p_j + 2\tau \\ 1 & \text{if} \quad p_i \leq \min\{p_j - 2\tau, 1-\tau\} \\ \frac{2(1-p_i)+2\tau}{4\tau} & \text{if} \quad 1-\tau < p_i < p_j - 2\tau \\ \frac{2\tau - p_i + p_j}{4\tau} & \text{otherwise.} \end{cases} \tag{9}$$

In the first three cases, only one firm produces because the price differences are too large. When firm i has the lower price, it will serve the entire market only if $p_i \leq 1 - \tau$. In the fourth case, price differences are low, and demand is determined by the consumer who is indifferent between both firms. (8) and (9) imply Assumptions 1 and 2, as well as concavity of profits and stability. Using (9), the entrant's reaction function is[32]

$$p_1 = \frac{2\tau + p_0}{2}. \tag{10}$$

A.2.2 Proof of Proposition 1

Existence and Characterization of PSE (i)-(iii)

(i) At $p_0^* = 1 - \tau$, the captive market is exactly covered, as the net utility of the consumer with $v_1 = 1 - \tau$ equals zero. Thus, the indifferent switcher also has non-negative utility. As $\tau > \frac{1}{7}$ implies $|p_1^* - p_0^*| < 2\tau$, both firms serve switchers. Using (10), $p_1^* = \frac{\tau+1}{2}$ is a best response to $p_0^* = 1-\tau$. Using (8) and (9), a deviation of the incumbent to $p_0 < p_0^*$ gives profit $p_0\left(\mu + (1-\mu)\frac{p_1 - p_0 + 2\tau}{4\tau}\right)$. This is not a profitable deviation because $\tau \geq \frac{3(1-\mu)}{9-\mu}$. An upward deviation to p_0 which leaves the switcher market entirely to the entrant ($p_0 \geq \frac{5\tau+1}{2}$) would give profits $\mu p_0 \frac{2(1-p_0)+2\tau}{4\tau}$ by (8). For $\tau < 1/3$ such a deviation is dominated by $p_0^* = 1 - \tau$.

(ii) In the proposed equilibrium, the captive market is covered, and even the captive consumer with $v_0 = 1 - \tau$ obtains a positive surplus as $\tau < \frac{3-3\mu}{9-\mu}$ implies $p_0^* < 1 - \tau$. By (8), the

[32] This follows from the entrant's FOC, taking into account that, as long as $\tau < 1$, $p_0 \in [0, 1-\tau]$ implies $p_1 = \frac{2\tau+p_0}{2} \in [0, 1-\tau]$.

incumbent thus has a locally inelastic captive demand of μ. When $\mu < 0.6$, which is implied by the parameter restrictions of (ii), $0 < p_0^* - p_1^* < 2\tau$. Thus, both firms face positive switcher demand. The incumbent's demand from switchers thus is $(1-\mu)\left(\frac{p_1 - p_0 + 2\tau}{4\tau}\right)$. His profit in the equilibrium and after local deviations is thus $\Pi_0 = p_0\left(\mu + (1-\mu)\left(\frac{p_1 - p_0 + 2\tau}{4\tau}\right)\right)$. By (9), the entrant's profit is $\Pi_1 = p_1(1-\mu)\left(\frac{p_0 - p_1 + 2\tau}{4\tau}\right)$. The proposed equilibrium candidate prices $p_0^* = \frac{2\tau(3+\mu)}{(3-3\mu)}$ and $p_1^* = \frac{2\tau(3-\mu)}{(3-3\mu)}$ thus fulfill the firm's F.O.C.'s. Next, consider deviations by the incumbent to $p_0 > p_0^*$ where he only serves the captive market, obtaining a profit of $p_0 \min\{\frac{2(1-p_0)+2\tau}{4\tau}, 1\}$. $\tau < 1/3$ implies that the best such deviation involves $p_0 = 1-\tau$. When $\tau > \frac{9\mu - 9\mu^2}{15\mu - 8\mu^2 + 9}$, no profitable deviation of this type exists. To see this, first note that, for $\tau > \frac{3\mu - 3}{11\mu - 15} > \frac{9\mu - 9\mu^2}{15\mu - 8\mu^2 + 9}$, the price $1-\tau$ is so low that the incumbent would still serve switchers: The highest net utility of a switcher is $1 - \tau - (1-\tau) = 2\tau$ when buying from the deviating incumbent, whereas at the entrant it is $1 - \tau - \frac{2\tau(3-\mu)}{3-3\mu}$. $\tau > \frac{3\mu-3}{11\mu-15}$ implies $2\tau > 1 - \tau - \frac{2\tau(3-\mu)}{3-3\mu}$. Thus suppose $\tau < \frac{3\mu-3}{11\mu-15}$, so that the deviation is feasible. Comparing the deviation profit $\mu(1-\tau)$ with the profit in the candidate equilibrium, $\Pi_0 = p_0^*\left(\mu + (1-\mu)\left(\frac{p_1^* - p_0^* + 2\tau}{4\tau}\right)\right)$, one can show that the net benefit from deviation is negative as long as $\tau > \frac{9\mu - 9\mu^2}{15\mu - 8\mu^2 + 9}$.

