# PHY 117 HS2024

Today:
Cross product
Torque
Static equilibrium
Center of mass
Stability vs. rotation
Moment of inertia

Week 4, Lecture 1
Oct. 8th, 2024
Prof. Ben Kilminster

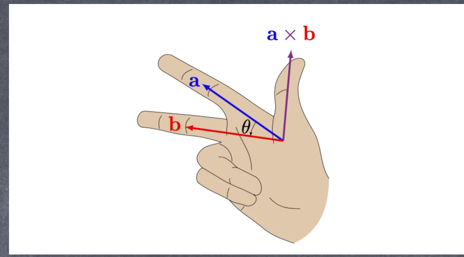
vector product or the cross product

$$\bar{c} = \bar{a} \times \bar{b}$$

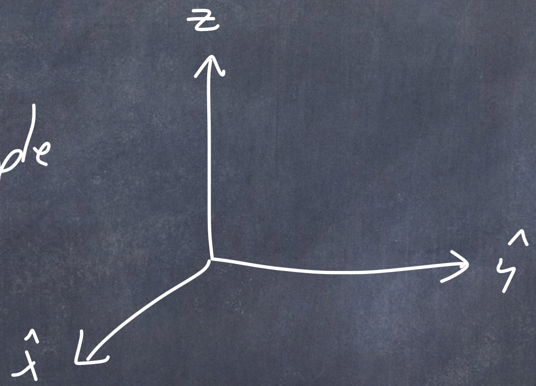
$\bar{c}$ is $\perp$ to both $\bar{a}$ and $\bar{b}$

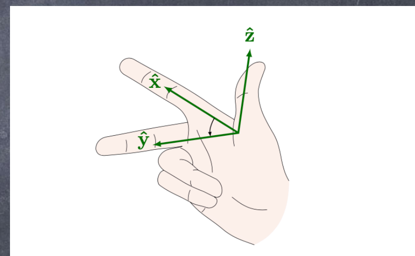
$$\bar{c} = \bar{a} \times \bar{b} = |\bar{a}|\,|\bar{b}|\sin\theta$$

$\theta$ is angle from $\bar{a}$ to $\bar{b}$




Example




what is $\hat{x} \times \hat{y}$ ? $\Rightarrow$

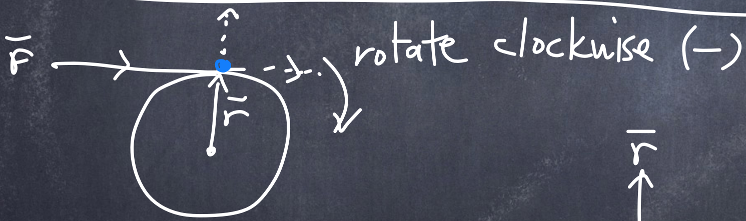
what is $\hat{y} \times \hat{z}$ ? $\Rightarrow$

what is $\hat{y} \times \hat{x}$ ? $\Rightarrow$

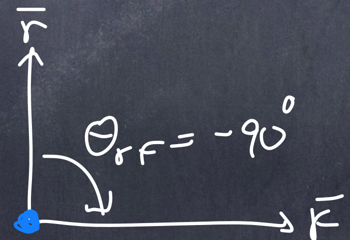
Torque:

- a force applied at a distance from a fixed point
- tends to cause the object to rotate.
- symbol, $\bar{\tau}$ vector
- units $F \cdot x = [N \cdot m]$

$$\boxed{\text{torque} = \bar{\tau} = \bar{r} \times \bar{F} = |\bar{r}||\bar{F}| \sin\theta_{rF} = \underbrace{rF}_{\text{magnitudes}} \underbrace{\sin\theta_{rF}}_{\substack{\text{angle from } \bar{r} \text{ to } \bar{F} \\ \text{angle can be } (-) \text{ or } (+)}}}$$




Draw the $\bar{r}$ and $\bar{F}$ vectors so that they start at the same point.




