Efficient Matching and Merging with Sector-Antenna Showers

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TPP Seminar
05/10/2021

Outline

1) Antenna showers on sectorised phase spaces [Brooks, CTP, Skands 2003.00702]

2) Efficient (CKKW-L-style) merging with sector showers [Brooks, CTP 2008.09468]

- (POWHEG also possible but not shown here; see [Höche, Mrenna, Payne, CTP, Skands 2106.10987])

3) Towards NNLO+PS matching with sector showers [Campbell, Höche, Li, CTP, Skands 2108.07133]

VINCIA overview

C. T. Preuss, **P. Skands**, R. Verheyen

- **originally** developed as plug-in to PYTHIA 8.2 (started $\sim$ 2007 by P. Skands)
- **now** part of PYTHIA 8.3 (since October 2019) as one of three showers:
  - "simple" shower (PYTHIA's $p_{\perp}$-ordered DGLAP shower)
  - VINCIA
  - DIRE
- full-fledged **antenna** shower for ISR, FSR, coloured resonances (top)
- **exact** treatment of **mass corrections** (phase space and antenna functions)
- full **helicity dependence** in shower and MECs
- dedicated **default tuning** (similar to PYTHIA's Monash tune)

Recent and ongoing developments:

- interleaved resonance decays for top, Z, W (P. Skands, R. Verheyen)
- interleaved coherent QED multipole shower (P. Skands, R. Verheyen)
- full-fledged (collinear) EW shower module (P. Skands, R. Verheyen)
- QCD **sector showers** $\rightarrow$ this talk
- **efficient merging** (CKKW-L) with **sector showers** $\rightarrow$ this talk
- automated **matrix element corrections** (MECs) $\rightarrow$ this talk
- **(N)NLO matching** with sector showers $\rightarrow$ this talk

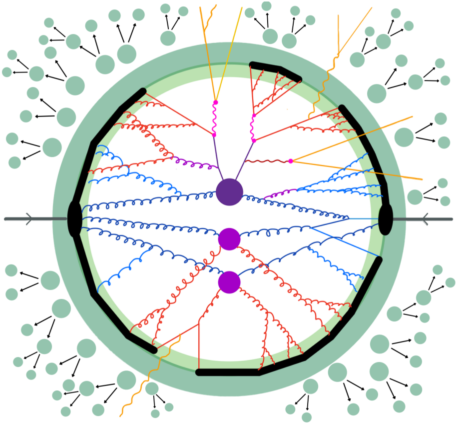
What is an event generator?

Particle-level event generators aim at **simulating** high-energy particle collisions in **full detail** by dividing events into three **energy regimes**:

Hard regime
(multiple) high-energy $2 \to n$ processes with small n

Soft regime
forming and fragmentation of (visible) hadrons at low energies

Transition regime
QCD bremsstrahlung (+ QED/EW emissions)



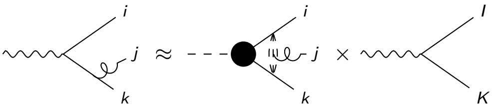
Sketch by Peter Skands

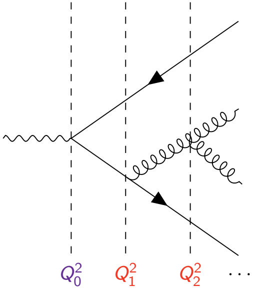
The "big players": PYTHIA, SHERPA, HERWIG

Parton showers

Parton showers dress a LO calculation with **additional radiation**, describing the evolution from **parton level** (quarks, gluons, ...) to the **particle level** (hadrons).

- amplitudes **factorise** in limits where emissions are **soft** ($E_j \to 0$) or **collinear** ($\vartheta_{jk} \to 0$)



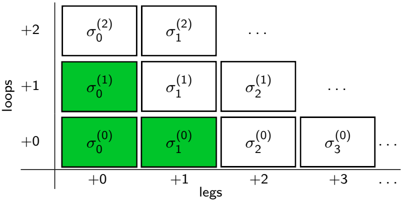


- starting from a **high scale** Q_0^2, further radiation is modelled under the assumption that it is **soft**/**collinear** and **ordered** ($Q_0^2 > Q_1^2 > Q_2^2 > \cdots > Q^2_{\mathrm{had}}$)
- evolves event from hard scale Q_0^2 to soft scale Q^2_{had} but introduces **logarithms** of the form