(iii) At the proposed prices, the incumbent serves all captive consumers; with a net surplus of zero if $v_0 = 1 - \tau$. The entrant serves all switchers at the highest price where all switchers buy from it: the switcher at position zero is indifferent between buying from the entrant and buying from the incumbent. By (8), the incumbent's profit is $\mu(1-\tau)$; by (9), the entrant's profit is $(1-\mu)(1-3\tau)$. As $\tau < 1/7 < 1/3$, the incumbent cannot benefit from an increase of the prices to captive consumers. Arguing as in (ii), if the incumbent lowers his prices to some $p_0 < p_0^*$, his profit becomes $\Pi_0 = p_0\left(\mu + (1-\mu)\left(\frac{p_1 - p_0 + 2\tau}{4\tau}\right)\right)$. Such a deviation is not profitable when $\tau \geq \frac{1-\mu}{3\mu+1}$. The entrant's profit is $\Pi_1 = p_1(1-\mu)\left(\frac{p_0 - p_1 + 2\tau}{4\tau}\right)$. The entrant has no incentive to set $p_1 < p_1^*$. Setting $p_1 > p_1^*$ is not profitable when $\tau \leq \frac{1}{7}$.

Non-Existence of other Pure-Strategy Equilibria

First, consider equilibria where not even the the switcher market is completely covered. A firm that marginally increases prices would thus not lose demand to the competitor. Hence, its optimal price would correspond to the price $p = \frac{1+\tau}{2}$ of a monopolist with demand $D_M(p, \tau) = \frac{2(1-p)+2\tau}{4\tau}$. Incomplete coverage would require that the consumer with $v_1 = v_0$ would not buy (so that $p = \frac{1+\tau}{2} > 1$ or $\tau > 1$, violating $\tau < 1/3$). Second, consider an equilibrium where only the captive market is not completely covered. First assume that there is market partition. Then the price of the incumbent must correspond to the optimum on the monopolistic captive market. As $\tau < 1/3$ by assumption, this price is inconsistent with incomplete coverage. Thus any equilibrium with incomplete coverage in the captive market must have both firms competing actively in the switcher market. Profits would be $\Pi_0 = p_0 \left(\mu \left(\frac{2(1-p_0)+2\tau}{4\tau}\right) + (1-\mu)\left(\frac{p_1-p_0+2\tau}{4\tau}\right)\right)$. The entrant's profit would be $\Pi_1 = p_1(1-\mu)\left(\frac{p_0-p_1+2\tau}{4\tau}\right)$. This leads to an equilibrium candidate $p_0 = \frac{6\tau+4\mu-2\tau\mu}{5\mu+3}$ and $p_1 = \frac{6\tau+\mu+4\tau\mu}{5\mu+3}$. As $p_0 < 1-\tau$ for $\tau < 1/3$, there would be full coverage of the captive market, a contradiction. Third, there can be no equilibrium with full coverage in both markets and market partition in which, unlike in (iii), the marginal consumer earns positive surplus, as the incumbent could profitably raise prices. Thus, any equilibrium with full coverage and market partition must give zero surplus to the marginal captive consumers, as in (iii). This fixes both prices as as in (iii). Equilibria with full coverage, but without market partition, give zero surplus to marginal captive consumers as in (ii) or a positive surplus as in (iv). In the former case, the zero surplus requirement determines both prices as in (ii). In the latter case, the equilibrium conditions must necessarily hold for both firms.

Existence of Mixed-Strategy Equilibria

The region where none of the conditions (i)-(iii) holds is given by $\tau < \frac{1-\mu}{1+3\mu}$ and $\tau < \frac{9\mu(1-\mu)}{15\mu-8\mu^2+9}$. As the demand functions (8) and (9) (and thus the profit functions) are continuous, and strategy spaces are non-empty and compact, the existence of an MSE immediately follows from Theorem 3 in Dasgupta and Maskin (1986).[33]

[33] Recall that we can restrict price choices to $[0, 1+\tau]$. For homogeneous goods, $\tau = 0$, the result does not apply because the payoff functions are not continuous. However, it is possible to calculate an MSE directly.

