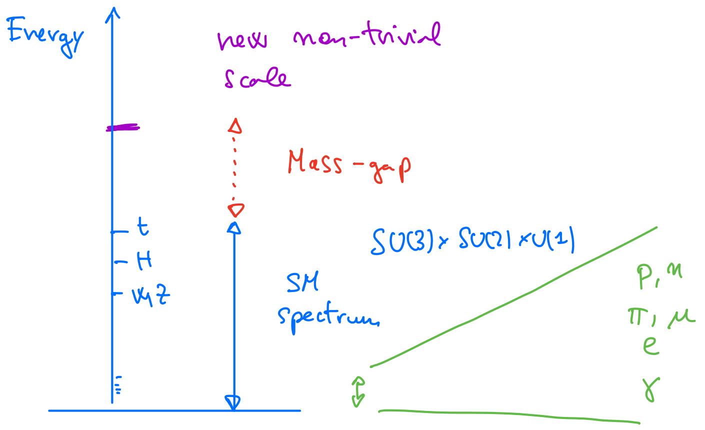
8. The SM as an effective theory

8.1 Generalities

The modern point of view on the SM is that this is the low-energy limit of a theory containing more degrees of freedom



low-energy limit of the SM

$\underbrace{U(1)_Q}$ QED $\oplus$ $\underbrace{SU(3)_c}$ QCD + Fermi theory

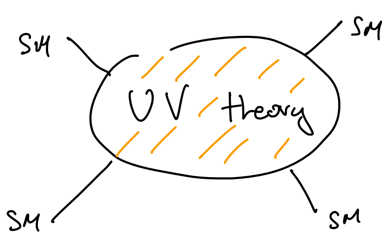
- Non-renormalizable interaction written in terms of the light-states
- $G_F \sim M_{heavy}^{-2}$

Similarly to the case of the low-energy limit of the SM, we can think there is a theory "above" the SM (the UV completion of the SM)

UV theory
(unknown)

↙ light d.o.f → Ψ_{SM}, Φ

↘ unbroken symm. → $SU(3)_c \times SU(2)_L \times U(1)_Y$



low-energy limit $\Rightarrow$ $\times$ $= \mathcal{L}_{SM}$ + "tower" of higher-dim. ops. suppressed by Λ^{-n}

$$\mathcal{L}_{SM-EFT} = \underbrace{\mathcal{L}_{gauge} + \mathcal{L}_Y + \mathcal{L}_{Fermion}^{Kin.}}_{\text{only } d=4 \text{ ops.}} + \mathcal{L}_\phi + \sum_{d>4}\sum_i \frac{c_i^{[d]}}{\Lambda^{d-4}} O^i(\psi_{SM}, \phi, A^a)^{[d]}$$

only $d=4$ ops. $\downarrow$ adim. couplings.

$\mathcal{L}_\phi \uparrow \mu^2 \phi^\dagger \phi$

Infinite list of operators (finite at any fixed order in d) satisfying

- Gauge symm. $SU(3)_c \times SU(2)_L \times U(1)_Y$
- Local interactions of $\psi_{SM} \oplus \phi \oplus A^a$
- Lorentz invariance
- Global/approximate symm. (?)

Which is the (lowest) value of Λ, and which is the structure of the $c_i^{[d]}$ are the big open questions in particle physics

N.B.: If we probe processes at $E \ll \Lambda$ this construction is predictive (as it is the Fermi theory at low energies), given finite order of operators ($\leftrightarrow$ couplings) at finite d ($\leftrightarrow$ precision in E/Λ)

8.2 The Higgs hierarchy problem

All the SM couplings, except the Higgs mass term, are dimension-less couplings

$\Rightarrow$ Logaritmic evolution under RGE

$$\mu^2 \frac{d}{d\mu^2} g_i = \beta_{ij}(\{g\})\, g_j \longrightarrow g_i(v) \approx g_i(\Lambda) + \underset{\uparrow}{\beta_{ij}}\, g_j \log(v^2/\Lambda^2)$$

$$O\left(\frac{g_i^2}{16\pi^2} = \frac{\alpha_i}{4\pi}\right)$$

$$\text{gauge}\begin{cases} g_3(v) \approx 1 \\ g_2(v) \approx 2/3 \\ g_1(v) \approx 1/3 \end{cases}$$

