8. The SM as an effective theory

Summary:

$\begin{cases} * \text{ light d.o.f.: } \psi_{SM}, \phi \\ * \text{ unbroken symm.: } SU(3)_c \times SU(2)_L \times U(1)_Y \end{cases}$

low-energy limit $\longrightarrow$ $\mathcal{L}_{SM-EFT}$

$$\mathcal{L}_{SM-EFT} = \underbrace{\mathcal{L}_{gauge} + \mathcal{L}_Y + \mathcal{L}_{Fermion}^{kin.}}_{\text{Only } d=4 \text{ ops.}} + \underset{\mu^2 \phi^\dagger \phi}{\underset{\uparrow}{\mathcal{L}_\phi}} + \sum_{d>4} \sum_i \frac{c_i^{[d]}}{\Lambda^{d-4}} \underset{\uparrow}{\mathcal{O}^i(\psi_{SM}, \phi, A^a)^{[d]}}$$

$\begin{cases} \text{Local interactions} \\ \text{Lorentz invariance} \\ SU(3)_c \times SU(2)_L \times U(1)_Y \end{cases}$

- $\mathcal{L}^{d=2}$: 1 operator $\mu^2 \phi^\dagger \phi$ [Higgs mass term]

  $m_h^2 \propto \left[\mu^2(\Lambda) + \frac{c^2}{16\pi^2}\Lambda^2 + \dots\right]$ Higgs hierarchy problem $\rightarrow$ "low" Λ (not far from v)

- $\mathcal{L}^{d=3}$: no ops. allowed (by SM gauge + field content)

- $\mathcal{L}^{d=4}$: dimension-less coupl. (most of the SM coupl.) $\rightarrow$ logarithmic RG evolution $\rightarrow$ stable

  (RG = Renormalization Group)

- $\mathcal{L}^{d=5}$: 1 operator $\frac{g_\nu}{\Lambda_{LN}} (L_L^T \phi_c^* \phi_c L_L)$ [neutrino masses] $\boxed{\text{violates total lepton number}}$

  $m_\nu \sim \frac{g_\nu v^2}{\Lambda_{LN}} \Big|_{g_\nu \sim 1} \sim \sqrt{\Delta m^\nu_{atm}}$ $\longrightarrow$ high effective scale, connected to violation of L

  $\Lambda_{LN} \sim 10^{15}$ GeV

$\mathcal{L}^{d=6}$: long list of operators ...

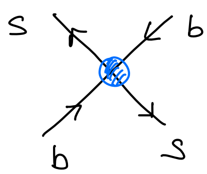
most of are connected to the breaking of flavor symm.

8.4 Flavor-violating operators & the "flavor problem"

E.g. $\quad (\bar{Q}^i_L \gamma^\mu Q^j_L)^2 \qquad Q^i_L = \begin{pmatrix} u^i_L \\ d^i_L \end{pmatrix} \qquad i=1,2,3$

$(\bar{Q}^i_L \gamma^\mu Q^j_L)(\bar{L}^k_L \gamma_\mu L^e_L)$

The $\Delta F=2$ bounds:



$$\mathcal{L}^{(SM)}_{eff}(\Delta B=2) = (\bar{b}_L \gamma^\mu s_L)^2 \frac{y_t^2}{16\pi^2 M_W^2} (V^*_{tb} V_{ts})^2 \cdot \underbrace{f\left(\frac{m_t^2}{m_W^2}\right)}_{\mathcal{O}(1)}$$

$$\mathcal{L}^{d=6} \supset \frac{C_{bs}}{\Lambda^2} (\bar{b}_L \gamma^\mu s_L)^2$$

$$\mathcal{A}(B\text{-}\bar{B}) = \mathcal{A}_{SM} + \mathcal{A}_{NP} = M_{SM}\Big[1 + \underbrace{\frac{16\pi^2 M_W^2}{\Lambda^2} \frac{C_{bs}}{(V^*_{tb} V_{ts})^2} \cdot \mathcal{O}(1)}_{\text{exp} \Rightarrow \ \lesssim 10\%} \Big]$$

A similar structure (and corresponding similar bounds) holds also for $\bar{B}_s - B_s$ mixing and for $K-\bar{K}$ mixing

$\Downarrow$

$$\frac{16\pi^2 M_W^2}{\Lambda^2} \frac{C_{ij}}{(V^*_{ti} V_{tj})^2} \lesssim 10^{-1} \quad \longrightarrow \quad \frac{\Lambda^2}{C_{ij}} \gtrsim \frac{(1\,\text{TeV})^2}{|V^*_{ti} V_{tj}|^2} \cdot 10^{-1}$$

$\hookrightarrow$ for $K-\bar{K}$ mixing $|V^*_{ts} V_{td}|^2 \sim 10^{-8}$!

