

# A Panel Data Approach for Spatial and Network Selection Models

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## 1 Introduction

### Motivation:

- Common features of economic data: **spatial/network pattern** (cross-sectional interdependence), non-randomly missing observations (**sample selection/treatment selection**), panel-data
- No model exists to deal with all three simultaneously!
- Neglecting selection and/or spatial/network correlation results in **biased** coefficient estimates!

	Cross Section	Panel
Non-Spatial/ Non-Network	Heckman (1976, 1979)	Wooldridge (1995)
Spatial/ Network	McMillen (1995), Flores-Lagunes, Schnier (2012), Doğan, Taşpinar (2017)	This paper!

### Example: Export-Wage Premium

- Empirical and theoretical evidence that **exporters pay higher wage/worker** than non-exporting firms (**treatment effect** of exporter status)
- Exporting decision as well as wage/worker depends on latent export profitability → **treatment ≠ random**
- Wages may have a **spatial pattern** due to local labor markets, commuting, etc. → Shocks to wages are **correlated across firms!**
- Profitability of exporting may have **network pattern** due to input/output linkages or industry affiliation → Shocks to profitability are **correlated across firms!**

### This paper:

Develop **two-step approach** towards selection on unobservables akin to Heckman (1976, 1979) and Wooldridge (1995) but for **panel-data with spatial or network interdependencies** in both the selection and the outcome equation.

## 2 Econometric Model

### Selection equation

- Fixed effects (Mundlak 1978, Wooldridge 1995)

$$y_{ti}^{A*} = x_{ti}^A \beta^A + e_{ti}^A, \quad \forall t = 1, \dots, T; i = 1, \dots, N,$$

with  $e_{ti}^A = \bar{x}_i^A \delta^A + \underbrace{\mu_i^A + \varepsilon_{ti}^A}_{=\xi_{ti}^A}$

$$y_{ti}^A = 1[y_{ti}^{A*} > 0],$$

- Panel SAR process (Kapoor, Kelejian, and Prucha, 2007)

$$\begin{aligned} e_{ti}^A &= \rho^A \sum_{j=1}^N w_{tj} e_{tj}^A + \bar{x}_i^A \delta^A + \xi_{ti}^A \\ e_{ti}^A &= \sum_{j=1}^N r_{tj}^A \bar{x}_j^A \delta^A + \underbrace{\sum_{j=1}^N r_{tj}^A \xi_{tj}^A}_{=u_{ti}^A}, \quad \text{using } R_t^A = (I_N - \rho^A W_t)^{-1} = (r_{tj}^A) \end{aligned}$$

$$\text{Selection equation restated: } y_{ti}^{A*} = x_{ti}^A \beta^A + \sum_{j=1}^N r_{tj}^A \bar{x}_j^A \delta^A + u_{ti}^A$$

### Outcome equation

- Fixed effects + panel SAR process
- Correct for selection bias by making use of joint normality assumption of spatial/network error components

#### Spatial/Network Sample Selection

$$E[y_{ti}^B | y_{ti}^A = 1, x^{A0}, x^B] = x_{ti}^B \beta^B + \sum_{j=1}^N r_{tj}^B \bar{x}_j^B \delta^B + E[u_{ti}^B | y_{ti}^A = 1, x^{A0}, x^B]$$

#### Spatial/Network Treatment Selection

$$E[y_{ti}^B | y_{ti}^A, x^{A0}, x^B] = \alpha y_{ti}^A + x_{ti}^B \beta^B + \sum_{j=1}^N r_{tj}^B \bar{x}_j^B \delta^B + E[u_{ti}^B | y_{ti}^A, x^{A0}, x^B]$$

- Spatially/network adjusted (generalized) Inverse Mills' Ratio (=Correction Function):

$$\begin{aligned} E[u_{ti}^B | y_{ti}^A = 1, x^{A0}, x^B] &= \frac{\sigma_{\xi^B A} \sum_{j=1}^N r_{tj}^B r_{tj}^A \phi(z_{ti})}{\sqrt{\sigma_{\xi^A}^2 \sum_j^N (r_{tj}^A)^2} \Phi(z_{ti})} = \tau \psi_{ti} \lambda_{ti} \\ E[u_{ti}^B | y_{ti}^A, x^{A0}, x^B] &= \frac{\sigma_{\xi^B A} \sum_{j=1}^N r_{tj}^B r_{tj}^A \phi(z_{ti})}{\sqrt{\sigma_{\xi^A}^2 \sum_j^N (r_{tj}^A)^2}} \frac{y_{ti}^A - \Phi(z_{ti})}{\Phi(z_{ti}) [1 - \Phi(z_{ti})]} = \tau \psi_{ti} \lambda_{ti}^g \end{aligned}$$

## 3 Estimation Strategy (Outline)

Step 1: Estimate selection equation using **Pooled Bayesian Spatial/Network Error Probit** model to obtain  $\hat{\theta}_A = \{\hat{\beta}^A, \hat{\delta}^A, \hat{\rho}^A\}$ , where  $\tilde{\beta}^A = \frac{\beta^A}{\sigma_{\xi^A}}$ ,  $\tilde{\delta}^A = \frac{\delta^A}{\sigma_{\xi^A}}$ .

