Non-leptonic B-decays at two loops in QCD

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Based on

- Bell, TH, [arXiv:1410.2804], JHEP
- Bell, Beneke, Li, TH, [arXiv:1507.03700], PLB
- Kränkl, TH, [arXiv:1503.00735], JHEP
- Kränkl, Li, TH, [arXiv:1606.02888], JHEP

Theory Seminar, Zürich, October 4^{th}, 2016

Outline

- Introduction
- Theoretical framework
- Two-loop penguin amplitudes
- The decay $B \to D\,\pi$ at two loops
- Conclusion

Introduction to non-leptonic B decays

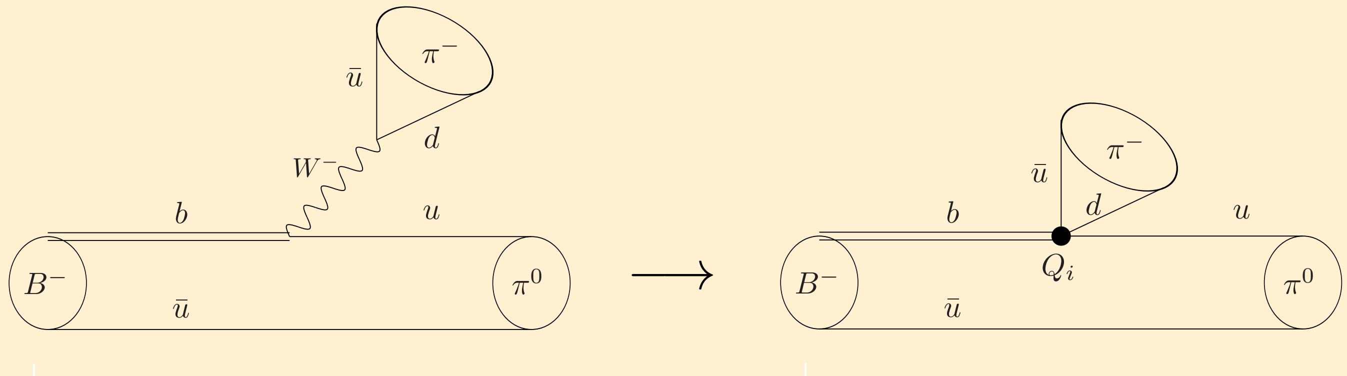
- Non-leptonic B decays offer a rich and interesting phenomenology
  - Large data sets from B-factories, Tevatron, LHCb, in future Belle II
  - $\mathcal{O}(100)$ final states
  - Numerous observables:
    - branching ratios
    - CP asymmetries
    - polarisations
    - Dalitz plot analyses
    - Combinations thereof
- Test of CKM mechanism (CP violation)
- Indirect search for New Physics
  - Not as sensitive as rare or radiative B decays, but large data sets

Introduction to non-leptonic B decays

- Theoretical description complicated by purely hadronic initial and final state
  - QCD effects from many different scales

- Theory approaches
  - Factorisation approaches: PQCD *[Keum,Li,Sanda'00]*, QCDF *[Beneke,Buchalla,Neubert,Sachrajda'99-'01]*
    * Disentangle long and short distances
  - QCD Factorisation
    * Systematic framework to all orders in α_s and leading power in Λ/m_b
    * Problems with factorisation of power suppressed and annihilation contributions. Endpoint divergences
  - Flavour symmetries: Isospin, U-Spin ($d \leftrightarrow s$), V-Spin ($u \leftrightarrow s$), Flavour SU(3)
    * Only few a priori assumptions about scales needed
    * Implementation of symmetry breaking difficult
  - Dalitz plot analysis. Applied to 3-body decays
    * Mostly a fit to data, but also QCD-based predictions possible *[Kränkl,Mannel,Virto'15]*

Effective theory for B decays



- M_W, M_Z, $m_t \gg m_b$: integrate out heavy gauge bosons and t-quark

- Effective Hamiltonian: *[Buras,Buchalla,Lautenbacher'96; Chetyrkin,Misiak,Münz'98]*

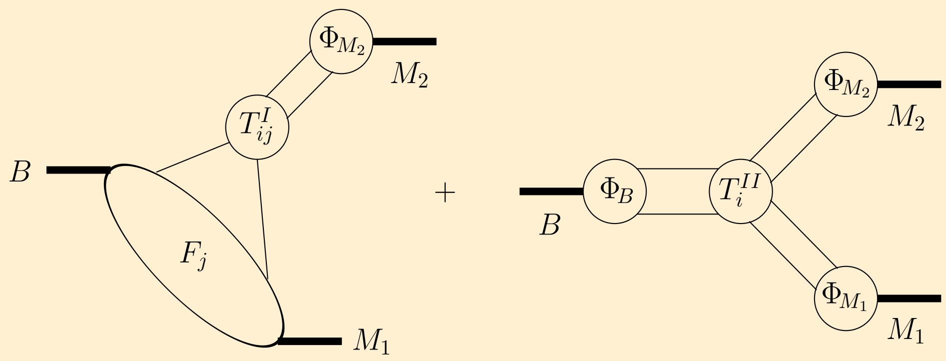
$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left[C_1 Q_1^p + C_2 Q_2^p + \sum_{k=3}^{6} C_k Q_k + C_8 Q_8 \right] + \text{h.c.}$$

$$Q_1^p = (\bar{d}_L \gamma^\mu T^a p_L)(\bar{p}_L \gamma_\mu T^a b_L) \qquad Q_4 = (\bar{d}_L \gamma^\mu T^a b_L) \textstyle\sum_q (\bar{q}\gamma_\mu T^a q) \qquad Q_8 = -\frac{g_s}{16\pi^2} m_b \, \bar{d}_L \, \sigma_{\mu\nu} G^{\mu\nu} b_R$$

$$Q_2^p = (\bar{d}_L \gamma^\mu p_L)(\bar{p}_L \gamma_\mu b_L) \qquad Q_5 = (\bar{d}_L \gamma^\mu \gamma^\nu \gamma^\rho b_L) \textstyle\sum_q (\bar{q}\gamma_\mu \gamma_\nu \gamma_\rho q)$$

$$Q_3 = (\bar{d}_L \gamma^\mu b_L) \textstyle\sum_q (\bar{q}\gamma_\mu q) \qquad Q_6 = (\bar{d}_L \gamma^\mu \gamma^\nu \gamma^\rho T^a b_L) \textstyle\sum_q (\bar{q}\gamma_\mu \gamma_\nu \gamma_\rho T^a q) \qquad \lambda_p = V_{pb} V_{pd}^*$$

QCD factorisation



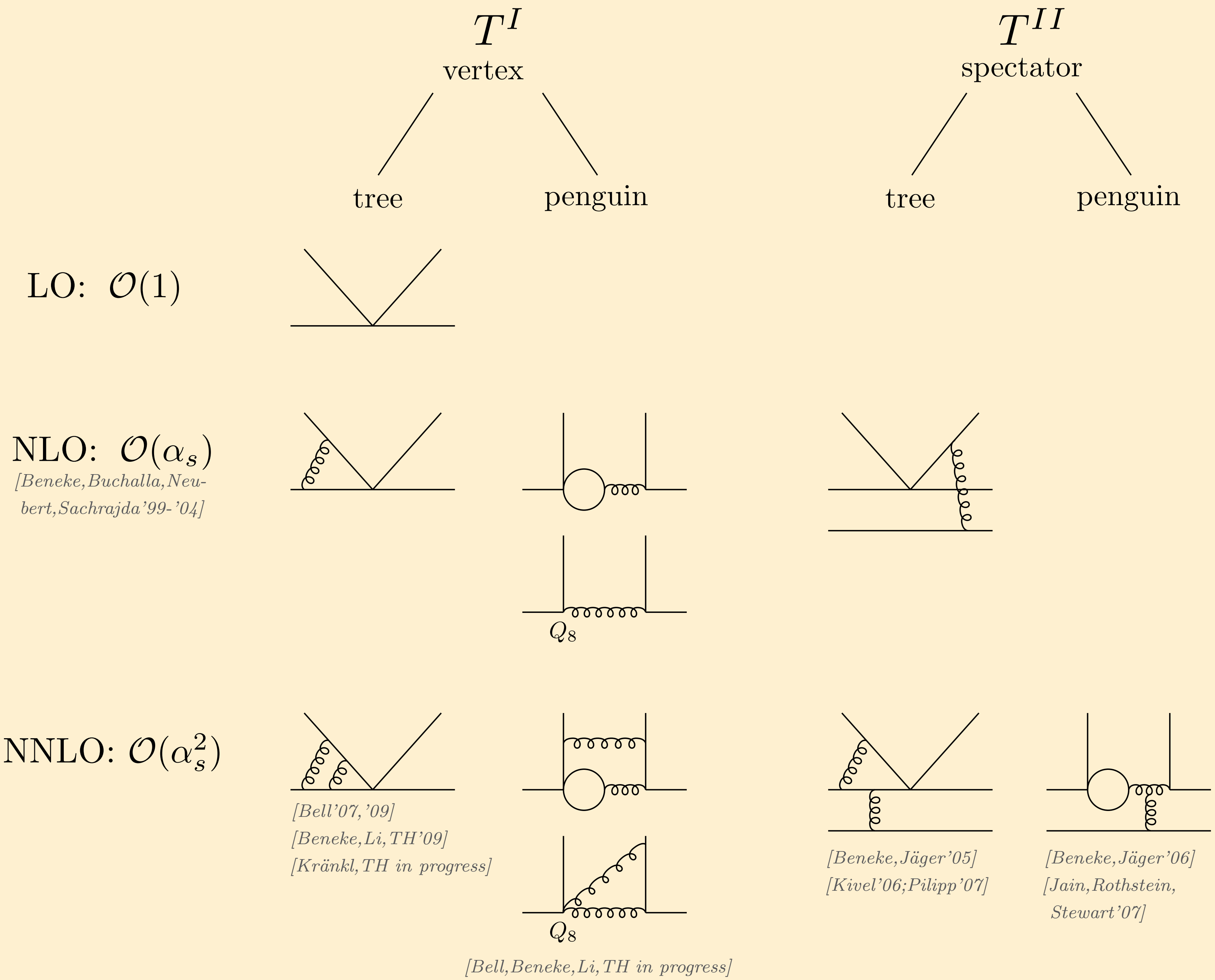
- Amplitude in the limit $m_b \gg \Lambda_{\rm QCD}$ *[Beneke,Buchalla,Neubert,Sachrajda'99-'04]*