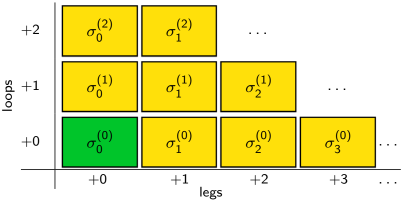
$$\alpha_{\mathrm{S}}^n \to \alpha_{\mathrm{S}}^n \log^m \left(\frac{Q_0^2}{Q^2} \right), n \leq 2m, \quad \textbf{large if } Q^2 \ll Q_0^2$$

Parton showers vs fixed-order calculations

NLO



LO+PS

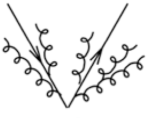


Fixed-order calculations → hard jets

- reliable at **high scales** if **no large scale hierarchies** are present
- **accurate** predictions for **limited number** of legs (+ loops)
- determines **perturbative accuracy** (LO, NLO, NNLO, . . .)

Showers → jet substructure

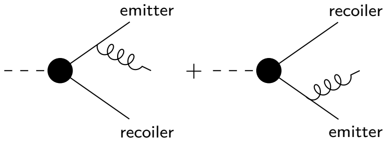
- reliable in **soft/collinear** regions if **large scale hierarchies** are present
- **approximate** predictions for **many** particles
- determines **logarithmic accuracy** (LL, NLL, NNLL, . . .)



⇒ largely **complementary**, so ideally **combine them**!

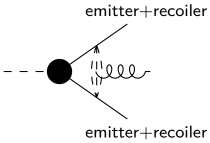
Many ways to skin a cat...

Dipoles



- e.g. SHERPA CSS, HERWIG dipole, DIRE
- **distinguish** emitter and recoiler
- **two** branching kernels per colour dipole
- partition **soft eikonal**
- related to **NLO dipole subtraction** [Catani, Seymour hep-ph/9605323]

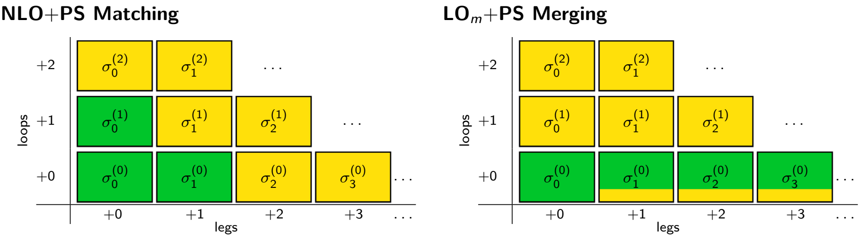
Antennae



- e.g. ARIADNE, VINCIA
- **both parents** absorb transverse recoil
- **one** branching kernel per colour dipole
- partition **collinear singularities**
- related to **(N)NLO antenna subtraction** [Campbell, Cullen, Glover hep-ph/9809429] [Gehrmann-de Ridder, Gehrmann, Glover hep-ph/0505111]

Not mentioned here: **DGLAP** showers (PYTHIA $p_\perp$, HERWIG $\tilde{q}$)

Combining showers and fixed-order calculations



Some disambiguation:

Matching combine a fixed-order (typically NLO) calculation with a parton shower, **avoiding double-counting** in **overlap regions**

Merging combine **multiple** inclusive (N)LO event samples into a **single** inclusive one with additional **shower radiation**, **accounting** for **Sudakov suppression** and **avoiding double-counting** in **overlap regions** (typically via **phase-space slicing**)

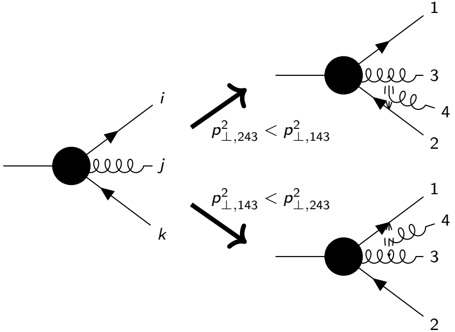
⇒ can we define a shower **designed** for **matching and merging**?

Part I: Sector-Antenna Showers

Sector showers [Brooks, CTP, Skands 2003.00702; Lopez-Villarejo, Skands 1109.3608]

Idea: combine antenna shower with deterministic jet-clustering algorithm

- let shower only generate emissions that would be clustered by a $(3 \mapsto 2)$ jet algorithm ($\sim$ ARCLUS [Lönnblad Z.Phys.C 58 (1993)])



$$p^2_{\perp,ijk} = \frac{2(p_i \cdot p_j)(p_j \cdot p_k)}{p_i \cdot p_j + p_j \cdot p_k + p_i \cdot p_k}$$

⇒ **softest gluon** always regarded as the emitted one

⇒ only **one** (most singular) splitting kernel contributes per phase space point

Since PYTHIA 8.304: **full-fledged*** implementation of sector showers in VINCIA

*including FSR, ISR, resonance-decay showers

Interlude: why sector showers?