Step 2: Use estimated parameters to construct spatially/network adjusted (generalized) Inverse Mills' Ratio.

Step 3: Add estimated spatially/network adjusted (generalized) Inverse Mills' Ratio in outcome equation and estimate using **Pooled Non-linear Least Squares** to obtain  $\hat{\theta}^B = \{\hat{\beta}^B, \hat{\delta}^B, \hat{\tau}, \hat{\rho}^B\}$ , or  $\hat{\theta}^B = \{\hat{\alpha}, \hat{\beta}^B, \hat{\delta}^B, \hat{\tau}, \hat{\rho}^B\}$ .

## 4 Monte Carlo Evidence (Results)

### Case 1: Medium Spatial/Network Correlation

		$\tilde{\beta}_1^A$	$\tilde{\beta}_2^A$	$\tilde{\delta}_1^A$	$\tilde{\delta}_2^A$	$\rho^A$	$\beta_1^B$	$\beta_2^B$	$\delta_1^B$	$\tau$	$\rho^B$
N=250	SNSS	True	0.707	0.707	0.707	0.707	0.5	1	3	0.707	0.5
	Mean	0.732	0.735	0.736	0.734	0.449	0.999	3.011	0.699	0.487	
	Bias	0.025	0.028	0.029	0.027	-0.051	-0.001	0.011	-0.009	-0.013	
	RMSE	0.091	0.084	0.198	0.185	0.131	0.095	0.192	0.173	0.109	
	WPS	Mean	0.653	0.670	0.681	0.708	0.964	3.038	0.679		
	Ignore spatial/ network correlation	Bias	-0.054	-0.037	-0.026	0.001	-0.036	0.038	-0.028		
	RMSE	0.096	0.081	0.140	0.134	0.102	0.192	0.189			
	NLLS	Mean					0.987	2.988		0.421	
	Ignore sample selection	Bias					-0.013	-0.012		-0.079	
	RMSE					0.095	0.189		0.149		
N=500	SNSS	Mean	0.716	0.719	0.721	0.711	0.482	0.999	3.001	0.702	0.497
	Bias	0.009	0.012	0.014	0.004	-0.018	-0.001	0.001	-0.005	-0.003	
	RMSE	0.055	0.060	0.134	0.140	0.074	0.061	0.141	0.127	0.061	
	WPS	Mean	0.660	0.657	0.686	0.671	0.982	3.222	0.808		
	Ignore spatial/ network correlation	Bias	-0.047	-0.050	-0.022	-0.036	-0.018	0.222	0.101		
	RMSE	0.069	0.074	0.098	0.109	0.064	0.264	0.170			
	NLLS	Mean					0.990	2.987		0.508	
	Ignore sample selection	Bias					-0.010	-0.013		0.008	
	RMSE					0.061	0.141		0.064		

### Case 2: No Spatial/Network Correlation

		$\tilde{\beta}_1^A$	$\tilde{\beta}_2^A$	$\tilde{\delta}_1^A$	$\tilde{\delta}_2^A$	$\rho^A$	$\alpha$	$\beta_1^B$	$\beta_2^B$	$\delta_1^B$	$\tau$	$\rho^B$
	True	0.707	0.707	0.707	0.707	0	1	1	3	0.707	0	
N=250	SNTS	Mean	0.739	0.740	0.743	0.744	-0.114	1.005	1.000	3.006	0.697	-0.009
	Bias	0.032	0.033	0.036	0.037	-0.114	0.005	0.000	0.006	-0.010	-0.009	
	RMSE	0.092	0.085	0.193	0.177	0.245	0.112	0.051	0.132	0.106	0.137	
	WPS	Mean	0.715	0.716	0.719	0.718	1.000	0.999	3.009	0.706		
	Ignore spatial/ network correlation	Bias	0.008	0.009	0.011	0.011	0.000	-0.001	0.009	-0.001		
	RMSE	0.082	0.075	0.143	0.140	0.112	0.050	0.130	0.106			
	NLLS	Mean					1.350	0.942	2.973		0.067	
	Ignore sample selection	Bias					0.350	-0.058	-0.027		0.067	
	RMSE					0.362	0.076	0.130		0.148		
N=500	SNTS	Mean	0.721	0.723	0.730	0.718	-0.066	1.003	0.999	3.002	0.701	-0.006
	Bias	0.014	0.016	0.023	0.011	-0.066	0.003	-0.001	0.002	-0.006	-0.006	
	RMSE	0.055	0.059	0.133	0.133	0.176	0.086	0.033	0.099	0.076	0.087	
	WPS	Mean	0.709	0.711	0.714	0.706	1.001	0.999	3.002	0.705		
	Ignore spatial/ network correlation	Bias	0.002	0.004	0.007	-0.001	0.001	-0.001	0.002	-0.002		
	RMSE	0.086	0.033									