PHY117 HS2024

Today: Electric potential
Capacitance
Dielectrics
Electrodynamics $\begin{cases} \text{Electric current} \\ \text{Resistance} \end{cases}$

Note: sometimes we refer to a potential difference ΔV as V

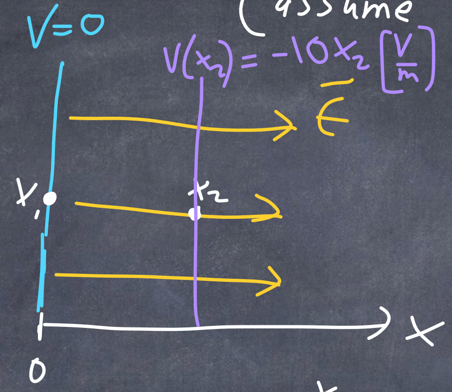
Be careful with E_0 + ε_0: (two different things)

Week 9, Lecture 2
Nov. 13th, 2024
Prof. Ben Kilminster

what is $V(x)$ if $\bar{E} = 10\frac{N}{C}\,\hat{x}$?

$E = 10\frac{N}{C} = 10\frac{V}{m}$

(assume that $V=0$ at $x=0$)



$$dV = -\bar{E}\cdot d\bar{l}$$

$$dV = -10\frac{V}{m}\,\hat{x}\cdot dx\,\hat{x}$$

$$dV = -10\,dx\left[\frac{V}{m}\right]$$

$$V(x_2) - V(x_1) = \int_{x_1}^{x_2} dV = \int_{x_1}^{x_2} -10\,dx\left[\frac{V}{m}\right] = -10x\Big]_{x_1}^{x_2}\left[\frac{V}{m}\right]$$

$$= 10\frac{V}{m}(x_1 - x_2)$$

we are told that $V=0$ at $x=0$

so $V(x_2) - V(x_1) = 10\frac{V}{m}(x_1 - x_2)$

since $V(x_1=0) = 0$

$$V(x_2) - 0 = 10\frac{V}{m}(0 - x_2) \Rightarrow V(x_2) = -10x_2\left[\frac{V}{m}\right]$$

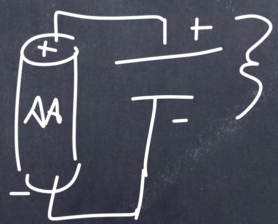
we saw that $\underset{\substack{\text{energy} \\ [J]}}{\Delta U} = \underset{\substack{\text{charge} \\ [C]}}{q}\ \underset{\substack{\text{potential} \\ [V]}}{\Delta V}$

A convenient unit of energy is the electron volt $[e \cdot V]$

example

$$\Delta U = 1\underset{\substack{\text{charge} \\ \text{of electron}}}{e}V = 1\,(1.6\mathrm{E}{-19}\,C)(1\,V) = 1.6\mathrm{E}{-19}\,J$$

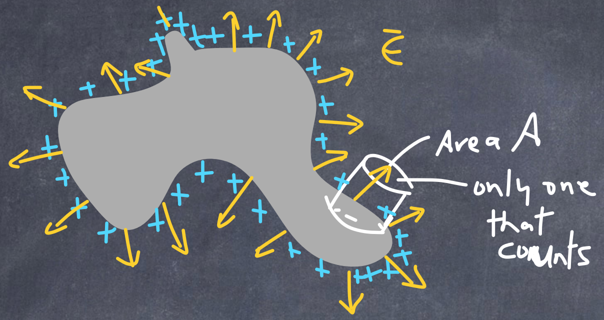
battery



$\Delta V = 1.5\,V \implies$ An electron moving through 1.5 V of potential difference will gain 1.5 eV of energy

The Large Hadron Collider (LHC) at CERN has protons with energy 6.5 teV = $6.5\mathrm{E}12$ eV

What is $\bar{E}$ close to a conductor?