$\theta_{rF}$ is the angle that rotates $\bar{r}$ into $\bar{F}$

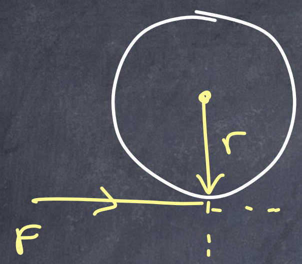
$$\tau = rF\sin(-90^\circ)$$

$$\tau = -rF$$

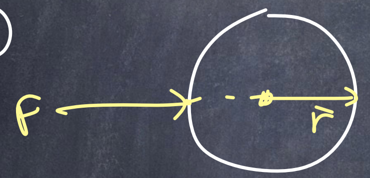
(−) clockwise

calculate the torque & direction for each case

1)




2)

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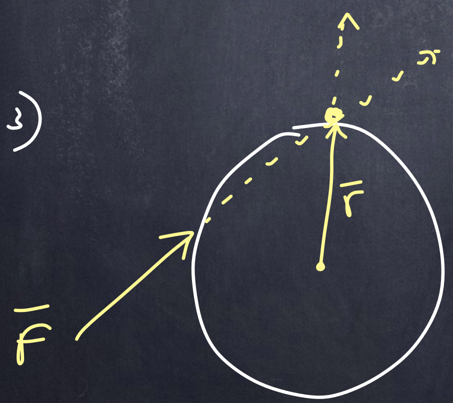
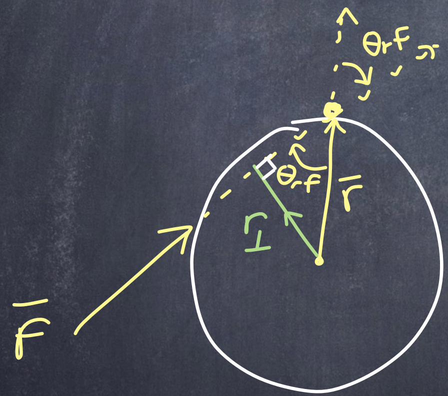


Torque can be also thought of as the product of the force with the "lever arm" $= r_\perp =$ the part or component of $\bar{r}$ that is perpendicular to the force.

$$\tau = r_\perp F$$




$r_\perp$: component of $\bar{r}$ that is $\perp$ to $\bar{F}$

$$\tau = r_\perp F = (r \sin\theta_{rF}) F$$

add direction $\left[ \tau = (r \sin\theta_{rF}) F \right.$ in clockwise $(-)$ direction.

same as previous answer.

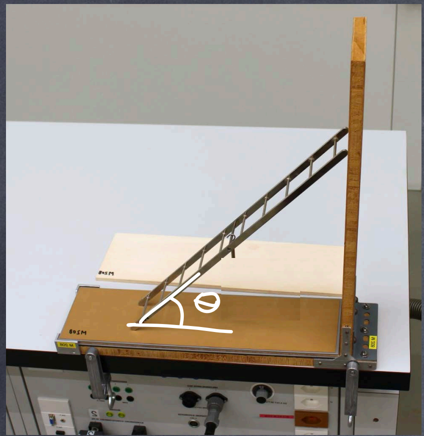
$\bar{\tau}$ vector points into page $\otimes$

# static equilibrium:

2 conditions: $\sum \bar{F} = 0$ (so no acceleration)

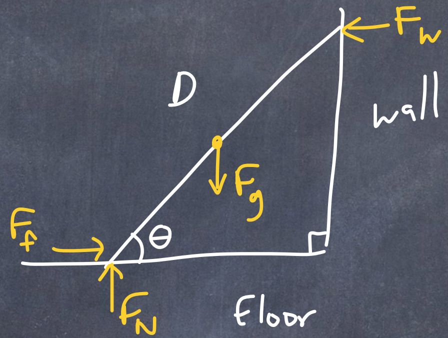
$\sum \bar{\tau} = 0$ $(\tau_{cw} = \tau_{ccw})$

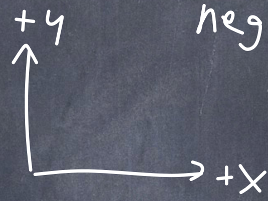
Trick: for calculating, the point of rotation matters (the right point makes the problems easier to solve)




What is the smallest angle, $\theta_{min}$, such that the ladder does not fall?

Ladder has mass, $M$
length, $D$
angle, $\theta$

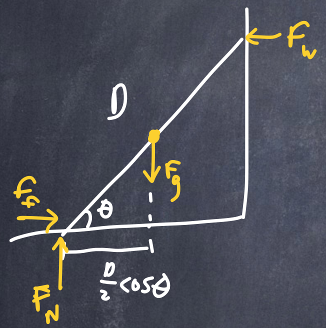


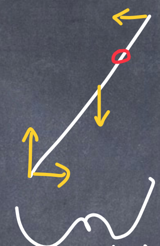




For now, we neglect friction of the ladder on wall.

