A unified approach to comparative statics puzzles in experiments

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ABSTRACT: The paper shows that several game-theoretic solution concepts provide similar comparative statics predictions over a wide class of games. I start from the observation that, in many experiments, behavior is affected by parameter shifts that leave the Nash equilibrium unchanged. I explain the direction of change with a heuristic *structural approach*, using properties such as strategic complementarities and increasing differences. I show that the approach is consistent with general comparative statics results for (i) the Nash equilibrium of a game with perturbed payoff functions, (ii) the quantal response equilibrium, (iii) level-k reasoning. I also relate the structural approach to equilibrium selection concepts.

Keywords: comparative statics, supermodularity, strategic complementarity, quantal-response equilibrium, level-k reasoning.

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1   Introduction

Laboratory experiments have cast doubt on the predictive value of the Nash equilibrium and its refinements. At least the joint hypothesis that monetary payoffs are maximized and the Nash equilibrium is played is often in conflict with the facts.[1] Nevertheless, as argued by Samuelson (2005), even when point predictions do not hold, comparative statics predictions may still be borne out in the lab.[2] However, in an insightful contribution, Goeree and Holt (2001), henceforth GH, report the results of ten pairs of experiments where the Nash equilibrium is the same in both cases, but nevertheless subjects behave differently. Thus, not only the point predictions are wrong, but even the comparative statics implication that behavior should not be affected by the parameter change fails to hold.

This paper presents a unified explanation of the treatment effects in several GH puzzles, *without making any attempt to provide point predictions*.[3] I start from the simple observation that 6 of the 10 pairs of experiments analyzed by GH share important structural properties. First, for suitable partial orders on strategy spaces they are games with strategic complementarities (GSC): Both players' best responses are weakly increasing in the actions of the other player. Second, increasing differences (ID) holds: In one of the treatments (H), for each initial strategy profile, the incremental payoff from increasing the own action is weakly higher than in the other one (L). These

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[1] For instance, subjects only rely on iterated elimination of dominated strategies to a limited extent (Beard and Beil 1994). Deviations from the Nash prediction also occur in games where social preferences matter, including public goods games (Ledyard 1995), ultimatum games (Güth et al. 1982) and trust games (Fehr et al. 1993).

[2] Samuelson himself points out the limitations of his statement, mentioning bargaining experiments of Ochs and Roth (1989) where the effects of the discount factor and the length of the game are inconsistent with standard predictions.

[3] GH provide various possible explanations for some of the observed deviations from Nash behavior, for instance, social preferences, maximin behavior, and a noisy theory of introspection. Related papers explain similar phenomena using selection theories based on risk dominance and potential maximization (Anderson et al. 2001 and Goeree and Holt 2005).

two properties combined give a clear intuition why players are likely to choose higher actions in H than in L. First, because incremental payoffs are higher in H than in L, incentives to increase actions are higher in H for fixed behavior of the other player. Second, if players accordingly believe that the opponents will choose higher actions in game H, this reinforces the tendency to choose high actions by GSC. Based on these two structural properties of the game, it is therefore intuitive to predict that actions are weakly higher for H than for L, even though direct calculation of Nash equilibria predicts no change.

The direction of change in the six GH puzzles satisfying strategic complementarities (SC) and ID is always predicted correctly in this fashion. In addition, a similar structure-based prediction in another GH example that is not a GSC is confirmed by the data.[4] In the remaining three cases, this heuristic *structural approach* does not yield the wrong predictions. It is not applicable, because the games are too complex to allow for comparative statics results that are based purely on the structural properties of the game.

The very fact that the structural approach is intuitive and provides correct predictions for 7 out of 10 GH examples (and many other similar experiments) might be regarded as a sufficient justification for its use. Nevertheless, I offer several possible foundations. All these foundations build from a well-known *monotone comparative statics* results of Milgrom and Roberts (1990) and Vives (1990), according to which the Nash equilibrium of a parameterized GSC satisfying ID is weakly increasing in the parameter.[5] In addition, they provide reasons why the equilibrium might increase *strictly* in the parameter.

First, I suppose that players are not playing according to the monetary payoffs, but instead have payoffs resulting from a perturbation of monetary payoffs (for instance, because of social preferences). The perturbation does not have to be small, as long as it does not destroy the basic structural properties. Then, the Nash equilibrium of the perturbed game satisfies the

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[4]This example (generalized matching pennies) is not a GSC, but is simple because the parameter only enters the payoffs of one player.

[5]These results state more generally that the smallest and largest Nash equilibrium of such games are weakly increasing in the parameter.

weak comparative statics predicted by the structural approach.

Second, I consider the QRE of Mc Kelvey and Palfrey (1995), which does not presuppose that players choose best responses to the expected behavior of others, but allows for the possibility of errors. I show that it satisfies the same weak comparative statics as the Nash equilibrium: In supermodular games satisfying ID, if the parameter increases, the equilibrium weakly increases in the sense of first-order stochastic dominance if the error distribution is invariant under the parameter change. This is interesting because the QRE has frequently been used to explain analogous observations in repeated settings [6],[7]

Third, I consider level-k reasoning according to which some types of players (level-0 players) randomize, some types (level-1 players) best respond agains level-0 players, some (level-2 players) best respond against level-1 players, and so on.[8] Again, for supermodular games satisfying ID, level-k reasoning leads to action profiles that are weakly increasing in the parameter.

Finally, I show that, in symmetric games with ID and multiple parameter-independent equilibria, the comparative statics predictions implied by equilibrium selection via risk dominance or potential maximization are consistent with the approach proposed here.

To sum up, several different theoretical approaches can all rationalize existing comparative statics puzzles.[9] This obviously makes it hard to discriminate between these theories. However, it is good news in the sense that, for an important class of games, we can quite confidently predict the direction of treatment effects, because the predictions can be based on a wide

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[6]See Capra et al. (1999), Anderson et al. (2002), Goeree et al. (2003).

[7]Another promising approach to understanding the GH paradoxes was provided by Eichberger and Kelsey (2007) who appeal to ambiguity aversion to explain the deviations from equilibrium behavior.

[8]See, e.g., Stahl and Wilson (1995).

[9]In the working paper, I also discuss the relation to adjustment dynamics (see Milgrom and Roberts 1990, Vives 1990, Milgrom and Shannon 1994, Echenique 2002).

variety of different arguments.

