## 7.7 The $\Delta F = 2$ amplitudes in the gauge-less limit

As we have seen last time:

$$\mathcal{L}_{\text{eff}}^{B^0\to\bar{B}^0} = (\bar{b}_L \gamma^\mu d_L)^2 \frac{g^4}{m_W^2} (V_{tb}^* V_{td})^2 \propto \frac{m_t^2}{16\pi^2 m_W^2}$$

$m_t$ – enhanced contribution
( "GIM" cancellation )

$$\boxed{\phi_{B_d} = 2 \cdot \arg(V_{tb}^* V_{td})}$$

This result has some peculiar features:

- non-decoupling effect of the heavy mass inside the loop
- what happens if $m_t \to \infty$ ?
- " " if $m_W \to 0$ and/or $g \to 0$ ?

To better understand the result, it is worth to repeat the calculation in a special limit: the gauge-less limit of the SM Lagrangian.

Since flavor dynamics originates from the Yukawa sector, we should be able to understand the main features of flavor-changing processes in this limit.

gauge-less limit: $\begin{cases} g \to \emptyset \\ m_W \to \emptyset \\ v \neq 0 \end{cases}$

The Higgs sector contains
1 massive state ( $h^0$ )
+
3 Goldstone bosons ( $\pi^0, \pi^\pm$ )

$$\mathcal{L}_Y^{(up)} = \bar{Q}_L Y_u u_R \phi_c$$

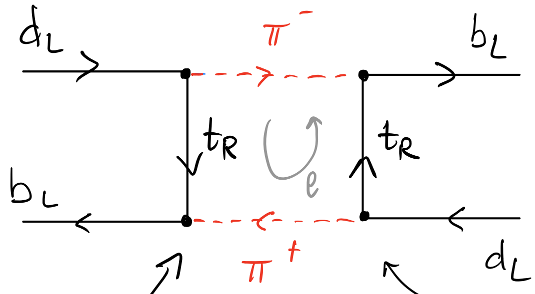
$(\bar{u}_L \quad \bar{d}_L)$

$$\begin{pmatrix} \phi^{0*} \\ \phi^- \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} v + h + i\pi^0 \\ \pi^- \end{pmatrix}$$

charged interaction term : $\frac{1}{\sqrt{2}}\, d_L^i \left(V^+ \lambda_u\right)_{ij} u_R^j \; \pi^-$

diag $(Y_u, Y_c, Y_t)$

$\approx$ diag $(0, 0, Y_t)$




$(Y_u^+)_{31} \approx Y_t V_{td}$

$(Y_u)_{33} \approx V_{33}^+ Y_t \approx V_{tb}^* Y_t$

$$M_{12} = \left[(Y_u)_{33}\,(Y_u^+)_{31}\right]^2 \frac{1}{4}\int \frac{d^4\ell}{(2\pi)^4}\left(\bar{b}_L \frac{1}{\not{\ell}-m_t} d_L\right)\left(\bar{b}_L \frac{1}{\not{\ell}-m_t} d_L\right)\frac{1}{(\ell^2)^2}$$

$$= \left[(Y_u)_{33}\,(Y_u^+)_{31}\right]^2 \frac{1}{16\pi^2}\,\frac{1}{4}\,\left(\bar{b}_L \gamma^\mu d_L\right)^2 * \frac{1}{m_t^2}$$

$$= \frac{\left(V_{tb}^* V_{td}\right)^2}{16\pi^2}\,\frac{1}{4}\,\left(\bar{b}_L \gamma^\mu d_L\right)^2 \frac{Y_t^4}{m_t^2} \searrow \frac{4\, m_t^2}{v^4} \qquad Y_t = \frac{\sqrt{2}\, m_t}{v}$$

This is the correct result obtained from the full $\mathcal{L}_{SM}$ in the limit $m_t \gg m_W$. Using $m_W^2 = g^2 v^2/4$ we get indeed

