An Economic Analysis of Takings

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This article identifies economically efficient rules for governing compensation when the state takes private property. Despite a variety of informational and behavioral assumptions, a basic principle emerges: a fully efficient rule entails compensation based on the gains society enjoys from the taking (either its actual gains or its expected gains). Moreover, in many takings situations this principle can be implemented in more than one way, providing society some flexibility with which to achieve its other goals without sacrificing economic efficiency.

1. Introduction

The state has long had the authority to deprive a citizen of the benefit of his or her private property. It has, for instance, a right of eminent domain, which entitles it to take physical possession of private property and put it to public use (e.g., putting a highway where a farm once stood). Similarly, it has a right to regulate, which entitles it to restrict what citizens do with their property (e.g., causing a brewer to shut her brewery by prohibiting the manufacture of alcoholic beverages). Such actions have come to be known as *takings*. Integrally connected to the state's authority to "take" has been an obligation, in certain instances, to compensate the citizen. Despite a long judicial history and much scholarly analysis,[1] questions remain about when the state should be required to pay compensation and how much it should pay. This article addresses these questions, with a focus on the following:

(i) Previous work (see, e.g., Kaplow, 1986) has established that regulatory

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1. A *partial* list of previous scholarship includes Blume, Rubinfeld, and Shapiro (1984), Blume and Rubinfeld (1984), Farber (1992), Fischel and Shapiro (1989), Kaplow (1986), Michelman (1967), Rose (1984), and Rubenfeld (1993). The Rubenfeld article contains a nice summary of the judicial history of takings law in the United States and the unsettled nature of the compensation question, particularly as it applies to regulatory takings.

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takings and physical takings are economicallyequivalent[2]—does economic efficiency then dictate that their compensatory treatment be the same?

(ii) What is (are) the economically efficient compensation rule(s) to adopt for takings, both regulatory and physical?

(iii) How and to what extent does the answer to question (ii) depend on assumptions about the strategic behavior of the state and the information available to both the state and citizen?

Question (i) is important for two reasons. One, were the answer yes, then current policy, which basically treats the two types of takings differently, would arguably need to be changed. Two, in current policy debates, the *presumed* answer seems to be yes. This has led some to call for the two types to enjoy the same compensatory treatment.[3] Although the same treatment may be compelling to some on the basis of fairness or other moral grounds, some of its appeal may follow from the presumption that this is necessary for economic efficiency. Others—who see an important moral distinction between regulatory and physical takings—have responded to this presumption by either denying economic equivalence or by denying that economic efficiency is an appropriate criterion for judging takings policy (see, e.g., Rubenfeld, 1993).

I would argue, however, that both sides of this policy debate are guilty of presuming incorrectly. Economic equivalence would imply the same compensatory treatment only if there were just one economically efficient compensation rule. As I show, however, there is more than one efficient rule for any given takings situation. One can, then, choose among these efficient rules based on the moral (i.e., political or philosophical) issues of the specific situation.

I derive this result and others using a model similar to the one used by Blume, Rubinfeld, and Shapiro (1984): A citizen invests in her property. The private benefit she then enjoys is, at least in part, determined by this investment. After investing, but before she can (completely) enjoy her private benefit, the state may take her private benefit (this could be either a regulatory or physical taking). A taking results in some social benefit being realized. The criteria for efficiency in this situation have two parts: The state takes the citizen's private benefit only if it is less than the social benefit; and, given this taking rule, the

2. An example easily conveys the intuition: What is the *economic* difference between the state physically taking, via eminent domain, a citizen's old-growth forest to create a spotted-owl sanctuary and the state accomplishing the same objective through a regulatory taking that prohibits logging in the citizen's forest when logging represents the only economical use of the land?

3. See, e.g., "Endangered Property Rights," *The Wall Street Journal*, September 12, 1994, p. A14; or "Is Taking Stealing?" *The Economist*, March 6, 1993, p. 24. This reasoning may also have served, in part, to justify the introduction of legislation that would require compensation when the federal government's regulatory actions reduce property values by at least 25 percent or $10,000 ("Endangered Property Rights," *id.*) or be, in part, behind Arizona's Proposition 300, which, *inter alia*, will help property owners receive compensation for regulations that affect their property values ("Arizona's Proposition 300 Looms Large in Property Owners' War on Regulations," *The Wall Street Journal*, October 24, 1994, p. A16). Of course, the politics behind these two measures are more complicated than this; so I am not suggesting that this reasoning is the sole motivation for these measures.

citizen invests the *socially* optimal amount in her property. As I show, to achieve full efficiency, the citizen's compensation must be tied to *society's* benefit from the taking. That is, the citizen must be compensated based on what society gains from the taking rather than on what she loses from the taking. The reason is an old one in economics: To induce an agent to act in a socially efficient manner (e.g., invest correctly), the agent's objective must be equivalent, on the appropriate margin, to society's objective. This can be accomplished only by incorporating the social benefit into the agent's reward function. This why current compensation practices, which do not tie the citizen's compensation to the social benefit, fail to achieve efficiency.[4]

Although this argument demonstrates that the citizen's compensation must be tied to the social benefit, it does not say how it should be tied to the social benefit. It turns out that there are essentially two equally effective ways to tie a citizen's compensation to the social benefit: one, she can be paid the social benefit if her private benefit is taken; or, two, she can be charged the social benefit if she retains her private benefit. At one level, this is nothing but the Coase theorem (Coase, 1960): In an externality problem—which essentially is what a takings problem is—the property right can reside with the citizen, so she is compensated for what is taken; or the property right can reside with the state, so the citizen pays for the privilege of enjoying her private benefit. At another level, however, this goes beyond the Coase theorem, because it requires determining not only who is compensated (i.e., who has the property right), but, as I will show, also *how* compensation is paid.

The analysis that follows also goes beyond the Coase theorem because of the need to consider both strategic behavior and asymmetric information [recall question (iii)]. The previous literature typically has assumed that the state acts benevolently to maximize social welfare. Recalling, however, that a large impetus for the Fifth Amendment was the danger of the state acting tyrannically, the assumption of a benevolent state clearly is not always appropriate. This article, therefore, also analyzes the takings problem when the state acts nonbenevolently.

As I show, whether the state's benevolence matters depends on the information structure. Information has not received much attention from the previous literature. Typically, the citizen's benefit from retaining the property and the state's (society's) benefit from taking the property have been assumed to be

4. Current practices can be summarized *roughly* as follows: The state owes compensation only if the citizen's property is physically invaded, but not if the citizen is simply deprived of the benefit of her property by regulation. If the state owes compensation, then the compensation should fully compensate the citizen for her lost benefit.

The requirement of compensation in cases of physical invasion was established by the Fifth Amendment of the U.S. Constitution. For regulatory takings, an important precedent is *Mugler v. Kansas,* 123 U.S. 623 (1887), in which the Supreme Court ruled that Kansas did not owe Mugler compensation despite forcing him to shut his brewery when it prohibited the manufacture of alcoholic beverages. Admittedly, other decisions, most recently *Lucas v. South Carolina Coastal Council,* 112 S. Ct. 2886 (1992), have called this precedent into question, at least in some instances. On the other hand, there is no evidence of a full-blown retreat from *Mugler.*

commonly known (i.e., there is symmetric information).[5] However, one can imagine situations in which one or both of these benefits is the beneficiary's private knowledge (i.e., there is asymmetric information). Regulatory takings, for instance, often involve asymmetric information, since the state typically enacts regulations without knowing exactly who will be affected and, therefore, without knowing exactly what benefit has been taken from each affected citizen. Asymmetric information could also describe some eminent domain cases; for example, the citizen's property is idiosyncratic or the state is driven by national security motives. In this article, I therefore consider various assumptions concerning the information structure.