While the results on the Nash equilibrium of the perturbed game and on equilibrium selection are very closely related to existing results, the comparative statics result for the QRE and for level-k reasoning are new and hopefully interesting in their own right: They provide comparative statics predictions for these concepts that hold independently of the calibration of the underlying parameters.

In Section 2, I will sketch three of the GH examples. In Section 3, I will introduce the structural approach as a heuristic. Sections 4, 5 and 6 relate the approach to the Nash equilibrium the QRE and level-k reasoning, respectively. Section 7 discusses the relation to selection theories. Section 8 concludes.

2   Introductory examples

I shall first sketch three of the ten GH examples.

(i) In the *Kreps game*, players choose actions from $X_1 = \{0, 1\}$ and $X_2 = \{0, 1, 2, 3\}$, respectively. Table 1 gives payoffs, where $\theta \in \mathbb{R}^+$.

	$x_2 = 0$	$x_2 = 1$	$x_2 = 2$	$x_2 = 3$
$x_1 = 0$	$200, 50$	$0, 45$	$10, 30$	$20, -250$
$x_1 = 1$	$0, -250$	$10, -100$	$30, 30$	$\theta + 50, \frac{6}{5}\theta + 40$

Table 1: Kreps Game

For all $\theta \in \Theta$, there are two pure Nash equilibria $((0,0)$ and $(1,3))$. In addition, there is a mixed-strategy equilibrium where player 1 chooses $x_1 = 0$ with probability $30/31$, and player 2 chooses $x_2 = 0$ with probability $1/21$ and $x_2 = 1$ with probability $20/21$. Thus, an increase of θ does not affect the equilibrium structure. However, GH report the following results. For $\theta = 0$, 32% of the subjects in the role of player 1 chose the high action 1; whereas 96% did so for $\theta = 300$. For $\theta = 0$, no subject in the role of player 2

chose $x_2 = 3$, but 84% did so for $\theta = 300$. Thus, the experimental evidence suggests that, as θ increases, more subjects choose high actions.

I am interested in this particular comparative statics observation of GH.[10] One could of course explain it with selection arguments, based for instance on payoff dominance. However, my goal is to find an explanation of treatment effects that also applies to games with unique parameter-independent Nash equilibria such as the following.

(ii) In the *Traveler's Dilemma*,[11] two players $i = 1, 2$ simultaneously choose integers $x_i \in \{180, ..., 300\}$. Each player is paid the minimum of the chosen numbers; in addition, the player with the lower number receives a transfer $R > 1$ from the player with the higher number. Therefore, defining $\theta = -R$,

$$\pi_i (x_i, x_j; \theta) = \min (x_i, x_j) + \theta \cdot sign (x_i - x_j). \tag{1}$$

The dots on the lines in Figure 1 give the reaction functions for any $\theta \in \Theta = (-\infty, -1)$. Thus, for all θ the game has a unique Nash equilibrium $x_1 = x_2 = 180$.[12] GH considered $\theta = -5$ and $\theta = -180$.[13] For $\theta = -180$, 80% of the subjects chose actions between 180 (the minimum) and 185, whereas 80% choose actions between 295 and 300 (the maximum) for $\theta = -5$. Thus, as in the *Kreps Game*, even though the Nash equilibrium is independent of θ, a parameter increase induces higher actions.

(iii) In the *common-interest proposal game* (GH, Figure 3), two players move sequentially, according to the game tree in Figure 2.[14] Thus, the strategy spaces are $X_1 = X_2 = \{0, 1\}$. The parameter space is $\Theta = (0, 60)$. For all $\theta \in \Theta$, the unique subgame perfect equilibrium is $x_1 = x_2 = 0$. GH considered $\theta = 0$ and $\theta = 58$. For $\theta = 0$, 84% of the subjects in the role

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[10]GH emphasize that for $\theta = 0$ many subjects (68%) choose $x_2 = 2$, the only action that is neither part of a pure-strategy equilibrium nor of a mixed-strategy equilibrium.

[11]The game goes back to Basu (1994).

[12]This equilibrium is also the unique rationalizable strategy profile.

[13]Similar results have been obtained by Capra et al. (1999) for other parametrizations.

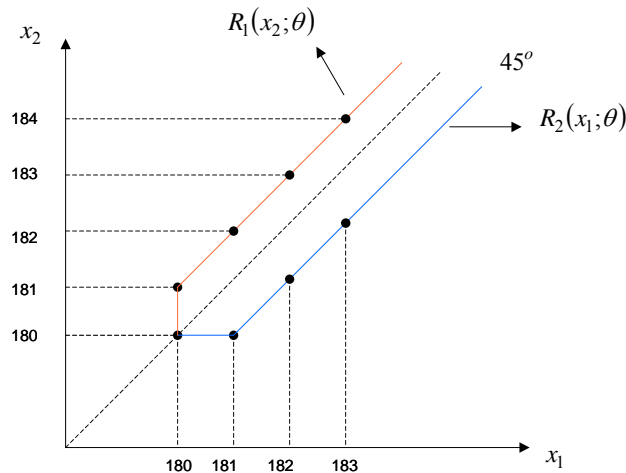[14]I use the name "common-interest proposal game", because $(0, 0)$ is the optimal outcome for both players.

Figure 1: Traveler's Dilemma

of player 1 and all the subjects in the role of player 2 chose the equilibrium actions $x_i = 0$. For $\theta = 58$, however, the corresponding figures are only 46% and 75% respectively. Hence, higher parameter values lead to higher actions.

Summing up, the following cases arise in the examples: (i) multiple pure-strategy equilibria, (ii) a unique pure-strategy equilibrium, or (iii) a unique subgame-perfect equilibrium. In all the examples, however, the set of pure-strategy equilibria is parameter-independent, but there are nevertheless clear treatment effects.

