8 Measurement of the Gravitational Constant G

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The most precise published results for the gravitational constant G have claimed uncertainties of about 100 ppm. Such measurements are usually done by some variant of the Cavendish torsion balance [1], but other methods have also been tried. During the last thirty years quite a few experiments have been performed, however without significant progress. What makes things worse, the results show large and unexplained discrepancies indicating that there are not well understood systematic effects.

To overcome these difficulties, we have developed a new method for measuring G. The idea arose from our previous experiment at the Gigerwald storage lake [2,3]. The method has become feasible due to recent progress in the construction of beam balances with extremely high sensitivity. The principle of the experiment will be explained in the following section.

The experiment is set up and running. A first result for G with an uncertainty of 230 ppm was published recently [4] and will briefly be described in this report. Presently, preparations are made for measurements with higher accuracy.

8.1 The principle of the experiment

A schematic view of the experiment is shown in Fig. 8.1. The essential components of the set-up are a single-pan beam balance, two test masses and two large field masses. The test masses (1 kg each) are suspended with thin tungsten wires and alternately connected to the balance. The difference of their weights is measured with high precision and taken as the signal. The balance and the test masses are inside a vacuum system. The field masses are cylindrical in shape and have a central bore such that the test masses can pass through. By moving the field masses between the two positions shown, their gravitational force on the test masses changes the signal. From the difference of the signal for the two states and from

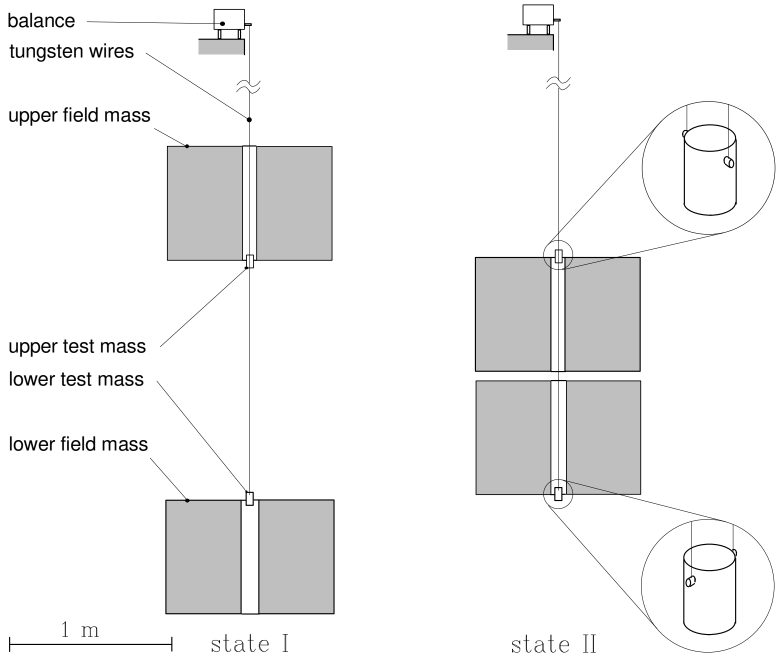

Figure 8.1: *Schematic view of the experiment to measure the gravitational constant. See main text for explanation.*

known values for the masses, densities, and distances, the gravitational constant G can be computed.

There are several features which make this principle of measurement very promising. The test masses are placed at positions where the gravitational force of the field masses has a local extremum. Therefore the relative position of the masses is quite uncritical. The value of G is computed from a double difference and many disturbing forces and drift effects cancel in the result. The field masses are vessels made of stainless steel with a volume of 500 l each. They can be filled with a liquid, either water or mercury. Compared with solids, a liquid is much to be preferred. This is because the density of a liquid is homogeneous, provided its temperature is homogeneous and this can be measured. The effect of compressibility is nearly negligible and can accurately be taken into account. The effect of the vessel walls can be measured with empty vessels and subtracted if necessary. Finally, the balance can be accurately calibrated with the weight of a standard mass.

8.2 First results

The experiment has been set up in a 4.8 m deep pit at the Paul-Scherrer-Institute. This place turned out to be well suited with good mechanical and thermal stability. The beam balance has been provided by Mettler-Toledo. It is the most precise balance, commercially available for comparing 1-kg masses. Since we need to measure only small weight differences, the sensitivity of the balance could be further improved. Detailed descriptions of the experiment have been given in previous annual reports.

The dimensions of the two vessels were measured on a Zeiss precision measuring table. The masses of the vessels were determined with an accuracy of 8 ppm using a precision balance with a measuring range of 500 kg. Hence G could be measured also with the empty vessels, which provided a valuable consistency check (see below).

First measurements of G with water-filled vessels were performed during last year. The procedure for filling in the water was as follows. First, water of high purity was put into containers with a volume of 20 l each and the containers were weighed. Then the water was pumped through a closed system from the containers into the vessels. To prevent air bubbles from being locked in the vessels, the vessels were first evacuated and again just before the last container was filled in. To determine the mass of the water in the vessels, the empty containers were weighed again. The amount of water remaining in the filling system was found similarly. In that way the mass of the water in the vessels could be determined with an uncertainty of 24 ppm.