The rest of the article is organized as follows. In Section 2, I formulate an alternative version of the Blume et al. (1984) model and review the economic equivalence of regulatory takings and physical takings. In Section 3, I consider optimal takings policies under the assumption that the state acts to maximize social welfare. When the state is "benevolent" like this, first-best efficiency is easily achieved, regardless of the information structure. I also review the Blume et al. results concerning the *inefficiency* of both a full-compensation rule (the citizen is paid exactly her private benefit if her property is taken) and a no-compensation rule (the citizen is not compensated at all). In Section 4, I consider the situation where the state is not benevolent; rather it is concerned only with the well-being of the majority (the rest of society) to the exclusion of the citizen's well-being.[6] In this situation, the information structure matters considerably more. Furthermore, although first-best takings policies exist, they fare considerably less well with respect to noneconomic criteria—in particular, they may require that the citizen sometimes pay the state compensation for not taking her property. Moreover, if one excludes takings policies in which the citizen pays the state, then no first-best takings policy exists. I then characterize the second-best policy given this restriction. I conclude in Section 5.

2. Model

There are two periods. In the first, a risk-neutral citizen invests I dollars in some property (e.g., a farm or factory).[7] Only the citizen knows how much she

5. On the other hand, as a referee pointed out, some of the earlier literature can be seen as implicitly assuming that the state's motive for a taking was its private information, because otherwise the courts could have simply forbid inefficient takings.

6. Alternative interpretations include the state acting in the interest of the monarch, the state acting on behalf of a special interest, or the state acting on behalf of some government agency with a semiprivate agenda.

7. Following Kaplow (1986), the assumption of a risk-neutral citizen can be justified by noting that property owners have an incentive—even in the *absence* of a possible taking—to insure themselves against risk. For example, factory owners sell shares in their factories to diversify their wealth. Given such market-based means of dissipating risk, one ultimately can view the state as taking property from a risk-neutral, decision-making "citizen" (e.g., the factories' shareholders), who is also the residual claimant.

It is incorrect, however, to conclude from this—as Kaplow seems to—that government compensation is unnecessary when private means of insurance exist. Whether the *residual* claimant (the citizen) is compensated will have an impact on the residual claimant's decision making. If the invests. In the second period, she receives a private benefit of b dollars if her property is not taken.[8] This benefit is randomly drawn from the interval $[0, B]$. Denote the probability that her private benefit is b or less by $G(b; I)$. Assume that the more the citizen invests, the greater is her probability of receiving large private benefits; that is, $\partial G/\partial I \leq 0$. Assume, however, that there are decreasing returns to her investment; that is, $\partial^2 G/\partial I^2 \geq 0$. Finally, let $g(b; I)$ be the density function associated with $G(b; I)$.

Were the citizen always to enjoy the benefit of her property—that is, were takings impossible—the amount she would invest would maximize her expected private benefit minus her investment cost; that is,

$$\mathbf{E}_b\{b \mid I\} - I = \int_0^B bg(b; I)\, db - I \tag{1}$$

(where $\mathbf{E}_b\{b \mid I\}$ is the expected private benefit given an investment of I). Here and throughout I assume positive solutions (i.e., $I > 0$) to all such maximization programs.

After the first period, but prior to the second, the state can take the property from the citizen by right of eminent domain or through its right to regulate. If the property is taken, the citizen loses her private benefit. Society gains a social (dollar) benefit, s, if the property is taken. The value of s is unknown before the end of the first period, but becomes known (by the state at least)before the state decides to take the property. Assume that s is drawn from the interval $[0, S]$. Denote the probability that the social benefit is s or less by $F(s)$. Denote the corresponding density function by $f(s)$, where $f(s) > 0$ for all $s \in (0, S)$. The assumption of a stochastic social benefit reflects the reality that citizens often make investments when they are uncertain whether future circumstances will lead to a taking. For example, will future suburban sprawl lead to the area around a citizen's farm being rezoned? Or will this sprawl lead to highways being built on her farm? I allow for the possibility that a likely outcome is that society receives no benefit from taking the citizen's property; that is, $F(0)$ could be strictly positive. Assume, however, that a socially beneficial taking is always possible; that is, $F(B) < 1$. Society as a whole is risk neutral.

The social benefit, s, has two possible interpretations in this model. It could be the harm suffered by society if the citizen uses her property (e.g., s could represent the nuisance from the citizen's operating her factory in a residential neighborhood). Alternatively, s could be the benefit from society's use of the citizen's property (e.g., from flooding her farm as a dam reservoir). Under the first interpretation, social welfare (the sum of private and social benefits) is $b - s$ if the citizen retains her property and 0 if it is taken. Under the second interpretation, social welfare is b if the citizen retains her property and s if

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residual claimant is to make socially optimal decisions, some government compensation scheme will be necessary (see Proposition 1 below).

8. More generally, b represents the difference between the citizen's utility if there is no taking and her utility if there is a taking.

it is taken. It is readily shown, however, that these two interpretations are economically equivalent (see the Appendix).[9,10]

Social welfare is maximized by the state's taking the property only when the social benefit exceeds the private benefit; that is, when $s > b$. Under this rule, expected social welfare can be written (up to an additive constant) as

$$\mathrm{E}_{b,s}\{SW \mid I\} - I = \int_0^B \left(bF(0) + \int_0^B (b-s) f(s)\, ds \right) g(b; I)\, db - I, \qquad (2)$$

(where $\mathrm{E}_{b,s}\{SW \mid I\}$ is expected social welfare given an investment of I). Maximizing the social problem, Expression (2), yields a smaller optimal level of investment than does maximizing the private problem, Expression (1). Intuitively, when a taking is possible, the private benefit will, with positive probability, be forgone in exchange for the social benefit, and this affects the *socially* optimal level of investment.[11] Or, in other words, because there is a possibility that the investment will be wasted, the socially optimal investment is less than it would be were the citizen always to enjoy its benefit.

3. The Benevolent State

Here, I assume that the state acts to maximize social welfare: It wishes to take the property only if the social benefit, s, exceeds the private benefit, b. I relax this assumption in Section 4.

To begin, I review the Blume et al. (1984) results concerning two compensation rules often employed in practice.[12] The first, often used when property is taken for use, is *full compensation*—the citizen is paid b for the property. The second, often used when property is taken to avoid harm, is *no compensation*—the citizen is paid 0 for the property. Despite their real-world prevalence, neither rule achieves full efficiency.

Proposition 1 (Blume et al., 1984). Assume that there is a continuous distribution of social benefits and that, with positive probability, social benefits will exceed private benefits. Then neither full compensation nor no compensation

9. This equivalence, however, would disappear if the citizen's investment affected the distribution of the social benefit (e.g., if her infrastructure investments for logging made her land more valuable as a park). Some results would still hold were the distribution of the social benefit dependent on the citizen's investment. For instance, the "citizen-as-perfect-monopolist" rule (Proposition 2) and the "buy-back" rule (Proposition 3) would still hold, although the citizen-as-perfect-monopolist rule would then be for physical takings only and the buy-back rule for regulatory takings only. More important, perhaps, would be the change in the information structure: the citizen's investment would become her private signal of the social benefit. A complete analysis under this alternative assumption is, however, outside the scope of this article.