3   The structural approach

I will now introduce the heuristic *structural approach* to predict treatment effects even when the set of Nash equilibria is independent of treatments, as in the above examples. To repeat, the approach makes no attempt to explain why the observed play corresponds closely to the equilibrium in one case, but not in the other; it merely predicts the direction of change in behavior across treatments, not the relation to the equilibrium in any single experiment.
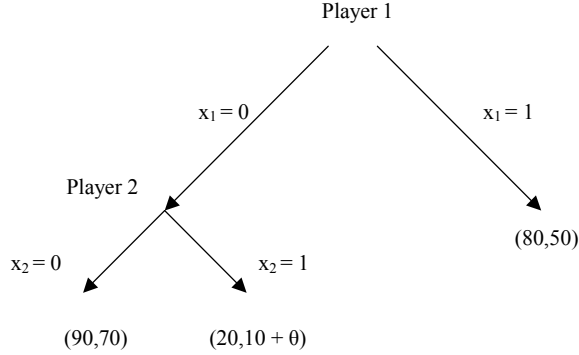
Figure 2: A Common-Interest Proposal Game

3.1 Defining the structural approach

In all the examples, there are players $i = 1, 2$, strategy spaces X_i and payoff functions $\pi_i(x_i, x_j, \theta)$, where $\theta \in \Theta$, a partially ordered set, such that:

1. X_i is independent of θ;

2. X_i is a finite set;[15]

3. X_i is equipped with a partial order $\geq$ that is independent of θ, with respect to which X_i forms a lattice.[16]

The following properties of the game are crucial.

Definition 1 *(i)* π_i *satifies increasing differences in* $(x_i; \theta)$,*(ID), if*

$$\Delta_i \left(x_i^H, x_i^L; x_j; \theta \right) \equiv \pi_i \left(x_i^H, x_j; \theta \right) - \pi_i \left(x_i^L, x_j; \theta \right)$$

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[15] This assumption can be weakened considerably at the cost of greater technicalities. For the purposes of interpreting the experimental evidence, the set-up is sufficiently general.

[16] A lattice requires that the infimum and supremum of each pair of elements exists in X_i. In the following, the lattice structure will typically come from a complete order on a finite set.

is weakly increasing in θ*, that is,* $\Delta_i\left(x_i^H, x_i^L; x_j; \theta^H\right) \geq \Delta_i\left(x_i^H, x_i^L; x_j; \theta^L\right)$
for all $x_i^H, x_i^L \in X_i$*,* $x_j \in X_j$*,* $\theta \in \Theta$ *(* $i = 1, 2$*,* $j \neq i$*) such that* $x_i^H > x_i^L$*,*
$\theta^H > \theta^L$*.*

(ii) π_i *is supermodular (SUP) if* $\Delta_i\left(x_i^H, x_i^L; x_j; \theta\right)$ *is weakly increasing in* x_j
for all $x_i^H, x_i^L \in X_i$*,* $x_j \in X_j$*,* $\theta \in \Theta$ *(* $i = 1, 2$*,* $j \neq i$*) such that* $x_i^H > x_i^L$*.*

By (i), an increase in θ has the direct effect of weakly increasing the incremental payoff for each player. Thus, for fixed behavior of the other player, increasing own actions becomes weakly more attractive, so that reaction functions are weakly increasing in θ.[17] By (ii), the payoff increase from increasing x_i is non-decreasing in x_j for $j \neq i$. Thus, the optimal response of player i is weakly increasing in x_j, that is, the game is a GSC. The positive direct effects of higher θ on x_i and the induced indirect effects on x_j are mutually reinforcing. Together, ID and SUP therefore suggest a (weakly) positive effect of θ on actions. This leads to the main hypothesis:

Hypothesis (Structural Approach): *When ID and SUP hold, the frequency distribution of observed play for* θ^H *weakly dominates the corresponding distribution for* $\theta^L < \theta^H$ *according to first-order stochastic dominance (FOSD).*[18]

An important implicit assumption in this heuristic approach is that player i's beliefs about the direction in which x_j changes with θ or x_i are fully determined by whether θ and x_i increase or decrease incremental payoffs Δ_j: Positive effects on Δ_j necessarily translate into expecting higher actions of player j. While this appears to be a fairly weak constraint on beliefs, it is still a constraint which may sometimes be implausible.[19]

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[17] A formal version of this statement relies on Lemma 1 in the Appendix.

[18] In a finite game, this states that, as θ increases, the fraction of players choosing an action up to and including any predetermined level of x_i weakly decreases.

[19] See the example in Section 8.

3.2 Experimental evidence

The first justification for such structure-based predictions is that they are confirmed in many examples. As an illustration, take the Kreps game. Straightforward derivations show that this game satisfies SUP and ID with respect to the standard (total) orders on X_1, X_2 and θ.[20] The structure-based prediction is thus that for $\theta^H = 300$ players tend to choose higher actions than for $\theta^L = 0$. This is precisely the observed outcome.[21]

This argument illustrates the central message of the paper: By ignoring details of the game and focusing instead on basic structural properties, one often obtains a weak prediction of treatment effects that is consistent with the evidence. The structural approach is a powerful tool for explaining comparative statics puzzles. For instance, the same logic can be applied to five other GH examples. Most immediately, the common-interest proposal game, the related *conflicting-interest proposal game*[22] and the *extended coordination game* also satisfy SUP and ID with respect to suitable parameters and partial orders.[23] In all three cases, like in the Kreps game, there are clear treatment effects, even though the equilibrium set is independent of θ.

Two other GH games, the traveler's dilemma and an auction game, are not supermodular, but nevertheless GSC. To illustrate, consider the traveler's dilemma. The game is not supermodular, because $\Delta_i \left(x_i^L + 1, x_i^L; x_j; \theta^H \right) = 0$ when $x_j \leq x_i^L - 1$, but $\Delta_i \left(x_i^H, x_i^L; x_j; \theta^H \right) = \theta < 0$ when $x_j = x_i^L$.[24] Because

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[20] As to ID, for both players, an increase in θ raises the benefit from choosing the highest action ($x_1 = 1$ and $x_2 = 3$) rather than any other one, whereas there is no relation between θ and the benefit for player 2 from increasing x_2 from 0 to 1 or 2, or from 1 to 2. As to SUP, for instance for player 1, the incremental payoffs increase from -200 to 10, 20 and finally $\theta + 30$ as player 2 increases his actions from 0 to 3.

[21] The overly strong independence prediction obtained by simple comparison of Nash equilibria for different parameter values is a boundary case of the structure-based prediction that the equilibrium is weakly increasing in θ.

[22] I use this term for the game described in Figure 4 of GH.

[23] Details of the arguments are available upon request.

[24] For all other constellations $\Delta_i \left(x_i^L + 1, x_i^L; x_j; \theta^H \right)$ is independent or increasing in x_j.