Near surface, $\bar{E} \parallel \hat{n}$

Also, for a conductor, charge is on surface,

$$\oint_S \bar{E} \cdot \hat{n}\, dA = \frac{Q}{\varepsilon_0}$$

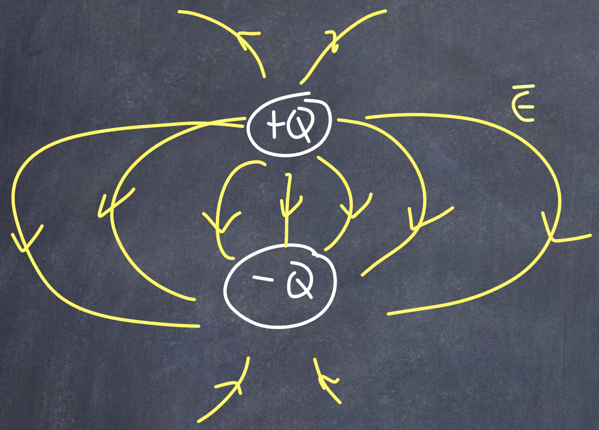
$$\sigma = \frac{Q}{A} \qquad Q = \sigma A$$

$$E \cancel{A} = \frac{\sigma \cancel{A}}{\varepsilon_0}$$

$$\boxed{\bar{E} = \frac{\sigma}{\varepsilon_0}\hat{n} \quad \text{normal to surface}}$$

Capacitors:

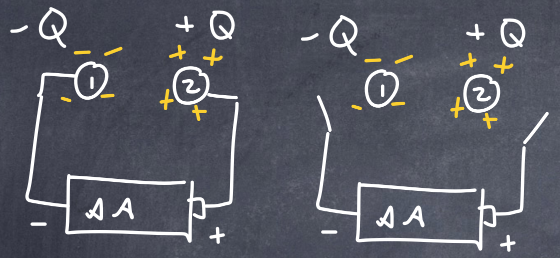
Any two oppositely charged conductors form a capacitor



In a circuit diagram, the symbol for a capacitor is  or 

We can charge a capacitor by connecting each conductor to the opposite terminals of a battery.

Let's charge a capacitor:



we charge with
$\Delta V = 1.5\,V$

Q on 1 is opposite to Q on 2.
There is a potential difference between the conductors, ΔV

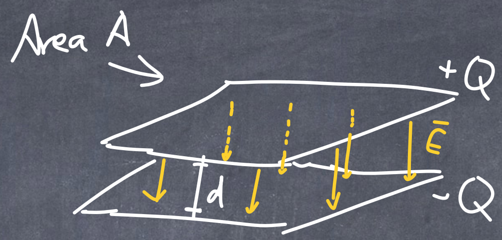
we define the capacitance as $\dfrac{Q}{|\Delta V|} \equiv C$

The capacitance is the ability of 2 conductors to store electrical energy.

The units of capacitance are
$1\ \text{Farad} = 1\,F = 1\,\dfrac{C}{V}$

Energy stored in a capacitor:
$$U = \frac{1}{2} C (\Delta V)^2$$

what is the capacitance of a parallel-plate capacitor, with area A, separation d, and a charge $+Q$, $-Q$



we know that $E = \frac{\sigma}{\varepsilon_0}$, $\sigma = \frac{Q}{A}$ $\Rightarrow$ $E = \frac{Q}{\varepsilon_0 A}$ ①

The Field is uniform between the 2 plates.

$$|\Delta V| = \left| -\int_0^d \bar{E} \cdot d\bar{\ell} \right| = E\ell \Big]_0^d = Ed \overset{①}{=} \frac{Qd}{\varepsilon_0 A}$$

$$C = \frac{Q}{|\Delta V|} = \frac{\cancel{Q}}{\frac{\cancel{Q}\, d}{\varepsilon_0 A}} = \frac{\varepsilon_0 A}{d}$$

$$\boxed{C = \frac{\varepsilon_0 A}{d}}$$

capacitance only depends on the dimensions.

