VBF Higgs production at NNLO
... and beyond

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This work

- first semi-differential N^3LO calculation of VBF Higgs production in the structure function approach
- first fully differential NNLO calculation of VBF Higgs production using a novel "projection-to-Born" method
- Some (very) preliminary results on MiNLO improved VBF Hjjj production

What to expect

- N^3LO corrections are at the permille level and well within the scale uncertainty band of the NNLO prediction
- the associated scale uncertainty is reduced by a factor 5 at N^3LO
- NNLO corrections are sizeable, $\mathcal{O}(10\%)$, and outside NLO band
- the corrections are (almost everywhere) negative
- only moderate shrinkage of NNLO bands compared to NLO bands
- the "projection-to-Born" method is quite general and can be extended to other similar processes

Reasons to study VBF
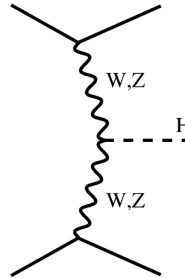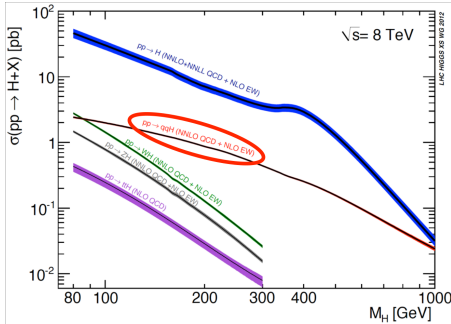
- largest cross section at tree-level and second-largest of all channels
- distinct signature of two forward jets
- tagging reduces backgrounds (eg $H \to b\bar{b}$)
- non-zero Higgs transverse momentum at lowest order
- sensitive to CP properties of the Higgs through correlation of forward jets
- sensitive to trilinear Higgs self-coupling through loop-corrections

Reasons to study VBF

| Production process	ATLAS+CMS	ATLAS	CMS
μ_{ggF}	$1.03^{+0.17}_{-0.15}$	$1.25^{+0.24}_{-0.21}$	$0.84^{+0.19}_{-0.16}$
μ_{VBF}	$1.18^{+0.25}_{-0.23}$	$1.21^{+0.33}_{-0.30}$	$1.13^{+0.37}_{-0.34}$
μ_{WH}	$0.88^{+0.40}_{-0.38}$	$1.25^{+0.56}_{-0.52}$	$0.46^{+0.57}_{-0.54}$
μ_{ZH}	$0.80^{+0.39}_{-0.36}$	$0.30^{+0.51}_{-0.46}$	$1.35^{+0.58}_{-0.54}$
μ_{ttH}	$2.3^{+0.7}_{-0.6}$	$1.9^{+0.8}_{-0.7}$	$2.9^{+1.0}_{-0.9}$

- largest cross section at tree-level and second-largest of all channels
- distinct signature of two forward jets
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Inclusive NNLO VBF Higgs Production

Until recently VBF Higgs production was only known inclusively at NNLO.

[Bolzoni et al. (2010)]
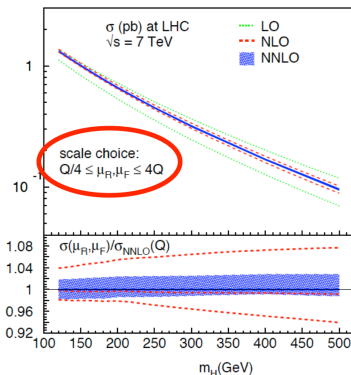
- the calculation suggests tiny renormlisation and factorisation scale variations ($\sim 1-2\%$)

- NNLO results well within NLO band

- result obtained in the structure function approach

No cuts can be applied to the calculation, as it is totally inclusive over hadronic final states with the same vector boson momenta.

VBF Cuts

Due to huge QCD backgrounds a set of very selective cuts have to be applied. Typical cuts are:

- jets defined with anti-k_t, $R = 0.4$ and $p_t > 25$ GeV
- two hardest jets within $|y| < 4.5$
- high dijet invariant mass, $M_{j_1 j_2} > 600$ GeV, and separation, $\Delta y_{j_1 j_2} > 4.5$
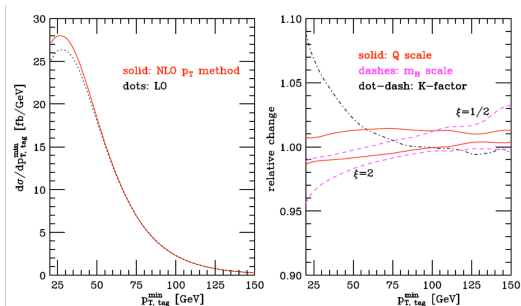- hardest jets in opposite hemispheres, $y_{j_1} y_{j_2} < 0$
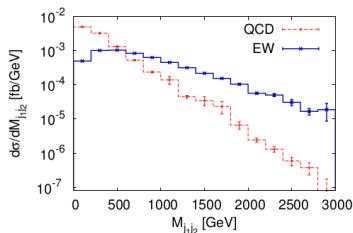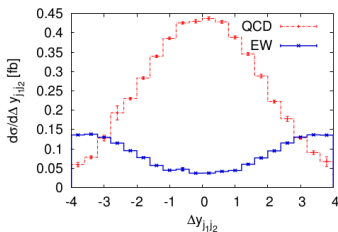
[Jäger, Zanderighi (2011)]

Exclusive NLO VBF Higgs Production

To enable the application of realistic VBF cuts one has to be fully differential. Until recently differential VBF Higgs production was known to NLO(+PS). [Figy, Oleari, Zeppenfeld (2003)]

Calculation suggests small uncertainties from missing higher order corrections ($\sim 2\%$).

Structure Function Approach

One can think of VBF Higgs production as a double Deep Inelastic Scattering (DIS×DIS) with no cross-talk between the upper and lower sectors.

[Han, Valencia, Willenbrock (1992)]

- this picture is accurate to more than 1%
  [Bolzoni et al. (2012)], [Ciccolini, Denner, Dittmaier (2008)], [Harlander et al. (2008)], [Andersen et al. (2008)]
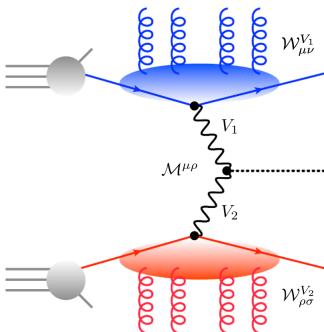
- the factorisation of the two sectors is exact if one imagines two copies of QCD, QCD_1 and QCD_2 , respectively for the upper and lower sectors.
- all DIS coefficients are known to to N^3LO.
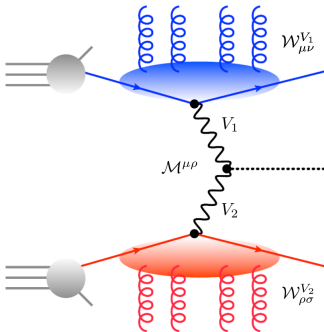- as the DIS coefficients are inclusive over the hadronic final state, the calculation cannot provide differential results.

Structure Function Approach

$$d\sigma_{VBF} = \frac{G_F^2}{s} M_{V_1}^2 M_{V_2}^2 \Delta_{V_1}^2(Q_1^2) \Delta_{V_2}^2(Q_2^2) \times$$
$$\mathcal{W}_{\mu\nu}^{V_1}(x_1, Q_1^2) \mathcal{M}^{\mu\rho} \mathcal{M}^{*\nu\sigma} \mathcal{W}_{\rho\sigma}^{V_2}(x_2, Q_2^2) d\Omega_{VBF}$$

and

$$\mathcal{M}^{\mu\nu} = 2\sqrt{\sqrt{2}G_F} M_V^2 g^{\mu\nu}$$

Hadronic Tensor

$$\mathcal{W}_{\mu\nu}^{V}(x_i, Q_i^2) =$$
$$\left(-g_{\mu\nu} + \frac{q_{i,\mu} q_{i,\nu}}{q_i^2}\right) F_1^V(x_i, Q_i^2)$$
$$+ \frac{\hat{P}_{i,\mu} \hat{P}_{i,\nu}}{P_i \cdot q_i} F_2^V(x_i, Q_i^2)$$
$$+ i\epsilon_{\mu\nu\rho\sigma} \frac{P_i^\rho q_i^\sigma}{2P_i \cdot q_i} F_3^V(x_i, Q_i^2)$$

Structure Function Approach

The Structure Functions can be expressed as convolutions of the short distance DIS coefficient functions and PDFs

$$F_i^V = \sum_{a=q,g} C_{i,a}^V \otimes f_a, \quad i = 2, L, 3, \quad V = Z, W^+, W^-,$$

where DIS coefficient functions are known to the third order in α_s. [Moch, Rogal, Vermaseren, Vogt (2005-2008)]

Conceptually N^3LO not more complicated than NNLO, but non-trivial flavour structures start appearing at N^3LO
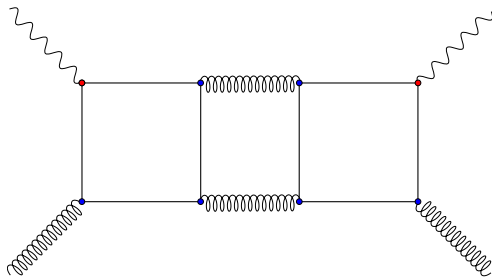
What about missing N^3LO PDFs?