ID still holds,[25] a reduction in the transfer parameter R, or equivalently, an increase in θ, increases incremental payoffs. Hence, even though θ has no effect on the reaction function in the specific example, the game structure suggests that player i's reaction to x_j is weakly increasing in θ.[26] The traveler's dilemma corresponds to the boundary case where the reaction functions are unaffected by the parameter change even though ID holds. Ignoring all details of the game structure except ID and SC suggests that a parameter increase has the direct effect of increasing actions for both players, and that these effects are mutually reinforcing, so that actions should increase with θ, as required by the structure-based prediction.

For later reference, note that in the two sequential games, the supermodularity condition for player 2 is actually superfluous, because player 2 only acts for $x_1 = 0$: As θ increases, ID implies that the payoff increase of player 2 from higher actions increase. Because π_1 is supermodular, this increases the payoff increase for player 1 from higher choices.

Beyond the GH examples, many authors have investigated coordination games, which can be addressed similarly. Consider an *effort coordination game* with payoffs

$$\pi_i(x_i, x_j; \theta) = \min(x_i, x_j) + \theta \cdot x_i,$$

where $x_i \in \{0, 1, ..., M\}$ and $\theta = -c$ for some effort cost parameter $c \in (0, 1)$. For $c < 1$, the set of pure-strategy equilibria is the diagonal ($x_1 = x_2$). Thus, if one uses the set of pure-strategy equilibria to predict responses to parameter changes, increases in costs should have no effect on equilibrium effort. The comparative statics become more counter-intuitive if one allows for mixed-strategy equilibria. For instance, for $X_i = \{0, 1\}$, there is an equilibrium such that each player chooses $x_i = 1$ with probability c. Thus, as

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[25]To see this, first note that, because of the term $\min(x_i, x_j)$ in the payoff function, there is an incentive to choose high actions. The term $\theta \cdot sign(x_i - x_j)$ acts as a counterbalance, but less so as θ approaches zero from below. Therefore, the incremental payoff from increasing x_i is non-decreasing in θ.

[26]Again, Lemma 1 in the Appendix provides the formal justification of this argument.

costs increase, agents put more weight on the high effort level, so that, paradoxically, effort *increases* with costs. Unsurprisingly, experimental results (van Huyck et al. 1990; Goeree and Holt 2005) show that for lower c more subjects choose higher effort. The structural approach resolves the tension between theoretical predictions and empirical observations. Effort coordination games are supermodular, because the net benefit from increasing effort is $1 - c > 0$ if the original effort level is smaller than the effort of the other player, and $-c < 0$ otherwise. Also, π_i satisfies ID. Therefore, the structural prediction is that actions are weakly increasing in θ.[27]

Another application concerns public goods experiments (e.g., Ledyard 1995). For example, a typical two-player version would have

$$\pi_i\left(x_i, x_j; \theta\right) = p\left(z_i - x_i\right) + \frac{\theta}{2} \sum_{j=1}^{2} x_j,$$

where z_i, p and θ are positive constants and $\frac{\theta}{2} < p < \theta$, so that $x_1 = x_2 = 0$ is the only Nash equilibrium and is inefficient. Nevertheless, increases in θ typically lead to higher choices.[28] The games satisfy ID, and SUP holds trivially because payoffs are additively separable.[29]

The very fact that the predictions of the structural approach are consistent with the experimental evidence is a strong argument in its favor. In

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[27] Several authors have analyzed the effects of changing various parameters in other 2×2-coordination games satisfying (SUP) and (ID). For instance, in the experiments of Huettel and Lockhead (2000), Schmidt et al. (2003), and most of the experiments of Guyer and Rapoport (1972), the comparative-statics predictions correspond exactly to those obtained from the structural approach, and the arguments are similar as in the following discussion of effort coordination games. The propositions of this paper are not applicable for the "Benefit-to-other"-treatment of Guyer and Rapoport, because (ID) does not hold.

[28] z_i is the endowment of player i; x_i is interpreted as a contribution to a public good; higher θ corresponds to an increase in the return on investment in the public good.

[29] Another example is provided by the imperfect price competition game with inelastic demand analyzed by Capra et al. (2002). In this game, subjects earn exactly the price they set if it is the minimum of the two prices; otherwise they earn only a fraction. The equilibrium is independent of the fraction, but observed play is increasing. The game can be shown to satisfy (SUP) and (ID), so that the structural approach applies.

addition, the intuition for this observation is straightforward. Even subjects who, for whatever reason, do not display Nash behavior, are likely to understand the two basic structural properties: (i) High incremental payoffs make high actions attractive for given actions of the other player; (ii) incremental payoffs increase with the other player's action. If players understand these two properties, and if they believe that other players do so, too, then they should choose high actions for high parameter values.

Before turning to more precise justifications of the structural approach, I note that a slight modification of the idea can be used to explain a seventh GH example, the *generalized matching pennies* game, along similar lines, even though it is not a GSC (See Appendix 2).[30]

As summarized in Table 2, the structural approach can thus explain the evidence in seven of the ten examples provided by GH. In the remaining cases, it does not provide a false prediction. It is simply not applicable because the games do not have suitable structural properties. Loosely speaking, the direct and indirect effects of parameter changes are not mutually reinforcing, so that general comparative statics results cannot be derived.

4   Nash equilibria of perturbed games

Many approaches to explaining deviations from Nash equilibria rely on the idea that actual payoffs differ from monetary payoffs, for instance, because players have social preferences. Specifically, suppose that instead of the mon-

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[30]In this example, with an appropriate order on strategy spaces, a higher parameter increases the equilibrium action of one player, but leaves the action of the other player constant; while observed actions of both players are affected. Any order on the strategy space implies that the actions are SC for one player, but strategic substitutes (SS) for the other one. However, because the parameter only affects one of the two payoff functions, an intuitive structure-based prediction of treatment can be given even so, and this intuitive prediction can be justified as in the GSC case.