$$\langle M_1 M_2 | Q_i | \bar{B} \rangle \simeq m_B^2 \, F_+^{B\to M_1}(0)\, f_{M_2} \int_0^1 du \;\, T_i^I(u)\, \phi_{M_2}(u)$$

$$+ f_B\, f_{M_1}\, f_{M_2} \int_0^1 d\omega dv du \;\, T_i^{II}(\omega, v, u)\, \phi_B(\omega)\, \phi_{M_1}(v)\, \phi_{M_2}(u)$$

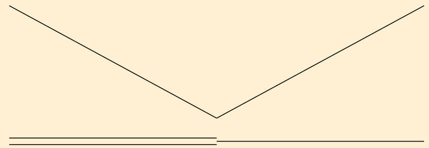
- $T^{I,II}$: Hard scattering kernels, perturbatively calculable
- F_+ : $B \to M$ form factor
  f_i : decay constants
  ϕ_i : light-cone distribution amplitudes
  } Universal. From Sum Rules, Lattice
- Strong phases are $\mathcal{O}(\alpha_s)$ and/or $\mathcal{O}(\Lambda_{\rm QCD}/m_b)$

Anatomy of QCD factorisation

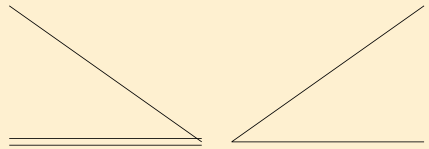


Classification of amplitudes

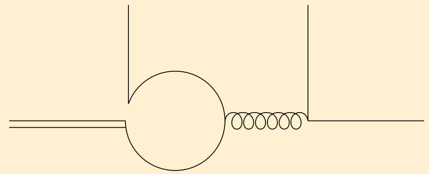
- α_1 : colour-allowed tree amplitude



- α_2 : colour-suppressed tree amplitude



- $\alpha_4^{u,c}$: QCD penguin amplitudes



$$\begin{aligned}
\sqrt{2}\,\langle \pi^-\pi^0 | \mathcal{H}_{eff}\,| B^-\rangle &= A_{\pi\pi}\,\lambda_u\big[\alpha_1(\pi\pi)+\alpha_2(\pi\pi)\big] \\
\langle \pi^+\pi^- | \mathcal{H}_{eff}\,| \bar{B}^0\rangle &= A_{\pi\pi}\,\big\{\lambda_u\big[\alpha_1(\pi\pi)+\alpha_4^u(\pi\pi)\big]+\lambda_c\,\alpha_4^c(\pi\pi)\big\} \\
-\,\langle \pi^0\pi^0 | \mathcal{H}_{eff}\,| \bar{B}^0\rangle &= A_{\pi\pi}\,\big\{\lambda_u\big[\alpha_2(\pi\pi)-\alpha_4^u(\pi\pi)\big]-\lambda_c\,\alpha_4^c(\pi\pi)\big\}
\end{aligned}$$

$$\begin{aligned}
\langle \pi^-\bar{K}^0 | \mathcal{H}_{eff}\,| B^-\rangle &= A_{\pi\bar{K}}\,\Big[\lambda_u^{(s)}\,\alpha_4^u+\lambda_c^{(s)}\,\alpha_4^c\Big] \\
\langle \pi^+K^- | \mathcal{H}_{eff}\,| \bar{B}^0\rangle &= A_{\pi\bar{K}}\Big[\lambda_u^{(s)}\,(\alpha_1+\alpha_4^u)+\lambda_c^{(s)}\,\alpha_4^c\Big]
\end{aligned}$$

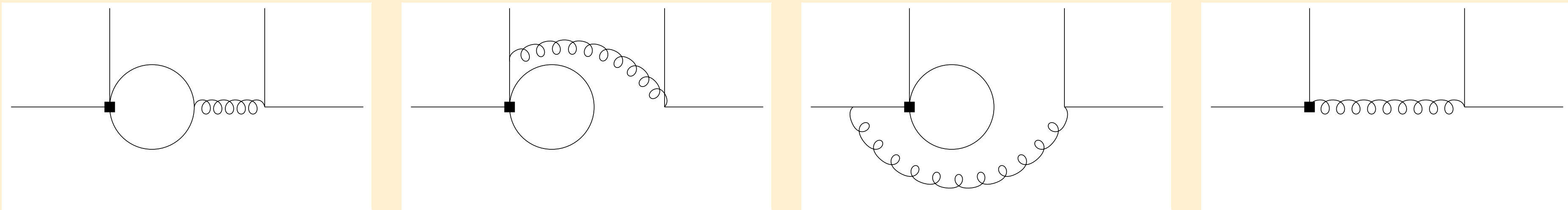
[Beneke,Neubert'03]

- Tree amplitudes α_1 and α_2 known analytically to NNLO

[Bell'07'09; Beneke,Li,TH'09]

Penguin amplitudes a_4^u and a_4^c to NLO

- NLO:



$$\alpha_4^u(\pi\pi) = -0.029 - [0.002 + 0.001i]_V + [0.003 - 0.013i]_P + [?? \; + \; ??\; i]_{\mathcal{O}(\alpha_s^2)}$$

$$+ \left[\frac{r_{\rm sp}}{0.485}\right] \left\{[0.001]_{\rm LO} + [0.001 + 0.000i]_{HV+HP} + [0.001]_{\rm tw3}\right\}$$

$$= (-0.024^{+0.004}_{-0.002}) + (-0.012^{+0.003}_{-0.002})i$$

$$\alpha_4^c(\pi\pi) = -0.029 - [0.002 + 0.001i]_V - [0.001 + 0.007i]_P + [?? \; + \; ??\; i]_{\mathcal{O}(\alpha_s^2)}$$

$$+ \left[\frac{r_{\rm sp}}{0.485}\right] \left\{[0.001]_{\rm LO} + [0.001 + 0.001i]_{HV+HP} + [0.001]_{\rm tw3}\right\}$$

$$= (-0.028^{+0.005}_{-0.003}) + (-0.006^{+0.003}_{-0.002})i$$

Motivation for NNLO

- Direct CP asymmetries start at $\mathcal{O}(\alpha_s)$
  - Large (scale) uncertainties
  - NNLO is only first perturbative correction
  - NNLO is NLO for direct CP asymmetries!
- NLO results for tree amplitudes

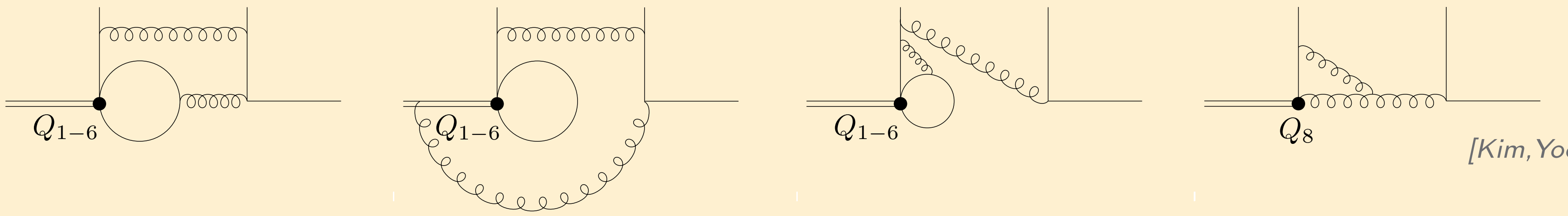
$$\alpha_1(\pi\pi) = 1.009 + [0.023 + 0.010\,i]_{\rm NLO} - \left[\tfrac{r_{\rm sp}}{0.445}\right]\Big\{[0.014]_{\rm LOsp} + [0.008]_{\rm tw3}\Big\} = 1.010 + 0.010i$$

$$\alpha_2(\pi\pi) = 0.220 - [0.179 + 0.077\,i]_{\rm NLO} + \left[\tfrac{r_{\rm sp}}{0.445}\right]\Big\{[0.114]_{\rm LOsp} + [0.067]_{\rm tw3}\Big\} = 0.222 - 0.077i$$

  - Large cancellation in LO + NLO in α_2. Particularly sensitive to NNLO
- Problems with colour-suppressed, tree-dominated decays (e.g. $\bar{B}^0 \to \pi^0\pi^0$)
  - However: New preliminary result by Belle: $\mathcal{B}(\bar{B}^0 \to \pi^0\pi^0) = (0.90 \pm 0.16) \cdot 10^{-6}$
- Does factorisation hold at NNLO?