All approximations aside, we can't truely claim N^3LO accuracy without also having N^3LO PDFs available. Naive estimate

$$\delta_A^{PDF} = \frac{1}{2}\left|\frac{\sigma_{NNLO\text{-}PDF}^{NNLO} - \sigma_{NLO\text{-}PDF}^{NNLO}}{\sigma_{NNLO\text{-}PDF}^{NNLO}}\right| = 1.1\%,$$

and slightly more sophisticated ($Q_0 = 8$ GeV)

$$f^{N^3LO,approx.}(x,Q) = f^{NNLO}(x,Q)\frac{F_2^{NNLO}(x,Q_0)}{F_2^{N^3LO}(x,Q_0)},$$

$$\delta_B^{PDF}(Q_0) = \left|\frac{\sigma^{N^3LO} - \sigma_{rescaled}^{N^3LO}(Q_0)}{\sigma^{N^3LO}}\right| = 7.9\text{‰}.$$

What about missing N^3LO PDFs?
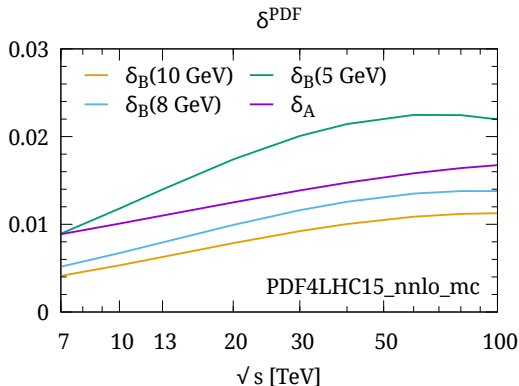
- the value of $Q_0 = 8$ GeV is found by requiring that the method is reliable in estimating NNLO corrections to PDFs
- the uncertainty associated with varying the scale in the DGLAP evolutions is found to be below the permille level
- $\rightarrow$ could obtain approximate N^3LO PDFs using just N^3LO coefficient functions

Scale variations at N^3LO

We use RGE methods to evaluate the structure functions at arbitrary renormalisation scales

$$\alpha_S(Q) = \alpha_S(\mu_R) + \alpha_S^2(\mu_R)\beta_0 L_{RQ} + \alpha_S^3(\mu_R)(\beta_0^2 L_{RQ}^2 + \beta_1 L_{RQ}) + \mathcal{O}(\alpha_S^4(\mu_R))$$

$$L_{RQ} = \ln\left(\frac{\mu_R^2}{Q^2}\right), \quad L_{FQ} = \ln\left(\frac{\mu_F^2}{Q^2}\right)$$

$$C_i = \sum_{k=0}\left(\frac{\alpha_S(Q)}{2\pi}\right)^k C_i^{(k)} =$$

$$C_i^{(0)} + \frac{\alpha_S(\mu_R)}{2\pi}C_i^{(1)} +$$

$$\left(\frac{\alpha_S(\mu_R)}{2\pi}\right)^2\left(C_i^{(2)} + 2\pi\beta_0 C_i^{(1)} L_{RQ}\right) +$$

$$\left(\frac{\alpha_S(\mu_R)}{2\pi}\right)^3\left[C_i^{(3)} + 4\pi\beta_0 C_i^{(2)} L_{RQ}\right.$$

$$\left. + 4\pi^2 C_i^{(1)} L_{RQ}(\beta_1 + \beta_0^2 L_{RQ})\right] + \mathcal{O}(\alpha_S^4)$$

$$f(x,Q) = f(x,\mu_F)\left(1 - \frac{\alpha_S(\mu_R)}{2\pi}L_{FQ}P^{(0)}\right.$$

$$- \left(\frac{\alpha_S(\mu_R)}{2\pi}\right)^2 L_{FQ}\left[P^{(1)} - \frac{1}{2}L_{FQ}(P^{(0)})^2\right.$$

$$\left. - \pi\beta_0 P^{(0)}(L_{FQ} - 2L_{RQ})\right]$$

$$- \left(\frac{\alpha_S(\mu_R)}{2\pi}\right)^3 L_{FQ}\left[P^{(2)} - \frac{1}{2}L_{FQ}(P^{(0)}P^{(1)}\right.$$

$$+ P^{(1)}P^{(0)}) + \pi\beta_0(L_{FQ} - 2L_{RQ})\times$$

$$(L_{FQ}(P^{(0)})^2 - 2P^{(1)}) + \frac{1}{6}L_{FQ}^2(P^{(0)})^3$$

$$+ 4\pi^2\beta_0^2 P^{(0)}(L_{RQ}^2 - L_{FQ}L_{RQ} + \frac{1}{3}L_{FQ}^2)$$

$$\left. - 2\pi^2\beta_1 P^{(0)}(L_{FQ} - 2L_{RQ})\right] + \mathcal{O}(\alpha_S^4)\right)$$

N^{3}LO results

|	$\sigma^{(13\,\mathrm{TeV})}$ [pb]	$\sigma^{(14\,\mathrm{TeV})}$ [pb]	$\sigma^{(100\,\mathrm{TeV})}$ [pb]
LO	$4.099\,^{+0.051}_{-0.067}$	$4.647\,^{+0.037}_{-0.058}$	$77.17\,^{+6.45}_{-7.29}$
NLO	$3.970\,^{+0.025}_{-0.023}$	$4.497\,^{+0.032}_{-0.027}$	$73.90\,^{+1.73}_{-1.94}$
NNLO	$3.932\,^{+0.015}_{-0.010}$	$4.452\,^{+0.018}_{-0.012}$	$72.44\,^{+0.53}_{-0.40}$
N^{3}LO	$3.928\,^{+0.005}_{-0.001}$	$4.448\,^{+0.006}_{-0.001}$	$72.34\,^{+0.11}_{-0.02}$

- We study pp collisions with PDF4LHC_nnlo_mc and electroweak parameters fixed to their PDG values
- the central renormalisation and factorisation scale is set equal to Q_1, Q_2 and varied independently by a factor 2 up and down
- N^{3}LO corrections are tiny ($\sim 2\text{‰}$) but predictions well within NNLO scale uncertainty

N³LO results
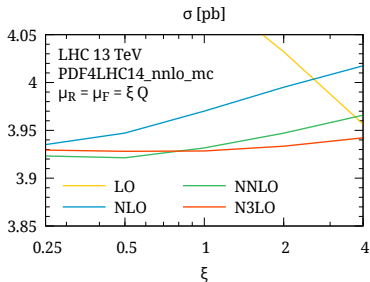
- the N³LO corrections are tiny over a large range of energies and stay well within the scale uncertainty band of the NNLO prediction
- cross section becomes extremely stable under the variation of renormalisation and factorisation scales
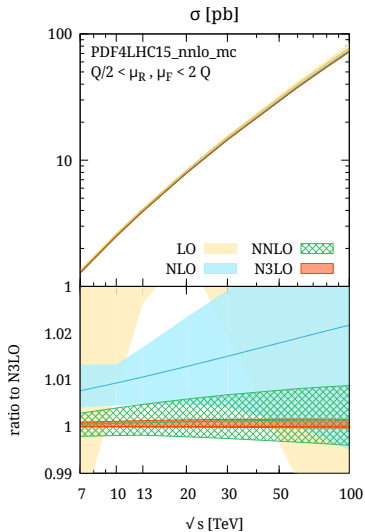
N^3LO results