10. This equivalence has occasionally been missed by the legal literature (see, e.g., Rubenfeld, 1993).

11. This result is proved formally in the Appendix (it is a corollary of Proposition 1).

12. Despite their many similarities, this model and the model in Blume et al. (1984) also have their differences. A major difference is that by treating the citizen's benefit as stochastic, this model is better suited to considering alternative informational assumptions.

will achieve full efficiency; indeed, both will lead to overinvestment relative to the first-best investment level.

Consider the full-compensation rule first. The citizen receives b regardless of whether there is a taking. So, *from the citizen's perspective*, it is as if there were no possibility of a taking. Consequently, she will set her investment to maximize Equation (1). Since that level of investment exceeds the socially optimal level of investment, full compensation leads to overinvestment.

Consider the no-compensation rule. The citizen receives b if there is no taking and 0 if there is. Therefore, her expected utility is

$$\mathbf{E}_b\{b \mid b \geq s, I\} - I = \int_0^B bF(b)g(b; I)\,db - I, \tag{3}$$

where $\mathbf{E}_b\{b \mid b \geq s, I\}$ is the expected value of the private benefit given investment I and given that the private benefit exceeds the social benefit. As proved in the Appendix, the citizen's utility-maximizing investment under the no-compensation rule exceeds the socially optimal level of investment. Intuitively, when the citizen decides how much to invest under the no-compensation rule, she is motivated by two considerations: one, her expected benefit; and, two, influencing the state not to take her property (lowering the probability of a taking). Since she can influence the state not to take her property only by realizing a private benefit greater than the social benefit, this influence consideration encourages her to invest more than the socially optimal amount.[13,14]

What, then, are economically efficient compensation rules? The answer can be found by invoking a fundamental principle of economics: To make an agent act in the social interest, align the agent's interests with society's. Here, this means inducing the citizen to maximize (2).

It turns out that there are many compensation rules that achieve this goal. The simplest rule, perhaps, is to pay the citizen the *social* benefit, s, generated by the taking. Under this rule, the citizen's expected utility is readily shown to be

$$\int_0^B \left(bF(0) + \int_0^b (b-s)f(s)\,ds\right) g(b; I)\,db - I + \mathbf{E}_s\{s\}, \tag{4}$$

where $\mathbf{E}_s\{s\}$ is the *expected* social benefit. Expression (4) is (2) plus an additive constant that is independent of the amount invested; hence, the same level of investment maximizes both expressions. Intuitively, since the citizen receives the larger of b and s, she is receiving social welfare (or the equivalent thereof) in all states. This, in turn, means she will wish to invest so as to maximize

13. One might think that this influence effect exists only when the citizen's benefit is "large" relative to the taking. This, however, is not right: it is not the absolute size of the citizen's benefit that matters, but rather the *marginal* increase that matters. Provided $f(s) > 0$ for s in $[0, B]$, this influence effect exists (see the proof of Proposition 1 in the Appendix).

14. The idea that influence activities can lead to inefficient distortions is well established in the contract theory literature (among other literatures). See, e.g., Milgrom (1988) or Meyer, Milgrom, and Roberts (1992).

expected social welfare. Since this is what one wants her to do, this rule, therefore, induces a citizen to invest in the socially optimal manner.

A possible objection to compensating the citizen by paying her the social benefit is that this would unfairly enrich her. For example, the value of building a dam that floods the citizen's farm could greatly exceed the value of her farm. Alternatively, the social benefit may represent the harm that the citizen is causing, and it may seem unjust to reward her for not causing harm. Fortunately, there exist other compensation rules or modifications of this rule that are less objectionable on these grounds. A simple modification is to impose an upper limit, L, on what the citizen receives, so the citizen receives the smaller of the social benefit and this limit (i.e., she receives $\min\{s, L\}$).[15] Provided this limit is *not less* than the maximum possible private benefit, B, this modified rule will still achieve the first-best outcome (this is proved in the Appendix). Intuitively, since the citizen cannot affect the likelihood of being compensated with a social benefit greater than B, capping her compensation at B or greater can have no impact on her decision making.

So far the analysis has (implicitly) assumed complete information. For the compensation rule under consideration, however, such an assumption is unnecessary.[16] Since the state is here assumed to be benevolent, the state can be trusted to reveal what the social benefit, s, is. That is, there is no loss of generality in assuming that the social benefit is common knowledge (this is not true, however, when the state is not benevolent, as in Section 4). If the citizen's private benefit is common knowledge, it is clear how to implement the compensation rule. If it is not common knowledge, there are two equivalent ways to implement the rule: One, the state could offer to pay the citizen $\min\{s, L\}$ if the citizen would agree to the taking (alternatively, agree to cease causing harm). The citizen will agree provided her benefit is less than what she is offered—that is, if $b \leq \min\{s, L\}$—and she will refuse otherwise. Clearly, the citizen is making the efficient decision and, by doing so, is implementing the optimal compensation and taking rule. Alternatively, the state can ask the citizen to name her price for agreeing to the taking or for ceasing to cause harm. The state will pay the citizen's price if it does not exceed $\min\{s, L\}$. Since the citizen may be assumed to know s, she will set her price at $\min\{s, L\}$ if that exceeds her private benefit, and she will name a price in excess of $\min\{s, L\}$ if $\min\{s, L\}$ does not exceed her private benefit. Again, the citizen is making the efficient decision and, by doing so, is implementing the optimal compensation and taking rule. Effectively, what is happening is that the citizen is being put in

15. In practice, setting an upper limit could be difficult (i.e., it may be difficult to determine B). In these situations, the upper bound could be ignored (e.g., set equal to S). Presumably, some means of setting the upper limit could be chosen that would avoid excessive litigation. In any case, setting the upper limit is likely to be an easier task than determining the private benefit itself, which is what current policy essentially demands (see, e.g., Reilly, 1992, for a discussion of the practical difficulties associated with current policy in this regard).

16. The same is not true of the full-compensation or the no-compensation rule. How to implement these rules such that a taking occurs if and only if $b \geq s$ is unclear when the citizen's private benefit is her private information.

the position of being able to sell to the state as a perfectly price-discriminating monopolist. To summarize the analysis to this point:

Proposition 2 (the citizen-as-perfect-monopolist rule). Consider a benevolent state. Then the first-best outcome can be achieved by using the compensation rule that pays the citizen $\min\{s, L\}$ in the event of a taking, where $L \geq B$. This rule can be implemented either by requiring the state to bid $\min\{s, L\}$ to the citizen and giving the citizen the right to refuse; or by requiring the citizen to name her price and giving the state the right to refuse if the price named exceeds $\min\{s, L\}$.