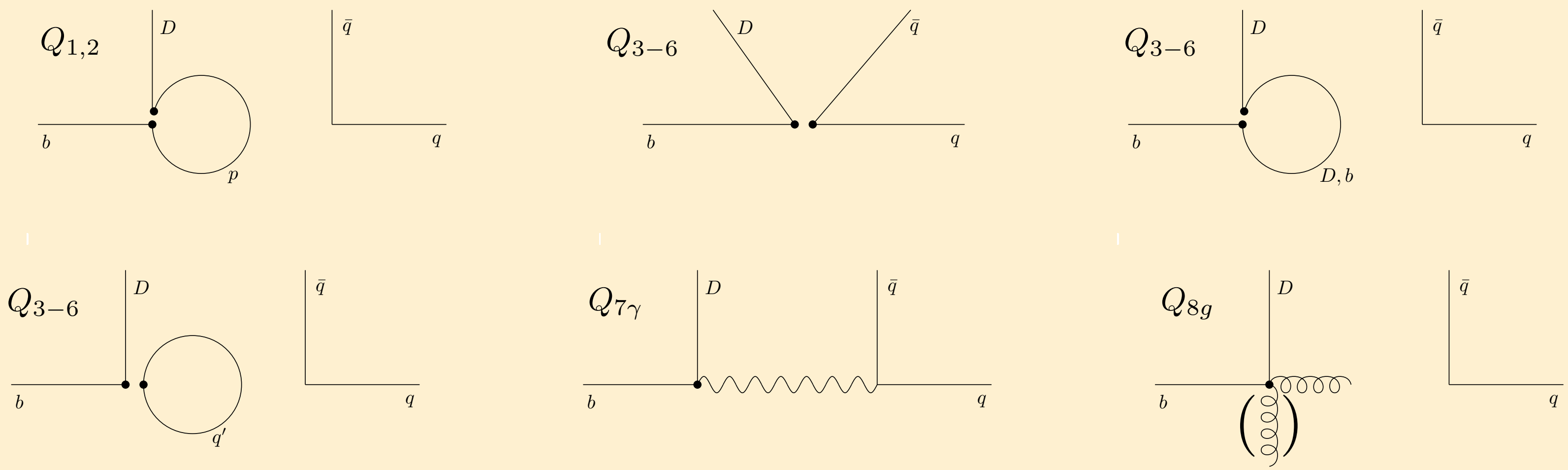
Penguin amplitudes at two loops

- $\mathcal{O}(70)$ diagrams at NNLO.



[Kim,Yoon'11]

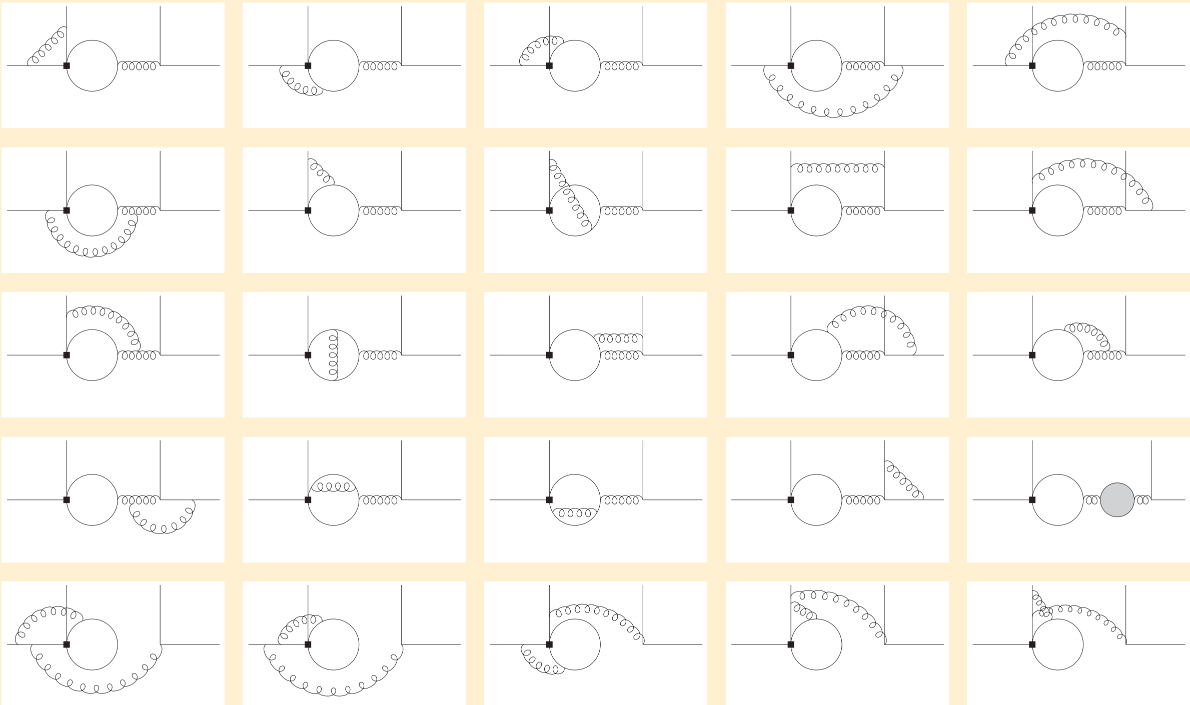
- Quite some book-keeping due to various insertions



- !!! Focus on $Q_1^{u,c}$ and $Q_2^{u,c}$ insertions !!!