From the knowledge of Q_1 and Q_2 it is trivial to reconstruct the momentum of the Higgs. The calculation is therefore fully differential in the Higgs kinematics.
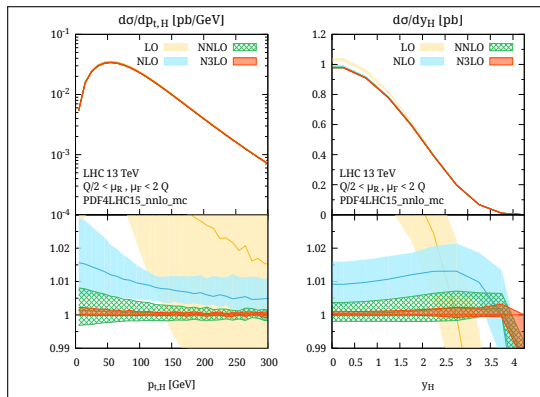
- the corrections are almost flat throughout the entire spectrum
- the N^3LO prediction completely contained within the scale uncertainty band of the NNLO prediction
- only differential in the momenta of the proton remnants, and hence no real information on the tagging jets
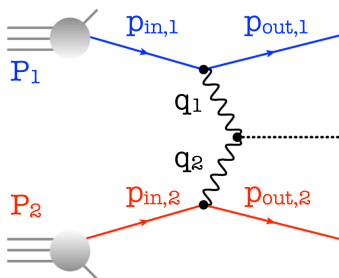
Limitations of the structure function approach

- gluon exchanges between the upper and lower hadronic sectors, which appear at NNLO, but are kinematically and colour suppressed; These contributions along with the heavy-quark loop induced contributions have been estimated to contribute at the permille level [Bolzoni et al. (2012)]

- t-/u-channel interference which are known to contribute $\mathcal{O}(5‰)$ at the fully inclusive level and $\mathcal{O}(0.5‰)$ after VBF cuts have been applied [Ciccolini, Denner, Dittmaier (2008)]

- contributions from s-channel production, which have been calculated up to NLO. At the inclusive level these contributions are sizeable but they are reduced to $\mathcal{O}(5‰)$ after VBF cuts [Ciccolini, Denner, Dittmaier (2008)]

- single-quark line contributions, which contribute to the VBF cross section at NNLO. At the fully inclusive level these amount to corrections of $\mathcal{O}(1\%)$ but are reduced to the permille level after VBF cuts have been applied [Harlander et al. (2008)]

- loop induced interference between VBF and ggH production. These contributions have been shown to be much below the permille level [Andersen et al. (2008)]

Beyond the Structure Function Approach

This work: We eliminate the limitations of the Structure Function Approach.

If the scattering is Born like, then the vector boson momenta q_i, and on-shell conditions, fix the incoming and outgoing parton momenta:

$$p_{in,i} = x_i P_i$$

$$p_{out,i} = x_i P_i - q_i$$
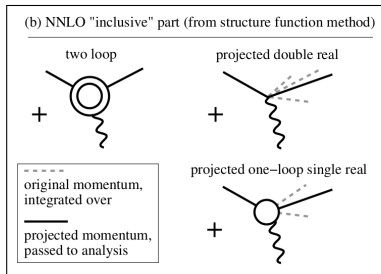
$$x_i = \frac{q_i^2}{2 q_i P_i}$$

Beyond the Structure Function Approach

The calculation is based on two ingredients:

1. *An "inclusive" contribution*
   - use the Structure Function Approach and use four-vectors q_1, q_2 to assign Born-like kinematics using the equations below
   - use the projected Born-like momenta to compute differential distributions

$$p_{in,i} = x_i P_i$$

$$p_{out,i} = x_i P_i - q_i$$
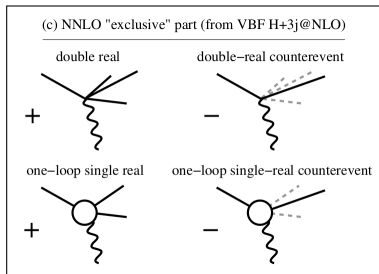
$$x_i = \frac{q_i^2}{2q_i P_i}$$

Beyond the Structure Function Approach

The calculation is based on two ingredients:

2. *An "exclusive" contribution*

   - use the electroweak $H + jjj$ NLO calculation in the
     factorized approximation [Figy et al. (2007)], [Jäger et al. (2014)]

(c) NNLO "exclusive" part (from VBF H+3j@NLO)

double real   double–real counterevent

+   −

one-loop single real   one–loop single–real counterevent

+   −

- for each parton, keep track of whether it belongs to the upper or lower sector, and compute vector-boson momenta q_1, q_2
- for each event add counter-event with projected Born kinematics and opposite weight

The counter-events cancel identically with the projected terms from the "inclusive" contribution.

Beyond the Structure Function Approach

Schematically we express the "projection-to-Born" (P2B) method as

$$d\sigma = \int d\Phi_B (B + V) + \int d\Phi_R R$$

$$= \underbrace{\int d\Phi_B (B + V) + \int d\Phi_R R_{P2B}}_{\text{"inclusive" contribution}} + \underbrace{\int d\Phi_R R - \int d\Phi_R R_{P2B}}_{\text{"exclusive" contribution}}$$

- from the "exclusive" ingredient we get the full double-real and one-loop single-real contributions.

- when integrated over phase-space, the counter-events cancel the projected double-real and one-loop single-real contributions from the "inclusive" ingredient
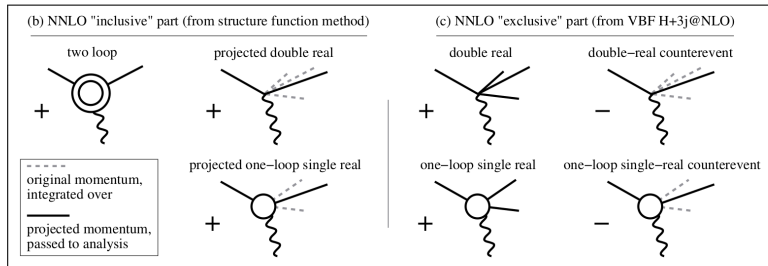
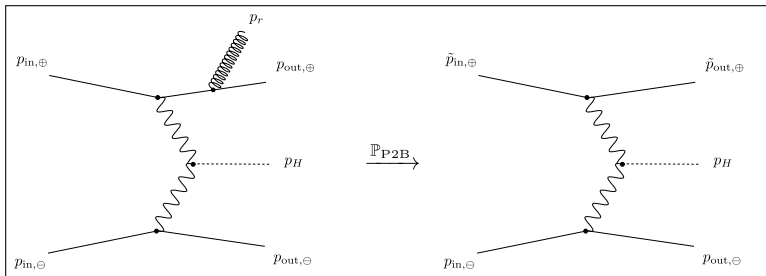Hence the sum of the two contributions gives the complete, fully differential NNLO result.

