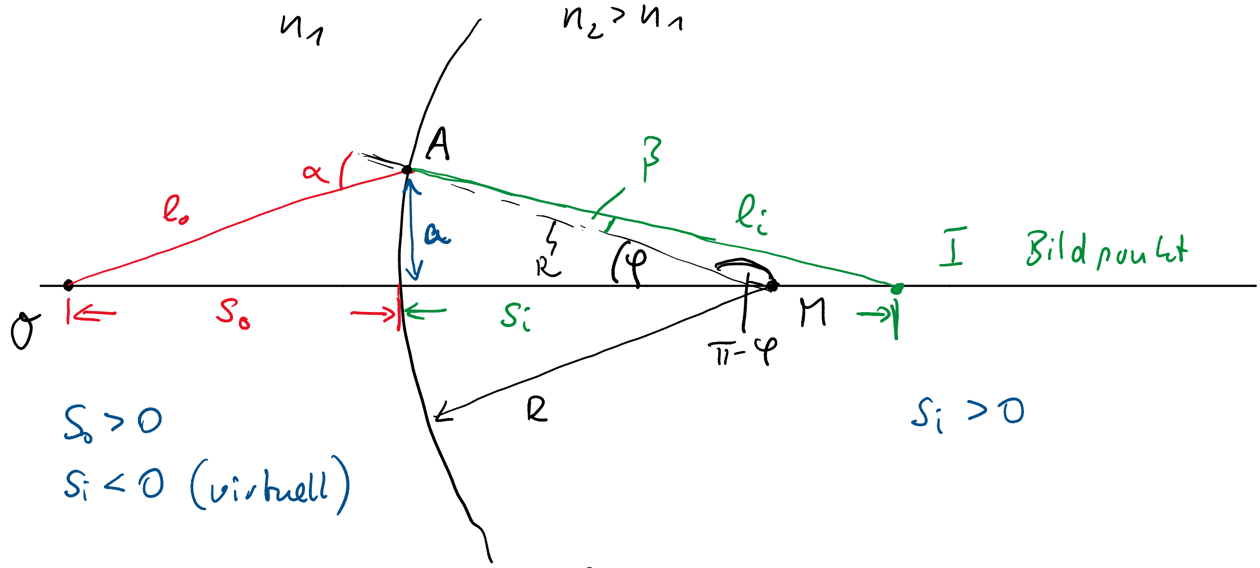
Die Linsengleichungen



$S_o > 0$

$S_i < 0$ (virtuell)

$S_i > 0$

Parameter: φ: $A(\varphi)$

OWL: $S = n_1 \ell_o + n_2 \ell_i$

ΔOAM: Kosinussatz, $\ell_o^2 = R^2 + (s_o+R)^2 - 2R(s_o+R)\cos\varphi$

ΔAMI: $\ell_i^2 = R^2 + (s_i - R)^2 - 2R(s_i-R)\cos(\pi-\varphi)$

$= R^2 + (s_i-R)^2 + 2R(s_i-R)\cos\varphi$

$$\frac{d(\ell_o^2)}{d\varphi} = \frac{d(\ell_o^2)}{d\ell_o} \cdot \frac{d\ell_o}{d\varphi} = 2\ell_o \frac{d\ell_o}{d\varphi}$$

$$= -2R(S_o+R)\,(-\sin\varphi)$$

$$\frac{d(\ell_i^2)}{d\varphi} = 2\ell_i \frac{d\ell_i}{d\varphi}$$

$$= 2R(s_i-R)\,(-\sin\varphi)$$

Fermat'sches Prinzip

$$\frac{ds}{d\varphi} \stackrel{!}{=} 0 = n_1 \cdot \frac{d\ell_o}{d\varphi} + n_2 \frac{d\ell_i}{d\varphi}$$

$$= \frac{n_1}{2\ell_o} \quad 2R(S_o+R)\sin\varphi - \frac{n_2}{2\ell_i}\, 2R(S_i-R)\sin\varphi$$

$$= R\sin\varphi \underbrace{\left(\frac{n_1}{\ell_o}(S_o+R) - \frac{n_2}{\ell_i}(S_i-R)\right)}_{=0} \overset{!}{=} 0$$

$$\Rightarrow \quad \frac{n_1 S_o}{\ell_o} - \frac{n_2 S_i}{\ell_i} + R\left(\frac{n_1}{\ell_o} + \frac{n_2}{\ell_i}\right) = 0$$

$$\Rightarrow \quad \frac{n_1}{\ell_o} + \frac{n_2}{\ell_i} = -\frac{1}{R}\left(\frac{n_1 S_o}{\ell_o} - \frac{n_2 S_i}{\ell_i}\right)$$