$Y_t \simeq 1 \qquad \lambda \approx 0.12$

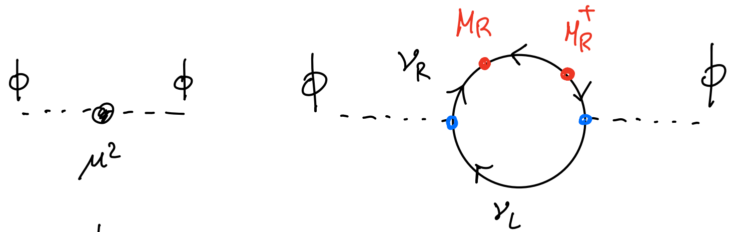
$\Rightarrow$ relatively small change over several orders of magnitude

The only exception is the Higgs mass term:

$$\mu^2(v) = \mu^2(\Lambda) + \underbrace{\frac{c^2}{16\pi^2}(\Lambda^2 - v^2)}_{\text{quadratic sensitivity to the cut-off}} + \log \text{ terms}$$

The quadratic cut-off dependence signals a quadratic dependence of the renormalized value of μ^2 at low energies from possible physical high scales in the theory

E.g. $\mathcal{L}^{NP} = \mathcal{L}^{SM} + \underbrace{\mathcal{L}_{\nu_R}}_{\text{heavy RH neutrino}}$ $\qquad \mathcal{L}_{\nu_R} = \bar{\Psi}_R \not{D} \Psi_R - M_R \nu_R^T \nu_R + \underbrace{Y_\nu \bar{\nu}_L \phi \nu_R}_{\text{Yukawa interaction}}$



$$\Downarrow \qquad\qquad\qquad\qquad \Downarrow$$

$$\mu^2(\Lambda) \qquad \delta\mu^2(\Lambda) \sim \frac{Y_\nu^2}{16\pi^2}\left[|M_R|^2 \log\left(\frac{M_R^2}{\Lambda^2}\right) + \Lambda^2 \right] + \dots$$

If $\frac{Y_\nu M_R}{4\pi} \gg m_H$ we need to fine-tune the values in the Lagrangian in order to reproduce the observed value of m_H

$$\Downarrow$$

General argument why we expect some form of New Physics not far from the TeV scale $\rightarrow$ stabilization of the Higgs sector

8.3 Classifying the ops. of $d>4$ in the SMEFT

1) $\phi^n \rightarrow (\phi^\dagger\phi)\ ,\ (D_\mu\phi D^\mu\phi^\dagger) \rightarrow d \geq 6$

2) $\psi^n \rightarrow (\bar\psi_L \gamma^\mu \psi_L)^k \rightarrow d \geq 6$

4) $A^n \rightarrow F^{\mu\nu}\ ,\ D_\mu F^{\mu\nu} \rightarrow d \geq 6$

5) $A^n, \phi^m \rightarrow (\phi^\dagger\phi)\ ,\ F_{\mu\nu}\ ,\ D_\mu \rightarrow d \geq 6$

6) $A^n, \psi^m \rightarrow \cancel{\bar\psi_L \sigma_{\mu\nu}\psi_R F^{\mu\nu}}$ not G.I $\rightarrow \bar\psi_L \sigma^{\mu\nu}\psi_R\, \phi\, F_{\mu\nu} \rightarrow d \geq 6$

7) $\psi, \phi \rightarrow$ one (and only one) $d=5$ term $\boxed{L_L^T\, \phi_c^*\phi_c\, L_c}$

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM}^{(d\leq 4)} + \mathcal{L}_\nu^{(d=5)} + \mathcal{L}^{(d\geq 6)}$$

$$\uparrow$$

$$\frac{g_\nu}{\Lambda_{LN}}\left(L_L^T\, \phi_c^*\phi_c\, L_L\right) \rightarrow \underline{\text{neutrino masses!}}$$

$$m_\nu \sim \frac{g_\nu v^2}{\Lambda_{LN}}\bigg|_{g_\nu\sim 1} \sim \sqrt{\Delta m^\nu_{atm}} \sim 10^{-1}\ eV$$

$$\Lambda_{LN} \sim 10^{15}\ GeV$$

The only allowed $d=5$ op. in $\mathcal{L}_{SMEFT}$ is a bit special since it violates total lepton number (<u>exact symmetry</u> of $\mathcal{L}_{SM}^{d\leq4}$)

It's possible to conceive lower scales of NP, which do not alter the structure of $\mathcal{L}^{d=5}$ ($\rightarrow$ small m_ν), provided LN remains unbroken.