Beyond the Structure Function Approach

Schematically we express the "projection-to-Born" (P2B) method as

$$d\sigma = \int d\Phi_B (B + V) + \int d\Phi_R R$$

$$= \underbrace{\int d\Phi_B (B + V) + \int d\Phi_R R_{P2B}}_{\text{"inclusive" contribution}} + \underbrace{\int d\Phi_R R - \int d\Phi_R R_{P2B}}_{\text{"exclusive" contribution}}$$

(b) NNLO "inclusive" part (from structure function method)

two loop   projected double real

(c) NNLO "exclusive" part (from VBF H+3j@NLO)

double real   double−real counterevent

- - - - - original momentum, integrated over

projected momentum, passed to analysis

projected one−loop single real

one−loop single real   one−loop single−real counterevent

The projection

$$\Omega_R = (p_{\text{in},\oplus}, p_{\text{in},\ominus}, p_H, p_{\text{out},\oplus}, p_{\text{out},\ominus}, p_r)$$

$$\mathbb{P}_{\text{P2B}} \Omega_R = \Omega_{\text{P2B}}$$
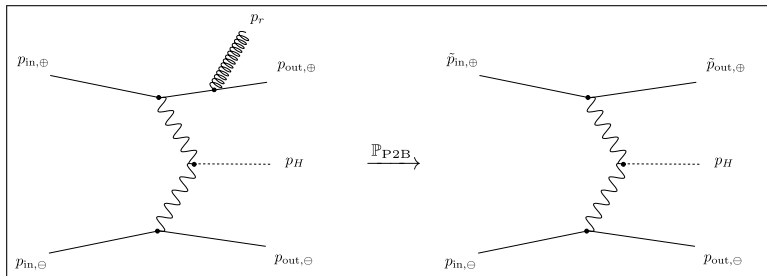$$= (\tilde{p}_{\text{in},\oplus}, \tilde{p}_{\text{in},\ominus}, p_H, \tilde{p}_{\text{out},\oplus}, \tilde{p}_{\text{out},\ominus})$$

Use lightcone coordinates:

$$p = (p^x, p^y, p^-, p^+) \qquad \text{with} \qquad p^\pm = \frac{1}{\sqrt{2}} (p^E \pm p^z)$$

The projection

$$\Omega_{\text{R}} = (p_{\text{in},\oplus}, p_{\text{in},\ominus}, p_H, p_{\text{out},\oplus}, p_{\text{out},\ominus}, p_r)$$

$$\mathbb{P}_{\text{P2B}} \, \Omega_{\text{R}} = \Omega_{\text{P2B}}$$
$$= (\tilde{p}_{\text{in},\oplus}, \tilde{p}_{\text{in},\ominus}, p_H, \tilde{p}_{\text{out},\oplus}, \tilde{p}_{\text{out},\ominus})$$

Such that

$$p_{\text{in}} = (0, 0, 0, p_{\text{in}}^+) \qquad\qquad \tilde{p}_{\text{in}} = (0, 0, 0, \tilde{p}_{\text{in}}^+)$$

$$p_{\text{out}} = (p_{\text{out}}^x, p_{\text{out}}^y, p_{\text{out}}^-, p_{\text{out}}^+) \qquad \tilde{p}_{\text{out}} = (\tilde{p}_{\text{out}}^x, \tilde{p}_{\text{out}}^y, \tilde{p}_{\text{out}}^-, \tilde{p}_{\text{out}}^+)$$

$$p_r = (p_r^x, p_r^y, p_r^-, p_r^+)$$

The projection
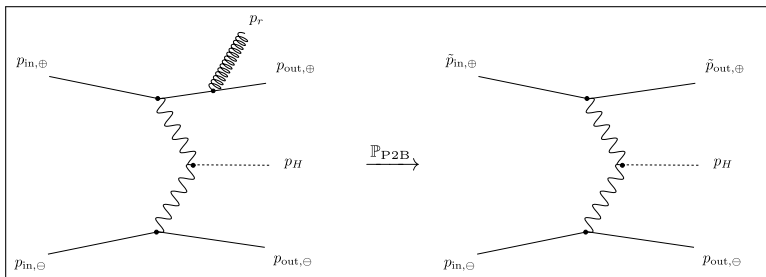
By momentum conservation we have

$$p_{in} - p_{out} - p_r = \tilde{p}_{in} - \tilde{p}_{out} \quad \Rightarrow \quad \begin{cases} \tilde{p}_{out}^{x} = p_{out}^{x} + p_{r}^{x} \\ \tilde{p}_{out}^{y} = p_{out}^{y} + p_{r}^{y} \\ \tilde{p}_{out}^{-} = p_{out}^{-} + p_{r}^{-} \\ \tilde{p}_{in}^{+} = p_{in}^{+} - p_{out}^{+} - p_{r}^{+} + \tilde{p}_{out}^{+}. \end{cases}$$

$$(\tilde{p}_{out})^2 = 0$$
$$\Rightarrow \quad (\tilde{p}_{out}^{x})^2 + (\tilde{p}_{out}^{y})^2 - 2\tilde{p}_{out}^{-}\tilde{p}_{out}^{+} = 0$$
$$\Rightarrow \quad \tilde{p}_{out}^{+} = \frac{(p_{out}^{x} + p_{r}^{x})^2 + (p_{out}^{y} + p_{r}^{y})^2}{2(p_{out}^{-} + p_{r}^{-})},$$

Implementation

1. "inclusive" code
   - matrix elements coded with structure functions using parametrised versions of the DIS coefficient functions evaluated by HOPPET
   - phase-space taken from POWHEG's VBF_H generator
2. "exclusive" code
   - start with the VBF_HJJJ calculation in POWHEG (based on vbfnlo)
   - extend POWHEG's tags to uniquely associate radiation with each sector (upper or lower line)
   - for each event map the kinematics onto Born-like kinematics and determine the vector-boson momenta q_1, q_2 using the equations on p.7.

- we have tested the "inclusive" code against a private version of the structure function calculation (thanks to Marco Zaro) and the structure functions themselves against APFEL 2.4.1.
- we have tested that the "exclusive" code reproduces the original VBF_HJJJ result. The sum of "inclusive" and "exclusive" at NLO agrees with VBF_H
- tagging tested by checking that the probablity of assigning a parton to the wrong sector decreases as the rapidity between the two hardest jets increases

Phenomenology

We study 13 TeV LHC collisions with $M_H = 125$ GeV and NNPDF3.0_nnlo_as118. We use the following VBF cuts:

- Jets defined with anti-k_t, $R = 0.4$ and $p_t > 25$ GeV
- Two hardest jets within $|y| < 4.5$
- High dijet invariant mass, $M_{j_1 j_2} > 600$ GeV, and separation, $\Delta y_{j_1 j_2} > 4.5$
- Hardest jets in opposite hemispheres, $y_{j_1} y_{j_2} < 0$

We choose a central scale which approximates well $\sqrt{Q_1 Q_2}$ and symmetrically vary by a factor 2 up and down

$$\mu_0^2(p_{t,H}) = \frac{M_H}{2} \sqrt{\left(\frac{M_H}{2}\right)^2 + p_{t,H}^2}$$

Phenomenology

	$\sigma^{(\text{no cuts})}$ [pb]	$\sigma^{(\text{VBF cuts})}$ [pb]
LO	$4.032\,^{+0.057}_{-0.069}$	$0.957\,^{+0.066}_{-0.059}$
NLO	$3.929\,^{+0.024}_{-0.023}$	$0.876\,^{+0.008}_{-0.018}$
NNLO	$3.888\,^{+0.016}_{-0.012}$	$0.826\,^{+0.013}_{-0.014}$

- NNLO corrections tiny ($\sim 1\%$) without cuts and sizeable with VBF cuts ($\sim 5\%$)
- NNLO results outside NLO band (also true when using NLO PDFs)
- corrections tend to be dominated by the extra real radiation. The effect is softer jets and hence fewer events pass the cuts

Phenomenology
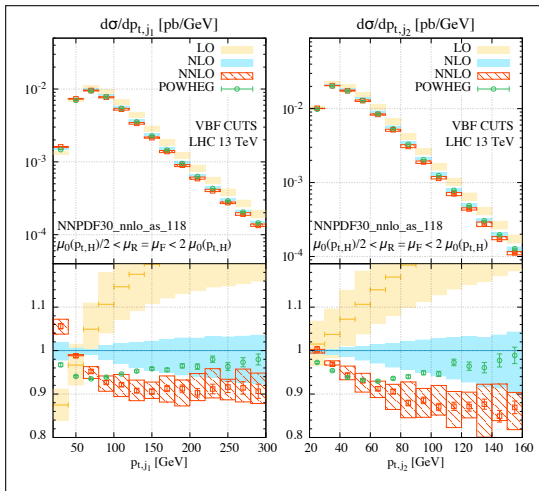
- NNLO corrections can be large $\mathcal{O}(10\%)$ and are often outside the NLO band
- the NNLO corrections tend to be dominated by extra real radiation. These appear to make the jets softer
- NOTE: NNLO PDF used everywhere. Similar results hold when using LO/NLO PDFs
- expanding the scale variation from 3-point to 7-point doesn't change the size of the NLO bands noticeably