A variety of showers has been developed to date, so why another one?

- conceptually **simpler structure** (only a single branching per PS point)
- clear **scale definitions** due to deterministic clusterings
- simple removal of **overlaps**

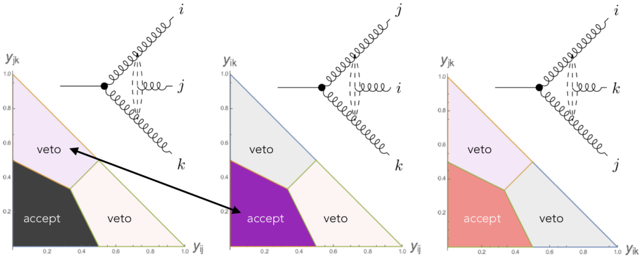
$\Rightarrow$ **full numerical control** over shower domain & evolution

$\Rightarrow$ sector showers are **designed** for matching & merging

Phase space sectors

Branching phase space gets divided into **non-overlapping sectors**.

- e.g. first emission in $H \to gg$:



- branchings in the shower are accepted **if and only if** they correspond to the **correct sector**
- sectors defined by minimal $p_\perp$ in event, but always contain:
  - soft endpoint
  - "full" collinear region for qg
  - "half" of the collinear region for gg with boundary at $z = \frac{1}{2}$

Note: in general, non-trivial sector boundaries away from the singular limits!

Sector antenna functions

Splitting kernels have to incorporate **full** single-unresolved limits for given PS point (Kosower subtraction terms [Kosower PRD 57 (1998) 5410, PRD 71 (2005) 045016])

- e.g. (FF) $qg \mapsto qgg$ ($s_{ij} = 2p_i \cdot p_j$):

$$A^{\mathrm{sct}}_{qg\mapsto qgg}(i_q, j_g, k_g) \to \begin{cases} \frac{2s_{ik}}{s_{ij}s_{jk}} & \text{if } j_g \text{ soft} \\ \frac{1}{s_{ij}}\frac{1+z^2}{1-z} & \text{if } i_q \parallel j_g \\ \frac{1}{s_{jk}}\frac{2(1-z(1-z))^2}{z(1-z)} & \text{if } j_g \parallel k_g \end{cases}$$

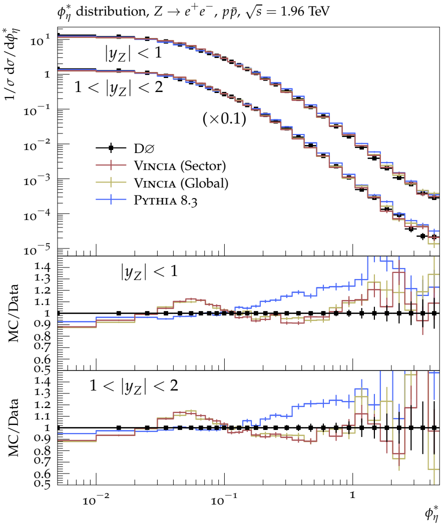
Compare to **global** antenna functions (a.k.a. *sub-antenna* functions):

- only "half" of the $j_g \parallel k_g$ limit contained in the splitting kernel:

$$A^{\mathrm{gl}}_{qg\mapsto qgg}(i_q, j_g, k_g) \to \begin{cases} \frac{2s_{ik}}{s_{ij}s_{jk}} & \text{if } j_g \text{ soft} \\ \frac{1}{s_{ij}}\frac{1+z^2}{1-z} & \text{if } i_q \parallel j_g \\ \frac{1}{s_{jk}}\frac{1+z^3}{1-z} & \text{if } j_g \parallel k_g \end{cases}$$

- "rest" of the jk-collinear limit reproduced by neighbouring antenna ($z \leftrightarrow 1 - z$)

Sector showers vs global showers



The sector approach is merely an **alternative way** to fraction singularities, so **formal accuracy** of the shower should be **retained**.