← capacitance is $C = \frac{\varepsilon_0 A}{d} = \frac{Q}{|\Delta V|}$

$$|\Delta V| = \frac{Q d}{\varepsilon_0 A}$$

If we put Q on our capacitor, and increase d, then ΔV increase

Energy stored in a capacitor

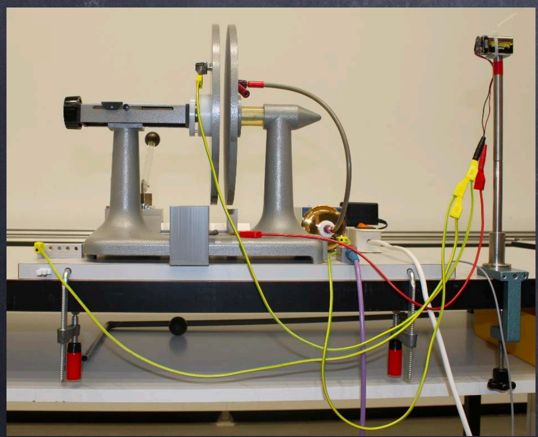
$$U = \frac{1}{2} C (\Delta V)^2$$

$$U = \frac{1}{2} \frac{\varepsilon_0 A}{\cancel{d}} E^2 d^{\cancel{2}}$$

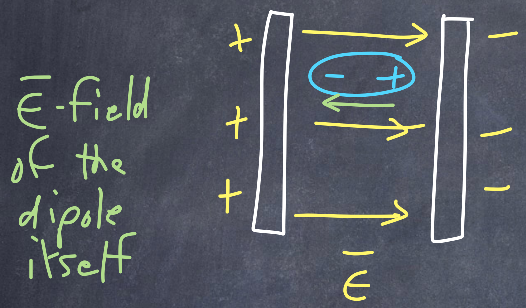
$$U = \frac{1}{2} \varepsilon_0 A E^2 d$$

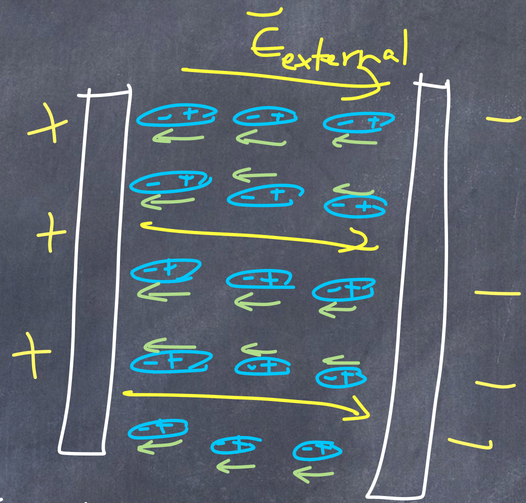
$$\Delta U = \frac{1}{2} \varepsilon_0 A E^2 (\Delta d) = -W$$

we do work to increase the stored energy of the capacitor



Dielectrics - In a non-conducting material, we know that dipoles tend to align opposite to the E-field





The $\bar{E}$-fields of the dipoles sum up

$$\text{total } \bar{E} \text{ inside} = \bar{E}_{external} - \bar{E}_{dipoles}$$

we call the non-conducting material a dielectric.

Each material has its own dielectric constant, K_d that weakens the electric field, such that $E = \dfrac{E_0}{K_d}$