A.2.3 Profits and Entry

Proposition 1 directly implies second-stage equilibrium profits and thus entry conditions:

Proposition 2. *The profits in the PSE regions are given as follows:*

(i) CFE: $\Pi_0 = \frac{\tau}{9}\frac{(3+\mu)^2}{1-\mu}$ *and* $\Pi_1 = \frac{\tau}{9}\frac{(3-\mu)^2}{1-\mu}$

(ii) LSE: $\Pi_0 = \frac{7\tau+\mu+t\mu-1}{3\tau}(1-\tau)$ *and* $\Pi_1 = \frac{1-\mu}{16}\frac{(\tau+1)^2}{t}$

(iii) MPE: $\Pi_0 = \mu(1-\tau)$ *and* $\Pi_1 = (1-\mu)(1-3\tau)$

Entry takes place in any of these regions if and only if $\Pi_1 = \frac{\tau}{9}\frac{(3-\mu)^2}{1-\mu} \geq F$.

Proof. Proposition 1 gives the equilibrium prices. Inserting these prices into the profit expressions using the demand functions (8) and (9) gives the results. ■

A.2.4 Consumer surplus

We now provide expressions for consumer surplus in the PSE of the discrete-choice model. They support the claims made in Section 4.2.1, in particular Corollary 2. Standard calculations yield the following results, noting that, in each of the three regions, the surplus of consumers of firm $i = 0, 1$ is given as $\frac{1}{2\tau}\left(\int_{1+(p_i-p_j)/2}^{1-\tau}(v_i - p_i)\right) dv_i$ for $j \neq i$.

Lemma 6. *Consumer surplus in each equilibrium region is given by the following terms.*

(i) In the LSE region,

$$CS = \mu\tau + (1-\mu)\frac{2\tau + 33\tau^2 + 1}{32\tau}$$

(ii) In the CFE region,

$$CS = \mu\frac{3 - 6\tau - 3\mu - \mu}{3(1-\mu)} + \frac{18 - 27\tau - 36\mu + 13\tau\mu + 18\mu^2 + 13\tau\mu^2}{18(1-\mu)}$$

(iii) In the MPE region,

$$CS = \mu\tau + (1-\mu)3\tau$$

In each case, the first term is the surplus of captive consumers, and the second one is the surplus of switchers. These sums are decreasing in μ, implying Corollary 2(i). Corollary 2(ii) follows because if, in CFE, an increase in μ affects market structure, then it induces entry, whereas it deters entry in the LSE and MPE regions. As the incumbent never charges more than the monopoly price in any of the three regions, and the entrant charges prices strictly below, the statement follows.

A.2.5 Appendix: Large Heterogeneity

We now describe the equilibrium with large heterogeneity ($\tau > 1/3$).

Proposition 3. *Suppose $\mu < 1$ and $1/3 < \tau \leq \frac{2\mu+3}{\mu+6}$. Then there exists a PSE where only the switcher market is fully covered. Prices are:*

$$p_0^* = \frac{2\left(3\tau + 2\mu - \tau\mu\right)}{5(\mu+3)}, \; p_1^* = \frac{2\left(3\tau + \mu + 2\tau\mu\right)}{5(\mu+3)}.$$

Proof. The incumbents profit is $p_0\left(\mu\left(\frac{2(1-p_0)+2\tau)}{4\tau}\right) + (1-\mu)\left(\frac{p_1-p_0+2\tau}{4\tau}\right)\right)$ in and near the equilibrium. Using his FOC and (10) yields p_0^* and p_1^* as equilibrium candidates. For $\tau < 1/3$, the captive market is not covered, and there is no profitable downward deviation to prices covering the market. $\tau \leq \frac{2\mu+3}{\mu+6}$ guarantees that, by contrast, the switcher market is covered, and it is not profitable to increase prices so that the switcher market is no longer covered.

■

A.3 Appendix: Taste-dependent share of captive consumers

We now formalize the statements made in Section 4.1.3, where we allowed the share of captive consumers to depend on consumer preferences. We consider equilibria where, as in the CFE, there are consumers who are indifferent between the products of both firms with valuation difference v^* in the interior of the support of G and obtain positive net valuation. We obtain the relevant demand functions as follows:

$$D^M(p_0;\tau,\mu) = \int_{-\infty}^{2(1-p_0)} g(\nu;\tau)\,d\nu$$

$$D_0(p_0,p_1;\tau,\mu) = \int_{-\infty}^{2(1-p_0)} L(\nu;\mu)\,g(\nu;\tau)\,d\nu + \int_{-\infty}^{p_1-p_0} (1-L(\nu;\mu))\,g(\nu;\tau)\,d\nu$$

$$D_1(p_1,p_0;\tau,\mu) = \int_{p_1-p_0}^{\infty} (1-L(\nu;\mu))\,g(\nu;\tau)\,d\nu$$

The first-order conditions for the two firms are then

$$\int_{-\infty}^{2(1-p_0)} L(\nu;\mu)\,g(\nu;\tau)\,d\nu + \int_{-\infty}^{p_1-p_0} (1-L(\nu;\mu))\,g(\nu;\tau)\,d\nu$$
$$-p_0\big((l(2(1-p_0);\mu)\,g(2(1-p_0);\tau) + (1-l(p_1-p_0;\mu))\,g(p_1-p_0;\tau))\big) = 0$$

$$\int_{p_1-p_0}^{\infty} (1-L(\nu;\mu))\,g(\nu;\tau)\,d\nu + p_1\left((1-l(p_1-p_0;\mu))\,g(p_1-p_0;\tau)\right) = 0$$

For this kind of equilibrium, we can provide a condition under which entry profits are increasing in the share of captive consumers. The result extends Lemma 2(i) to the context with preference-dependent shares of captive consumers.