Since $\min\{s, L\}$ is greater than b when the property is taken, it follows that the citizen receives compensation in excess of her loss. This may seem odd, since Proposition 1 showed that full compensation was inefficient. What must be remembered, however, is that what matters is not the total amount of the citizen's compensation, but rather how her compensation varies on the relevant margin.[17]

This last insight suggests the following alternative to the rule just discussed: The state informs the citizen that she may retain her property (continue to cause harm) if she pays the state the social benefit (which, recall, is either common knowledge or has been truthfully revealed by the *benevolent* state). If she chooses to surrender her property (cease causing harm), she pays the state nothing. Rationally, she will choose to retain her property only if her private benefit exceeds the social benefit. Consequently, her expected utility maximization problem is

$$\int_0^B \left(bF(0) + \int_0^b (b - s) f(s)\, ds\right) g(b; I)\, db - I.$$

This, however, is just (2); hence, the citizen will invest in the socially optimal manner. This establishes the following:

Proposition 3 (the buy-back rule). With a benevolent state, the first-best outcome can be achieved by using the compensation rule in which the citizen retains her property only if she pays the state s.

Note that the buy-back rule works regardless of whether the state knows the citizen's private benefit at the time it initiates a taking. Note, too, that the buy-back rule offers an alternative to the citizen-as-perfect-monopolist rule in which the citizen is *not* enriched by a taking. In a sense, then, these two rules allow society some flexibility with which to achieve efficiency without completely sacrificing its other objectives, such as fairness.[18]

17. This insight disproves Kaplow (1986) and Rubenfeld's (1993) assertions that correcting the overinvestment problem implies compensation that is less than the citizen's loss.

18. An earlier version of this article explored other rules that offered intermediate levels of transfers between the state and the citizen relative to the two rules considered here. These other rules, however, depended on the citizen's private benefit being common knowledge.

The existence of two rules might, at first, seem to create difficulties: which rule to use when? Since the two rules are efficient, there is no need to choose the rule in advance: as long as the citizen knows that *some* efficient rule will be employed ex post, she will make the correct investment and the ex post allocation will be efficient. Admittedly, to minimize ex post litigation, it would probably be beneficial to set the criteria for choosing the taking rule in advance.[19] Of course, current policy also has multiple rules—rules, moreover, that are much harder to implement in practice than the two rules proposed here would be (see, e.g., Reilly, 1992, for a discussion of the often complicated ways in which compensation is currently calculated).

4. The Nonbenevolent State

In legal writing, one motive for compensating a citizen for taken property is to restrain the state from the tyrannical use of its rights of regulation or eminent domain.[20] That is, the state is assumed not to act benevolently but to act on behalf of the interest of the majority (i.e., the rest of society) while essentially ignoring the interest of the individual property owner.

Now assume the state initiates a taking if and only if the social benefit exceeds what the state must pay the citizen (i.e., if and only if $s \geq p$, where p is the payment made to the citizen).

From an efficiency perspective, assuming that the state has "selfish" interests matters only if the state cannot be constitutionally constrained to act in a socially efficient manner. In turn, the state can escape being so constrained only if it cannot be compelled to reveal the social benefit. This logic is summarized by the following corollary (the proof is straightforward, given the earlier analysis, and is therefore omitted):

Corollary 1. Assume that the state can be compelled to reveal the social benefit, s (i.e., the social benefit is known by the citizen). Then the first-best outcome is attainable using either the citizen-as-perfect-monopolist rule (Proposition 2) or the buy-back rule (Proposition 3).

In contrast, assume, henceforth, that the state cannot be compelled to reveal the social benefit; that is, the social benefit is the state's private information. To appreciate the consequences of this assumption, suppose first that the citizen could set her compensation for the taking and the state had the right to accept or refuse the citizen's offer. Since the citizen now does not know the social benefit, she is in the position of a monopolist who faces a downward-sloping

19. However, there is probably no hope of eliminating all litigation. For instance, the line between regulatory takings and physical takings is not always clear-cut. Consider, e.g., two recent Supreme Court decisions: *Nollan v. California Coastal Commission*, 483 U.S. 825, 97 L. Ed. 2d 677, 107 S. Ct. 3141 (1987), and *Dolan v. City of Tigard*, 1994 U.S. LEXIS 4826; 129 L. Ed. 2d 304 (1994).

20. "... the constitutions of the United States and of this state, and of most of the other states of the Union, have imposed a great and valuable check ... by declaring, that private property should not be taken for public use without just compensation" (James Kent, *Commentaries on American Law*, quoted in Rubenfeld, 1993: 1082).

demand curve. That is, if β is the compensation for which she asks, then one can interpret her probability of "making a sale" at β, $1 - F(\beta)$, to be the demand she faces at that "price." Demand clearly is decreasing in price (i.e., $d[1 - F(\beta)]/d\beta = -f(\beta) < 0$). The well-known problem of monopoly pricing will therefore arise: The citizen will set a price in excess of her private benefit (her marginal cost); hence, with positive probability, a taking that would be efficient will not occur (i.e., when $b < s < \beta$).[21] In other words, the familiar result that monopoly pricing leads to a loss in social welfare obtains.

The problem is no better if the state is allowed to set compensation and the citizen has the right to accept or refuse the state's offer. If the state knows the citizen's private benefit, then the state will offer b when $b \leq s$ and it will offer less when $b > s$. Although this leads to efficient takings, the compensation rule is the full-compensation rule, which means the citizen will invest inefficiently. If the state does not know the citizen's private benefit, then a monopoly-pricing problem will again exist: The state will offer less than s in the hopes of capturing some of the surplus.[22] Offering less than s means, however, that some efficient takings will be forgone with positive probability (i.e., when the citizen's benefit lies between what the state offers and the social benefit).

Fortunately, these problems can be overcome by using a more sophisticated mechanism. Suppose that after the state initiates a taking, the citizen sets her "price" of β. The state, then, decides whether to take the property at that price. If there were nothing more to the mechanism, then, as just seen, the resulting monopoly-pricing problem could lead to socially desirable takings being forgone. To remedy this, the mechanism must have a second "price": a base transfer, $A(\beta)$, that the state must pay the citizen *regardless* whether the state ends up actually taking the property.[23] Since the state must pay $A(\beta)$ regardless, only β matters for the state's decision to take the property.

To ensure efficient takings, this mechanism must induce the citizen to set β equal to her private benefit, b. This can be accomplished by having $A(\beta)$ equal (plus or minus a constant) the *state's* expected surplus if it were to take the citizen's property at a price of β. To see why this is so, note that if $A(\beta)$ equals the expected surplus of the state, then the citizen is capturing, in expectation, total welfare (recall, the citizen also gets either b or β). She therefore has an incentive to choose her price, β, to maximize expected total welfare. A well-known result is that total welfare is maximized by pricing at marginal cost.

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21. The citizen will choose β to maximize $[1 - F(\beta)] \cdot (\beta - b)$. The first-order condition is

$$1 - F(\beta) - f(\beta)(\beta - b) = 0,$$

which can be met only if $\beta > b$.

22. Let σ be what the state offers. The state will choose σ to maximize $(s - \sigma) \cdot G(\sigma; I_0)$, where I_0 is the citizen's equilibrium level of investment in this situation. The first-order condition is

$$g(\sigma; I_0) \cdot (s - \sigma) - G(\sigma; I_0) = 0,$$

which can be met only if $\sigma < s$.

23. To be precise, $A(\beta)$ could be negative (the citizen pays the state), but more on this later.

Marginal cost in this context is the citizen's private benefit, b. Consequently, the citizen does best to set $\beta = b$.