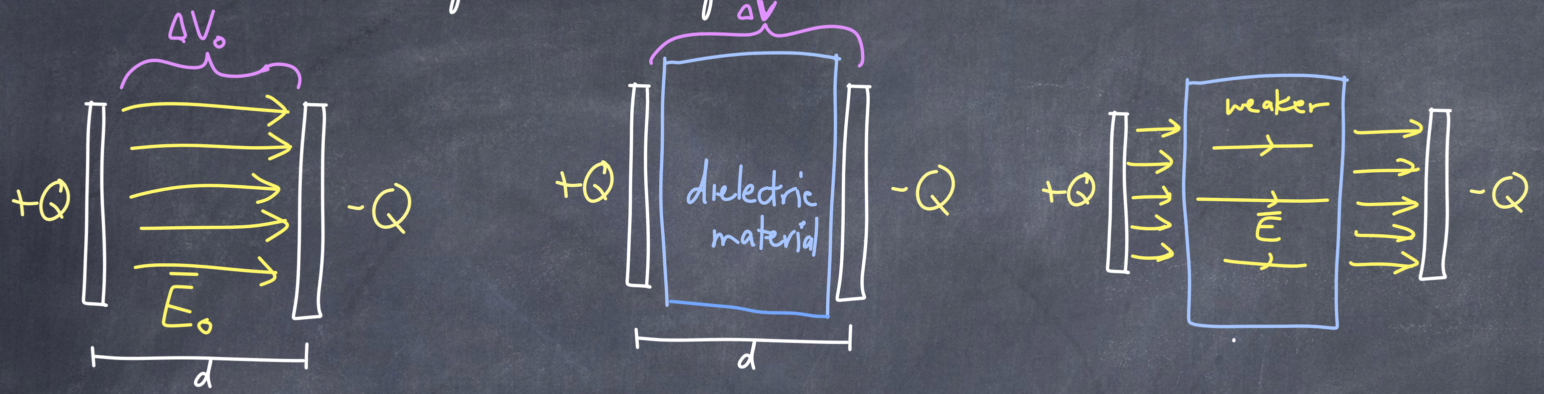
E_0 : E-field with no dielectric

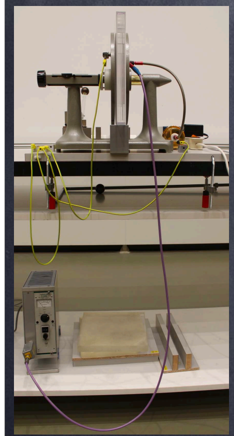
E : E-field with dielectric

materials	K_d Dielectric constant (no units)
air	1.00059
water	80
paper	3.7
parafin	2
plexiglass	3.4

note: $K_d > 1$

Parallel-plate capacitor with a dielectric





Initially, we have ΔV_0 + E_0 with no dielectric

$$|\Delta V_0| = E_0 d$$

And we know that $E = \dfrac{E_0}{K_d}$, so

we can get $\Delta V = E d = \dfrac{E_0 d}{K_d} = \dfrac{|\Delta V_0|}{K_d} \Rightarrow$ Adding dielectric decrease ΔV

So $$C = \frac{Q}{|\Delta V|} = \frac{Q\,K_d}{V_0} = K_d C_0$$

For our parallel plate capacitor:

$$C_0 = \frac{\varepsilon_0 A}{d} \rightarrow C = \frac{K_d \varepsilon_0 A}{d}$$

In general:

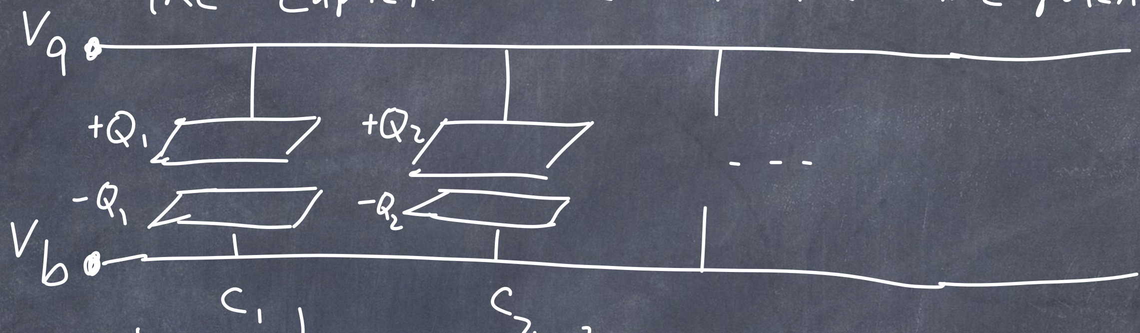
$C = Q/V$

$Q = CV$

Combining capacitors:

when we combine capacitors in parallel, the capacitors are at the same potential

$V \equiv V_a - V_b$



$Q_1 = C_1 V$ $\quad$ $Q_2 = C_2 V$ $\quad$ . . .