Nash Prediction	Game	Observed Actions	Reason
Unique pure Nash equilibrium independent of θ	Traveler's dilemma (Capra et al. 1999, GH) Public goods games (Ledyard 1995)	Increasing in θ	SC + ID SUP + ID
Unique SPE independent of θ	Proposal games (GH Fig. 3 and 4)	Increasing in θ	SUP + ID
Unique mixed equilibrium: increasing in θ for player 2, constant for player 1	Matching pennies (Ochs 1995, GH)	Player 2: increasing Player 1: decreasing	SC/SS ID
Unique Bayesian Equilibrium independent of θ	Auction game (GH)	Increasing in θ	SC + ID
Multiple pure equilibria; mixed equilibrium is independent of θ	Kreps game (GH) Extended coordination game (GH)	Increasing in θ	SUP + ID
Multiple pure Nash equilibria where mixed equilibrium is decreasing in θ	Effort coordination (Van Huyck et al. 1990, Goeree and Holt 2005) Wolf's dilemma (Huettel-Lockhead 2000)	Increasing in θ	SUP + ID
Period-2 equilibrium independent of first-period play	Capacity game (Brandts et al. 2003)	Period-2 actions increasing in own period-1 action, decreasing in opponent's.	SUP + ID

Table 2: Summary of Results

etary payoff functions π_i, players have objective functions as follows:

$$\widehat{\pi}_i (x_i, x_j; \theta) = \pi_i (x_i, x_j; \theta) + g_i (x_i, x_j; \theta) \qquad (2)$$
$$\pi_i \text{ and } \widehat{\pi}_i \text{ satisfy } SUP \text{ and } ID$$

Then we obtain:

Remark 1: *If the game with payoff functions $\widehat{\pi}_i$ as in (2) has a unique Nash equilibrium before and after the parameter increase, this equilibrium is weakly increasing in θ.*

This is an immediate implication of a more general comparative statics result of Milgrom and Roberts (1990). The intuition is similar to the one given for the structural approach: As θ increases, the incremental benefits from higher actions increase weakly for both players, and these effects are mutually reinforcing. This is true for the perturbed as well as for the unperturbed objective functions.

Of course, to obtain point predictions and to show that the equilibrium is strictly increasing, the perturbation would have to be specified. To understand weak comparative statics, however, there is no need to do so. As long as both actual payoffs $\widehat{\pi}_i$ and monetary payoffs π_i satisfy SUP and ID, the weak comparative statics conclusions for π_i translates to $\widehat{\pi}_i$. However, only suitable perturbations guarantee that the equilibrium for $\widehat{\pi}_i$ is increasing in θ for suitable parameters, rather than merely non-decreasing.

To illustrate the idea, first consider the game with payoffs π_i given by the following matrix, where $\theta \in \{0, 1\}$.

	$x_2 = 0$	$x_2 = 1$
$x_1 = 0$	$10, 10 - 9\theta$	$8 + \theta, 8 - 4\theta$
$x_1 = 1$	$8 + \theta, 8 - 4\theta$	$7 + \theta, 9 - \theta$

Table 3: Payoffs in the Perturbed Effort Coordination Game Game

Clearly, π_i satisfies ID and SUP, and the unique Nash equilibrium is $(0, 0)$, independent of θ. Now suppose players are altruistic, with payoff functions

$\pi_i + 0.5\pi_j$. It is straightforward to show that the perturbation $g_i = 0.5\pi_j$ satisfies SUP and ID for $i = 1, 2$; $j \neq i$, and that the equilibrium for the game with payoff functions $\widehat{\pi}_i = \pi_i + 0.5\pi_j$ is increasing in θ, namely $(0,0)$ for $\theta = 0$ and $(1,1)$ for $\theta = 1$. Thus, while the structural approach predicts weakly increasing actions both for the unperturbed and for the perturbed game, actions are increasing only in the perturbed game.

Next, consider the common-interest proposal game (Figure 2). Suppose that player 2 cares about the difference between his payoff and the payoff of player 1. Thus, we introduce a perturbation term $g_2(x_2, x_1) = k(\pi_2(x_2, x_1) - \pi_1(x_1, x_2))$, $k > 0$. Hence, $g_2(0, 1) = g_2(1, 1) = -30k$; $g_2(0, 0) = -20k$; $g_2(1, 0) = k(\theta - 10)$. Clearly, g_2 satisfies ID and $g_1 \equiv 0$ trivially satisfies SUP. By the arguments in Section 3.2 for the sequential GH examples, this is sufficient to apply the structural approach, which predicts that the equilibria of the modified game are still weakly increasing in θ between 0 and 60. Direct calculation of the SPE of the modified game shows that player 2 chooses $x_2 = 1$ if and only if $\theta \geq \frac{60-10k}{1+k}$. Hence, for $k < 6$, the best response of player 2 jumps upwards at some critical level of θ. Anticipating this, player 1 chooses $x_1 = 1$ if and only if $\theta \geq \frac{60-10k}{1+k}$. Thus, there is a positive effect of suitable parameter changes on the equilibrium outcome.

5 The quantal response equilibrium

The *quantal response equilibrium* (QRE) introduced by Mc Kelvey and Palfrey (1995) does not presuppose best responses; instead players can make errors. Consider a finite game with strategy spaces $X_i = \{x_i^0, ..., x_i^{N_i}\}$. Denote the probabilities with which player i chooses action x_i as $p_{x_i}^i$. Let $\boldsymbol{\varepsilon}_i = (\varepsilon_{i1}, ..., \varepsilon_{iN_i})$ be a vector of perturbations for player i, drawn from a joint density f_i. Then, by assumption, player i chooses $\nu \in X_i$ if and only if ν maximizes the sum of the expected payoff and the perturbation, that is,

$$\sum_{x_j \in X_j} p_{x_j}^j \pi_i(\nu, x_j; \theta) + \varepsilon_{i\nu} \geq \sum_{x_j \in X_j} p_{x_j}^j \pi_i(x_i, x_j; \theta) + \varepsilon_{ix_i} \ \forall x_i \neq \nu. \quad (3)$$

Using this condition, one immediately arrives at the *stochastic best-response function* or *quantal response function* that assigns to each probability vector $\mathbf{p}^j$ for player j the probability vector $\mathbf{p}^i = \mathbf{p}^i(\mathbf{p}^j; \theta)$ of choices for player i defined by the requirement that $\boldsymbol{\varepsilon}_i$ satisfies (3). A QRE requires that each player's own error distribution is consistent with stochastic best response.

The next comparative statics result shows that the similarity in the predictions of the structural approach and the QRE is not a coincidence.