Penguin amplitudes at two loops

- For $Q_{1,2}^{u,c}$ only a subset of ~ 25 diagrams contributes

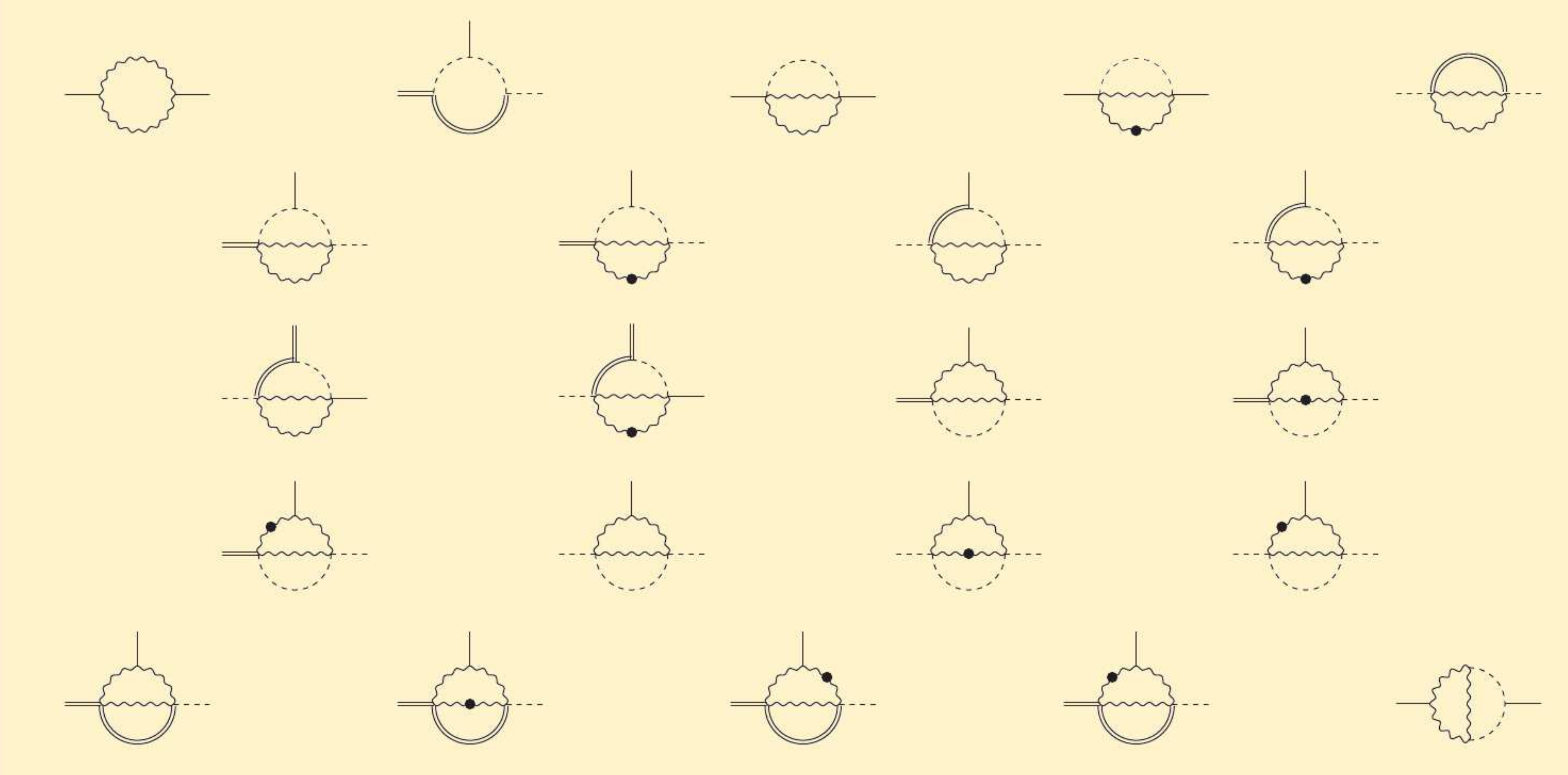


- Regularize UV and IR divergences dimensionally. Poles up to $1/\epsilon^3$
- Reduction: Integration-by-parts relations, Laporta algorithm

  [Tkachov'81; Chetyrkin,Tkachov'81] [Laporta'01; Anastasiou,Lazopoulos'04; Smirnov'08; Studerus,von Manteuffel'10,'12]

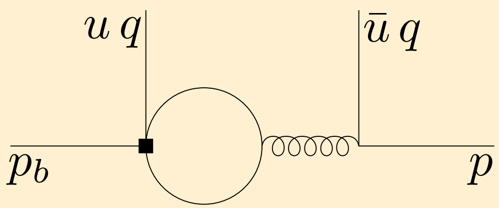
  – Obtain a set of 29 master integrals

Master integrals



- Double: m_b^2 , wavy: m_c^2 , solid: $\bar{u}\, m_b^2$, dashed: 0 .
- Most integrals are four-liners with three external legs
- Only one five-liner: Two-point function, one-scale integral

Kinematics



$$p^2 = q^2 = 0$$

$$p_b^2 = m_b^2$$

- Fermion loop with either $m = 0$ or $m = m_c$.
- Genuine two-scale problem: $\bar{u}$, m_c^2/m_b^2
- Threshold at $\bar{u} = 4m_c^2/m_b^2$
- Choice of suitable kinematic variables crucial

$$s = \sqrt{1 - 4z_c/\bar{u}}\,,\; r = \sqrt{1 - 4z_c} \quad \longleftrightarrow \quad \bar{u}\,,\; z_c = \frac{m_c^2}{m_b^2} \quad \longleftrightarrow \quad s_1 = \sqrt{1 - 4/\bar{u}}\,,\; r$$

$$\updownarrow$$

$$p = \frac{1 - \sqrt{u^2 + 4\bar{u}z_c}}{\bar{u}}\,,\; r$$

Computing the masters

- Use differential equations in canonical form *[Henn'13]*

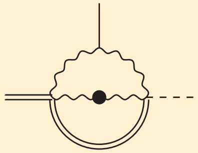
$$d\,\vec{M}(\epsilon, x_n) \;=\; \epsilon\;\; d\tilde{A}(x_n)\;\; \vec{M}(\epsilon, x_n)$$

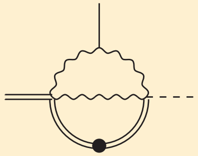
  - Factorises kinematics from number of space-time dimensions.
  - Together with boundary conditions, $\tilde{A}(x_n)$ completely fixes the solution

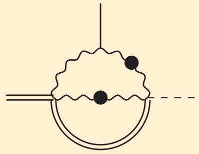
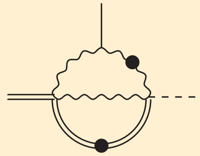
- Found canonical basis for all masters, including boundary conditions *[Bell,TH'14]*
  - First example of canonical basis in case of 2 different internal masses
  - Found analytical solution in terms of iterated integrals

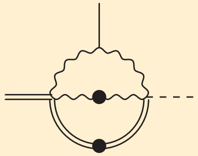
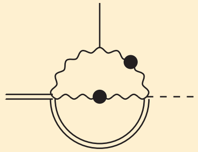
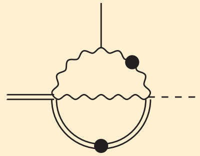
- Benefits of canonical basis
  - System disentangles order by order in ϵ
  - Homogeneous (pure) functions to all orders in ϵ
  - No fake higher weights
  - QCD amplitude much simpler, especially denominators of pre-factors of masters
  - Catalyses convolution with LCDA

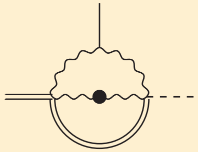
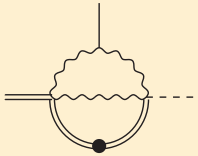
Canonical basis for master integrals I

$\frac{M_{18}}{u\epsilon^3} =$ 

$\frac{M_{19}}{u\epsilon^3} =$ 

$-\frac{2\,M_{20}}{u\bar{u}s\epsilon^2} =$  $+$ 

$\frac{M_{21}}{\epsilon^2} = \frac{2[(1+\bar{u})^2 z_c - \bar{u}^2]}{\bar{u}}$  $- \bar{u}s^2(1+\bar{u})\Big[$  $+$  $\Big]$

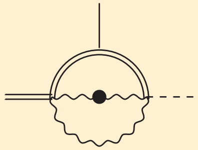
$+\frac{2\epsilon u}{m_b^2}\Big[$  $+$  $\Big]$

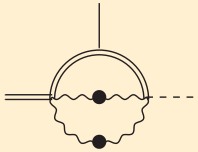
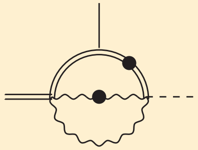
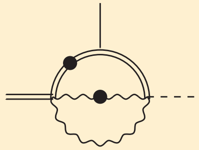
- Differential equation (sample)

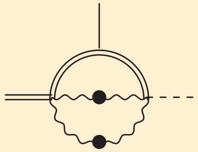
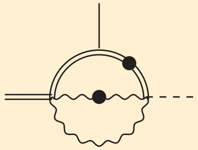
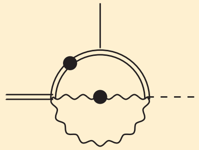
$$\frac{dM_{19}}{ds} = \frac{4\epsilon M_{18}\, r\,\left(r^2+1\right)}{((r^2+1)^2-4s^2)} - \frac{2\epsilon M_{19}\, r\,\left(r^2+s^2-2\right)}{(1-r^2)(r^2-s^2)} + \frac{4\epsilon M_{20}\, r\, s}{\left((r^2+1)^2-4s^2\right)} - \frac{\epsilon M_{21}\, r\,\left(r^2+1\right)}{((r^2+1)^2-4s^2)}$$

- Boundary conditions
  - M_{18} and M_{19} vanish in $s=r$ (i.e. in $u=0$)
  - M_{20} and M_{21} vanish in $s=+i\infty$ (i.e. in $u=1$)

Canonical basis for master integrals II

$$\frac{M_{23}}{u\epsilon^3} =$$ 

$$\frac{M_{24}}{\epsilon^2} = \frac{2(1+s_1)\sqrt{1+\frac{8z_c(1-s_1)}{(1+s_1)^2}}}{1-s_1} \Bigg[$$  $$+\,2$$  $$-\,\frac{2(1+s_1)}{1-s_1}$$  $$\Bigg]$$

$$\frac{M_{25}}{\epsilon^2} = \frac{2(1-s_1)\sqrt{1+\frac{8z_c(1+s_1)}{(1-s_1)^2}}}{1+s_1} \Bigg[$$  $$+\,2$$  $$-\,\frac{2(1-s_1)}{1+s_1}$$  $$\Bigg]$$

- Differential equation

$$\frac{dM_{23}}{ds_1} = \frac{2\,\epsilon\,M_{23}\,s_1\left(5-s_1^2\right)}{(1-s_1^2)\,(3+s_1^2)} - \frac{\epsilon\,M_{24}\,(3-s_1)}{4\,(1-s_1^2)\,\sqrt{1+\frac{8\,z_c(1-s_1)}{(1+s_1)^2}}} + \frac{\epsilon\,M_{25}\,(3+s_1)}{4\,(1-s_1^2)\,\sqrt{1+\frac{8\,z_c(1+s_1)}{(1-s_1)^2}}}$$