Note: same "global shower" tune in Vincia, no MECs here

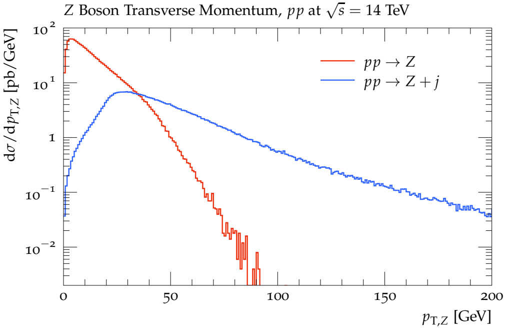
Part II: Efficient Merging with Sector Showers

Merging with traditional showers: illustration

Merging: introduce (arbitrary) **merging scale** and let each calculation populate the phase space where it does best:

Parton shower generates **soft/collinear** radiation → reject hard branchings ☺

Fixed-order calculation generates **hard** jet(s) → reconstruct "shower history" ☹



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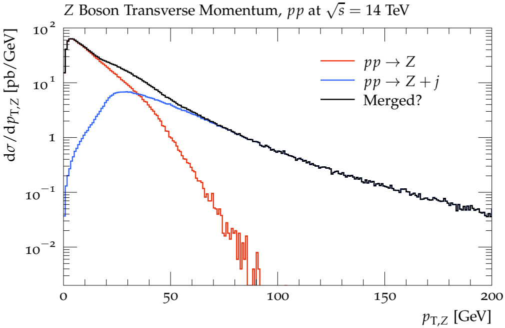

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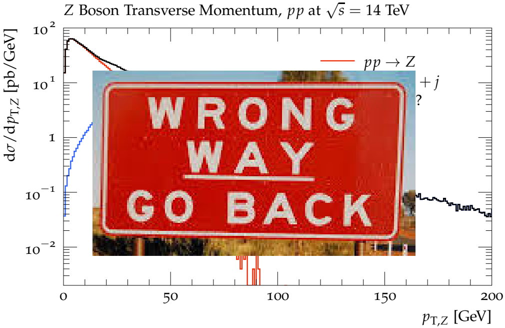

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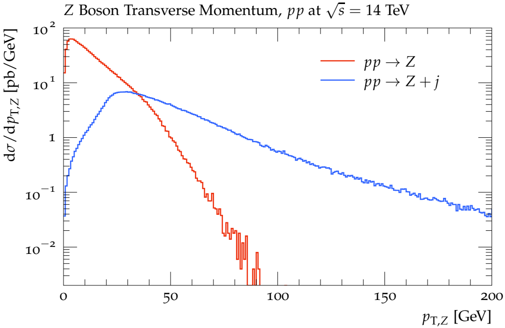

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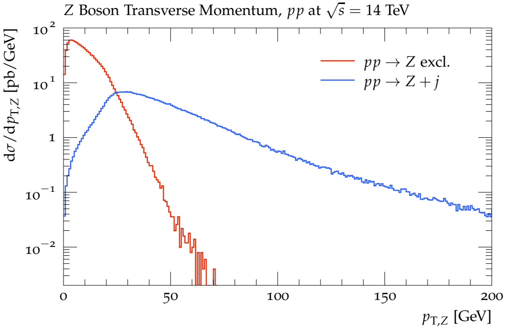

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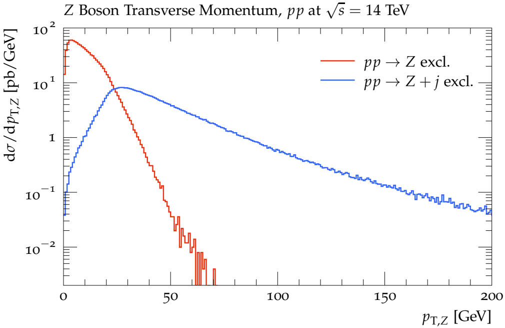

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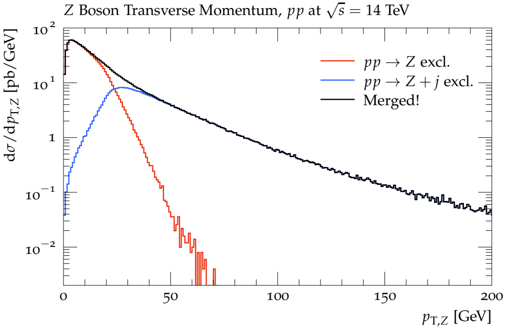
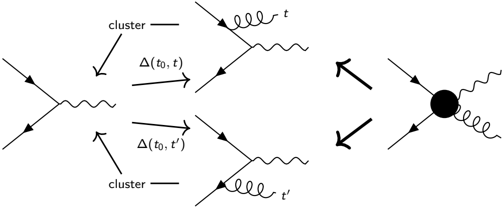