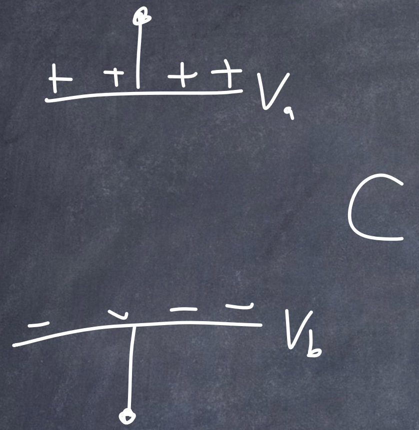
The total charge stored is $Q = Q_1 + Q_2 = C_1 V + C_2 V$

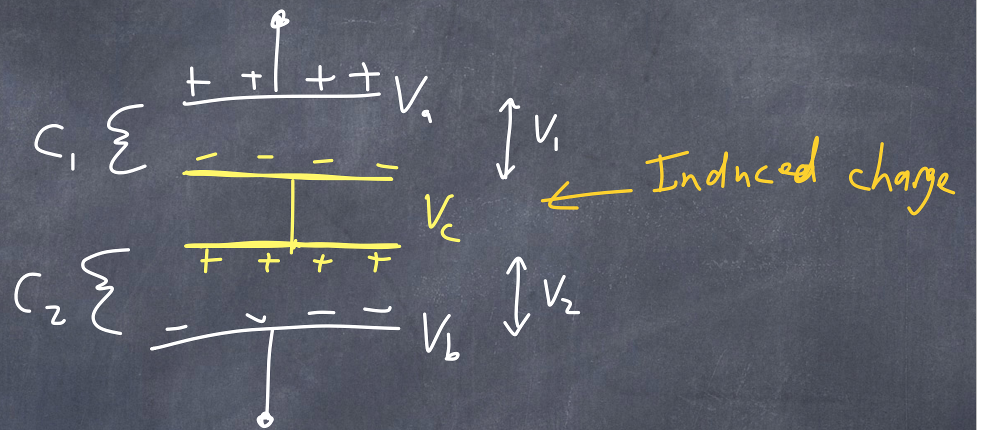
so $Q = (C_1 + C_2) V$

The equivalent capacitance of parallel capacitors is

$$\boxed{C_{eq} = C_1 + C_2 + \cdots = \frac{Q}{V}}$$

Combining capacitors in series:





$V \equiv V_a - V_b$

$V = \dfrac{Q}{C}$

$V_1 = V_a - V_c$ $\qquad$ $V_2 = V_c - V_b$

$V_1 = \dfrac{Q}{C_1}$ $\qquad$ $V_2 = \dfrac{Q}{C_2}$

The total potential difference is $V = V_a - V_b$

so $$V = V_a - V_b = (V_a - V_c) + (V_c - V_b) = \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$V = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \dots\right)$$

In general, $\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots}$

rule for adding capacitors in series.

If $C_1 = 2\mu F$ + $C_2 = 2\mu F$

stronger $\longrightarrow$ In parallel: $C_{eq} = C_1 + C_2 = 4\mu F$

weaker $\longrightarrow$ In series: $\frac{1}{C_{eq}} = \frac{1}{2\mu F} + \frac{1}{2\mu F} \Rightarrow C_{eq} = 1\mu F$

Since $U = \frac{1}{2}C\Delta V^2 \Rightarrow$ stored energy is more when capacitors are in parallel

Capacitors in parallel

vs.

in series

more energy,
more spark.

Electrostatics: $E=0$ in a conductor, V is the
(static charges) same everywhere in a conductor.

Electrodynamics:
(moving charges)

Current: rate of flow of electric charge.

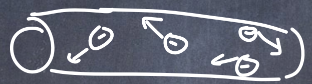
$$\underset{\text{current}\rightarrow}{I} = \frac{\Delta Q \leftarrow \text{charge}}{\Delta t \leftarrow \text{time}}$$

units $1\left[\frac{C}{s}\right] = 1\,A$

Amp = Ampere = A

consider a conductor

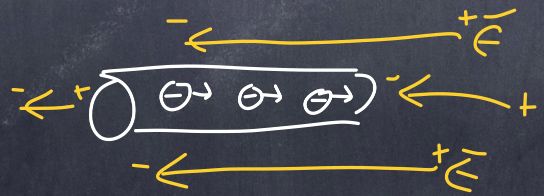
$\bar{E} = 0$



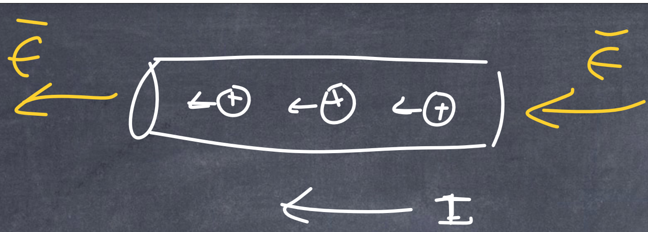
Free electrons in conductor move randomly due to thermal movement