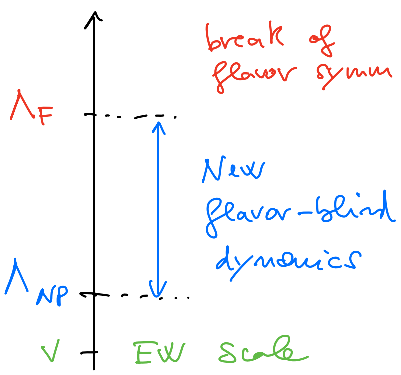
$$\Lambda \gtrsim \begin{cases} 10^4\ \text{TeV} \cdot (C_{sd})^{1/2} & \text{from } \bar{K}-K \text{ mixing} \\ 10^3\ \text{TeV} \cdot (C_{bd})^{1/2} & \quad " \quad \bar{B}-B \quad " \\ 10^2\ \text{TeV} \cdot (C_{bs})^{1/2} & \quad " \quad \bar{B}_s-B_s \quad " \end{cases}$$

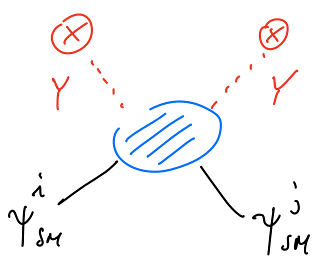
Stronger bounds are obtained for operators violating baryon number from the non-observation of proton decay ($\Lambda_{BV} \gtrsim 10^{16}$ GeV).

But baryon number is an exact accidental symmetry of the SM, hence we can simply assume that the scale of baryon number violating interactions is very high

How can we "protect" the couplings of the flavor-violating operators, if we want to have a low value of Λ (as suggested by m_H) ?
Not obvious, since flavor is not an exact symmetry of $\mathcal{L}_{SM}$
(we cannot proceed as for L & B).

8.5 The Minimal Flavor Violation hypothesis





We make the hypothesis that flavor-changing transitions are ruled by the SM Yukawa couplings also beyond the SM

Technically, this is achieved treating the SM Yukawa couplings as spurions of the flavor symm. of $\mathcal{L}^{SM}_{gauge}$

$$\mathcal{L}_{SM} = \mathcal{L}^{SM}_{gauge} + \mathcal{L}^{SM}_{Higgs}$$

Large global
Flavor Symmetry $\quad \psi_i \to V^{(\psi)}_{ij}\, \psi_j \qquad V V^\dagger = \mathbb{1} \qquad \psi = Q_L,\ u_R,\ d_R,\ L,\ e_R$

$$\boxed{U(3)^5 = U(3)_{Q_L} \times U(3)_{u_R} \times U(3)_{d_R} \times U(3)_{L_L} \times U(3)_{e_R}}$$

$$\mathcal{L}^Y = \underbrace{\bar{Q}_L^i}_{\bar{3}_{Q_L}} \underbrace{Y_u^{ij}}_{3_{Q_L} \times \bar{3}_{u_R}} \underbrace{u_R^j}_{3_{u_R}} + \bar{Q}_L \underbrace{Y_d}_{3_{Q_L} \times \bar{3}_{d_R}} d_R + \bar{L}_L \underbrace{Y_e}_{3_{L_L} \times \bar{3}_{e_R}} e_R + \text{h.c.} \qquad (*)$$

Spurion = fictitious field, with non-trivial transf. properties, "frozen" to some constant value $\quad [\, Y \to \langle 0 | Y(x) | 0 \rangle \,]$

If $Y_{u,d,e}$ have the transf. properties in $(*)$ $\to$ $\mathcal{L}^Y$ is invariant under $U(3)^5$