Proposition 1 *Suppose a finite game satisfies SUP and ID. Suppose that, for a fixed error distribution, a unique QRE $\mathbf{p}(\theta) = (\mathbf{p}^1(\theta), \mathbf{p}^2(\theta))$ exists for every θ. Then, an increase in θ shifts the equilibrium distribution $\mathbf{p}(\theta) = (\mathbf{p}^1(\theta), \mathbf{p}^2(\theta))$ weakly according to first-order stochastic dominance (FOSD).*[31]

Thus, a parameter increase in a game satisfying SUP and ID implies that higher choices become more likely for the QRE. The intuition for the result is quite similar to the intuition for Remark 1. As the parameter θ shifts upwards, increasing differences imply that, for fixed behavior of the opponent, players are more likely to respond with higher actions. Anticipating this, it becomes even more attractive for both players to increase their actions.[32]

Again, the interesting aspect of Proposition 1 is not that the comparative statics property holds for some QRE, but that it holds generally. Haile et al. (2008) have argued that, because of the degrees of freedom in specifying the error distributions, the QRE can explain any behavior in any given game. Nevertheless, as the authors themselves point out, the QRE may still put restrictions on possible comparative statics under the invariance assumption.[33]

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[31] The uniqueness property may not always be obvious to show, but uniqueness results exist, for instance, for the logit equilibrium (e.g., Anderson et al. 2001).

[32] The underlying invariance assumption for the error distribution is also used by Haile et al. (2008) and discussed there.

[33] In their Theorem 2, Haile et al. (2008) follow a different approach: They ask whether the behavior observed ex-post is consistent with agents putting greater weight on actions that, given the observed distribution of play, have become more attractive than others after the parameter increase.

Proposition 1 confirms this idea by giving comparative statics predictions purely on the basis of structural properties that are known ex ante.

A final remark concerns the invariance of the error distribution. It is clear that restrictions on beliefs are needed to generate the conclusion of Proposition 1: If for some reason players expect a higher value of θ to induce players to systematically make more errors where they choose low actions, then the result will no longer hold.[34] Conversely, however, if beliefs change in such a way that players are expected to make more errors where they choose high actions when θ increases, then the result should be reinforced.

6 Level-k reasoning

Next, we consider level-k reasoning. Let strategy spaces be $X_i = \left\{x_i^0, ..., x_i^{N_i}\right\}$. Suppose that, for each player $i \in \{1, 2\}$, there are different types, so-called level-k players, $k = 1, ..., K$, $K \leq \infty$. Suppose player i is of level k with some exogenous probability λ_i^k. Suppose further that all level-0 players choose $x_i \in X_i$ randomly according to some exogenous distribution $p^0(x_i, \theta)$. Level-k players choose $x_i \in X_i$ so as to maximize $\sum_{x_j \epsilon X_j} \pi_i(x_i, x_j, \theta) p^{k-1}(x_j; \theta)$.[35]

Proposition 2 *Suppose SUP and ID hold, and θ shifts p_i^0 according to weak FOSD. Then the expected action profile under level-k reasoning is weakly increasing in θ in the FOSD-sense.*

Usually, it is assumed that the actions of level-0 players are uniformly distributed. Our result not only holds for arbitrary distributions that are independent of θ, but even under the weaker condition that θ shifts the distribution upwards in the FOSD-sense. Intuitively, as long as there is no effect of θ on the distribution of level-0 players, level-1 players benefit from choosing higher actions as θ increases, because the incremental payoff

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[34]See the example at the end of Section 8.

[35]Level-k thinking was introduced by Stahl and Wilson (1995) and Nagel (1995); see also Crawford (2007).

is higher. If there is a positive effect of θ on the distribution of level-0 players, supermodularity reinforces the effect. Hence, as θ increases, level-1 players tend to choose higher actions. By inductive reasoning, higher level players choose higher actions.

Summing up, as for the Nash equilibrium and the QRE, level-k reasoning allows clear comparative statics predictions that are based only on ID and SUP.

7   Equilibrium Selection

As I show in more detail in the working paper, the structural approach fits nicely with selection criteria such as risk dominance (Harsanyi and Selten 1988). There, I consider *symmetric games* with $X_i = \{0, 1\}$ with pure-strategy equilibria $(0, 0)$ and $(1, 1)$.[36] I show that the comparative statics implied by the structural approach and by risk dominance coincide: If ID holds, risk dominance also tends to predict higher equilibria as θ increases. This extends a similar result of Goeree and Holt (2005) for effort games.

An alternative approach to equilibrium selection that generalizes to games with more than two players and continuous actions is available for potential games (Monderer and Shapley 1996, Goeree and Holt 2005). Such games are characterized by the existence of a potential $V(x_1, x_2; \theta)$ with the defining property that $\pi_1(x_1'', x_2; \theta) - \pi_1(x_1', x_2; \theta) = V(x_1'', x_2; \theta) - V(x_1', x_2; \theta)$ for all $x_1', x_1'' \in X_1$, $x_2 \in X_2$, $\theta \in \Theta$, and analogously for π_2.[37] Potential-maximizing strategy profiles are pure-strategy equilibria, but the converse is not necessarily true (Monderer and Shapley 1996): In games with multiple equilibria such as effort coordination games, there is typically a unique potential-maximizing profile which can be used for equilibrium selection. Monderer and Shapley

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[36] $(0, 0)$ is *risk dominant* if both players prefer 0 if they expect the other player to choose 0 and 1 with probability $1/2$ each.

[37] With continuously differentiable games, this boils down to the requirement that the partial derivatives of V with respect to each x_i coincide with those of $\pi_i(x_i, x_j; \theta)$.

(1996) have already argued that, in effort games, the observed effects of increasing costs can be explained using potential maximization, showing that, in the experiments of van Huyck et al. (1990), potential maximization selects the lowest equilibrium for high effort costs and the highest equilibrium for low effort costs. The working paper shows more generally that, when ID holds, potential maximization tends to select higher equilibria.

Summing up, the *structural approach* yields comparative statics predictions that are compatible with standard selection methods where they apply.

8   Conclusions

I have introduced a heuristic "structural" approach to predict treatment effects when Nash equilibria are the same in the different treatments. The resulting comparative statics predictions are supported by the experimental observations in all cases that I am aware of, in particular, in the GH examples. I have shown that the structural approach is consistent with the predictions of the QRE, level-k thinking and equilibrium selection theories.