There is still the issue of inducing the citizen to invest efficiently. This, however, is not a problem: Since the citizen is, for every value of b, effectively receiving total welfare, she automatically has the proper incentive to invest efficiently, that is, to maximize (2).

Formally,

$$A(\beta) = \int_{\beta}^{S} (s - \beta) f(s)\, ds - t, \tag{5}$$

where t is a positive constant.[24]

Efficiency dictates that the state always initiate a taking when the social benefit is positive (i.e., when $s > 0$), because otherwise a socially desirable taking will be forgone with positive probability (i.e., when $s > b \geq 0$). For this to be true, $A(b)$ must be strictly negative for some values of b. To see why, consider s as it nears zero. Conditional on b, the state's utility is then approximately $-A(b)$. For the state to initiate a taking, its expectation over $A(b)$ must be nonpositive. Since it is readily shown that $A(b)$ is decreasing in b, $A(b)$ cannot be a constant. It follows, then, that $A(b)$ must be strictly negative for some values of b. In other words, for this mechanism to work, the citizen must sometimes pay the state. To summarize:

Proposition 4. If the state is not benevolent and the social benefit is the state's private information, then the mechanism given by (5) above, with t set large enough to ensure that the state initiates a taking for all $s > 0$, will yield the first-best outcome.[25] However, this mechanism will entail that the citizen sometimes pays the state compensation.

Note that this solution to the takings problem effectively allows a nonbenevolent state to demand payments from its citizens in exchange for not taking their property. The danger in this is obvious. To a large extent the danger could be mitigated by rules that restrict how often the state is allowed to attempt a taking and by rules that require the state to show that its attempted taking is motivated by more than a desire to extort money from the citizen. Nevertheless, one might still expect objections to this mechanism. It therefore seems worth considering mechanisms in which the state cannot compel the citizen to pay it money.

The first result is that the first best is *not* attainable when the social benefit is the nonbenevolent state's private information and the citizen cannot be compelled to pay the state.

24. This mechanism, which is an extension of work by Riordan (1984) and others, was introduced into the literature by Hermalin and Katz (1993).

25. Admittedly, there could be practical difficulties in setting t to achieve this goal. Whether these difficulties would be greater than the difficulties encountered under current rules is, however, an open question (see note 15 *supra*).

Proposition 5. If the state is not benevolent, cannot be compelled to reveal the social benefit, and cannot demand payment from the citizen, then the first-best outcome is unattainable.

The intuition behind this result when the citizen's private benefit is common knowledge is as follows.[26] Given a realization of b, there can be only two levels of compensation: one if the property is taken and one if it is not. (If there were, say, two different levels when the property is taken, then which is paid would depend on what the state said about the social benefit, s. The state, of course, would make statements consistent with paying the lower level of compensation when it wished to take the property.) Since efficiency requires that the state take the property for all s greater than b, including s just barely greater than b, and since efficiency requires that the state not take the property for all s less than b, including s just barely smaller than b, it follows that compensation when the property is taken cannot exceed b. It also follows, by the same reasoning, that the difference in compensation between when the property is taken and when it is not taken cannot exceed b. Put this together with the restriction that all compensation be nonnegative (the citizen cannot pay the state), and we find that the compensation when the property is taken is b and the compensation when the property is not taken is 0. This, however, is the full-compensation rule, which leads the citizen to invest *in*efficiently.

What, then, is the second-best mechanism in this setting? The answer is the following:

Proposition 6. If the state is not benevolent, the social benefit is the state's private information, the citizen's private benefit is common knowledge, and the state cannot demand payment from the citizen, then, in the second-best mechanism, the state can take the citizen's property by paying an amount that depends solely on the realization of the private benefit (if the state does not take the citizen's property, then there is no transfer between the citizen and the state). In addition, if condition (M) is satisfied, that is,

$$g(b; I) = \eta(I)g^1(b) + (I - \eta(I))g^0(b), \qquad \text{(M)}$$

where $\eta(\cdot)$ is an increasing and concave function,
$I \in [I_0, I_1]$, $0 \leq \eta(I_0) < \eta(I_1) \leq 1$, and
$g^1(b) - g^0(b)$ is strictly increasing in b,

then there exists a $b^* \in (0, B)$ such that the citizen will be overcompensated (paid more than b) if $b < b^*$ and undercompensated (paid less than b) if $b > b^*$.

The intuition behind this result is as follows. The citizen's expected utility, conditional on her private benefit, is

$$U(b) \equiv b + \zeta(b)\{1 - F[b + \zeta(b)]\},$$

26. The intuition when the citizen's private information is her private benefit is somewhat more involved. The interested reader may wish to consult Myerson and Satterthwaite (1983).

where $b+\zeta(b)$ is the amount the state pays the citizen when it takes her property. Optimally, the function $\zeta(\cdot)$ should be chosen in such a way as to discourage overinvestment by the citizen (recall, from above, that this is the essence of the problem). To encourage the citizen to invest less, one wants to reward the citizen for realizing private benefits that are more likely given an appropriate level of investment (i.e., make $U(b) > b$ for values of b that are relatively more likely if the citizen does not overinvest) and to punish her for realizing private benefits that are more likely given overinvestment (i.e., make $U(b) < b$ for values of b that are relatively more likely if the citizen overinvests). Condition (M) is one of many conditions under which smaller values of the private benefit are evidence of appropriate investment, while larger values of the private benefit are evidence of overinvestment.[27] Consequently, when condition (M) is satisfied, the citizen is overcompensated for her property when her private benefit is relatively low (less than b^*) and she is undercompensated for her property when her private benefit is relatively high (greater than b^*). If condition (M) is not satisfied, then the second-best scheme would still entail overcompensating the citizen for some realizations of the private benefit and undercompensating her for other realizations; the difference is that there would no longer *necessarily* be a monotonic relation between the realization of the private benefit and whether the citizen is over- or undercompensated.[28]

To get some feel for the solution of the takings problem under Proposition 6, consider the following example. The social benefit is distributed uniformly on $[0, 5]$. The private benefit is 5/3 with probability $1 - .9\sqrt{2I}$ and is 10/3 with probability $.9\sqrt{2I}$, where I is restricted to lie in $[0, .6]$. In the first best, $I = 9/32 \approx .281$ and expected social welfare is approximately .559. Under a regime in which the citizen is fully compensated in the event of a taking—a regime that ensures an efficient allocation ex post—the citizen overinvests, $I = 3/5 = .6$. Social welfare is, therefore, .499. In the second-best mechanism—that is, under the constraint that the citizen never compensate the state—$\zeta(5/3) \approx .559$ and $\zeta(10/3) \approx -0.210$, which leads the citizen to invest $I \approx .326$. Expected social welfare under this second-best mechanism is approximately .546. Note that, despite the constraint that the citizen never pay the state, there is only a 2 percent loss in efficiency (versus an 11 percent loss in efficiency under the full-compensation rule). Note, too, that the mechanism risks very inefficient takings—when $b = 5/3 \approx 1.667$, the state fails to take the property even when s is as great as 2.226—in exchange for driving the citizen's investment close to the first-best level. It does so because the probability of suffering

27. If (i) the citizen's investment problem is globally concave in the investment level (i.e., if the first-order condition for this problem is sufficient as well as necessary) and (ii) $\partial g(b; I)/\partial I$ is strictly increasing in b, then this ordering will hold. In general, establishing global concavity (i.e., the validity of the "first-order approach") is difficult because of the endogeneity of the $\zeta(b)$ function (see, e.g., Jewitt, 1988 for a discussion in a related context). A mixing or spanning condition like (M), however, is generally sufficient (again, see Jewitt, 1988).