Näherung achsennaher Strahlen (engl. paraxial rays)

$$\Rightarrow \quad \varphi \approx 0 \qquad a \ll S_o, S_i, R$$

$$\Rightarrow \quad \ell_o \approx S_o \quad \wedge \quad \ell_i \approx S_i$$

$$\boxed{\frac{n_1}{S_o} + \frac{n_2}{S_i} = \frac{n_2 - n_1}{R}}$$

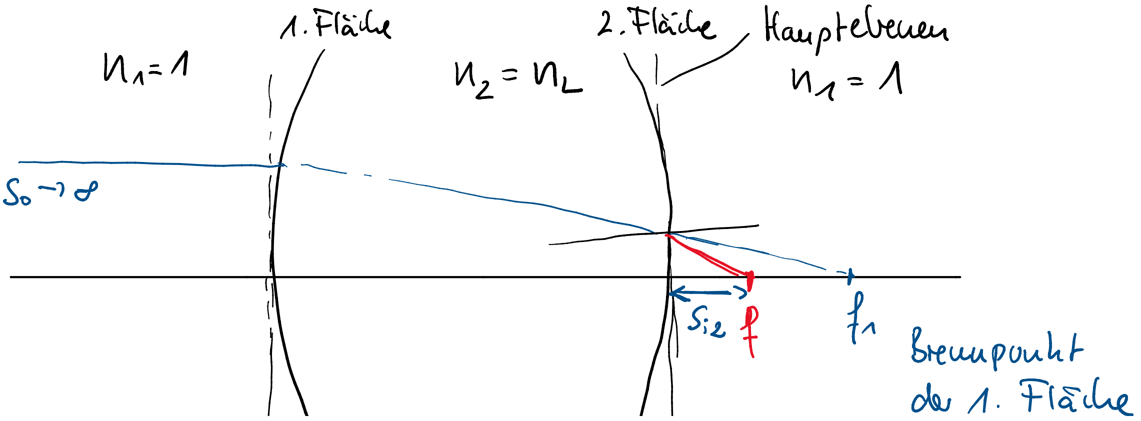
Brennweite f_i: $\quad S_o \to \infty \quad \Rightarrow \quad f_i := S_i(S_o \to \infty) = \frac{n_2}{n_2 - n_1} R$

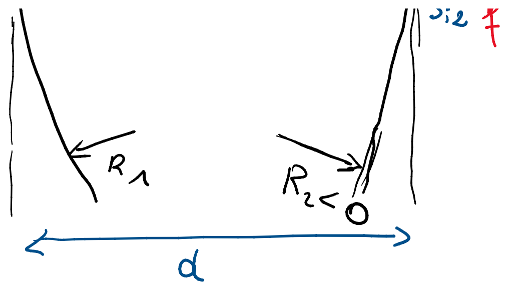
bildseitiger Brennpunkt.

f_o: $\quad S_i \to \infty \quad \Rightarrow \quad f_o := \frac{n_1}{n_2 - n_1} R$

gegenstandsseitiger Brennpunkt

Linse: 2 Grenzflächen





f_1 Brennpunkt der 1. Fläche

1. Fläche: $\frac{1}{s_{o1}} + \frac{n_L}{s_{i1}} = \frac{n_L - 1}{R_1}$

2. Fläche: $\frac{n_L}{s_{o2}} + \frac{1}{s_{i2}} = \frac{1 - n_L}{R_2}$

(+)

Bild der 1. Fläche = Gegenstand der 2. Fläche

$$s_{o2} = d - s_{i1}$$

$$\underbrace{\frac{1}{s_{o1}}}_{=\frac{1}{g}} + \frac{n_L}{s_{i1}} + \frac{n_L}{d - s_{i1}} + \underbrace{\frac{1}{s_{i2}}}_{=\frac{1}{b}} = (n_L - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Gegenstandsweite Bildweite

$$\frac{1}{g} + \frac{1}{b} = (n_L - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + \frac{n_L d}{s_i (d - s_{i1})}$$

$$\overset{d \to 0}{\approx} (n_L - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) =: \frac{1}{f} = \frac{1}{f_o} = \frac{1}{f_i}$$

Brennweite einer dünnen Linse

$$\boxed{\frac{1}{g} + \frac{1}{b} = \frac{1}{f}}$$ Abbildungsgleichung

$$\boxed{\frac{1}{f} = (n_L - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + \frac{(n_L - 1)^2}{n_L R_1 R_2} d}$$ Linsenmacher-

$$\boxed{\frac{1}{f} = (n_L-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) + \frac{(n_L-1)^2}{n_L R_1 R_2}\, d}$$ Linsenmacher-Gleichung