Proposition 4. *Consider any equilibrium* (p_0^*, p_1^*) *where there exists* ν^* *who is indifferent* *between both products, and* $L(\nu;\mu) < 1$*. Then* Π_1 *is locally increasing in* μ *if and only if*

$$\frac{dp_0^*}{d\mu} > \frac{\int_{p_1^*-p_0^*}^{\infty} \frac{\partial L(\nu;\mu)}{\partial \mu} g(\nu;\tau)\, d\nu}{g(p_1^*-p_0^*;\tau)\left(1-L(p_1^*-p_0^*;\mu)\right)},$$

where all quantities are given at the equilibrium.

Proof. As in the benchmark model, we obtain that Π_1 is increasing in μ if and only if $\frac{dp_0^*}{d\mu} > \frac{-\frac{\partial \Pi_1}{\partial \mu}}{\frac{\partial \Pi_1}{\partial p_0^*}}$. For $\Pi_1 = p_1^* \int_{p_1^*-p_0^*}^{\infty} (1-L(\nu;\mu))\, g(\nu;\tau)\, d\nu$, this expression corresponds to the right-hand side of the inequality in the proposition. ■

Intuitively, the numerator on the right-hand side captures the demand (and thus profit) loss from increasing the share of captive consumers. The denominator captures the effect of an incumbent price increase on own profits that is higher the more consumers are indifferent and the higher the current share of switchers is. To illustrate the implications of the result, suppose for simplicity that, as in Section 3, G is uniform. Then we obtain:

Corollary 4. *Suppose that* G *is uniformly distributed on* $[2\tau, 2\tau]$*. Suppose further that*

$$\left.\frac{dp_0}{d\mu}\right|_{\mu=0} > \int_{-2\tau}^{0} \frac{\partial L(0;\mu)}{\partial \mu} d\nu.$$

Then Π_1 *is non-monotone in* μ*.*

Proof. For $\mu = 0$, $\Pi_1 > 0$. Further, for $\mu = 1$, $D_1 = 0$ and hence $\Pi_1 = 0$. Next note that, for the uniform distribution, $\frac{\int_{-2\tau}^{p_1-p_0} \frac{\partial L(\nu;\mu)}{\partial \mu} d\nu}{1-l(p_1-p_0;\mu)}$ is the right hand side of the inequality in Proposition 4. For $\mu = 0$, $p_1 = p_0$, so that the expression simplifies to $\int_{-2\tau}^{0} \frac{\partial L(0;\mu)}{\partial \mu} d\nu$, implying that Π_1 is locally increasing in μ at $\mu = 0$. Therefore, there must be a region where profits are decreasing in μ so that $\Pi_1 = 0$ at $\mu = 1$. ■

In the model of Section 2, $L(\nu;\mu) \equiv \mu$. Using Proposition 1 to calculate $\left.\frac{dp_0}{d\mu}\right|_{\mu=0}$ and $p_1 - p_0$ in the CFE region, the condition in the corollary becomes $\frac{8\tau}{3} > 2\tau$, which always holds. If instead $L(\nu;\mu) = \mu - b\nu$ for some $b > 0$, the right-hand side of the condition in Proposition 4 is unaffected as $\frac{\partial L(\nu;\mu)}{\partial \mu} = 1$ continues to hold. As in the proof of Lemma 1,

$$\frac{dp_i}{d\mu} = \frac{\frac{\partial^2 \Pi_i}{\partial p_i \partial p_j}\frac{\partial^2 \Pi_j}{\partial p_j \partial \mu} - \frac{\partial^2 \Pi_i}{\partial p_i \partial \mu}\frac{\partial^2 \Pi_j}{\partial p_j^2}}{\frac{\partial^2 \Pi_i}{\partial p_i^2}\frac{\partial^2 \Pi_j}{\partial p_j^2} - \frac{\partial^2 \Pi_i}{\partial p_i \partial p_j}\frac{\partial^2 \Pi_j}{\partial p_j \partial p_i}},$$

if profit functions are continuous in b, so is $\frac{dp_0}{d\mu}$. Thus, at least for small values of b, the condition of Proposition 4 still holds.