The paper contributes to the literature as follows. First, it brings together two literatures that rarely speak to each other, namely the experimental literature and the literature on monotone comparative statics in games with strategic complementarities. Hopefully, this exercise contains potential for further cross-fertilization.[38] Second, the structural approach is more basic than the alternative suggestions: Without imposing a particular story about what subjects do for any given parameter value, it shows that structural properties of the game are useful to explain treatment effects. Third, I provide a unified explanation of seven of the ten GH examples which, to my knowledge, no other single approach does. Finally, though this was not detailed here, the approach can be applied to problems that do not concern comparative statics

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[38] A vaguely related experimental contribution of Chen and Gazzale (2004) demonstrates that learning in certain games with strategic complementarities, namely supermodular games, works particularly well. However, the authors do not treat comparative statics.

directly. For instance, Brandts et al. (2007) consider a two-stage game of capacity choice where the structural approach correctly predicts the effects of (endogenous) capacity choices on second-period actions.[39]

In spite of the large number of conceivable applications, it is important to recognize the limitations of the approach. First, obviously, it does not provide point predictions. Second, there are examples where the direct and indirect effects of parameter changes are not mutually reinforcing, so that no comparative statics predictions are possible without relying on the concrete specification. Third, I am convinced that cleverly designed experiments can show that there are some GSC satisfying ID, for which the observed actions are not increasing in the parameter. The challenge for future experimental work is to discover under which circumstances such violations of the structural approach will occur. A suggestion in this direction is the following. For $\theta \geq 0$, consider an asymmetric coordination game as follows:

	$x_2 = 0$	$x_2 = 1$
$x_1 = 0$	$5, 5$	$0, 0$
$x_1 = 1$	$0, 0$	$5.1, 5.1 + \theta$

Table 4: Payoffs in the Perturbed Effort Coordination Game

Clearly, this parameterized game satisfies SUP and ID, so that the structural approach would predict (weakly) higher actions as θ increases. But it is quite conceivable that, for large values of θ, players would avoid this action and believe that others do so to, either because of inequity aversion or simply because of the focal nature of the symmetric equilibrium.

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[39]For details, I refer the reader to the working paper.

9 Appendix

9.1 Appendix 1: Proofs

The following well-known monotone comparative statics (Topkis 1978) result will be helpful.

Lemma 1 *Let $f((x, \tau)$ be a real-valued function defined on $X \times T$, where X is a complete lattice and T is a partially ordered set. Suppose f satisfies increasing differences with respect to (x, τ). Then $g(\tau) \equiv \arg \max_{x \in X} f((x, \tau)$ is a weakly increasing correspondence.*[40]

In the following applications, X will correspond to the strategy set of one player; τ will be the strategy set of the other player or the parameter θ.

9.1.1 Proof of Proposition 1

Proposition 1 will be shown to be a simple corollary of the following result.

Lemma 2 *Suppose $\pi_i(x_i, x_j; \theta)$ satisfies SUP and ID. Then*
(i) For fixed choice probabilities of the opponent, $\mathbf{p}^j$, an increase in θ shifts the stochastic best response $\mathbf{p}^i(\mathbf{p}^j; \theta)$ according to FOSD.
(ii) The stochastic best response $\mathbf{p}^i(\mathbf{p}^j; \theta)$ is weakly increasing in $\mathbf{p}^j$.

Proof. (i) For $\nu \in \left\{ x_i^0, ..., x_i^{N_i} \right\}$, the probability that $x_i \leq \nu$ is chosen is

$$P_\nu(\theta) = prob \left(\max_{x_i \in X_i} \sum_{x_j \in X_j} p_{x_j}^j \pi_i(x_i, x_j; \theta) + \varepsilon_{ix_i} > \sum_{x_j \in X_j} p_{x_j}^j \pi_i(x_i', x_j; \theta) + \varepsilon_{ix_i'} \right)$$
$$\forall x_i' > \nu. \quad (4)$$

By ID,

$$\sum_{x_j \in X_j} p_{x_j}^j \pi_i(x_i, x_j; \theta) - \sum_{x_j \in X_j} p_{x_j}^j \pi_i(x_i', x_j; \theta)$$

---

[40] $g(\tau)$ is weakly increasing if $\tau^L < \tau^H$ implies $\min g(\tau^L) \leq \min g(\tau^H)$ and $\max g(\tau^L) \leq \max g(\tau^H)$, where the inequalities on X refer to some arbitrary partial order.

is weakly decreasing in θ for all $x_i' > \nu$. Because $\varepsilon_{ix_i'} - \varepsilon_{ix_i}$ is independent of θ by the invariance property, $P_\nu(\theta)$ is therefore weakly decreasing in θ.

(ii) Suppose $r < s \in \left\{ x_j^0, ..., x_j^{N_j} \right\}$. It suffices to show that replacing any $\mathbf{p}^j$ by $\mathbf{p}^{j\varepsilon} \equiv \left(\mathbf{p}_0^j, ..., \mathbf{p}_r^j - \varepsilon, ..., \mathbf{p}_s^j + \varepsilon, ..., \mathbf{p}_{n_j}^j \right)$ for $\varepsilon \in (0, \mathbf{p}_r^j]$ leads to an FOSD-shift in $\mathbf{p}^i$. This will be true if $P_\nu(\theta)$ is weakly larger for $\varepsilon = 0$ than for $\varepsilon > 0$ for all $\nu \in X_i$. This holds, because SUP implies

$$
\sum_{x_j \in X_j} p_{x_j}^{j\varepsilon} \pi_i (x_i, x_j; \theta) - \sum_{x_j \in X_j} p_{x_j}^{j\varepsilon} \pi_i (x_i', x_j; \theta) -
$$
$$
\sum_{x_j \in X_j} p_{x_j}^{j} \pi_i (x_i, x_j; \theta) + \sum_{x_j \in X_j} p_{x_j}^{j} \pi_i (x_i', x_j; \theta) = \tag{5}
$$
$$
\varepsilon \left(\pi_i (x_i, s; \theta) - \pi_i (x_i', s; \theta) - \pi_i (x_i, r; \theta) + \pi_i (x_i', r; \theta) \right) \leq 0. \tag{6}
$$

∎

To show that this implies Proposition 1, first note that, with FOSD as a partial order, $\mathcal{P}_i$ the set of distributions on X_i is a complete lattice (Echenique 2003, Lemma 1); this structure carries over to $\mathcal{P} = \mathcal{P}_i \times \mathcal{P}_j$. Further, by Lemma 2, the stochastic best response correspondence shifts out as θ increases. Denote the interval of probability vectors in $\mathcal{P}$ that are greater or equal to some $\mathbf{p}$ as $U(\mathbf{p})$. Since the best-response correspondence for $\theta^H > \theta^L$ is weakly increasing by part (ii) of the lemma, it maps $U(\mathbf{p}(\theta^L))$ into itself. Its fixed point must therefore satisfy $\mathbf{p}(\theta^H) \geq \mathbf{p}(\theta^L)$.