- Variable transformation to rationalize irrational factors:

$$t = \frac{1-s_1}{2} + \frac{1+s_1}{2}\sqrt{1+\frac{2(1-r^2)(1-s_1)}{(1+s_1)^2}} \qquad v = \frac{1+s_1}{2} + \frac{1-s_1}{2}\sqrt{1+\frac{2(1-r^2)(1+s_1)}{(1-s_1)^2}}$$

Completing the calculation

- Master formula for two-loop hard-scattering kernel

$$\widetilde{T}_i^{(2)} = \widetilde{A}_{i1}^{(2)\text{nf}} + Z_{ij}^{(1)}\,\widetilde{A}_{j1}^{(1)} + Z_{ij}^{(2)}\,\widetilde{A}_{j1}^{(0)} + Z_{\alpha}^{(1)}\,\widetilde{A}_{i1}^{(1)\text{nf}} + (-i)\,\delta m^{(1)}\,\widetilde{A}_{i1}^{\prime(1)\text{nf}} \\ + Z_{ext}^{(1)}\left[\widetilde{A}_{i1}^{(1)\text{nf}} + Z_{ij}^{(1)}\,\widetilde{A}_{j1}^{(0)}\right] - \widetilde{T}_i^{(1)}\left[C_{FF}^{(1)} + \widetilde{Y}_{11}^{(1)}\right] + \ldots$$

  - Renormalisation of UV divergencies
  - Subtraction of IR divergencies via matching onto SCET
    - ∗ Subtlety: Evanescent operators both on QCD and SCET side.

$$O_1 = \bar{\chi}\,\frac{\not{n}_-}{2}(1-\gamma_5)\chi\,\bar{\xi}\,\not{n}_+(1-\gamma_5)h_v\,,$$
$$\tilde{O}_n = \bar{\xi}\,\gamma_\perp^{\alpha}\gamma_\perp^{\mu_1}\gamma_\perp^{\mu_2}\ldots\gamma_\perp^{\mu_{2n-2}}\chi\,\bar{\chi}(1+\gamma_5)\gamma_{\perp\alpha}\gamma_{\perp\mu_{2n-2}}\gamma_{\perp\mu_{2n-3}}\ldots\gamma_{\perp\mu_1}h_v$$

- All poles cancel at first attempt ☺
- Obtain $\alpha_4^{u,c}$ via convolution with LCDA
  - Can get α_4^u completely analytically
  - α_4^c almost completely analytically

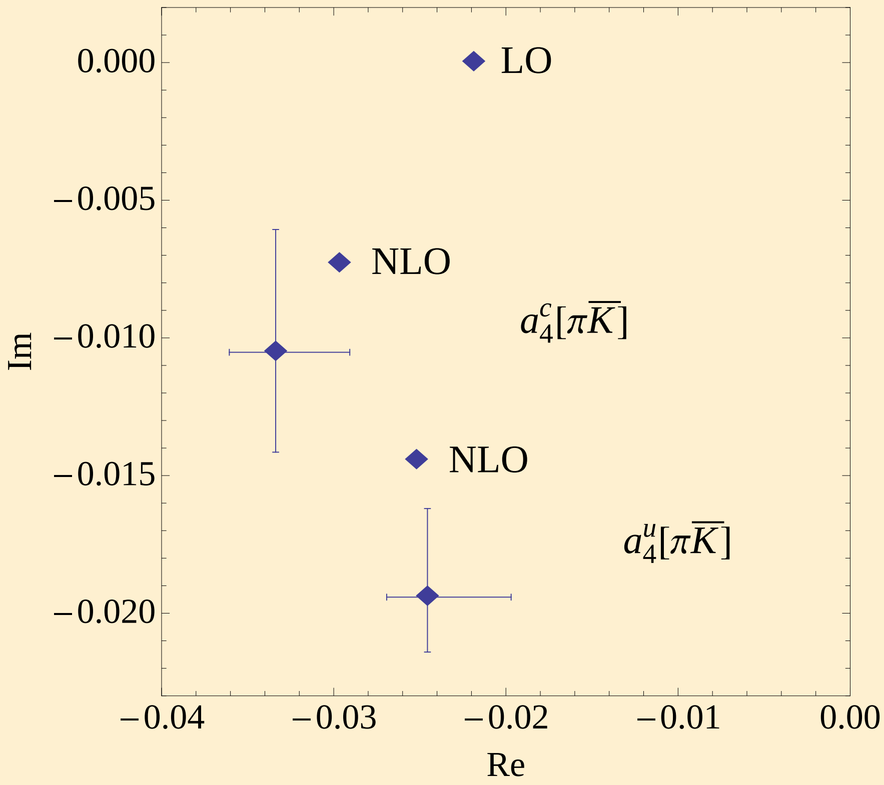
Results: Penguin Amplitudes

- Only $Q_{1,2}$ contribution. Inputs from *[Beneke,Li,TH'09]*

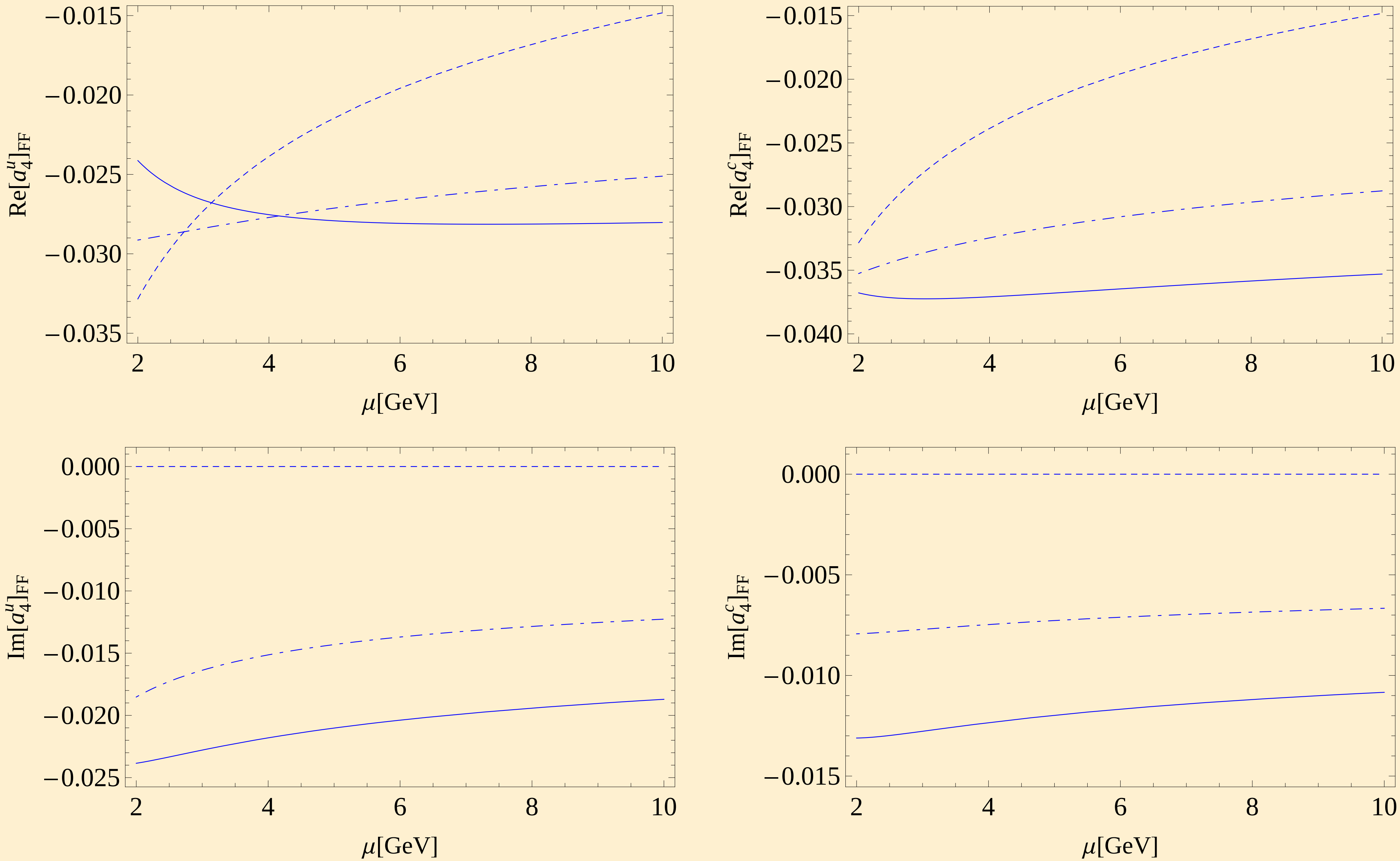
$$
\begin{aligned}
a_4^u(\pi\bar{K})/10^{-2} &= -2.87 - [0.09 + 0.09i]_{\mathrm{V}_1} + [0.49 - 1.32i]_{\mathrm{P}_1} - [0.32 + 0.71i]_{\mathrm{P}_2} \\
&\quad + \left[\frac{r_{\mathrm{sp}}}{0.434}\right]\Big\{[0.13]_{\mathrm{LO}} + [0.14 + 0.12i]_{\mathrm{HV}} - [0.01 - 0.05i]_{\mathrm{HP}} + [0.07]_{\mathrm{tw3}}\Big\} \\
&= (-2.46^{+0.49}_{-0.24}) + (-1.94^{+0.32}_{-0.20})i\,, \\
a_4^c(\pi\bar{K})/10^{-2} &= -2.87 - [0.09 + 0.09i]_{\mathrm{V}_1} + [0.05 - 0.62i]_{\mathrm{P}_1} - [0.77 + 0.50i]_{\mathrm{P}_2} \\
&\quad + \left[\frac{r_{\mathrm{sp}}}{0.434}\right]\Big\{[0.13]_{\mathrm{LO}} + [0.14 + 0.12i]_{\mathrm{HV}} + [0.01 + 0.03i]_{\mathrm{HP}} + [0.07]_{\mathrm{tw3}}\Big\} \\
&= (-3.34^{+0.43}_{-0.27}) + (-1.05^{+0.45}_{-0.36})i\,.
\end{aligned}
$$