As a consistency check, the volume of the filled in water was determined using the known density of pure water and compared with the independently measured volume of the vessels. The latter is presently known with 30 ppm uncertainty. Good agreement was found.

The gravitational signal was measured in the following way. The weight of one test mass was integrated for 150 s. Then the other test mass was connected to the balance and its weight measured over the same time interval. The difference of two such measurements is the signal, as defined before. These measurements were repeated for 270 min. Then the field masses were moved from one state to the other (see Fig. 8.1). Before that, the calibration of the balance was checked with a 1 g standard weight (in addition to a test mass). It turned out that the stability of the calibration was good enough and corrections for calibration drift were negligible. The sequence of events above was repeated many times. During the measurements, the temperature was monitored at 10 test points (vessels and vacuum enclosure) and kept constant to within 50 mK in space and time.

The total measuring time with water-filled vessels was three weeks. Two measurements

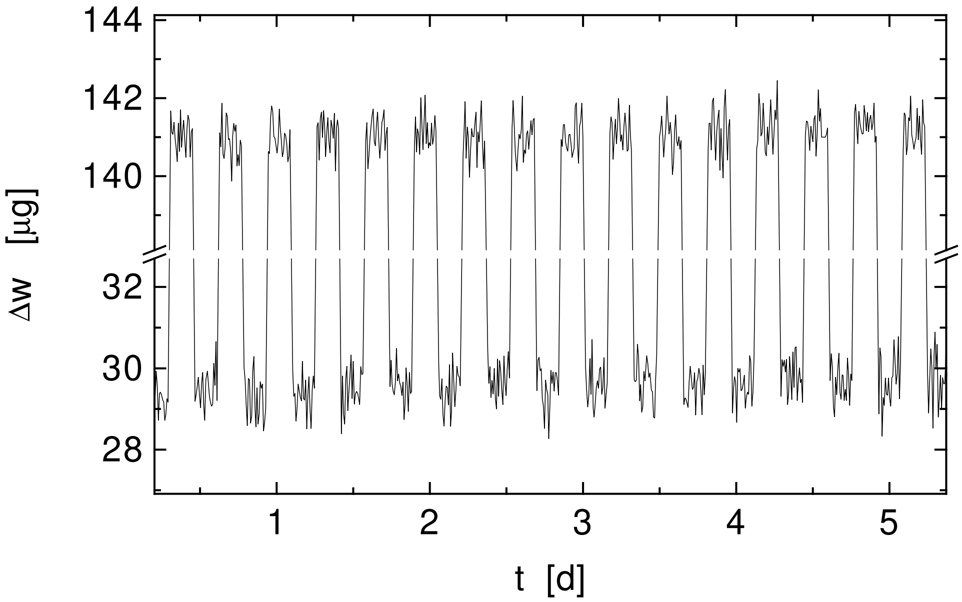

Figure 8.2: *A time series of the weight difference of the test masses, modulated by the gravitational force of the water-filled vessels. The vertical axis has been broken to show the noise.*

with empty vessels, each taking a similar amount of time, were performed before and after the measurements with water. Part of the data with water-filled vessels is shown in Fig. 8.2. The signal amplitude was approximately 111 μg as expected and could be determined with an statistical uncertainty of 8.3 ng or 75 ppm.

To determine G, the gravitational force on the test masses was computed by numerical integration of all movable parts of the field masses. The results from the three measurements

Uncertainties of G	ppm
Systematic effects of the balance	$\leq$ 130
Sorption effects	$\leq$ 45
Integration of mass distribution	$\leq$ 180
Mass of test mass	0.27
Test mass dimension	3
Test mass position	3
Weight of each vessel	8
Density of stainless steel vessels	10
Weight of water	8
Density of water	29
Mass calibration (1 g)	5
Local gravity (g)	0.06
Total	230

Table 8.1: *The systematic uncertainties of G as estimated for the measurement with water. The first group pertains to upper limits of systematic effects whose investigations are not yet completed.*