$V \sim 10^6\ m/s$

$\bar{E} \neq 0$



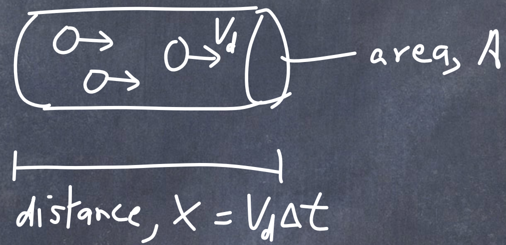
Electrons move in direction opposite to the E-field



By convention, the current I moves in the direction that <u>positive</u> charges would move.
(I points in same direction as $\bar{E}$)

Calculate speed of charge carriers:

$$n = \frac{\#\text{charge carriers}}{\text{volume}}$$



V_d : drift velocity

$$\text{volume} = A \cdot x = A V_d \Delta t$$

Total charge in this cylinder is:

$$\Delta Q = \frac{\#\text{charge carriers}}{\text{volume}} \cdot \text{volume} \cdot \text{charge per carrier}$$

$$= n \cdot \text{volume} \cdot e$$

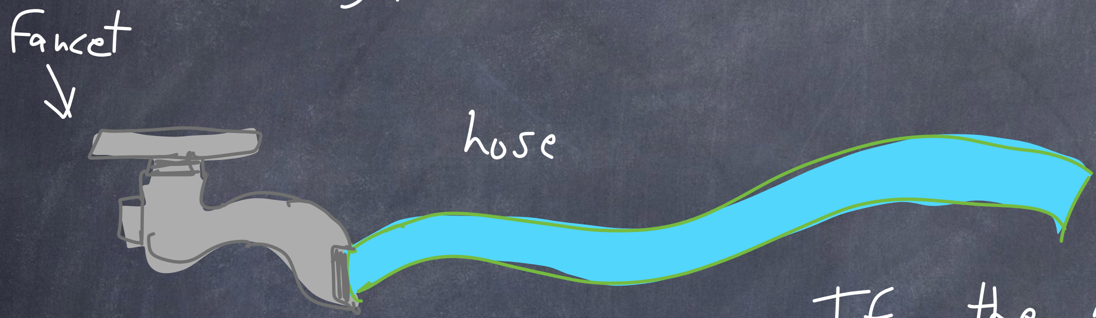
$$\Delta Q = n A V_d \Delta t e$$

so
$$I = \frac{\Delta Q}{\Delta t} = \frac{n A V_d \cancel{\Delta t} e}{\cancel{\Delta t}}$$

$$V_d = \frac{I}{nAe} \sim 10^{-5} \frac{m}{s}$$

If V_d is so slow, why does electricity seem so fast?

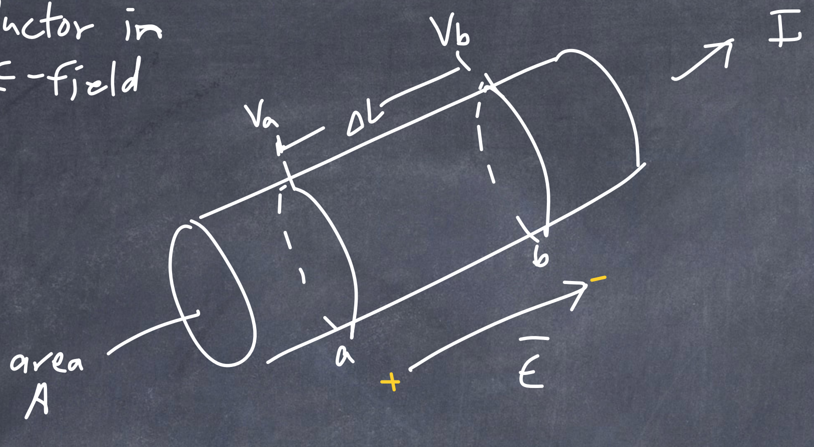
Analogy to a hose with water in it.