28. Since this is a normative article, one might ask how likely it is that condition (M) will be satisfied. Unfortunately, this is not a question that I, nor perhaps anyone, can answer.

these very inefficient takings is relatively small when $I \approx .326$ (it is .03). Therefore, the *threat* of very inefficient takings is used to keep the citizen from overinvesting, which in turn reduces the probability of actually suffering these very inefficient takings. Along the same lines, the ex post allocation is more nearly first-best efficient when the citizen's private benefit is 10/3, because $b = 10/3$ is by far the more likely realization (probability is .73). Although this is just a simple example, it does illustrate where the largest and smallest distortions in the ex post allocation of the property will be and why they will be there. Moreover, it shows that the expected loss from employing a second-best mechanism rather than a first-best mechanism *can be*—although, admittedly, need not be—small, because allocative inefficiencies can be "loaded" onto the least likely realizations of b and s.

5. Conclusions

Using a simple model, I have shown that there exist efficient (or nearly efficient) solutions to the takings problem despite a variety of complications, including asymmetric information and the state's nonbenevolence. Moreover, in many settings there exist multiple solutions. This multiplicity of solutions may help society to choose efficient rules that also meet noneconomic objectives, such as fairness. Conversely, it may be possible to achieve noneconomic objectives with little sacrifice of economic efficiency (see, e.g., the example in Section 4).

The preceding analysis is built on one basic insight: Efficiency requires compensating the citizen based not on what she loses, but rather on what society gains from the taking. Unless this is done, the citizen will not invest optimally in her property.

When society's benefit from a taking is common knowledge—because the state is benevolent and hence truthfully reveals it or is not benevolent but can be compelled to reveal it—there are two ways to align the citizen's objective with society's along the appropriate margin. Either the citizen is paid the social benefit of the taking in exchange for giving up her property (the citizen-as-perfect-monopolist rule),[29] or the citizen pays the state (society) the social benefit in exchange for keeping her property (the buy-back rule). As was shown, neither of these rules depends on whether the citizen's private benefit is commonly known or not. Moreover, both of these rules are simple to implement, a considerable improvement over current policy.

When the social benefit is not commonly known (which implies that the state is not benevolent), no "single-price" rule such as one of the ones just described will be efficient. Fortunately, there exists a "two-part tariff" that does achieve efficiency: The state's announcement that it wishes to take the property obligates the state and citizen to make a monetary transfer *regardless* of whether the property is taken. Moreover, the amount of this monetary transfer is fixed by the citizen's announcement of her private benefit, the "price" at

29. To be precise, her payment can be capped at some upper limit that is not smaller than her maximum possible private benefit.

which the state can then take the property. The relation between the monetary transfer and the citizen's private benefit is that the monetary transfer equals, on the margin, the expected social gain from the transfer if the property is taken at the citizen's announced private benefit. Consequently, the citizen receives, on the margin, expected social welfare. Since her objective coincides with the social objective, on the appropriate margin, she behaves in a socially desirable manner.

While the two-part tariff outlined above does achieve an efficient outcome, it does entail that the citizen sometimes pay the state. Although there are ways to guard against the state's potential abuse of such a rule, this fear or other, noneconomic objections might make this two-part tariff an unacceptable solution. However, if one imposes the condition that the citizen never pay the state, then no first-best efficient rule exists. The second-best rule under this condition requires that the citizen be undercompensated for her loss (receive less than her forgone private benefit) in some states and be overcompensated in other states. If the distribution of possible private benefits satisfies a spanning condition, then she is undercompensated when her private benefit is large and overcompensated when her private benefit is small.

Neither the first-best nor second-best rules coincide with traditional compensation rules. Roughly, these rules can be summarized as follows: fully compensate a citizen if her property is physically invaded, but pay her nothing if her private benefit is lost because of regulation. However, because these rules do not tie her compensation to society's gain, they cannot be efficient. This is not surprising, given the general point that compensation should be based on society's gain. I believe that policy-makers and future scholars would be well-advised to build on this general point in their design and analysis of takings law.

Appendix

The equivalence of the two interpretations of the social benefit. Suppose, given b, that the property is taken if $s \in \Sigma(b)$ and not taken if $s \in \Sigma^c(b)$ (where $\Sigma(b) \cup \Sigma^c(b) = [0, S]$). Then ex ante expected social welfare under the "nuisance" interpretation is

$$\int_0^B \left\{ \int_{\Sigma^c(b)} (b-s) f(s)\, ds \right\} g(b; I)\, db - I$$
$$= \int_0^B \left\{ \int_{\Sigma^c(b)} bf(s)\, ds + \int_{\Sigma(b)} sf(s)\, ds \right\} g(b; I)\, db - I - \mathrm{E}_s\{s\},$$

and the ex ante expected social welfare under the "use" interpretation is

$$\int_0^B \left\{ \int_{\Sigma^c(b)} bf(s)\, ds + \int_{\Sigma(b)} sf(s)\, ds \right\} g(b; I)\, db - I$$
$$= \int_0^B \left\{ \int_{\Sigma^c(b)} (b-s) f(s)\, ds \right\} g(b; I)\, db - I + \mathrm{E}_s\{s\},$$

where $\mathbf{E}_s\{s\}$ is the expected value of s (a constant). As claimed, the two interpretations are, thus, economically equivalent.

The citizen's investment problem. The citizen's investment problem is always of the form

$$\max_I \int_0^B h(b)g(b; I)\, db - I, \tag{A1}$$

where $h(\cdot)$ is some increasing function. The corresponding first-order condition is

$$\int_0^B h(b)g_I(b; I)\, db - I = 0. \tag{A2}$$

Consequently, the following lemma is useful for comparing solutions to various investment problems.

Lemma A1. Let $u(\cdot)$ and $v(\cdot)$ be two functions. If $u'(b) > v'(b) > 0$ for almost every b in $[0, B]$, then the I that solves

$$\int_0^B u(b)g_I(b; I)\, db - 1 = 0 \tag{A3}$$

is greater than the I that solves

$$\int_0^B v(b)g_I(b; I)\, db - 1 = 0. \tag{A4}$$

If, however, $u'(b) = v'(b)$ for almost every b in $[0, B]$, then the same I solves (A3) and (A4).

Proof of Lemma A1. If $u'(b) > v'(b)$, $u(b) - v(b)$ is increasing in b almost everywhere. Hence, since $\partial G/\partial I \leq 0$,

$$\int_0^B [u(b) - v(b)]g_I(b; I)\, db > 0 \Rightarrow \int_0^B u(b)g_I(b; I)\, db > \int_0^B v(b)g_I(b; I)\, db. \tag{A5}$$

Since the right-most term in (A5) is decreasing in I (recall $\partial^2 G/\partial I^2 > 0$), the result follows. If $u'(b) = v'(b)$, then $u(b) - v(b)$ equals a constant, Δ. Consequently,

$$\Delta = \int_0^B \Delta g(b; I)\, db \Rightarrow 0 = \int_0^B \Delta g_I(b; I)\, db$$
$$\Rightarrow \int_0^B u(b)g_I(b; I)\, db = \int_0^B v(b)g_I(b; I)\, db.$$

The result follows immediately. Q.E.D.