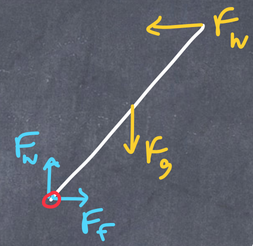
Next, consider $\sum \vec{\tau} = 0$ $\tau_{cw} = \tau_{ccw}$

what should be the rotation point



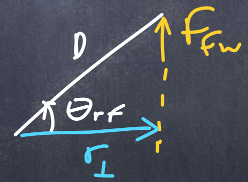



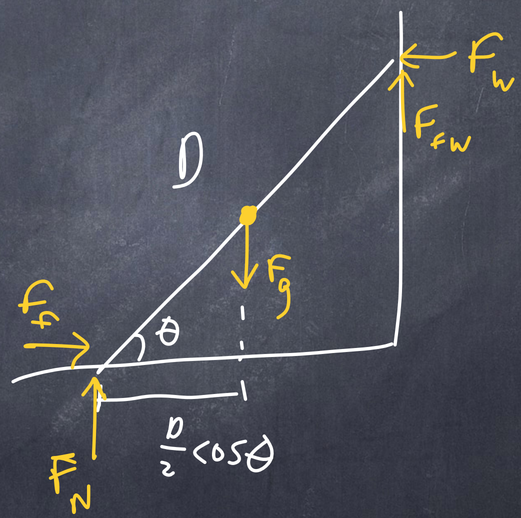
bad choice
all 4 forces
put $\tau$ on




No torque for $F_f$ + $F_N$ since they point through the axis of rotation.

What if there is friction on the wall?



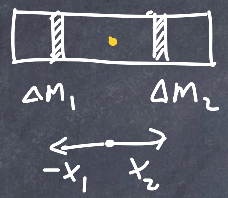




$$\tau_{ccw} = r_\perp F_{fw} = (D\cos\theta)\, F_{fw}$$

Center of mass: the point at which there is an equal amount of mass on all sides.

c.m.

for 2 slices of mass:

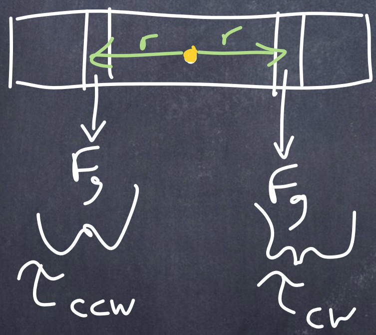



$$(\Delta m_1)(-x_1) + (\Delta m_2)(x_2) = 0$$

If $\Delta m_1 = \Delta m_2$, $(x_1 = x_2)$

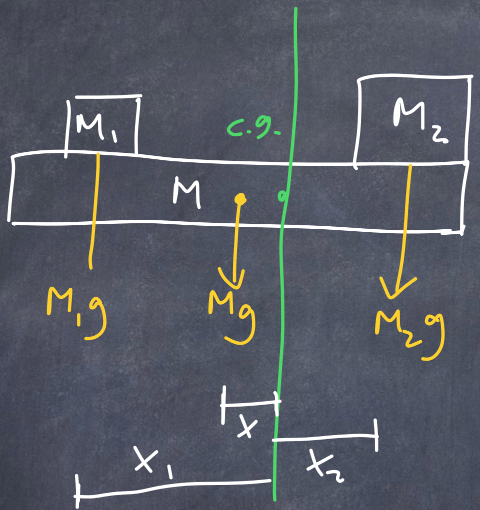
center of gravity: point at which all torques due to the force of gravity cancel out.

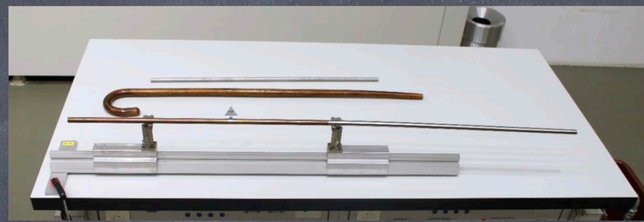
c.g.



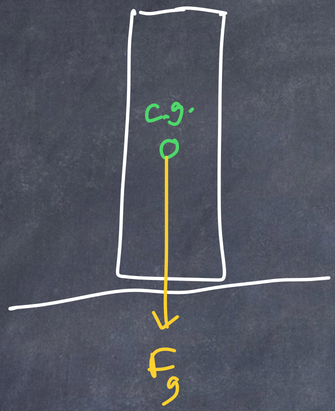

center of mass is equivalent to the center of gravity on earth.