If the hose is already filled with water, then when you turn it on, since the fluid is incompressible, the flow is instantaneous.

Electricity is ~ instantaneous despite the slow electrons.

Conductor in an $\mathcal{E}$-field



we know that

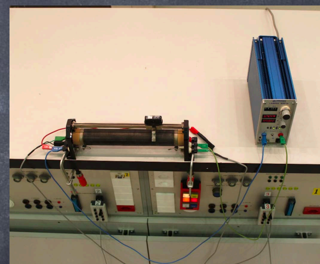
$$V = V_a - V_b = \mathcal{E}\,\Delta L$$

$$V_a - V_b > 0$$

V is the potential <u>difference</u>

For most materials, the current is proportional to the potential difference:

$$\boxed{I \propto V}$$



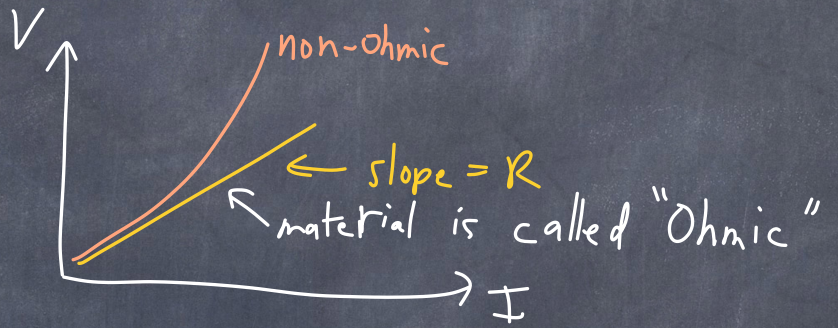
This constant of proportionality is $\dfrac{1}{R}$ where R is the resistance of the material

$$I = \left(\frac{1}{R}\right)V \implies \boxed{V = IR}$$

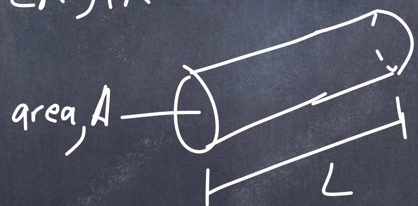
R has units of Ω (ohms)

where $1\,\Omega = 1\,\frac{V}{A} = \frac{1 \text{ volt}}{\text{Amp}}$

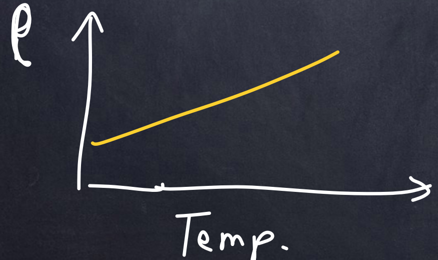
$V = IR$



The resistance R depends on the material, area, length



$R = \frac{L}{A}\rho$ ← "rho" is the resistivity



$$\rho = \rho_{20}\left[1 + \alpha\left(t_c - 20^\circ\right)\right]$$

ρ: resistivity

ρ_{20}: at 20°C

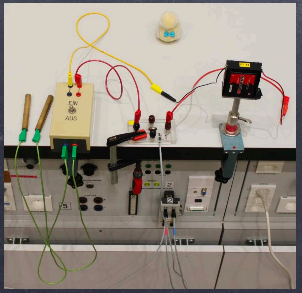
α: per material

t_c: Temp. in Celsius

material	$\rho_{20}\,[\Omega \cdot m]$	$\alpha\,\left[\frac{1}{C}\right]$
copper	1.7 E-8	3.9 E -3
aluminum	2.8 E-8	3.9 E -3
wood	$10^{8} - 10^{14}$	
glass	$10^{10} - 10^{14}$	

If R is small, then $I = \frac{V}{R}$ is big:

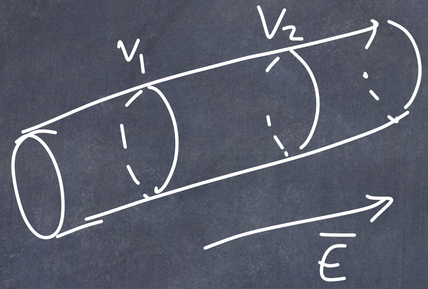
more current in a copper tube than in a wooden one.