More Phenomenology
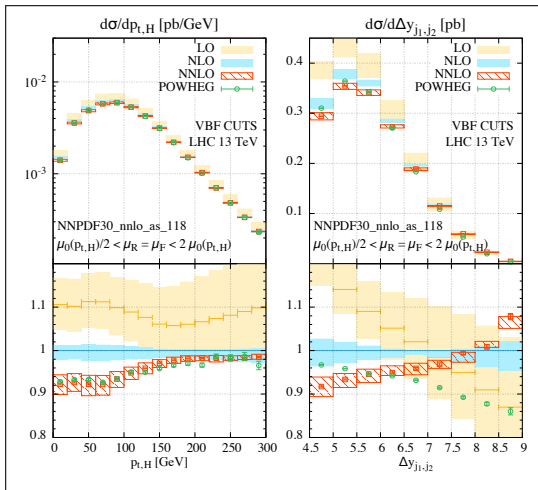
- in some cases NLO+PS agrees very well with the NNLO result (in particular $p_{t,H}$, M_{jj} and ϕ_{jj})
- in some cases not ($\Delta y_{j_1,j_2}$ and H_t)
- in general only modest shrinkage of bands from NLO to NNLO
- non-trivial kinematic dependence on k-factors (both LO/NLO and NNLO/NLO)

LHCHXWG YR4 results

We study LHC collisions with $M_H = 125$ GeV and PDF4LHC15_nnlo_100. We use the following setup:

- Jets defined with anti-k_t, $R = 0.4$ and $p_t > 20$ GeV
- Two hardest jets within $|y| < 5$
- Dijet invariant mass, $M_{j_1 j_2} > 130$ GeV, and separation, $\Delta y_{j_1 j_2} > 3$
- $\mu_R = \mu_F = M_W$ (varied a factor 2 up and down)
- Electroweak corrections obtained with HAWK [Ciccolini, Denner, Dittmaier (2008)]
- Photon PDF obtained from NNPDF2.3QED
- $\sigma^{VBF} = \sigma^{DIS}_{NNLOQCD}(1 + \delta_{EW}) + \sigma_\gamma$

$\sqrt{s}$[TeV]	σ^{VBF}[fb]	Δ_{scale}[%]	$\Delta_{PDF/\alpha_s/PDF\oplus\alpha_s}$[%]	$\sigma^{DIS}_{NNLOQCD}$[fb]	δ_{EW}[%]	σ_γ[fb]	$\sigma_{s\text{-channel}}$[fb]
7	1241	$^{+0.19}_{-0.21}$	$\pm 2.1/\pm 0.4/\pm 2.2$	1281	-4.4	17.1	584.5(3)
8	1601	$^{+0.23}_{-0.24}$	$\pm 2.1/\pm 0.4/\pm 2.2$	1656	-4.6	22.1	710.4(3)
13	3782	$^{+0.43}_{-0.33}$	$\pm 2.1/\pm 0.5/\pm 2.1$	3939	-5.3	51.9	1378.1(6)
14	4278	$^{+0.43}_{-0.34}$	$\pm 2.1/\pm 0.5/\pm 2.1$	4461	-5.4	58.5	1515.9(6)

| $\sqrt{s}$[TeV]	σ^{VBF}[fb]	Δ_{scale}[%]	$\Delta_{PDF/\alpha_s/PDF\oplus\alpha_s}$[%]	$\sigma^{DIS}_{NNLOQCD}$[fb]	δ_{EW}[%]	σ_γ[fb]	$\sigma_{s\text{-channel}}$[fb]
7	602.4(5)	$^{+1.3}_{-1.6}$	$\pm 2.3/\pm 0.3/\pm 2.3$	630.8(5)	-6.1	9.9	8.2
8	795.9(6)	$^{+1.3}_{-1.5}$	$\pm 2.3/\pm 0.3/\pm 2.3$	834.8(7)	-6.2	13.1	11.1
13	1975(1)	$^{+1.3}_{-1.2}$	$\pm 2.1/\pm 0.4/\pm 2.2$	2084(1)	-6.8	32.3	29.0
14	2236(3)	$^{+1.5}_{-1.3}$	$\pm 2.1/\pm 0.4/\pm 2.1$	2362(3)	-6.9	36.7	33.1

LHCHXSWG YR4 results
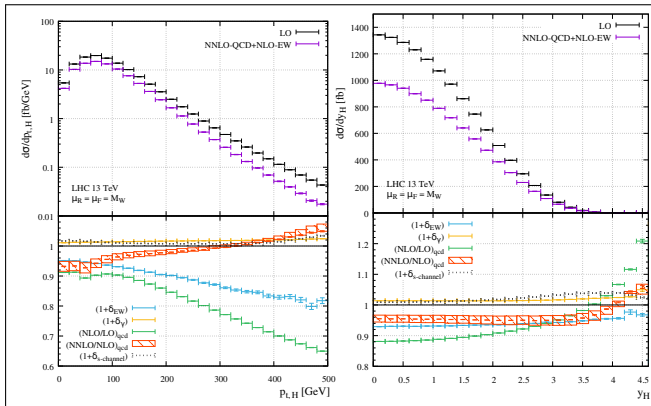
- NLO-QCD corrections can be very large (notice scale choice)
- NLO-EW corrections can be sizeable in the tails of distributions
- NNLO-QCD corrections of the same order as NLO-EW corrections
- Photon induced contribution very flat

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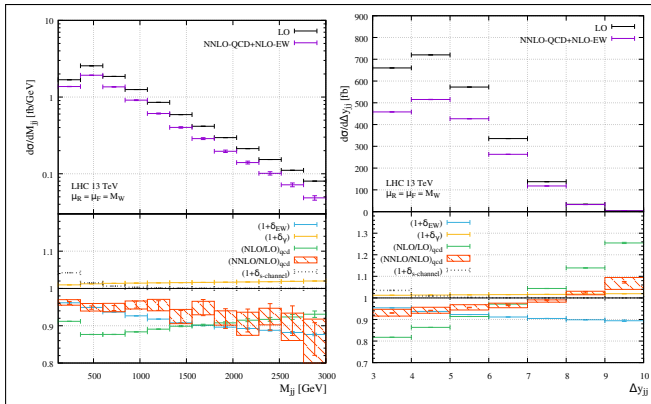
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VBF @ 100 TeV
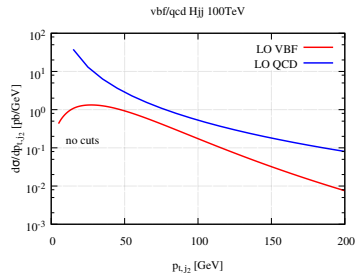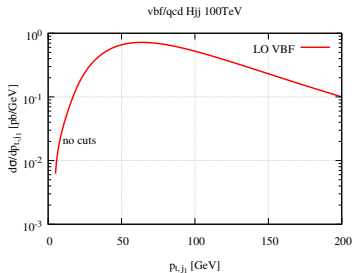
- due to the EW production mechanism of VBF, the jet spectra are mostly unchanged going from 14 TeV to 100 TeV
- in particular, the two tag jets have transverse momenta set by the vector boson mass whereas the QCD jets tend to peak at much lower transverse momenta
- VBF cuts should therefore still efficiently suppress the QCD background even at higher energies

VBF @ 100 TeV
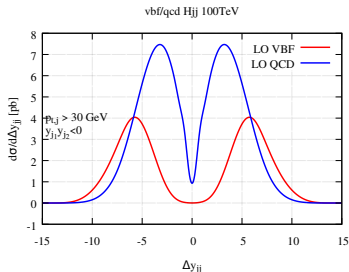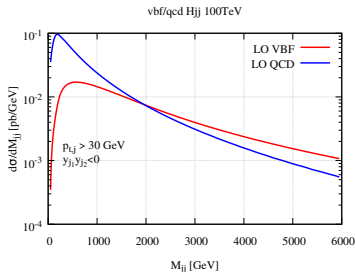
- after applying a soft transverse momentum cut on the tagging jets and requiring the jets to be in opposite dector hemispheres, we may try to optimise our VBF cuts
- it is clear that VBF production starts dominating for $\|\Delta y_{jj}\| > 6.5$
- after imposing this cut, the M_{jj} peak is shifted to around 2400 GeV
- in order to sufficiently suppress the QCD background while not cutting away the VBF peak we impose $M_{jj} > 1600$ GeV
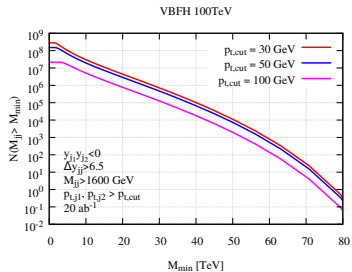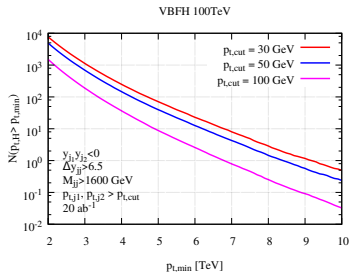
VBF @ 100 TeV