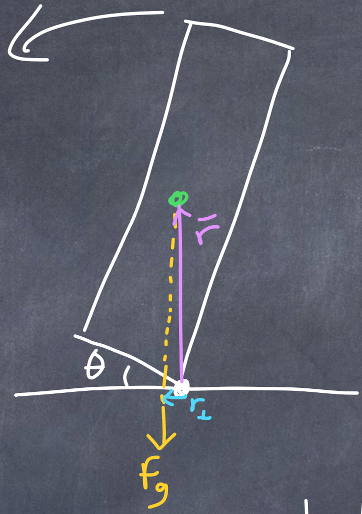
we can choose our origin of rotation at the center of gravity



# Stability



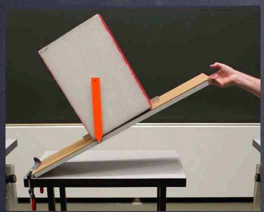




$\tau$ is CCW $\odot$ from $F_g$

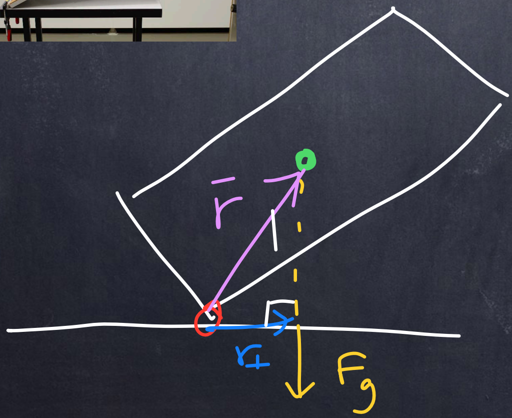
$$\bar{\tau} = \bar{r} \times \bar{F}$$

$\tau = r_\perp F$ (+) direction

An Object is stable when the torque due to gravity tends to restore the object to equilibrium.



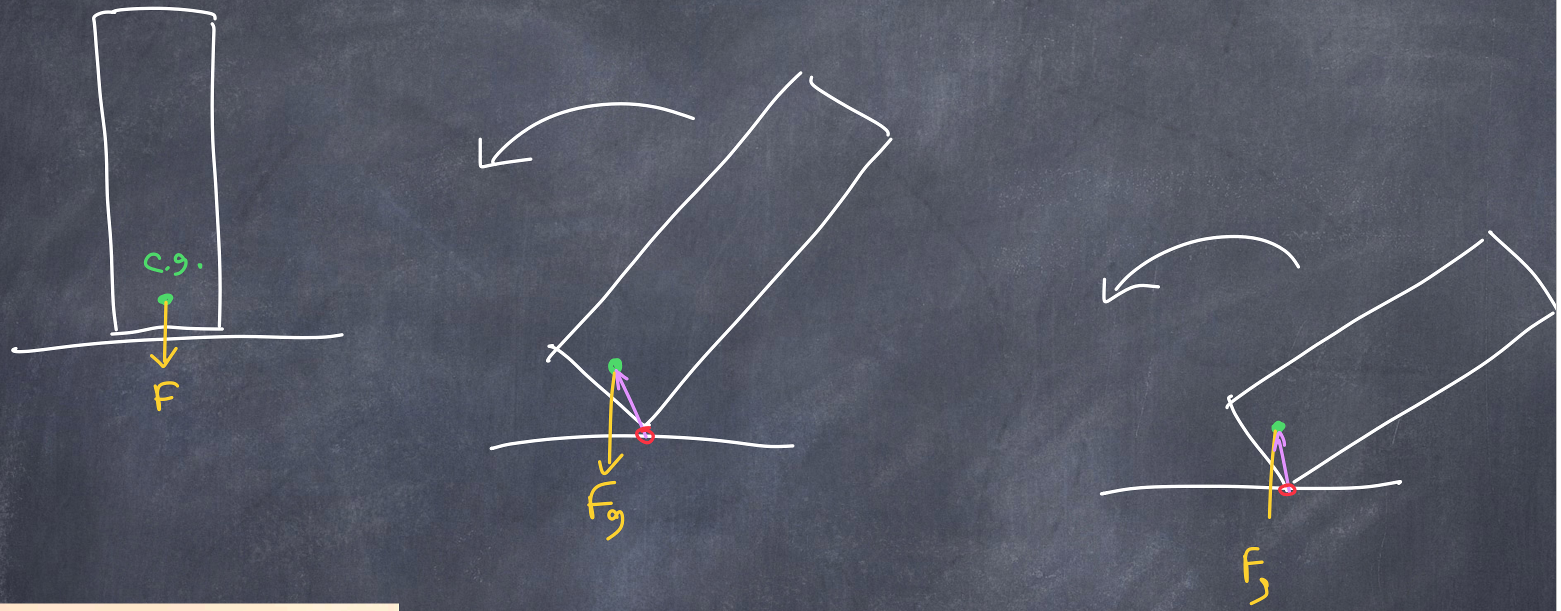
This depends on the direction of the torque with respect to the pivot point.