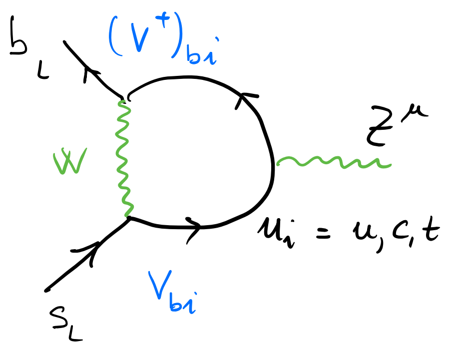
$$\frac{4\, m_t^2}{v^4} = \frac{g^4}{m_W^4}\, m_t^2 \longrightarrow \text{result obtained in last lecture}$$

N.B.:
- the non-decoupling behaviour for $m_t \to \infty$ is due to the fact that we need to send $Y_t \to \infty$ to obtain this limit.
- the dependence from $m_W$ is fictitious
- the GIM cancellation is also somehow fictitious

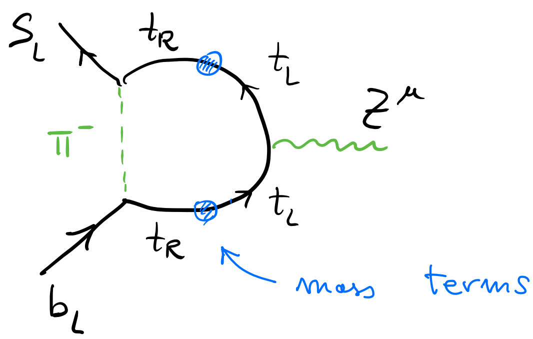
## 7.8 A closer look to FCNC processes

Having understood the usefulness of the gauge-less limit in flavor physics, it is worth to go back to analyze the structure of $\Delta F = 1$ FCNC amplitudes

### A) The "Z penguin"




$\Rightarrow$



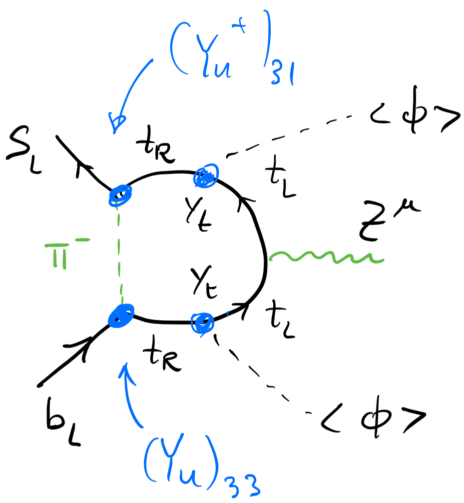

Similarly to the box amplitude in $\Delta F = 2$, also in this case we find an amplitude that grows with $y_t$

$$A \sim \frac{1}{16\pi^2 \cancel{m_t^2}} \, y_t^2 \, V_{tb}^* V_{ts} \, \cancel{m_t^2} \; \bar{b}_L \gamma^\mu s_L \; Z_\mu$$

$$\downarrow$$

$$\mathcal{L}_{eff} = \frac{1}{16\pi^2} \, y_t^2 \, V_{tb}^* V_{ts} \; \underbrace{\bar{b}_L \gamma^\mu s_L \; \phi^\dagger D_\mu \phi}_{d=6}$$

Indeed we do not modify the $d=4$ coupling of the Z boson to fermions (protected by gauge invariance) but we construct an effective $d=6$ operator with 2 Higgs fields, that after the spontaneous symmetry breaking effectively generates a flavor-changing coupling of the Z to quarks



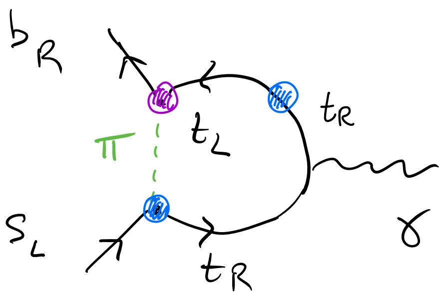

General argument: all non-trivial flavor-changing amplitudes generated beyond tree-level ($\Delta F = 2$, FCNC) correspond to $d=6$ eff. operators (generalised Fermi theory)