vbf/qcd Hjj 100TeV

vbf/qcd Hjj 100TeV
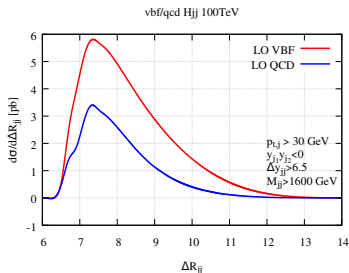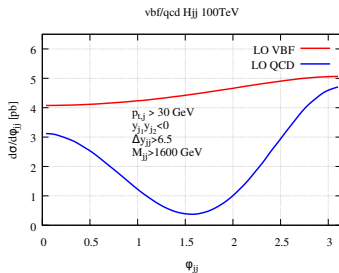
- further reduction possible by cutting in ϕ_{jj} whereas a cut in R_{jj} only minimally reduces the QCD background

VBF cuts	$\sigma(p_{t,j} > 30 \text{ GeV})$ [pb]	$\sigma(p_{t,j} > 50 \text{ GeV})$ [pb]	$\sigma(p_{t,j} > 100 \text{ GeV})$ [pb]
VBFH	14.1	7.51	1.08
QCD Hjj	5.04	1.97	0.331

| No cuts	$\sigma(p_{t,j} > 30 \text{ GeV})$ [pb]	$\sigma(p_{t,j} > 50 \text{ GeV})$ [pb]	$\sigma(p_{t,j} > 100 \text{ GeV})$ [pb]
VBFH	51.3	28.5	5.25
QCD Hjj	166	78.6	23.9

VBF @ 100 TeV

- at a 100 TeV collider gathering 20 ab^{-1} of data, we can expect very high reach in energy
- for the transverse momentum distribution of the Higgs, we can expect a reach of almost 10 TeV
- tagging jets will be produced with an invariant mass of up to 80 TeV
- without imposing cuts the reach is of course even greater

VBF @ 100 TeV
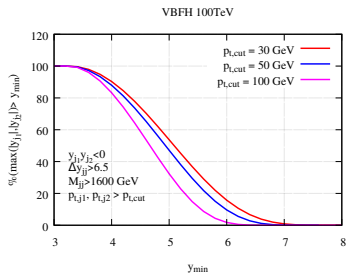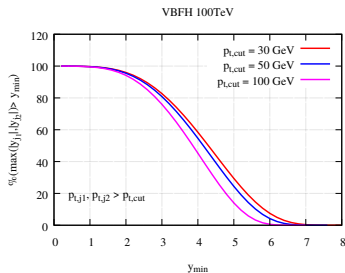
- studying VBF at a 100 TeV collider will require impressive detectors
- before VBF cuts are applied ATLAS/CMS would lose roughly 50% of the signal
- after VBF cuts it would be as much as 80% of the signal depending on the jet definition
- in order to retain more than 90% of the signal the detector would need a rapidity reach of about 6.5
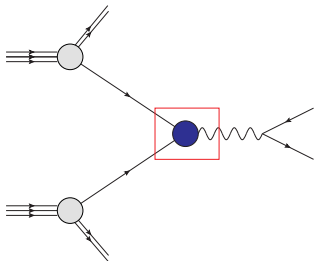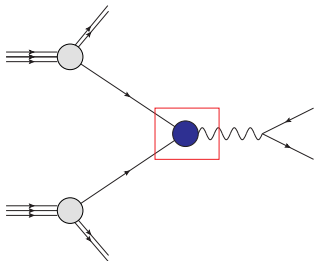
What about parton showers?
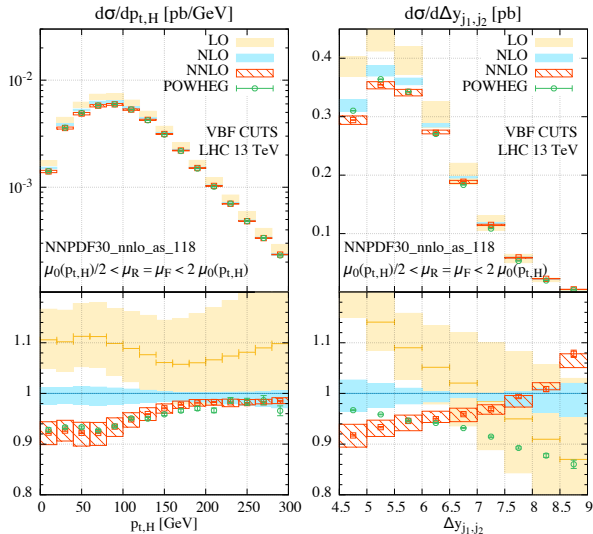
- parton shower: algorithm to resum (some classes of) collinear/soft logs in a "fully-exclusive" way.
- based on description of multiple soft-collinear real and virtual radiative corrections using a probabilistic language

$$d\sigma_{\rm SMC} = \underbrace{|\mathcal{M}_B|^2 d\Phi_B}_{d\sigma_B} \left\{ \right\}$$

Parton showers

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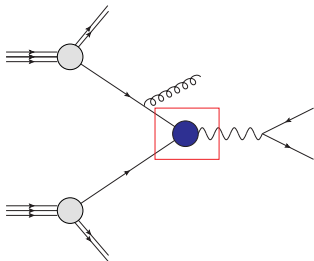$$d\sigma_{\mathrm{SMC}} = \underbrace{|\mathcal{M}_B|^2 d\Phi_B}_{d\sigma_B} \left\{ \Delta(t_{\max}, t_0) \right\}$$

$$\Delta(t_{\max}, t) = \exp\left\{ -\int_t^{t_{\max}} d\Phi_r' \, \frac{\alpha_s}{2\pi} \frac{1}{t'} P(z') \right\}$$

Parton showers

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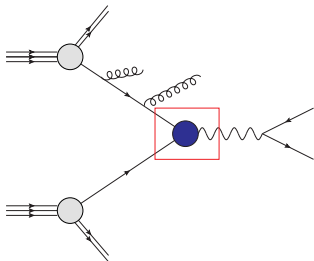

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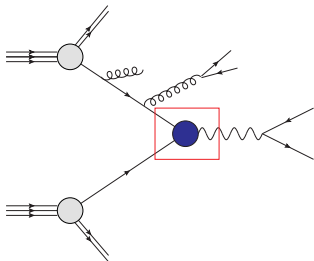

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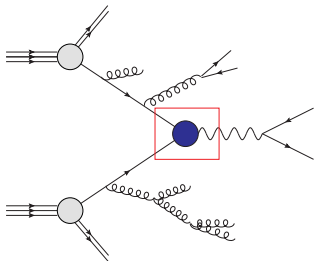

$$\Delta(t_{\max}, t) = \exp\left\{ -\int_t^{t_{\max}} d\Phi'_r \, \frac{\alpha_s}{2\pi} \frac{1}{t'} P(z') \right\}$$

Parton showers

- **parton shower**: algorithm to resum (some classes of) collinear/soft logs in a "fully-exclusive" way.
- based on description of multiple **soft-collinear** real and virtual radiative corrections using a **probabilistic language**

$$d\sigma_{\mathrm{SMC}} = \underbrace{|\mathcal{M}_B|^2 d\Phi_B}_{d\sigma_B} \left\{ \Delta(t_{\max}, t_0) + \Delta(t_{\max}, t) \underbrace{d\mathcal{P}_{\mathrm{emis}}(t)}_{\frac{\alpha_s}{2\pi} \frac{1}{t} P(z) \, d\Phi_r} \underbrace{\{\Delta(t, t_0) + \Delta(t, t') d\mathcal{P}_{\mathrm{emis}}(t')\}}_{t' < t} \right\}$$
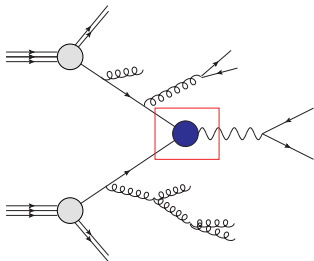