<u>Brechkraft</u> $D := \frac{1}{f}$ $\qquad [D] = \frac{1}{m} = 1\ \text{Dpt}$ Dioptrie

1. Fläche: $D_1 = \frac{1}{f_1} = (n_L-1)\frac{1}{R_1}$ (gewölbt.)

2. Fläche: $D_2 = \frac{1}{f_2} = (1-n_L)\frac{1}{R_2}$ (Bildseitige)

Gesamtbrechkraft $\boxed{D = \frac{1}{f} = D_1 + D_2 - \frac{d}{n_L} D_1 \cdot D_2}$

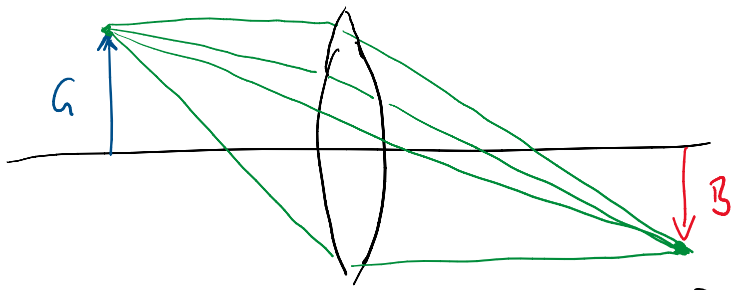
Korrektur

Brechkräfte addieren sich

Kombination zweier dünner Linsen im Abstand d

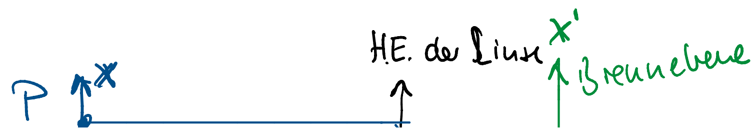
$$D = D_1 + D_2 - d\, D_1 \cdot D_2$$

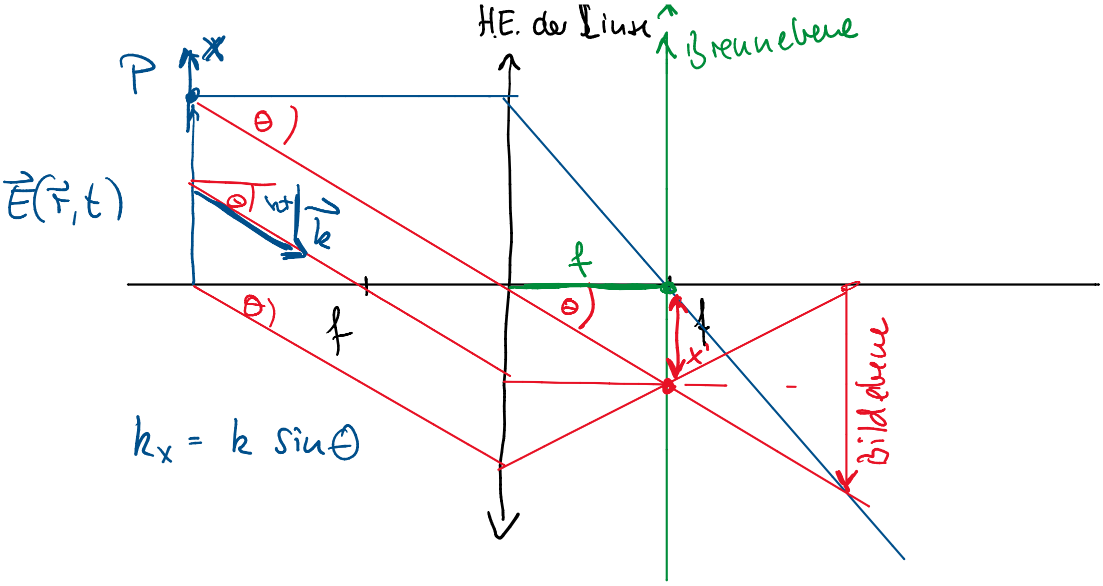
zB Brille, Kontaktlinse, Mikroskop



Fermat: Alle Strahlen von G nach B brauchen gleich viel Zeit!

Brennebene einer Linse





$x' = f \tan\theta \approx f \sin\theta$ für θ klein

$x' = \frac{f}{k} k_x = \underbrace{\frac{f\lambda}{2\pi}}_{\text{Konstante}} k_x$

Gegenstandsebene $\vec{E}(\vec{r},t)$

$\Rightarrow$ Brennebene $\Rightarrow$ $\boxed{\vec{E}(\vec{k}) = \text{FT der Verteilung in der Gegenstandsebene!}}$

Fourier-Ebene einer Linse.