Merging with traditional showers: CKKW-L

Basic CKKW-L idea [Catani, Krauss, Kuhn, Webber hep-ph/0109231], [Lönnblad hep-ph/0112284]

- construct **all possible** shower histories, choose **most likely**
- let (truncated) **trial showers** generate Sudakov factors
- re-weight event by Sudakov factors



- number of histories **scales factorially** with number of legs

Number of Histories for n Branchings

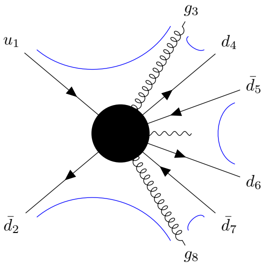
	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$
CS Dipole	2	8	48	384	3840	46080	645120
Global Antenna	1	2	6	24	120	720	5040

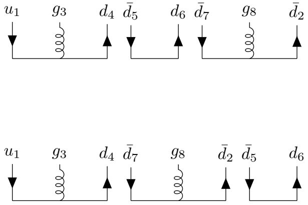
- quickly **increasing** complexity with multiplicity!

Merging with sector showers (MESS) [Brooks, CTP 2008.09468]

Tree-level merging with sector showers straight-forward:
start from CKKW-L and modify **history construction** (could be extended to NLO)

- sector showers have a **single** (!) history for **gluon emissions at LC**
- to account for **gluon splittings** $g \mapsto q\bar{q}$, find all viable quark permutations



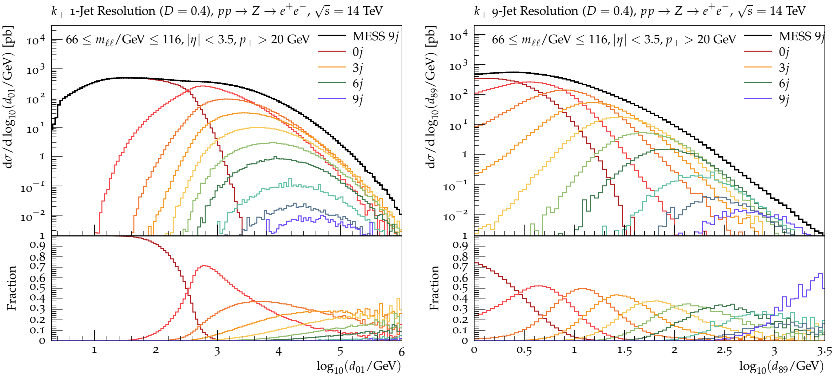


- for each colour-ordering, shower history again **uniquely** defined by sectors
- if multiple colour-orderings possible, choose one that **maximises** branching probability

Since PYTHIA 8.304: sector merging available with VINCIA

Merging with sector showers: validation

Parton-level results for merging in $pp \to Z$ with up to **9 jets**
(using HDF5 event samples from [Höche, Prestel, Schulz 1905.05120])

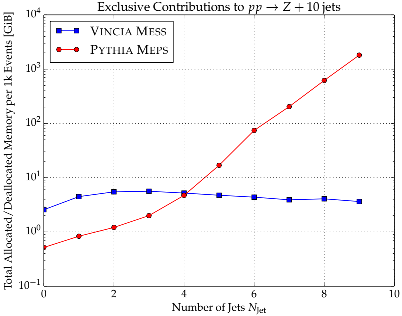


$\Rightarrow$ smooth transitions, no "sector effects" visible

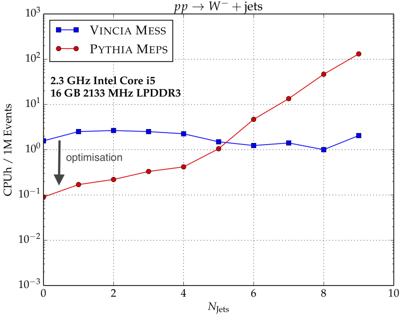
Merging with sector showers: efficiency

Gauge **efficiency gains** in $pp \to Z + 9j$ merging @ parton level
(using HDF5 event samples from [Höche, Prestel, Schulz 1905.05120]).

memory allocation/deallocation:



CPU time per event:

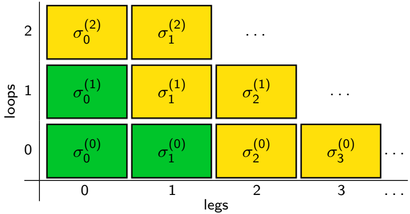


$\Rightarrow$ $\sim$ **constant** runtime and memory footprint in multi-jet merging