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$$\Delta(t_{\max}, t) = \exp\left\{ -\int_t^{t_{\max}} d\Phi_r' \frac{\alpha_s}{2\pi} \frac{1}{t'} P(z') \right\}$$

This is "LOPS"

- A parton shower changes shapes, not the overall normalization, which stays LO (*unitarity*)
- LL resummation is included in Sudakov form factors: easy to see that probability of having arbitrarily collinear emission becomes 0, instead of ∞

NLO+Parton Showers

- parton showers are only LO+LL: clearly including NLO corrections would be a big improvement. There are 2 methods to achieve this consistently:
- the POWHEG method:
  1. do these replacement

  $$B(\Phi_n) \Rightarrow \bar{B}(\Phi_n) = B(\Phi_n) + \frac{\alpha_s}{2\pi}\left[V(\Phi_n) + \int R(\Phi_{n+1})\,d\Phi_r\right]$$

  $$\Delta(t_m, t) \Rightarrow \Delta(\Phi_n; k_T) = \exp\left\{-\frac{\alpha_s}{2\pi}\int \frac{R(\Phi_n, \Phi'_r)}{B(\Phi_n)}\theta(k'_T - k_T)\,d\Phi'_r\right\}$$

  2. POWHEG "master formula" for the hardest emission:

  $$d\sigma_{\mathrm{POW}} = d\Phi_n\,\bar{B}(\Phi_n)\left\{\Delta(\Phi_n; k_T^{\min}) + \Delta(\Phi_n; k_T)\frac{\alpha_s}{2\pi}\frac{R(\Phi_n, \Phi_r)}{B(\Phi_n)}\,d\Phi_r\right\}$$

  [+ p_T-*vetoing subsequent emissions*, to avoid double-counting]

  3. properties:
     - inclusive observables: @NLO
     - first hard emission: full tree level ME
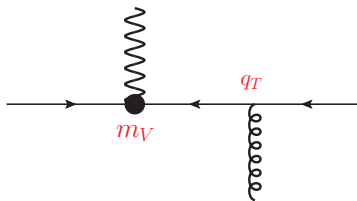   - (N)LL resummation of collinear/soft logs

---

- NLOPS has become the standard for LHC searches (at least for SM processes)

1. $V+j$ @ NLO, $V+jj$ @ LO $\Rightarrow$ use $V+j$ @ NLOPS (POWHEG)

$$d\sigma_{\mathrm{POWHEG}} = d\Phi_n \, \bar{B}_{\mathrm{NLO}}(\Phi_n) \left\{ \Delta(\Phi_n; k_{\mathrm{T}}^{\min}) + \Delta(\Phi_n; k_{\mathrm{T}}) \frac{\alpha_s}{2\pi} \frac{R(\Phi_n, \Phi_r)}{B(\Phi_n)} \, d\Phi_r \right\}$$

$$\bar{B}_{\mathrm{NLO}}(\boldsymbol{\Phi}_n) \, d\boldsymbol{\Phi}_n = \alpha_{\mathrm{S}}(\mu_R) \left[B + \alpha_{\mathrm{S}}^{(\mathrm{NLO})} V(\mu_R) + \alpha_{\mathrm{S}}^{(\mathrm{NLO})} \int d\Phi_{\mathrm{r}} R \right] d\boldsymbol{\Phi}_n$$

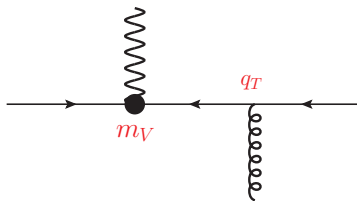$V+j$ is a 2-scales problem ($\rightarrow$ choice of μ not unique)

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$V+j$ is a 2-scales problem ($\rightarrow$ choice of μ not unique)

☞ want to reach NNLO accuracy for *e.g.* y_V, *i.e.* when fully inclusive over QCD radiation

- need to allow the 1st jet to become unresolved
- the above approach needs to be modified: as it stands, $\bar{B}_{\text{NLO}}(\Phi_n)$ is not finite when $q_T \rightarrow 0$!

2. integrate over phase space regions where V is produced with arbitrarily soft/collinear jet (i.e. finite results when integrating over all q_T spectrum)

MiNLO: Multiscale Improved NLO [Hamilton,Nason,Zanderighi, 1206.3572]

- original goal: method to a-priori choose scales in multijet NLO computation (where hierarchy among scales can spoil accuracy since resummation of logs is missing)
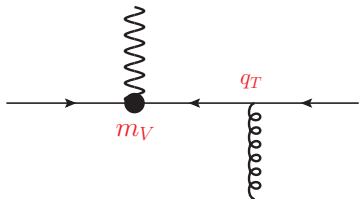- how: correct weights of different NLO terms with CKKW-inspired approach:

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- how: correct weights of different NLO terms with CKKW-inspired approach:
    - for all PS points, build the "more-likely" shower history that would have produced it (can be done by clustering kinematics with k_T-algo)
    - correct original NLO including α_S couplings evaluated at nodal scales and Sudakov FFs
    - make sure that NLO accuracy is not spoiled !

MiNLO

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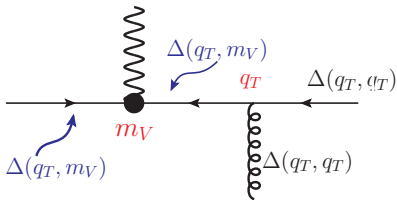
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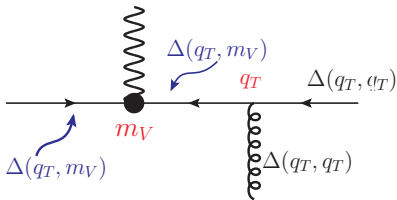(i.e. finite results when integrating over all q_T spectrum)

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$$\bar{B}_{\mathrm{MiNLO}} = \alpha_{\mathrm{S}}(q_T)\Delta_q^2(q_T,m_V)\Big[B\Big(1 - 2\Delta_q^{(1)}(q_T,m_V)\Big) + \alpha_{\mathrm{S}}^{(\mathrm{NLO})}V(\bar{\mu}_R) + \alpha_{\mathrm{S}}^{(\mathrm{NLO})}\int d\Phi_{\mathrm{r}}R\Big]$$

$\star\ \bar{\mu}_R = q_T$

$\star\ \log\Delta_{\mathrm{f}}(q_T,m_V) = -\int_{q_T^2}^{m_V^2}\frac{dq^2}{q^2}\frac{\alpha_{\mathrm{S}}(q^2)}{2\pi}\Big[A_f\log\frac{m_V^2}{q^2} + B_f\Big]$

$\star\ \Delta_{\mathrm{f}}^{(1)}(q_T,m_V) = -\frac{\alpha_{\mathrm{S}}^{(\mathrm{NLO})}}{2\pi}\Big[\frac{1}{2}A_{1,\mathrm{f}}\log^2\frac{m_V^2}{q_T^2} + B_{1,\mathrm{f}}\log\frac{m_V^2}{q_T^2}\Big]$

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☞ Sudakov FF included on $V+j$ Born kinematics

- VJ-MiNLO yields finite results also when 1st jet is unresolved ($q_T \to 0$)
- $\bar{B}_{\mathrm{MiNLO}}$ ideal to extend validity of $V+j$ POWHEG

MiNLO VBF Hjjj
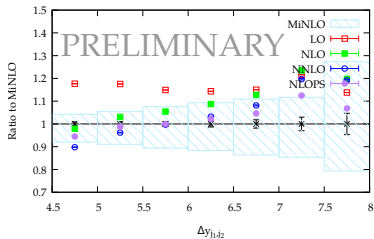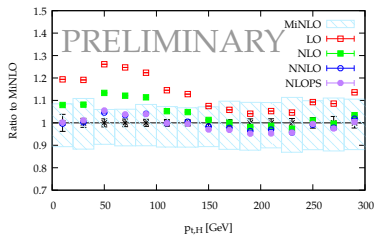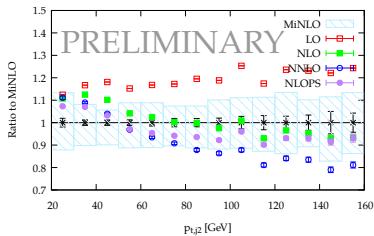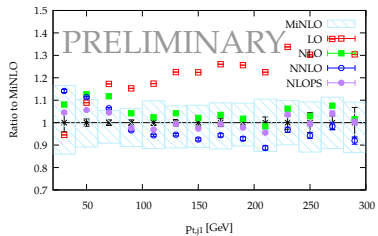
Conclusions

- inclusive N^3LO are tiny and well within scale uncertainty band at NNLO
- scale uncertainty band reduced significantly
- differential NNLO corrections are sizeable, $\mathcal{O}(10\%)$, and necessary for precision phenomenology
- only moderate shrinkage of NNLO bands compared to NLO bands
- NLO-EW corrections are comparable to NNLO-QCD corrections and should be included
- "projection-to-Born" method can be extended to compute fully differential VBF Higgs at N^3LO

A public code, `proVBFH`, will be released in the near future. Until then total cross sections and distributions with specific cuts can be provided.