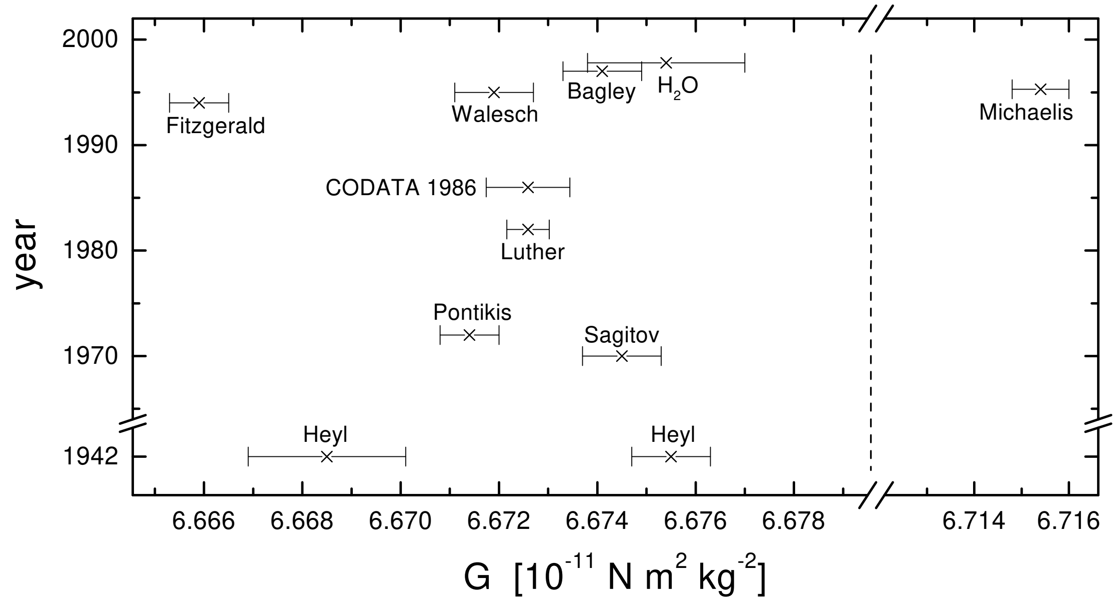

Figure 8.3: *Published results for G with a claimed uncertainty of less than* 300 *ppm [5,...,13]. The present standard value, recommended in 1986 by CODATA, is the value measured by Luther and Towler, but with doubled uncertainty. Our result is labeled 'H_2O'.*

were compatible within their statistical errors. The weighted average is

$$G = (6.6754 \pm 0.0005_{stat} \pm 0.0015_{sys}) \times 10^{-11} m^3 kg^{-1} s^{-2} \,.$$

The systematical uncertainties are given in Tab. 8.1. These values represent our present knowledge and we expect that the uncertainties can significantly be lowered in the future.

Our first result is compared in Fig. 8.3 with previous measurements. We have included the result of Michaelis *et al.* although likely to be wrong. This is of course well known by these authors. The amazing fact however is that despite much effort no cause could be found. Compared with the other results our value is quite high. In particular, it differs from the present standard value by 420 ppm (about $2\,\sigma$). At present, we see no reason in our experiment which could explain this discrepancy.

8.3 Preparation for measurements with mercury

The mercury was purchased in 400 flasks, each with a volume of 2.5 l. The total mercury mass is thus 13.6 tons. The specified purity is 99.99 %. Samples were taken from each flask so that later the density can be measured accurately. The mercury was filled into the vessels during last winter with basically the same procedure as previously with water. Each flask was weighed full and empty using a precision balance with a measuring range of 40 kg and a resolution of 10 mg. To minimize uncertainties due to a possible non-linearity of the balance, special calibration weights where used with approximately the same masses as a full or empty flask. About 3000 weighings were necessary, because the weight of each flask, full and empty, was determined from an average of three measurements and the calibration was checked after typically four flasks. In that way the mass of the filled in mercury could be determined with an uncertainty of 2 ppm.

A first measurement with mercury-filled vessels has been started recently. Some preliminary data are shown in Fig. 8.4. The signal amplitude is approximately 785 μg, about 7 times

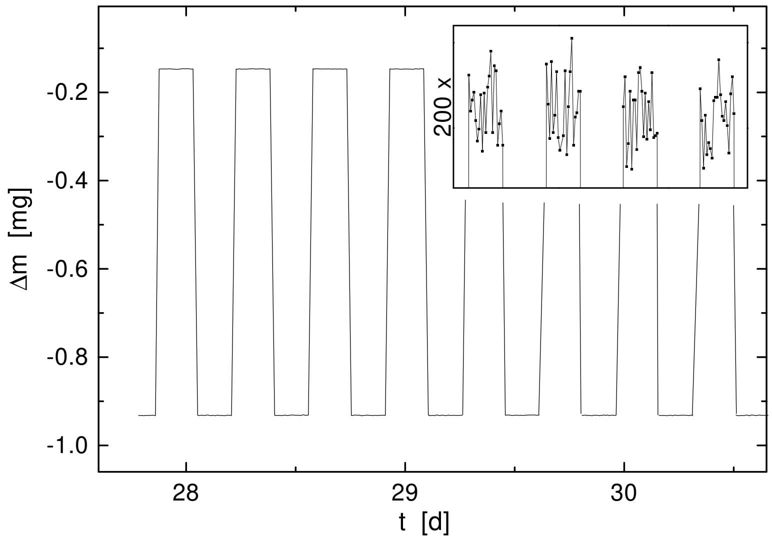

Figure 8.4: *A time series of the weight difference of the test masses modulated by the gravitational force of the mercury-filled vessels. The inset shows the signal with a vertical scale expanded by a factor 200.*

more than with water, and the signal is seen to be very clean. This makes us optimistic that we can achieve significant progress in the near future.

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