## B) The "$\gamma$ penguin"

The only allowed $d=6$ effective couplings between the $\gamma$ field and 2 quark fields (e.g. $b$ & $s$) are:

$$\bar{b}_R \sigma^{\mu\nu} s_L F^{\mu\nu} \phi^{+} \qquad \& \qquad \bar{b}_L \sigma^{\mu\nu} s_R F^{\mu\nu} \phi$$

Proceeding as before, we can identify the leading dependence from the Yukawa couplings:




$$\Rightarrow \quad \underset{\downarrow \; y_b}{(Y_d)_{33}} \quad \underset{\downarrow \; y_t V_{tb}^*}{(Y_u)_{33}} \quad \underset{\searrow \; y_t V_{ts}}{(Y_u^+)_{31}}$$

$$A \sim \frac{1}{16\pi^2 \cancel{m_t^2}} \underbrace{y_b \, \cancel{y_t^2} (V_{tb}^* V_{ts}) \, \bar{b}_R \sigma^{\mu\nu} s_L F^{\mu\nu} \phi}_{\text{VEV} \to m_b}$$

In this case the amplitude is <u>not</u> fully dominated by short-distance contributions (finite for $y_t \to \infty$)

$\hookrightarrow$ we can expect sizable corrections by the diagram where we replace $t$ with $c$ inside the loop (long-distance dynamics)

$$\Downarrow$$

The peculiar $y_t$ behaviour of $\Delta F = 2$ amplitudes & $Z$-penguin is <u>unique</u>

These are effective interactions whose strength is completely dominated by shor-distance physics (precisely calculable & highly sensitive to physics beyond SM)

## 7.9 Summary of accidental SM (global) symmetries

$$\mathcal{L}_{SM} = \mathcal{L}_{gauge} + \mathcal{L}_{Higgs}$$

$$\overbrace{\mathcal{L}_{YM}(A^a) + \underline{\mathcal{L}_\psi^{kin}(A^a, \psi^i)}}^{} + \overbrace{\underline{\mathcal{L}_H^{kin}(\phi, A^a) + \mathcal{L}_H^{pot}(\phi)} + \underline{\mathcal{L}_Y(\phi, \psi^i)}}^{}$$

Symmetry breaking terms

Flavor Symmetry

$$\boxed{U(3)^5 = U(3)_{Q_L} \times U(3)_{u_R} \times U(3)_{d_R} \times U(3)_{L_L} \times U(3)_{e_R}}$$

$$\psi_i \to V_{ij}^{(\psi)} \psi_j \qquad VV^\dagger = \mathbb{1}$$

$$\psi = Q_L, u_R, d_R, L_L, e_R$$

Custodial symmetry
(in the limit $g_Y \to 0$)

$$\boxed{SU(2)_L \times SU(2)_R \xrightarrow[VEV]{Higgs} SU(2)_{L+R}}$$

$$\Sigma = (\phi_c, \phi) \qquad \Sigma \to V_L \Sigma V_R^\dagger$$

N.B.:

- Some subgroups of these global symmetries are gauged:
  - $U(1)_Y$ = subgroup of both flavor & custodial symm.
  - $SU(2)_L$ = subgroup of custodial symm.

- The Yukawa interaction breaks both custodial & flavor symmetries. Beside the gauged $SU(2)_L \times U(1)_Y$, the only remaining exact (unbroken) global symmetries are

$$U(1)_B \times \underbrace{U(1)_{L_e} \times U(1)_{L_\mu} \times U(1)_{L_\tau}}_{}$$

baryon number — 3 separate lepton numbers (broken by neutrino masses)

- The smallness of the Yukawa couplings for fermions of the first 2 generations, and the smallness of the $3 \to 1,2$ elements of the CKM matrix are responsible for the smallness of FCNC $\Rightarrow$ almost exact residual $U(2)^5$ flavor symmetry acting on the light generations