Tagging
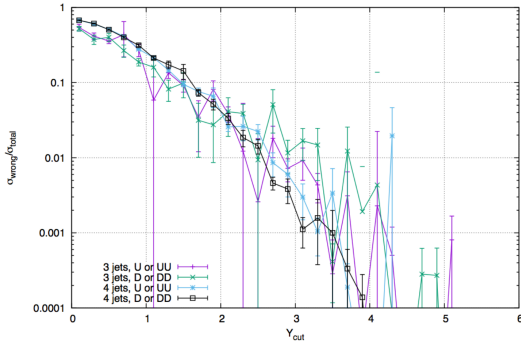
With no bug in the code, the probability of a tagged parton having wrong rapidity decreases with increasing rapidity separation between the two hardest jets.
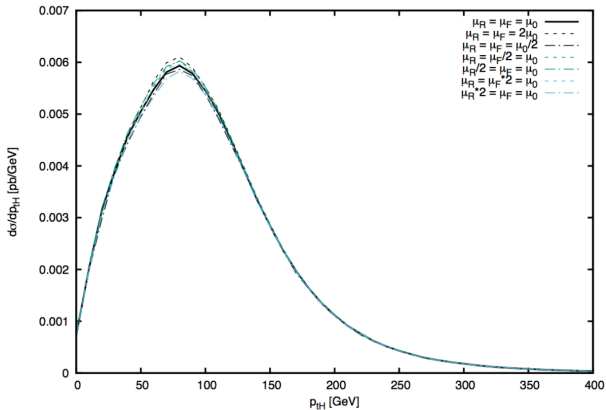
Tagging
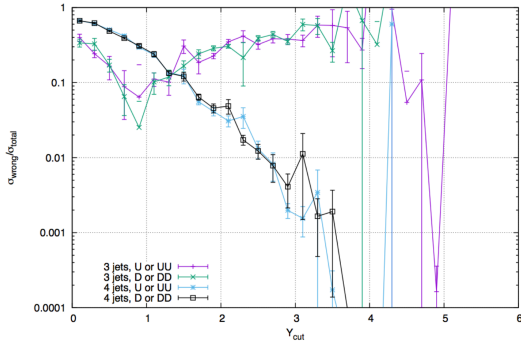
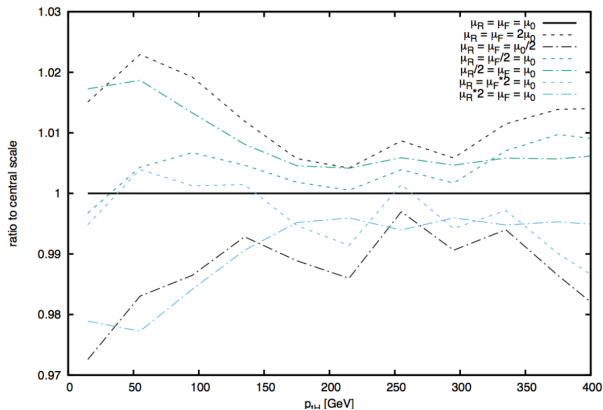With an $\mathcal{O}(1)$ bug in the code, this is clearly not the case any more.

3-point vs 7-point scale variations

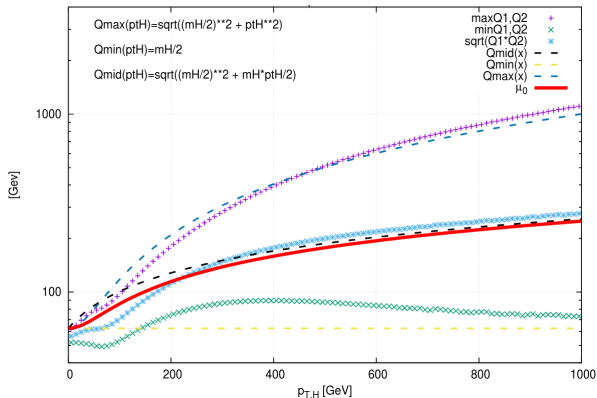3- and 7-point scale variations are very close to eachother.

3-point vs 7-point scale variations

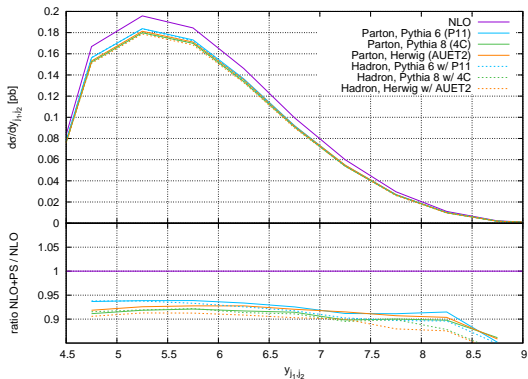3- and 7-point scale variations are very close to eachother.

Choice of scale

Our choice of $\mu_0^2(p_{t,H}) = \frac{M_H}{2}\sqrt{\left(\frac{M_H}{2}\right)^2 + p_{t,H}^2}$ is very close to a choice of $\mu = \sqrt{Q_1 Q_2}$.
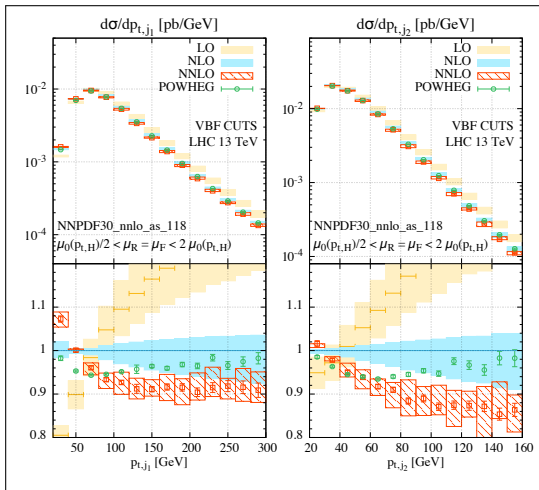
VBF and Parton Shower

- different parton showers give relatively similar results
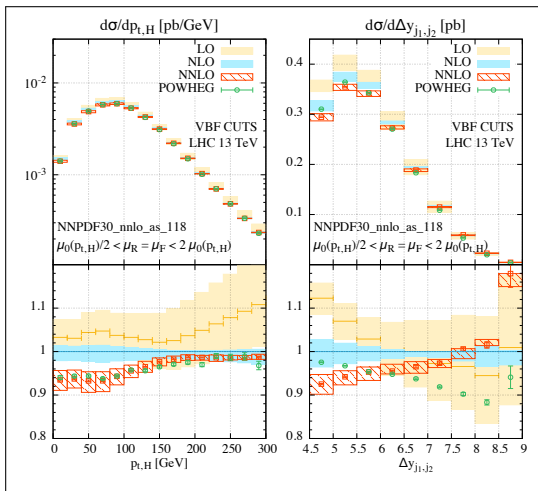- hadronisation effects are consistently small

NNLO/NLO/LO PDFs

- LO results with LO PDFs
- NLO results with NLO PDFs
- NNLO results with NNLO PDFs

NNLO/NLO/LO PDFs

- LO results with LO PDFs
- NLO results with NLO PDFs
- NNLO results with NNLO PDFs