Proof of Proposition 1. Consider full compensation first. As noted in the text, the citizen's problem is equivalent to her problem when there is no possibility of a taking. Corresponding to the formulation given in (A1), $h(b) = b$. The derivative of this with respect to b is 1. Compare this with Expression (2), where

$$h(b) = bF(0) + \int_0^B (b - s) f(s)\, ds.$$

The derivative of this with respect to b is $F(b)$, which is less than 1. It follows then, from Lemma A1, that the citizen overinvests relative to the first best under the full-compensation rule.

Consider no compensation next. As noted in the text, the citizen investment problem is given by expression (3). Consequently, $h(b) = bF(b)$. The derivative of this with respect to b is $F(b) + bf(b)$. Since this is greater than $F(b)$, it follows from Lemma A1 that the citizen overinvests relative to the first best under the no-compensation rule. Q.E.D.

Proof of Proposition 2. The citizen's investment problem is

$$\max_I \int_0^B \left(bF(b) + \int_b^S \min\{s, L\} f(s)\, ds \right) g_I(b; I)\, db - I. \tag{A6}$$

Differentiating the expression in large parentheses (i.e., $h(b)$) yields $F(b)$ (recall $L \geq B$, so $\min\{b, L\} = b$). Since this is the same as the derivative of the expression in large parentheses in expression (2), it follows from Lemma A1 that the solutions to maximizing (2) and (A6), are the same. So the citizen makes the first-best level of investment. It was shown in the text that the taking is efficient (i.e., occurs only if $s \geq b$) under this rule. Q.E.D.

Proof of Proposition 3. This was proved in the text.

Proof of Proposition 4. Suppose the state initiates a taking. The citizen's problem when choosing β is to maximize

$$\begin{aligned} &A(\beta) + bF(\beta) + \beta[1 - F(\beta)] \\ &\quad = \int_\beta^S (s - \beta) f(s)\, ds - t + bF(\beta) + \beta[1 - F(\beta)]. \end{aligned}$$

Differentiating, the first-order condition is

$$-[1 - F(\beta)] + bf(\beta) + 1 - F(\beta) - \beta f(\beta) = (b - \beta) f(\beta) = 0.$$

Clearly, the solution is $\beta = b$. So, since the state takes the property only if $s \geq \beta$, the takings under this rule will be efficient.

Consider the citizen's investment. Given her pricing strategy, she invests to maximize

$$\int_0^B \{b + A(b)\} g(b; I)\, db - I$$

$$= \int_0^B \left(b + \int_b^S (s - b) f(s)\, ds \right) g(b; I)\, db - I - t. \tag{A7}$$

Differentiating the expression in large parentheses in (A7) yields $F(b)$. Since this is the same as the derivative of the expression in large parentheses in expression (2), it follows from Lemma A1 that the citizen will invest the first-best amount.

Lastly, the state must initiate a taking whenever $s > 0$. If b is commonly known, this requirement translates into

$$\max\{0, s - b\} - A(b) \geq 0 \qquad \text{for all } (b, s) \in [0, B] \times (0, S].$$

Since

$$\max\{0, s - b\} - A(b) \geq -A(0) = t - \int_0^S \sigma f(\sigma)\, d\sigma,$$

This requirement is met only if $t \geq \mathrm{E}\{s\}$. Since $A(\cdot)$ is a decreasing function, it follows that there are values of b and s such that the citizen ends up paying the state.

If b is the citizen's private information, then the requirement that the state initiate a taking whenever $s > 0$ translates into

$$\int_0^B [\max\{0, s - b\} - A(b)] g(b; I^*)\, db \geq 0 \qquad \text{for all } s > 0$$

(where I^* is the first-best level of investment). A necessary condition for this requirement to be satisfied is that

$$t \geq \int_0^B \left(\int_b^S (s - b) f(s)\, ds \right) g(b; I^*)\, db.$$

Since $A(\cdot)$ is a decreasing function, it follows that there are values of b and s such that the citizen ends up paying the state. Q.E.D.

Proof of Proposition 5. Suppose, first, that the citizen's private benefit is commonly known. By the revelation principle, I can restrict attention to mechanisms in which (i) compensation and approval of the taking are functions of the state's announcement of the value of s, and (ii) the state's announcement is truthful *in equilibrium*. Let $x(\sigma, b)$ and $p(\sigma, b)$ be the taking rule and compensation rule, respectively, where σ is the state's announcement. If the mechanism is efficient, then $x(\sigma, b) = 1$ if and only if $\sigma \geq b$. Since the state does not have to initiate a taking, the mechanism must satisfy the following "participation" constraints:

$$sx(s, b) - p(s, b) \geq 0.$$

Moreover, truth-telling must be optimal for the state:

$$sx(s,b) - p(s,b) \geq sx(\sigma,b) - p(\sigma,b), \qquad \text{for all } \sigma \in [0,S].$$

Consider $s_0 < s_1 < b$. Using the truth-telling constraint, $p(s_0,b) = p(s_1,b)$. Consider $b < s_2 < s_3$. Using the truth-telling constraint, $p(s_2,b) = p(s_3,b)$. That is, there is one price when the property is not taken, $\underline{p}(b)$, and one price when the property is taken, $\bar{p}(b)$. Consider $s = b + \varepsilon$ and $\underline{s} = b - \varepsilon$. Letting $\varepsilon \to 0$, it follows, from the truth-telling constraint, that $\bar{p}(b) - \underline{p}(b) = b$. Since the state must initiate a taking (participate) when $s > b$—otherwise an efficient taking will not occur—it follows that $s - \bar{p}(b) \geq 0$ for all $s > b$. By continuity, $\bar{p}(b) \leq b$. Since $\underline{p}(b) \geq 0$, this implies $\bar{p}(b) = b$ and $\underline{p}(b) = 0$. An efficient mechanism, therefore, requires that the citizen receive $\bar{b}$ in compensation. But this a full-compensation rule, which leads to inefficient overinvestment.

Suppose the citizen's private benefit is her private knowledge. From Corollary 1 of Myerson and Satterthwaite (1983), an efficient mechanism cannot exist because both the state and citizen must be guaranteed nonnegative *expected* utility (the citizen, in fact, is guaranteed nonnegative ex post utility, since her worst outcome is to lose her property without compensation). Q.E.D.

Proof of Proposition 6. Define $V(s,b) = x(s,b)s - p(s,b)$. Consider $s_1 > s_0$. From the truth-telling constraints,

$$x(s_1,b)s_1 - p(s_1,b) = V(s_1,b) \geq x(s_0,b)s_1 - p(s_0,b)$$

and

$$x(s_0,b)s_0 - p(s_0,b) = V(s_0,b) \geq x(s_1,b)s_0 - p(s_1,b).$$

From these two expressions,

$$s_1[x(s_1,b) - x(s_0,b)] \geq p(s_1,b) - p(s_0,b) \geq s_0[x(s_1,b) - x(s_0,b)].$$

This last expression implies that $x(s,b)$ and $p(s,b)$ are both nondecreasing in s for all b. By adding $s_0x(s_0,b) - s_1x(s_1,b)$ to each term of this last expression and dividing through by $s_1 - s_0$, one obtains

$$x(s_0,b) \geq \frac{V(s_1,b) - V(s_0,b)}{s_1 - s_0} \geq x(s_1,b).$$

Taking limits as s_1 approaches s_0 establishes that $V_s(s,b) = x(s,b)$ almost everywhere. Consequently,

$$V(s,b) = V(0,b) + \int_0^s x(z,b)\,dz.$$

Since

$$V(0,b) = 0 \cdot x(0,b) - p(0,b).$$

participation requires $p(0,b) \leq 0$; but, since compensation cannot be negative, this entails $p(0,b) = 0$. Hence, $V(0,b) = 0$.