9.1.2   Proof of Proposition 2

For $k = 0$, the result corresponds to the assumption that θ shifts p_i^0 weakly according to FOSD. Lemma 1 implies that, to complete the induction, it suffices to show that $\sum_{x_j \in X_j} \pi_i(x_i, x_j, \theta) p^{k-1}(x_j; \theta)$, the expected payoff of a level-k player who assumes that player j is level $k-1$, satisfies increasing difference in $(x_i; \theta)$, provided the FOSD statement holds for level $k-1$. ID

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(increasing differences for π_i in (x_i, θ)) implies

$$\sum_{x_j \epsilon X_j} \left(\pi_i(x_i^H, x_j, \theta^H) - \pi_i(x_i^L, x_j, \theta^H) \right) p^{k-1}(x_j; \theta^L) \geq \qquad (7)$$

$$\sum_{x_j \epsilon X_j} \left(\pi_i(x_i^H, x_j, \theta^L) - \pi_i(x_i^L, x_j, \theta^L) \right) p^{k-1}(x_j; \theta^L)$$

for $\theta^H \geq \theta^L$. Because $p^{k-1}(x_j; \theta))$ satisfies FOSD in θ, $p^{k-1}(x_j; \theta^H)$ can be obtained from $p^{k-1}(x_j; \theta^L)$ by shifting mass $\varepsilon \geq 0$ from some value $x_j^{n_j'}$ to some $x_j^{n_j''}$ where $n_j'' > n_j'$, and iterating this procedure finitely many times. It thus suffices to show that

$$\sum_{x_j \epsilon X_j} \left(\pi_i(x_i^H, x_j, \theta^H) - \pi_i(x_i^L, x_j, \theta^H) \right) p^{k-1}(x_j; \theta^L) + \qquad (8)$$

$$\left(\pi_i(x_i^H, x_j^{n_j''}, \theta^H) - \pi_i(x_i^L, x_j^{n_j''}, \theta^H) - \left(\pi_i(x_i^H, x_j^{n_j'}, \theta^H) - \pi_i(x_i^L, x_j^{n_j'}, \theta^H) \right) \right) \varepsilon \geq$$

$$\qquad (9)$$

$$\sum_{x_j \epsilon X_j} \left(\pi_i(x_i^H, x_j, \theta^H) - \pi_i(x_i^L, x_j, \theta^H) \right) p^{k-1}(x_j; \theta^L)$$

This follows immediately from SUP.

9.2   Appendix 2: Generalized matching pennies

Even for asymmetric games that do not satisfy SC, the structural approach is sometimes useful. A case in point is *generalized matching pennies*, with $X_i = \{0, 1\}$ and $\Theta = \{44, 80, 320\}$ and payoffs as in Table 5.

	$x_2 = 0$	$x_2 = 1$
$x_1 = 0$	$\theta, 40$	$40, 80$
$x_1 = 1$	$40, 80$	$80, 40$

Table 5: Payoffs in the Generalized Matching Pennies Game

Identify a mixed strategy of player i, σ_i, with the probability of choosing action 1. For all $\theta \in \Theta = \{44, 80, 320\}$, the reaction correspondence for player 2 is given by the same dashed line $R_2(\sigma_1; \theta)$ in Figure 3, while it depends explicitly on θ for player 1. The unique mixed-strategy equilibrium
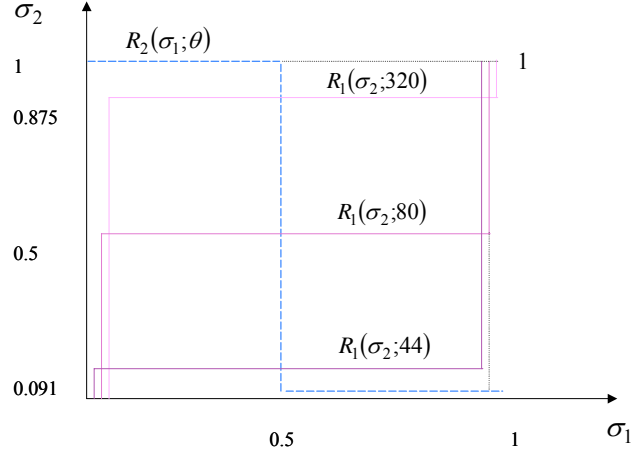
Figure 3: Generalized Matching Pennies

is $\sigma_1^* = \frac{1}{2}$, $\sigma_2^* = 1 - \frac{40}{\theta}$. Thus, unlike in the earlier examples, only player 1's equilibrium action is independent of θ: Player 2's choice x_2 is increasing in θ, as the probability with which $x_2 = 1$ is played increases in θ. As θ increases from 44 to 80 and 320, the percentage of subjects in the role of player 1 choosing the high action decreases from 92% to 52% and then to 4%, whereas the corresponding values for player 2 increase from 20% to 52% and then to 84%. Thus, contrary to the prediction of the mixed-strategy equilibrium *both* players' actions change as θ does.

To understand this, note that the game has the following properties:

(SUB$_2$) $\pi_2(x_2, x_1; \theta)$ is submodular in (x_2, x_1).[41]

(DD$_1$) $\pi_1(x_1, x_2; \theta)$ satisfies decreasing differences in (x_1, θ).[42]

(IND$_2$) $\pi_2(x_2, x_1; \theta)$ is independent of θ

Intuitively, by (DD$_1$), a direct effect of the increase in θ is that player 1's action should decrease. By SUB$_2$, this makes player 2 want to increase

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[41]This means that $\Delta_2\left(x_2^H, x_2^L; x_1; \theta\right)$ is weakly decreasing in x_1.

[42]This means that $\Delta_1\left(x_1^H, x_1^L; x_2; \theta\right)$ is weakly decreasing in θ.

his action. In the working paper, I provide a comparative statics result for games with the structural properties of generalized matching pennies that confirms these predictions for the Nash equilibrium.

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