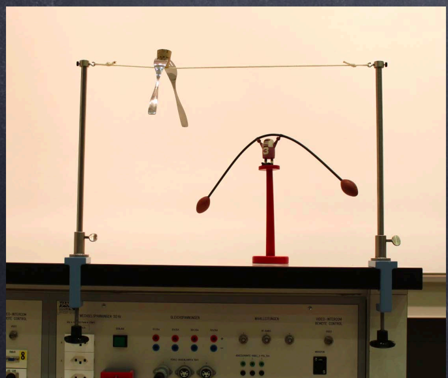



$\tau$ is CW, $\otimes$ from $F_g$

This is not stable

To Improve stability, lower the center of gravity.
(heavier at the bottom)

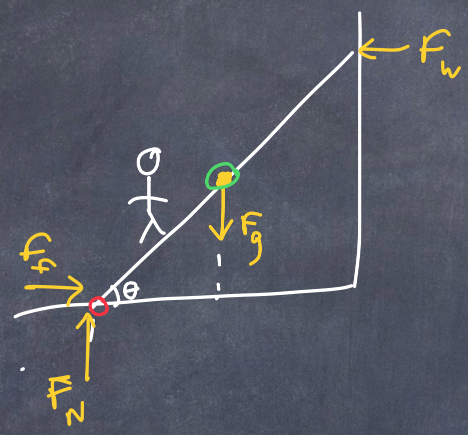


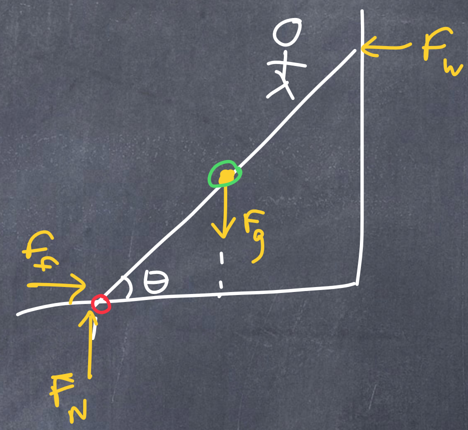



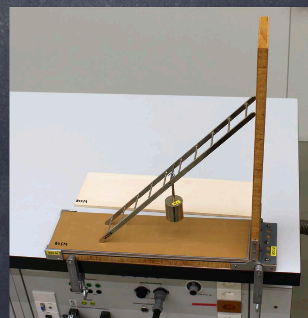
Ladder:

what if we add a person?

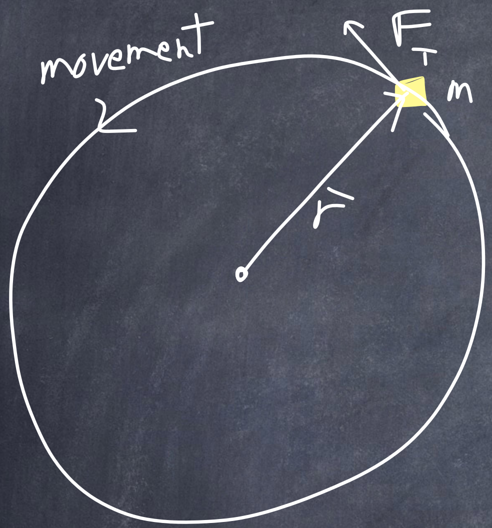
More or less stable?



Consider a tangential force on an object mass, m, constrained to move in a circle. (not centripetal force)




The force is tangential, $F_T$

The force is unbalanced, so

$$F_T = ma = m(r\alpha) \qquad a = r\alpha$$

we multiply both sides by r

$$rF_T = mr^2\alpha$$

$$\underbrace{r_\perp F}_{\tau} = I\alpha$$

$$\boxed{\tau \equiv I\alpha}$$

> $I \equiv mr^2$ : "moment of inertia" : this is for a particle of mass m at a radius r from the center of rotation

Note the parallels between linear + rotational motion

| linear motion | rotational motion |
| --- | --- |
| $F = ma$ | $\tau = I\alpha$ |

$I$ is kind of like mass $m$

$(I = mr^2)$

$$\boxed{\text{Newton's second law of rotation} \quad \Sigma\tau = I\alpha}$$