$\Rightarrow$ overall **optimisation** of the sector shower **possible**

Part III: Towards NNLO+PS Matching with Sector Showers

NLO+PS matching



Strategy developed $\gtrsim$ 20 years ago [Norrbin, Sjöstrand hep-ph/0010012] nowadays known as POWHEG matching [Nason hep-ph/0409146]

Alternative strategy: MC@NLO [Frixione, Webber hep-ph/0204244] (not discussed here)

- POWHEG master formula (for 2 Born jets):

$$\langle O \rangle^{\text{POWHEG}}_{\text{NLO+PS}} = \int \mathrm{d}\Phi_2 \, \mathrm{B}(\Phi_2) \underbrace{\boxed{k_{\text{NLO}}(\Phi_2)}}_{\text{local } K\text{-factor}} \underbrace{\boxed{\mathcal{S}_2(t_0, O)}}_{\text{shower operator}}$$

- **main trick**: matrix-element correction (MEC) in first shower emission

$$\mathcal{S}_2(t_0, O) = \Delta_2(t_0, t_c) O(\Phi_2) + \int_{t_c}^{t_0} \mathrm{d}\Phi_{+1} \frac{\mathrm{R}(\Phi_2, \Phi_{+1})}{\mathrm{B}(\Phi_2)} \Delta_2(t, t_c) O(\Phi_2)$$

where

$$\Delta_2(t, t') = \exp\left(- \int_{t'}^{t} \mathrm{d}\Phi_{+1} \, A_{2\mapsto 3}(\Phi_{+1}) w^{\text{MEC}}_{2\mapsto 3}(\Phi_2, \Phi_{+1}) \right) , \quad w^{\text{MEC}}_{2\mapsto 3} = \frac{\mathrm{R}(\Phi_2, \Phi_{+1})}{A_{2\mapsto 3}(\Phi_{+1}) \mathrm{B}(\Phi_2)}$$

Towards NNLO+PS [Campbell, Höche, Li, CTP, Skands 2108.07133]

Idea: "POWHEG at NNLO" (focus here on $e^+e^- \to 2j$)

$$\langle O\rangle_{\mathrm{NNLO+PS}} = \int \mathrm{d}\Phi_2\, \mathrm{B}(\Phi_2) \underbrace{\boxed{k_{\mathrm{NNLO}}(\Phi_2)}}_{\text{local } K\text{-factor}} \underbrace{\boxed{\mathcal{S}_2(t_0, O)}}_{\text{shower operator}}$$

Key aspect: 2-particle and 3-particle Sudakovs with (N)LO MECs in $\mathcal{S}_2$

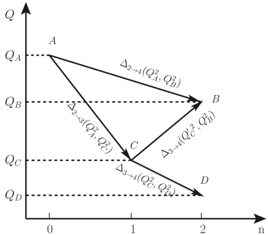
$$\Delta_2^{\mathrm{NLO}}(t_0,t) = \exp\left\{-\int_t^{t_0} \mathrm{d}\Phi_{+1}\, \mathrm{A}^{(0)}_{2\mapsto 3}(\Phi_{+1}) w^{\mathrm{NLO}}_{2\mapsto 3}(\Phi_2,\Phi_{+1})\right\}$$
$$\times \exp\left\{-\int_t^{t_0} \mathrm{d}\Phi^{>}_{+2}\, \mathrm{A}^{(0)}_{2\mapsto 4}(\Phi_{+2}) w^{\mathrm{LO}}_{2\mapsto 4}(\Phi_2,\Phi_{+2})\right\}$$
$$\Delta_3^{\mathrm{LO}}(t,t') = \exp\left\{-\int_{t'}^{t} \mathrm{d}\Phi'_{+1}\, \mathrm{A}^{(0)}_{3\mapsto 4}(\Phi'_{+1}) w^{\mathrm{LO}}_{3\mapsto 4}(\Phi_3,\Phi'_{+1})\right\}$$