This is why we put the label "LN" on the scale of the $d=5$ operator

A very similar argument holds for operator violating baryon number (appearing at $d=6$)

We can now look at the structure of the $d=6$ operators

$\Rightarrow$ Few independent electroweak structures (see tables) but <u>2499</u> new free couplings (mainly flavor sector)

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\varepsilon_{jk}(\bar{q}_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\varepsilon_{jk}(\bar{q}_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$		

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar{l}_p e_r\varphi)$
$Q_{\widetilde{G}}$	$f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}_p u_r\widetilde{\varphi})$
Q_W	$\varepsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star\left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}_p d_r\varphi)$
$Q_{\widetilde{W}}$	$\varepsilon^{IJK}\widetilde{W}_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$				

$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar{l}_p\sigma^{\mu\nu}e_r)\tau^I\varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{l}_p\gamma^\mu l_r)$
$Q_{\varphi\widetilde{G}}$	$\varphi^\dagger\varphi\,\widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar{l}_p\sigma^{\mu\nu}e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I\varphi)(\bar{l}_p\tau^I\gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar{q}_p\sigma^{\mu\nu}T^A u_r)\widetilde{\varphi}\, G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{e}_p\gamma^\mu e_r)$
$Q_{\varphi\widetilde{W}}$	$\varphi^\dagger\varphi\,\widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar{q}_p\sigma^{\mu\nu}u_r)\tau^I\widetilde{\varphi}\, W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{q}_p\gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu}B^{\mu\nu}$	Q_{uB}	$(\bar{q}_p\sigma^{\mu\nu}u_r)\widetilde{\varphi}\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I\varphi)(\bar{q}_p\tau^I\gamma^\mu q_r)$
$Q_{\varphi\widetilde{B}}$	$\varphi^\dagger\varphi\,\widetilde{B}_{\mu\nu}B^{\mu\nu}$	Q_{dG}	$(\bar{q}_p\sigma^{\mu\nu}T^A d_r)\varphi\, G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{u}_p\gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar{q}_p\sigma^{\mu\nu}d_r)\tau^I\varphi\, W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{d}_p\gamma^\mu d_r)$
$Q_{\varphi\widetilde{W}B}$	$\varphi^\dagger\tau^I\varphi\,\widetilde{W}_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar{q}_p\sigma^{\mu\nu}d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde{\varphi}^\dagger D_\mu\varphi)(\bar{u}_p\gamma^\mu d_r)$

The fact that a large number of free parameters is connected to the flavor sector emerges more clearly by the following table:

Imposing $U(3)^5$ but allowing only linear terms in the 3 Yukawa coupl. as breaking terms

Imposing exact $U(3)^5$ flavor symm.

No flavor symmetry for 3 or 1 gen.

Class	Operators	3 Gen.		1 Gen.		Exact		$\mathcal{O}(Y^1_{e,d,u})$	
1–4	X^3, H^6, H^4D^2, X^2H^2	9	6	9	6	9	6	9	6
5	$\psi^2 H^3$	27	27	3	3	–	–	3	3
6	$\psi^2 XH$	72	72	8	8	–	–	8	8
7	$\psi^2 H^2 D$	51	30	8	1	7	–	7	–
	$(\bar{L}L)(\bar{L}L)$	171	126	5	–	8	–	8	–
	$(\bar{R}R)(\bar{R}R)$	255	195	7	–	9	–	9	–
8	$(\bar{L}L)(\bar{R}R)$	360	288	8	–	8	–	8	–
	$(\bar{L}R)(\bar{R}L)$	81	81	1	1	–	–	–	–
	$(\bar{L}R)(\bar{L}R)$	324	324	4	4	–	–	–	–
total:		1350	1149	53	23	41	6	52	17

◯ = N. of independent real couplings (CP invariant)

◯ = N. of independent imaginary couplings (CP violation)

$\Downarrow$

Most of the SMEFT parameters appearing at $d=6$ control the breaking of the $U(3)^5$ flavor symmetry