- NNLO correction sizable, but no breakdown of perturbative expansion

Results: Penguin Amplitudes



Results: Scale dependence



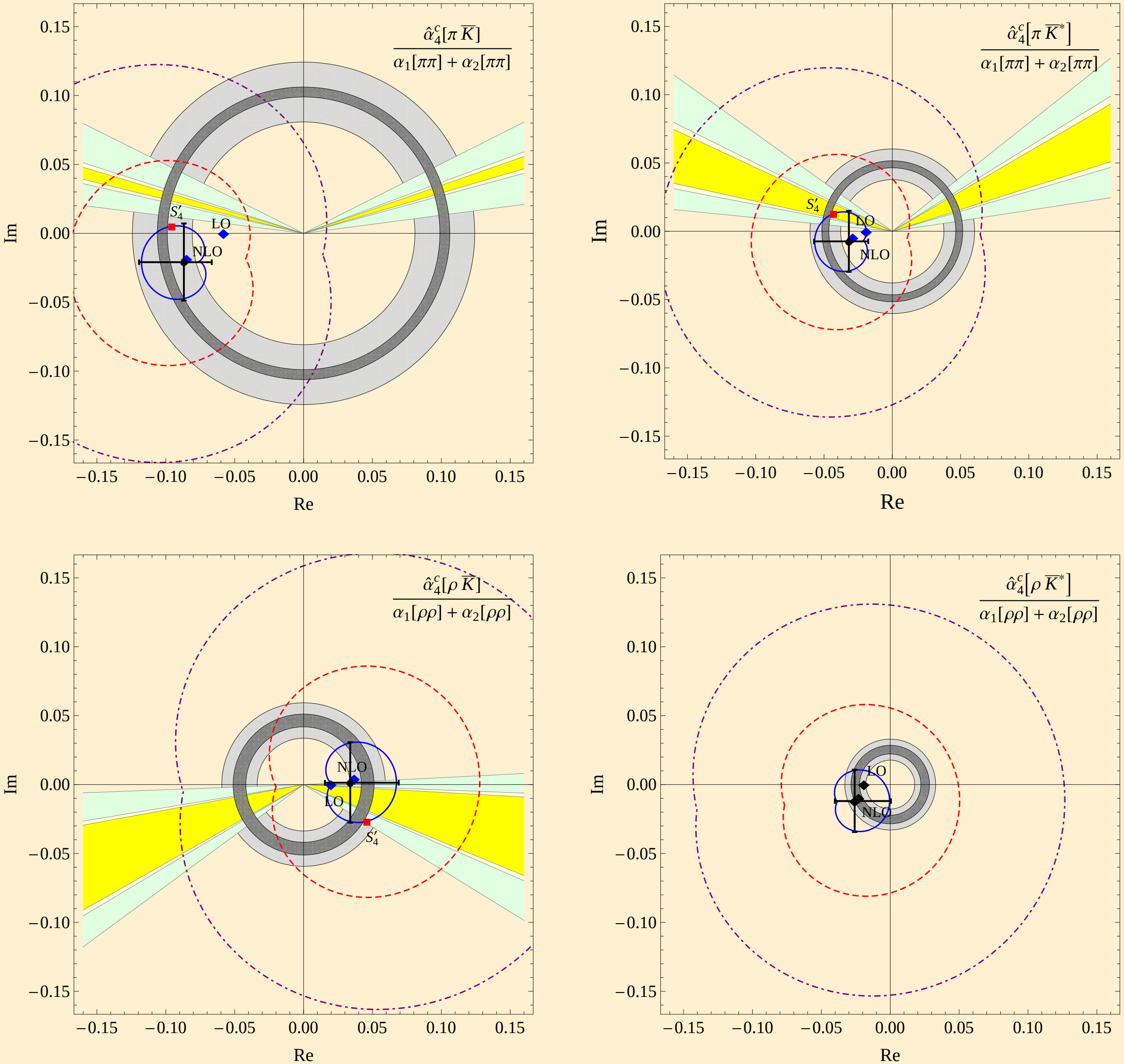
- Only form factor term, no spectator scattering

Results: Amplitude ratios

Ratio	NLO	NNLO
$\frac{P_{\pi\pi}}{T_{\pi\pi}}$	$-0.121 - 0.021i$	$-0.124^{+0.031}_{-0.060} + (-0.026^{+0.045}_{-0.046})i$
$\frac{P_{\rho\rho}}{T_{\rho\rho}}$	$-0.035 - 0.009i$	$-0.041^{+0.020}_{-0.016} + (-0.014^{+0.019}_{-0.018})i$
$\frac{P_{\pi\rho}}{T_{\pi\rho}}$	$-0.038 - 0.005i$	$-0.040^{+0.016}_{-0.030} + (-0.009^{+0.026}_{-0.026})i$
$\frac{P_{\rho\pi}}{T_{\rho\pi}}$	$0.040 + 0.002i$	$0.036^{+0.042}_{-0.023} + (-0.001^{+0.033}_{-0.033})i$
$\frac{C_{\pi\pi}}{T_{\pi\pi}}$	$0.317 - 0.040i$	$0.320^{+0.255}_{-0.142} + (-0.030^{+0.150}_{-0.091})i$
$\frac{C_{\rho\rho}}{T_{\rho\rho}}$	$0.165 - 0.064i$	$0.176^{+0.187}_{-0.133} + (-0.054^{+0.142}_{-0.104})i$
$\frac{C_{\pi\rho}}{T_{\pi\rho}}$	$0.219 - 0.064i$	$0.212^{+0.197}_{-0.112} + (-0.062^{+0.114}_{-0.079})i$
$\frac{C_{\rho\pi}}{T_{\rho\pi}}$	$0.092 - 0.080i$	$0.112^{+0.189}_{-0.144} + (-0.065^{+0.152}_{-0.115})i$
$\frac{T_{\rho\pi}}{T_{\pi\rho}}$	$0.821 + 0.016i$	$0.810^{+0.262}_{-0.200} + (\ 0.010^{+0.062}_{-0.062})i$
$\frac{\alpha_4^c(\pi K)}{\alpha_1(\pi\pi) + \alpha_2(\pi\pi)}$	$-0.085 - 0.019i$	$-0.087^{+0.022}_{-0.036} + (-0.021^{+0.029}_{-0.029})i$
$\frac{\alpha_4^c(\pi K^*)}{\alpha_1(\pi\pi) + \alpha_2(\pi\pi)}$	$-0.029 - 0.005i$	$-0.030^{+0.015}_{-0.026} + (-0.007^{+0.023}_{-0.023})i$
$\frac{\alpha_4^c(\rho K)}{\alpha_1(\rho\rho) + \alpha_2(\rho\rho)}$	$0.037 + 0.004i$	$0.034^{+0.039}_{-0.021} + (\ 0.001^{+0.030}_{-0.030})i$
$\frac{\alpha_4^c(\rho K^*)}{\alpha_1(\rho\rho) + \alpha_2(\rho\rho)}$	$-0.023 - 0.010i$	$-0.027^{+0.027}_{-0.016} + (-0.012^{+0.024}_{-0.023})i$

- Unpublished numbers.
  Only $Q_{1,2}$ contribution.
  Inputs from [Beneke,Li,TH'09].