Divide double-emission phase space into **strongly-ordered** and **unordered** region:

$$\mathrm{d}\Phi_{+2} = \underbrace{\mathrm{d}\Phi^{>}_{+2}}_{\text{u.o.}} + \underbrace{\mathrm{d}\Phi^{<}_{+2}}_{\text{s.o.}}$$

s.o. region: only **single-unresolved** limits

u.o. region: only **double-unresolved** limits



Towards NNLO+PS: MECs

Iterated tree-level MECs in **ordered** region [Giele, Kosower, Skands 1102.2126], [Fischer, Prestel 1706.06218]:

$$w^{\mathrm{LO}}_{2\mapsto3}(\Phi_2,\Phi_{+1}) = \frac{\mathrm{R}(\Phi_2,\Phi_{+1})}{\mathrm{A}^{(0)}_{2\mapsto3}(\Phi_{+1})\mathrm{B}(\Phi_2)}$$

$$w^{\mathrm{LO}}_{3\mapsto4}(\Phi_3,\Phi'_{+1}) = \frac{\mathrm{RR}(\Phi_3,\Phi'_{+1})}{\mathrm{A}^{(0)}_{3\mapsto4}(\Phi'_{+1})\mathrm{R}(\Phi_3)}$$

Tree-level MECs in **unordered** region:

$$w^{\mathrm{LO}}_{2\mapsto4}(\Phi_2,\Phi_{+2}) = \frac{\mathrm{RR}(\Phi_2,\Phi_{+2})}{\mathrm{A}^{(0)}_{2\mapsto4}(\Phi_{+2})\mathrm{B}(\Phi_2)}$$

NLO MECs for $+1j$ state [Hartgring, Laenen, Skands 1303.4974]:

$$w^{\mathrm{NLO}}_{2\mapsto3}(\Phi_2,\Phi_{+1}) = w^{\mathrm{LO}}_{2\mapsto3}(\Phi_2,\Phi_{+1}) \times \left(1 + w^{\mathrm{V}}_{2\mapsto3}(\Phi_2,\Phi_{+1})\right)$$

$$w^{\mathrm{V}}_{2\mapsto3}(\Phi_2,\Phi_{+1}) = \left(\frac{\mathrm{RV}(\Phi_2,\Phi_{+1})}{\mathrm{R}(\Phi_2,\Phi_{+1})} + \frac{\mathrm{I}^{\mathrm{NLO}}(\Phi_2,\Phi_{+1})}{\mathrm{R}(\Phi_2,\Phi_{+1})}\right.$$

NLO Born$+1j$

$$\left. + \int_0^t \mathrm{d}\Phi'_{+1} \left[\frac{\mathrm{RR}(\Phi_2,\Phi_{+1},\Phi'_{+1})}{\mathrm{R}(\Phi_2,\Phi_{+1})} - \frac{\mathrm{S}^{\mathrm{NLO}}(\Phi_2,\Phi_{+1},\Phi'_{+1})}{\mathrm{R}(\Phi_2,\Phi_{+1})}\right]\right)$$

NLO Born

$$- \left(\frac{\mathrm{V}(\Phi_2)}{\mathrm{B}(\Phi_2)} + \frac{\mathrm{I}^{\mathrm{NLO}}(\Phi_2)}{\mathrm{B}(\Phi_2)} + \int_0^{t_0} \mathrm{d}\Phi'_{+1} \left[\frac{\mathrm{R}(\Phi_2,\Phi'_{+1})}{\mathrm{B}(\Phi_2)} - \frac{\mathrm{S}^{\mathrm{NLO}}(\Phi_2,\Phi'_{+1})}{\mathrm{B}(\Phi_2)}\right]\right)$$

shower

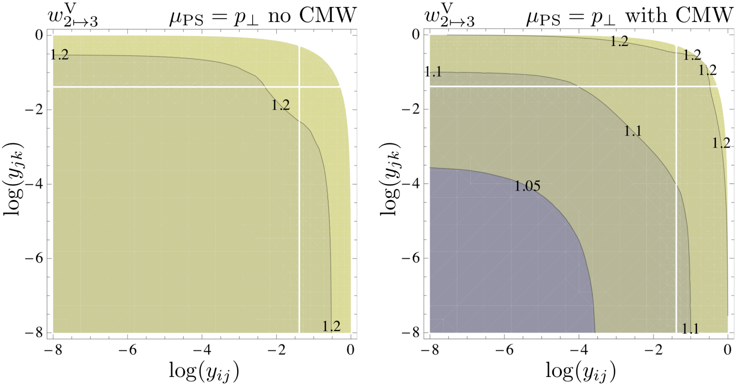
$$+ \left(\frac{\alpha_{\mathrm{S}}}{2\pi} \log\left(\frac{\kappa^2\mu^2_{\mathrm{PS}}}{\mu^2_{\mathrm{R}}}\right) + \int_t^{t_0} \mathrm{d}\Phi'_{+1}\, \mathrm{A}^{(0)}_{2\mapsto3}(\Phi'_{+1}) w^{\mathrm{LO}}_{2\mapsto3}(\Phi_2,\Phi'_{+1})\right)$$