There are two equivalent ways of expressing social welfare conditional on b and s:

$$x(s,b)s + [1 - x(s,b)]b = V(s,b) + U(s,b),$$

where $U(s,b)$ is the citizen's utility (gross of her investment). Using this last expression and substituting for $V(s,b)$, the citizen's expected utility conditional on b is

$$\int_0^S \left\{ x(s,b)s + [1 - x(s,b)]b - \int_0^s x(z,b)\,dz \right\} f(s)\,ds;$$

or, integrating by parts,

$$\int_0^S (\{x(s,b)s + [1 - x(s,b)]b\} f(s) - x(s,b)[1 - F(s)])\,ds.$$

The mechanism design problem is, thus,

$$\max_{x(\cdot,\cdot),I} \int_0^B \left\{ \int_0^S (x(s,b)s + [1 - x(s,b)]b) f(s)\,ds \right\} g(b;I)\,db - I$$

subject to the constraint that the desired investment maximize the citizen's expected utility; that is,

$$I \in \operatorname*{argmax}_{\hat{I}} \int_0^B \left\{ \int_0^S (\{x(s,b)s + [1 - x(s,b)]b\} f(s) - x(s,b)[1 - F(s)])\,ds \right\} g(b;I)\,db - \hat{I}.$$

Note that the objective function and the constraints are piecewise linear in $x(s,b)$, so the optimal $x(s,b)$ are, almost always, a corner solution (i.e., 0 or 1). Since $x(s,b)$ is nondecreasing in s, it follows that there exists an $s^*(b)$ such that $x(s,b) = 0$ if $s < s^*(b)$ and $x(s,b) = 1$ if $s > s^*(b)$. Since x takes only two values, it is readily shown—along the lines of Proposition 5—that there can be only two values of $p(s,b)$ for a given b and these two values differ by $s^*(b)$. Moreover, the smaller value is zero and the larger is $s^*(b)$. This completes the proof of the first half of Proposition 6.

Define $\zeta(b) = s^*(b) - b$. From the first part of the proposition, the mechanism-design problem can be recast as

$$\max_{I,\zeta(\cdot)} \int_0^B \left\{ bF(0) + \int_0^{b+\zeta(b)} bf(s)\,ds + \int_{b+\zeta(b)}^S sf(s)\,ds \right\} g(b;I)\,db - I$$

subject to the constraint

$$I \in \operatorname*{argmax}_{\hat{I}} \int_0^B \left\{ bF(0) + \int_0^{b+\zeta(b)} bf(s)\,ds + \int_{b+\zeta(b)}^S [s - H(s)] f(s)\,ds \right\} g(b;\hat{I})\,db - \hat{I},$$

where $H(s) = (1 - F(s))/f(s)$. A convenient change of variable is to define $i = \eta(I)$ and $I = c(i)$, where $c(\cdot) = \eta^{-1}(\cdot)$. By construction, the function $c(\cdot)$ is increasing and convex. Define

$$\bar{g}(b; i) = ig^1(b) + (1 - i)g^0(b).$$

Making this change in variable, the mechanism-design problem can be written as

$$\max_{i,\zeta(\cdot)} \int_0^B \left\{ bF(0) + \int_0^{b+\zeta(b)} bf(s)\,ds + \int_{b+\zeta(b)}^S sf(s)\,ds \right\} \bar{g}(b; i)\,db - c(i)$$

subject to the constraint

$$i \in \operatorname*{argmax}_{\hat{\imath}} \int_0^B \left\{ bF(0) + \int_0^{b+\zeta(b)} bf(s)\,ds + \int_{b+\zeta(b)}^S [s - H(s)]f(s)\,ds \right\} \bar{g}(b; \hat{\imath})\,db - \hat{\imath}.$$

Since $\partial^2 \bar{g}(b; i)/\partial i^2 \equiv 0$, it follows that the citizen's investment problem is globally concave (its second derivative is $-c''(i) < 0$). Consequently, the first-order condition can be substituted for the constraint. If λ is the Lagrange multiplier on the constraint, then the first-order conditions are

$$\int_0^B \left(\int_{b+\zeta(b)}^S [1 - F(s)]\,ds \right) \bar{g}_i(b; i)\,db - \lambda c''(i) = 0 \tag{A8}$$

and

$$\zeta(b)\bar{g}(b; i) + \lambda\{\zeta(b) - H[b + \zeta(b)]\}\bar{g}_i(b; i) = 0,$$

where I have used the first-order condition for the citizen's investment problem to simplify (A8).

By the mean-value theorem, there must exist a $b^* \in (0, B)$ such that $g_I(b^*; I) = 0$. Condition (M) ensures that b^* is unique and independent of i. From the second first-order condition, $\zeta(b^*) = 0$. Moreover, since b^* is unique and $\bar{g}(b; i)$ and $H(b)$ are strictly positive, $\zeta(b) = 0$ only if $b = b^*$. Because $\bar{g}(b; i)$ and $H(b)$ are continuous functions, it follows that $\zeta(b)$ is too. Consider $b \in (b^* - \varepsilon, b^* + \varepsilon)$. Since, for small ε, $\zeta(b) - H[b + \zeta(b)] < 0$, the sign of $\zeta(b)$ is the same as the sign of $\lambda\bar{g}_i(b; i)$. Consequently, since the sign of $\bar{g}_i(b; i)$ is the same as the sign of $b - b^*$, $\zeta(b)$ changes sign at b^*. Although it is natural to suppose that λ is negative—after all, the problem is to keep the citizen from *over*investing—this needs to be proved. To this end, suppose, instead, that $\lambda > 0$. Consequently, $\zeta(b) < 0$ if $b < b^*$ and $\zeta(b) > 0$ if $b > b^*$. It follows, then, that

$$\left\{ \int_{b+\zeta(b)}^S [1 - F(s)]\,ds \right\} \bar{g}_i(b; i) < \left\{ \int_b^S [1 - F(s)]\,ds \right\} \bar{g}_i(b; i)$$

for all b (except $b = b^*$). Since $\int_b^S [1 - F(s)]\, ds$ is a *de*creasing function of b,

$$\int_0^B \left\{ \int_{b+\zeta(b)}^S [1 - F(s)]\, ds \right\} \tilde{g}_i(b; i)\, db < \int_0^B \left\{ \int_b^S [1 - F(s)]\, ds \right\} \tilde{g}_i(b; i)\, db < 0.$$

But this and (A8) are inconsistent with $\lambda > 0$. Hence, $\lambda < 0$ and $\zeta(b) > 0$ if $b < b^*$ and $\zeta(b) < 0$ if $b > b^*$. Q.E.D.

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