When building higher-dim ops. we must respect this property

$$\cancel{(\bar{Q}_L^i \gamma^\mu Q_L^j)^2} \to \text{not invariant}$$

$$[\, \bar{Q}_L^i (Y_u Y_u^\dagger)_{ij} \gamma_\mu Q_L^j \,]^2 \to \text{invariant}$$

$$(Y_u Y_u^\dagger)_{ij} \sim 8_Q + 1_Q$$

$$\underbrace{3_Q \times \bar{3}_u \quad 3_u}_{} \times \bar{3}_Q$$

$$[\, \bar{Q}_L^i (Y_u Y_u^\dagger)_{ij} \gamma_\mu Q_L^j \,]^2 \to [\, \bar{u}_L^i (Y_u Y_u^\dagger)_{ij} \gamma_\mu u_L^j \,]^2 \xrightarrow[\text{up-basis}]{} [\, \bar{u}_L^i (\lambda_u^i)^2 \gamma^\mu u_L^i \,]^2$$

flavor-diagonal

$$[\, \bar{Q}_L^i (Y_u Y_u^\dagger)_{ij} \gamma_\mu Q_L^j \,]^2 \to [\, \bar{d}_L^i (Y_u Y_u^\dagger)_{ij} \gamma_\mu d_L^j \,]^2$$

$$\left[\bar d_L^i (Y_u Y_u^\dagger)_{ij}\gamma_\mu d_L^j\right]^2 \xrightarrow[\text{down-basis}]{} \bar d_L^i \left(V_{CKM}^\dagger \lambda_u^2 V_{CKM}\right)_{ij}\gamma^\mu d_L^j$$

$$\downarrow$$

$$\approx (V_{CKM}^\dagger)_{i3}\, y_t^2\, (V_{CKM})_{3j} = \underline{V_{ti}^* V_{tj}}\, y_t^2$$

$\uparrow$ some suppression we have in the SM for meson-antimeson mixing

Recall: (down-basis)

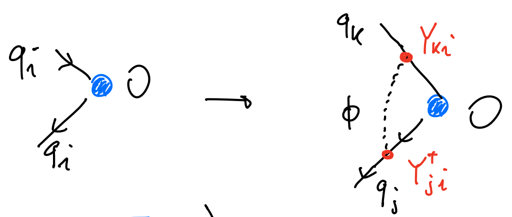
$$\begin{cases} Y_u = V_{CKM}^\dagger \lambda_u \\ Y_d = \lambda_d \end{cases}$$

The constraint we derived from meson mixing was

$$\frac{16\pi^2 M_W^2}{\Lambda^2}\frac{C_{ij}}{(V_{ti}^* V_{tj})^2} \lesssim 10^{-1} \quad \xrightarrow[C_{ij}\sim V_{ti}^* V_{tj}]{\text{within MFV}} \quad \Lambda \gtrsim \text{few TeV}$$

N.B.: It is not consistent to assume $c_{ij} = \delta_{ij}$ (exact flavor universality)

$\to$ symmetry broken within the SM $\to$ breaking induced by RG effects (by SM fields & SM dynamics)



$$\Rightarrow \quad c_{ij} = \delta_{ij} + \frac{O(1)}{16\pi^2}(YY^\dagger)_{ij}$$

($c_{ij} = \delta_{ij}$) as in MFV

The MFV hypothesis describes the minimal amount of flavor symm breaking we can have, in absence of tuned scenarios

Main predictions of MFV:

$$A(q_L^i \to q_L^j + X) = \underset{\substack{\uparrow\\ \text{loop-mediated}}}{} A_{SM}^{ij}\left[1 + \frac{16\pi^2 m_W}{\Lambda^2} * O(1)\right]$$

Flavor-universal corrections due to physics beyond SM

$\uparrow$ contrib. from $d=6$ built in terms of SM fields & Yukawa couplings

E.g.
$$\begin{cases} A(b\to s\gamma) = A(b\to s\gamma)^{SM}\left[1+\Delta_\gamma\right] \\ A(b\to d\gamma) = A(b\to d\gamma)^{SM}\left[1+\Delta_\gamma\right] \end{cases}$$
some relative NP correction

$$\begin{cases} M(\bar{B}_d - B_d) = M(\bar{B}_d - B_d)^{SM}\left[1+\Delta_{\Delta F=2}\right] \\ M(\bar{B}_s - B_s) = M(\bar{B}_s - B_s)^{SM}\left[1+\Delta_{\Delta F=2}\right] \end{cases}$$