Results: Amplitude ratios



Results: Direct CP asymmetries I

- Direct CP asymmetries in percent. Errors are CKM and hadronic, respectively.

f	NLO	NNLO	NNLO + LD	Exp
$\pi^- \bar{K}^0$	$0.71\,^{+0.13}_{-0.14}\,^{+0.21}_{-0.19}$	$0.77\,^{+0.14}_{-0.15}\,^{+0.23}_{-0.22}$	$0.10\,^{+0.02}_{-0.02}\,^{+1.24}_{-0.27}$	-1.7 ± 1.6
$\pi^0 K^-$	$9.42\,^{+1.77}_{-1.76}\,^{+1.87}_{-1.88}$	$10.18\,^{+1.91}_{-1.90}\,^{+2.03}_{-2.62}$	$-1.17\,^{+0.22}_{-0.22}\,^{+20.00}_{-\ \ 6.62}$	4.0 ± 2.1
$\pi^+ K^-$	$7.25\,^{+1.36}_{-1.36}\,^{+2.13}_{-2.58}$	$8.08\,^{+1.52}_{-1.51}\,^{+2.52}_{-2.65}$	$-3.23\,^{+0.61}_{-0.61}\,^{+19.17}_{-\ \ 3.36}$	-8.2 ± 0.6
$\pi^0 \bar{K}^0$	$-4.27\,^{+0.83}_{-0.77}\,^{+1.48}_{-2.23}$	$-4.33\,^{+0.84}_{-0.78}\,^{+3.29}_{-2.32}$	$-1.41\,^{+0.27}_{-0.25}\,^{+5.54}_{-6.10}$	1 ± 10
$\delta(\pi\bar{K})$	$2.17\,^{+0.40}_{-0.40}\,^{+1.39}_{-0.74}$	$2.10\,^{+0.39}_{-0.39}\,^{+1.40}_{-2.86}$	$2.07\,^{+0.39}_{-0.39}\,^{+2.76}_{-4.55}$	12.2 ± 2.2
$\Delta(\pi\bar{K})$	$-1.15\,^{+0.21}_{-0.22}\,^{+0.55}_{-0.84}$	$-0.88\,^{+0.16}_{-0.17}\,^{+1.31}_{-0.91}$	$-0.48\,^{+0.09}_{-0.09}\,^{+1.09}_{-1.15}$	-14 ± 11

$$\delta(\pi\bar{K}) = A_{\rm CP}(\pi^0 K^-) - A_{\rm CP}(\pi^+ K^-)$$

$$\Delta(\pi\bar{K}) = A_{\rm CP}(\pi^+ K^-) + \frac{\Gamma_{\pi^-\bar{K}^0}}{\Gamma_{\pi^+K^-}} A_{\rm CP}(\pi^-\bar{K}^0) - \frac{2\Gamma_{\pi^0K^-}}{\Gamma_{\pi^+K^-}} A_{\rm CP}(\pi^0 K^-) - \frac{2\Gamma_{\pi^0\bar{K}^0}}{\Gamma_{\pi^+K^-}} A_{\rm CP}(\pi^0\bar{K}^0)$$

Results: Direct CP asymmetries II

- Direct CP asymmetries in percent

f	NLO	NNLO	NNLO + LD	Exp
$\pi^-\bar{K}^{*0}$	$1.36\,^{+0.25}_{-0.26}\,^{+0.60}_{-0.47}$	$1.49\,^{+0.27}_{-0.29}\,^{+0.69}_{-0.56}$	$0.27\,^{+0.05}_{-0.05}\,^{+3.18}_{-0.67}$	-3.8 ± 4.2
$\pi^0 K^{*-}$	$13.85\,^{+2.40}_{-2.70}\,^{+5.84}_{-5.86}$	$18.16\,^{+3.11}_{-3.52}\,^{+\ 7.79}_{-10.57}$	$-15.81\,^{+3.01}_{-2.83}\,^{+69.35}_{-15.39}$	-6 ± 24
$\pi^+ K^{*-}$	$11.18\,^{+2.00}_{-2.15}\,^{+\ 9.75}_{-10.62}$	$19.70\,^{+3.37}_{-3.80}\,^{+10.54}_{-11.42}$	$-23.07\,^{+4.35}_{-4.05}\,^{+86.20}_{-20.64}$	-23 ± 6
$\pi^0\bar{K}^{*0}$	$-17.23\,^{+3.33}_{-3.00}\,^{+\ 7.59}_{-12.57}$	$-15.11\,^{+2.93}_{-2.65}\,^{+12.34}_{-10.64}$	$2.16\,^{+0.39}_{-0.42}\,^{+17.53}_{-36.80}$	-15 ± 13
$\delta(\pi\bar{K}^*)$	$2.68\,^{+0.72}_{-0.67}\,^{+5.44}_{-4.30}$	$-1.54\,^{+0.45}_{-0.58}\,^{+4.60}_{-9.19}$	$7.26\,^{+1.21}_{-1.34}\,^{+12.78}_{-20.65}$	17 ± 25
$\Delta(\pi\bar{K}^*)$	$-7.18\,^{+1.38}_{-1.28}\,^{+3.38}_{-5.35}$	$-3.45\,^{+0.67}_{-0.59}\,^{+9.48}_{-4.95}$	$-1.02\,^{+0.19}_{-0.18}\,^{+4.32}_{-7.86}$	-5 ± 45

$$\hat{\alpha}_4^p(M_1M_2) = a_4^p(M_1M_2) \pm r_\chi^{M_2} a_6^p(M_1M_2) + \beta_3^p(M_1M_2)$$

Results: Branching ratios

- Unpublished numbers. Only $Q_{1,2}$ contribution. Inputs from [Beneke,Li,TH'09].
- Branching ratios in 10^{-6}.

	NNLO	NLO	Experiment
$B^- \to \pi^-\pi^0$	$5.43^{+2.66}_{-2.14}{}^{+2.05}_{-1.73}{}^{+1.27}_{-0.57}{}^{+0.52}_{-0.50}$	5.33	$5.48^{+0.35}_{-0.34}$
$\bar{B}^0_d \to \pi^+\pi^-$	$7.47^{+3.15}_{-2.61}{}^{+3.36}_{-2.76}{}^{+0.30}_{-0.60}{}^{+1.18}_{-0.66}$	7.30	$5.10^{+0.19}_{-0.19}$
$\bar{B}^0_d \to \pi^0\pi^0$	$0.35^{+0.14}_{-0.11}{}^{+0.19}_{-0.11}{}^{+0.33}_{-0.09}{}^{+0.20}_{-0.10}$	0.33	$1.33^{+0.46}_{-0.46}$
$B^- \to \pi^-\bar{K}^0$	$16.03^{+0.79}_{-0.77}{}^{+9.66}_{-6.68}{}^{+0.87}_{-1.28}{}^{+13.51}_{-5.61}$	14.94	$23.79^{+0.75}_{-0.75}$
$B^- \to \pi^0 K^-$	$9.57^{+0.79}_{-0.74}{}^{+5.00}_{-3.50}{}^{+0.18}_{-0.39}{}^{+7.15}_{-3.01}$	8.97	$12.94^{+0.52}_{-0.51}$
$\bar{B}^0_d \to \pi^+ K^-$	$14.01^{+1.09}_{-1.03}{}^{+8.43}_{-5.76}{}^{+0.12}_{-0.26}{}^{+11.92}_{-4.92}$	12.88	$19.57^{+0.53}_{-0.52}$
$\bar{B}^0_d \to \pi^0\bar{K}^0$	$5.82^{+0.31}_{-0.31}{}^{+4.05}_{-2.72}{}^{+0.07}_{-0.16}{}^{+5.58}_{-2.26}$	5.31	$9.93^{+0.49}_{-0.49}$

- Errors are CKM, scale and inputs (masses, decay constants, FFs), Gegenbauer moments, power corrections

The decays $B \to D^{(*)} L$

[Kränkl,TH'15 and Kränkl,Li,TH in preparation]

- $L \in \{\pi, \rho, K^{(*)}, a_1(1260)\}$
- Only colour-allowed tree amplitude
  - No colour-suppressed tree amplitude, no penguins
  - Spectator scattering and weak annihilation power suppressed

$$BR(\bar{B}_0 \to D^+\pi^-) = \frac{G_F^2 (m_B^2 - m_D^2)^2 |\vec{q}|}{16\pi m_B^2} \tau_{\bar{B}^0} |V_{ud}^* V_{cb}|^2 \, |a_1(D\pi)|^2 \, f_\pi^2 \, F_0^2(m_\pi^2) + O\left(\frac{\Lambda_{QCD}}{m_b}\right)$$

- Applications
  - Ratios of non-leptonic decay widths

$$\frac{\Gamma(\bar{B}_d \to D^+\pi^-)}{\Gamma(\bar{B}_d \to D^{*+}\pi^-)} = \frac{(m_B^2 - m_D^2)^2 |\vec{q}|_{D\pi}}{4 m_B^2 |\vec{q}|^3_{D^*\pi}} \left(\frac{F_0(m_\pi^2)}{A_0(m_\pi^2)}\right)^2 \left|\frac{a_1(D\pi)}{a_1(D^*\pi)}\right|^2$$