Sometimes people use

$$\text{Conductivity} = \frac{1}{\text{resistivity}}$$

Energy is lost in a conductor as electrical energy is converted into thermal energy.



Potential decrease from V_1 to V_2

$$V = V_1 - V_2$$

Loss in energy $\Delta U = \Delta Q (V_2 - V_1)$

ΔQ is the amount of charge flowing from V_1 to V_2

In time Δt, $$\frac{\Delta U}{\Delta t} = \frac{\Delta Q(V)}{\Delta t}$$

V is the potential difference here

$\underbrace{\frac{\Delta Q}{\Delta t}}_{I}$

power [Watts]

[Amps] [volts]

$$1 \text{ Watt} = 1 \text{ Amp} \cdot \text{Volt} = 1 \, A \cdot V$$

$$\boxed{P = IV}$$

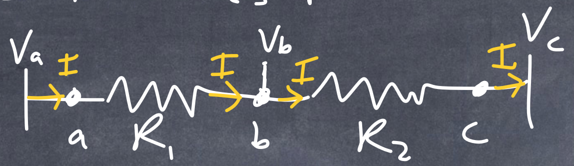
power

since we know $V = IR$

$$\boxed{P = I^2 R = \frac{V^2}{R}}$$

The energy loss depends on R.
At constant I, a higher R produces more heat generated.
At constant V, a higher R produces less heat.

Resistors in series:



what goes in, must come out

$I_a = I_b = I_c = I$

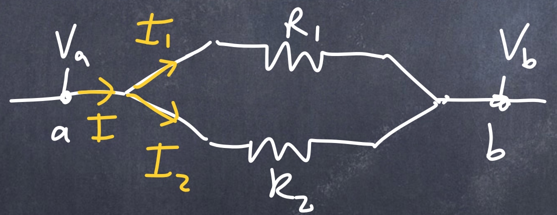
Equivalent resistance $R_{eq} = R_1 + R_2 + \dots$

$V_b = V_a - IR_1$

$V_c = V_a - IR_1 - IR_2$

potential decreases, but the current stays the same.

Resistors in parallel:



$I = I_1 + I_2$

Equivalent resistance decreases (more ways for the current to flow)

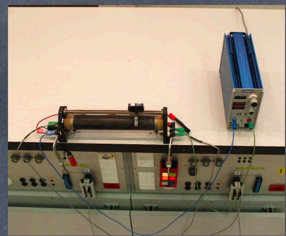
$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots}$$

voltage drop $V_a - V_b$ is the same across both pathways

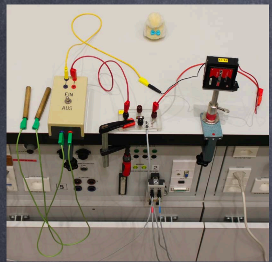
$V_{ab} = V_a - V_b = I_1 R_1 = I_2 R_2$

ES62

ES28



ES61



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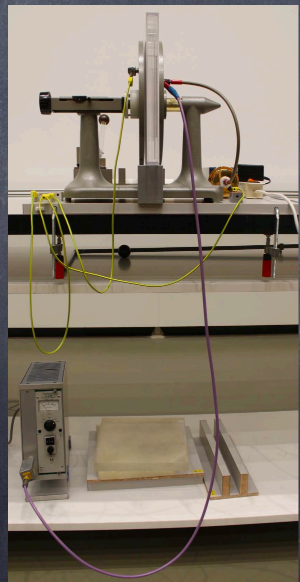

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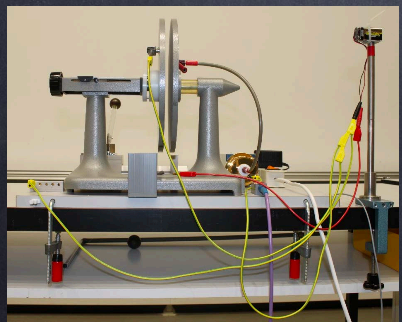

ES34