8.6 Beyond MFV: $U(2)^n$ flavor symmetries

The MFV hypothesis is very efficient in lowering the bounds on the effective scale of new physics from flavor-changing processes. However, it has some relevant drawbacks:

1) It does not address the origin of the observed mass hierarchies. (why $y_3 \gg y_{1,2}$?). Assuming MFV, this question is post-poned to higher energies

2) The MFV hypothesis does not help in suppressing bounds on Λ from flavor-conserving high-energy processes with light fermions, such as $u\bar{u} \longrightarrow e^+e^-$
These bounds have become very strong recently (thanks to LHC experiments), pushing the bounds on flavour-universal interactions (e.g. $\bar{Q}^i \gamma_\mu Q^i \bar{L}^j \gamma^\mu L^j$) above ~ 10 TeV

3) Given $y_t = O(1)$, an expansion of the effective operators in powers of Y_u is not well defined (large expansion parameter).

Let's start discussing the last point. Given Y_u is not small, when constructing higher-dimensional operators we must consider arbitrary powers of Y_u. Let's consider the quark bilinear $\bar{Q}_L (Y_u Y_u^\dagger)^m \gamma_\mu Q_L$
This bilinear induces an effective flavor-violating coupling among light quarks given by

$$\bar{d}_L^i (Y_u Y_u^\dagger)^m_{i\neq j} d_L^j \longrightarrow (Y_u Y_u^\dagger)^m_{i\neq j} \approx (V_{CKM}^\dagger)_{i3}\, Y_t^{2m}\, (V_{CKM})_{3j}$$

The coupling is still suppressed by $(V^\dagger_{CKM})_{i3}\,(V_{CKM})_{3j}$ as in the SM. From this observation we deduce that what "protects" flavor-changing processes is the smallness of the mixing between the third generation and the light ones in the Yukawa couplings ($\leftrightarrow$ smallness of off-diagonal entries in the CKM matrix), while the fact that y_t is large is not a problem.

This naturally leads us to consider smaller flavor symmetries acting only on the light fermions (= first two generations). In the quark sector we have $U(2)_{Q_L}\times U(2)_{u_R}\times U(2)_{d_R}$, which acts on the Yukawa couplings as follows:

$$Y_u = \left(\begin{array}{c|c} 2_{Q_L}\times\bar{2}_{u_R} & 2_{Q_L} \\ \hline \bar{2}_{u_R} & 1 \end{array}\right) \leftarrow U(2)_{Q_L} \qquad\qquad Y_d = \left(\begin{array}{c|c} 2_{Q_L}\times\bar{2}_{u_R} & 2_{Q_L} \\ \hline \bar{2}_{u_R} & 1 \end{array}\right) \leftarrow U(2)_{Q_L}$$

$$\uparrow\; U(2)_{u_R} \qquad\qquad\qquad\qquad \uparrow\; U(2)_{u_R}$$

In the limit of unbroken $U(2)_{Q_L}\times U(2)_{u_R}\times U(2)_{d_R}$ the Yukawa couplings are of the type $\mathrm{diag}(0,0,y_3)$, which is a very good approximation.

Postulating a minimal breaking of $U(2)_{Q_L}\times U(2)_{u_R}\times U(2)_{d_R}$ in the SMEFT (= minimal breaking to reproduce the observed Yukawa couplings), addresses the three shortcoming of MFV listed above:

1) A posteriori justification of why light masses are small: they are small because they break the symmetry (contrary to 3rd gen. masses).

2) All the breaking parameters are small (the largest is $|V_{ts}|\approx 4\times 10^{-2}$).

3) We can have new physics at the TeV scale, coupled mainly to 3rd gen., given the reduced flavor symmetry allows us to consistenty evade the strong LHC bounds on processes involving only light fermions.

In summary this hypothesis explains well what we see / do not see in present data, leaving open the possibility of new physics at the TeV scale addressing the Higgs hierachy problem.