Towards NNLO+PS: real-virtual corrections

Real-virtual correction factor

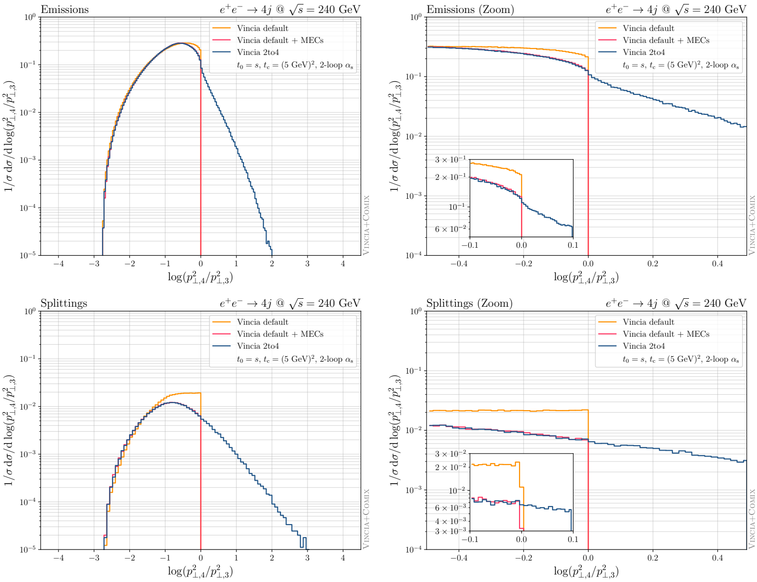
$$w^{\mathrm{NLO}}_{2\mapsto 3} = w^{\mathrm{LO}}_{2\mapsto 3}\left(1 + w^{\mathrm{V}}_{2\mapsto 3}\right)$$

studied in detail for $Z \to q\bar{q}$ in [Hartgring, Laenen, Skands 1303.4974]:



⇒ now: generalisation & (semi-)automation in VINCIA in progress
(using run-time interfaces to MCFM and SHERPA/COMIX)

Towards NNLO+PS: double-real corrections



⇒ sector showers allow for **clear separation** into **ordered** and **unordered** phase space

Conclusions

Sector showers combine shower **evolution** with jet **clustering** to become **maximally bijective**

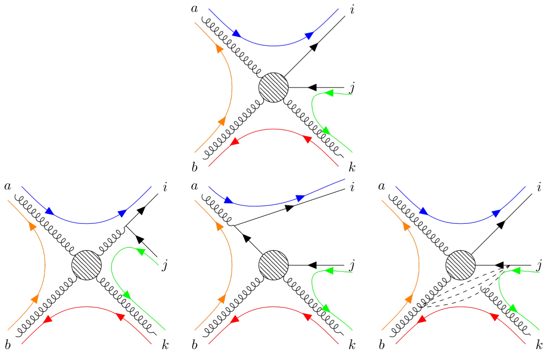
- "sectorised" VINCIA well validated against "global" VINCIA and PYTHIA
  (discontinuities? still searching...)
- sector merging has ∼**constant** overall **run time** and **memory usage**
- sector showers **default option** in VINCIA as of PYTHIA 8.304

This is just the beginning...

- sector **merging** easily extendable to **NLO**
  (lack of time that it hasn't been done yet...)
- sector decomposition facilitates inclusion of **NLO antenna functions** in shower evolution
  (including direct $2 \mapsto 4$ branchings covering double-unresolved limits)
- antenna-based **(N)NLO matching** and shower **evolution at NLO** ongoing developments
  (currently on a proof-of-concept level for $e^+e^- \to 2j$, but can be extended!)

Backup

Sector definitions



For massless particles, the sector resolution is defined by:

$$Q^2_{\mathrm{res},j} = \begin{cases} \frac{s_{ij}s_{jk}}{s_{ijk}} & \text{if } j \text{ is a } g \\ s_{ij}\sqrt{\frac{s_{jk}}{s_{ijk}}} & \text{if } (i,j) \text{ is a } q\bar{q} \text{ pair} \end{cases}$$

Sectors defined by:

$$\Theta_{\mathrm{sct},j} = \theta(\min\{Q^2_{\mathrm{res},i}\} - Q^2_{\mathrm{res},j})$$