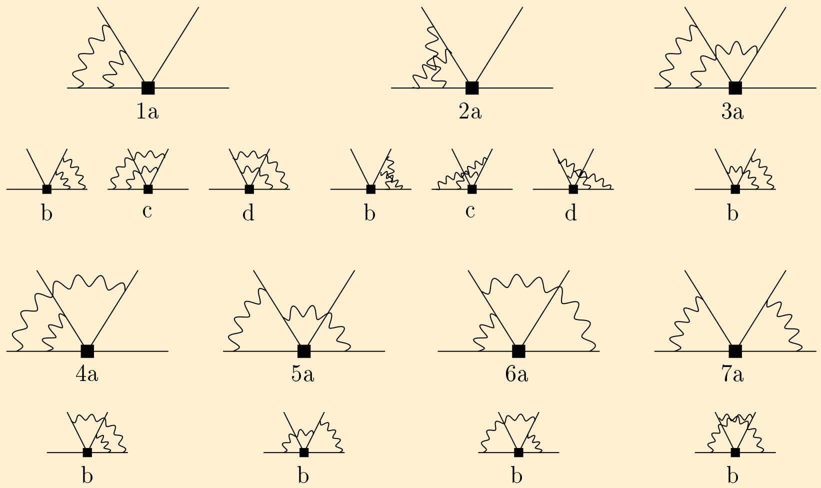
  - Test of factorisation via ratios to semi-leptonic decay

$$\frac{\Gamma(\bar{B}_d \to D^{(*)+}\pi^-)}{d\Gamma(\bar{B}_d \to D^{(*)+} l^- \bar{\nu})/dq^2\big|_{q^2 = m_\pi^2}} = 6\pi^2 |V_{ud}|^2 \, f_\pi^2 \, |a_1(D^{(*)}\pi)|^2$$

  - Estimate size of power corrections, test of QCD factorisation

NNLO calculation

- NLO correction small
  - Colour suppression
  - Small Wilson Coefficient
- At NNLO
  - Again around 70 diagrams

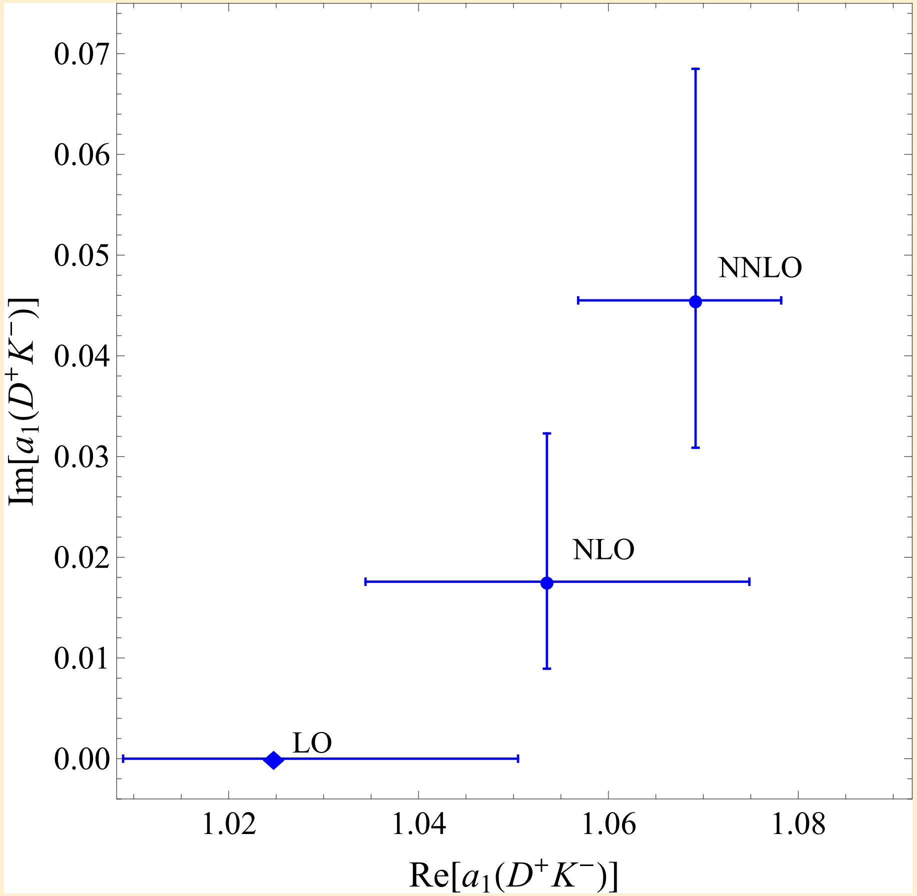


- Again
  - Genuine two-scale problem: u , $z_c \equiv m_c^2/m_b^2$ (but no threshold)
  - Regularize UV and IR divergences dimensionally. Poles up to $1/\epsilon^4$
  - IBP & Laporta reduction
  - Calculation of masters in a canonical basis *[Kränkl, TH'15]*
  - UV renormalisation, IR subtraction, matching onto SCET
  - Cancellation of poles
  - Convolution with LCDA

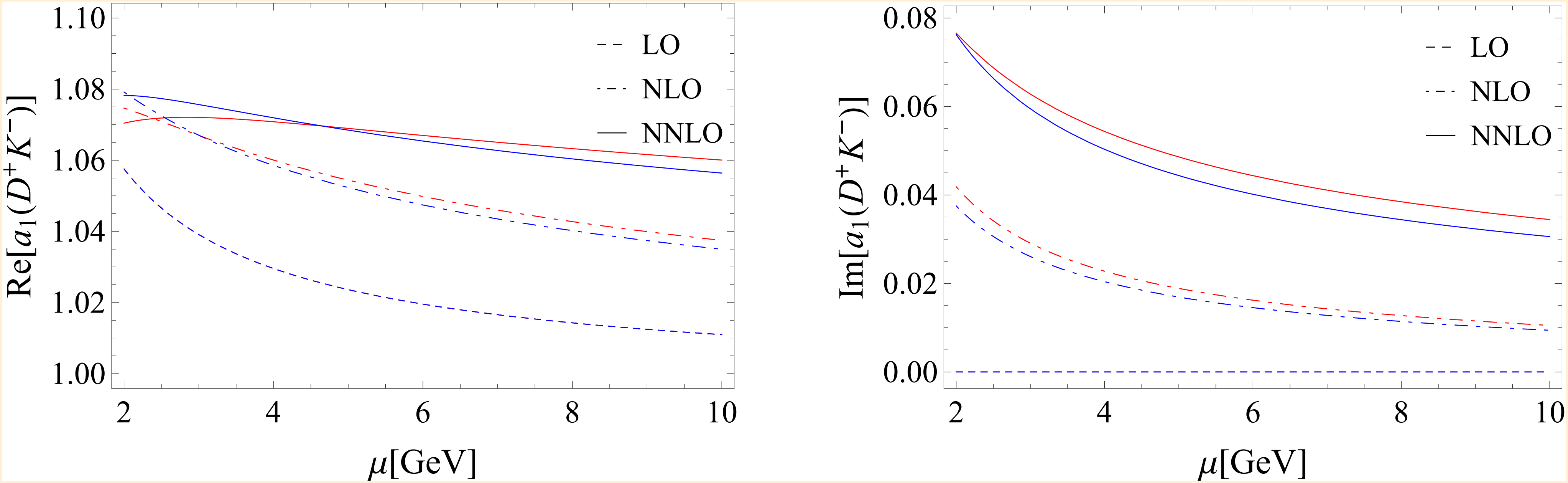
Result for $a_1(\bar{B}^0 \to D^+K^-)$

$$a_1(D^+K^-) = 1.025 + [0.029 + 0.018i]_{\rm NLO} + [0.016 + 0.028i]_{\rm NNLO}$$

$$= (1.069^{+0.009}_{-0.012}) + (0.046^{+0.023}_{-0.015})i$$



Scale dependence



Branching ratios in QCD factorisation I

$$\mathrm{BR}(\bar{B}^0 \to D^+\pi^-) = \frac{G_F^2 (m_B^2 - m_D^2)^2 |\vec{q}|_{D\pi}}{16\pi m_B} \tau_{\bar{B}^0} |V_{ud}^* V_{cb}|^2 \, |a_1(D\pi)|^2 f_\pi^2 F_0^2(m_\pi^2)$$

- Form factor parametrization from CLN *[Caprini,Lellouch,Neubert'97]*
  - Slope and normalization from fit to semileptonic data (HFAG)
- Calculation applies equally well to other $\bar{B}^0 \to D^{*+}L^-$ decays
- Branching ratios in 10^{-3}

Decay	Theory (NNLO)	Experiment
$\bar{B}^0 \to D^+\pi^-$	$3.93^{+0.43}_{-0.42}$	2.68 ± 0.13
$\bar{B}^0 \to D^{*+}\pi^-$	$3.45^{+0.53}_{-0.50}$	2.76 ± 0.13
$\bar{B}^0 \to D^+\rho^-$	$10.42^{+1.24}_{-1.20}$	7.5 ± 1.2
$\bar{B}^0 \to D^{*+}\rho^-$	$9.24^{+0.72}_{-0.71}$	6.0 ± 0.8

- NNLO central values about $20 - 30\%$ larger than experimental ones

Branching ratios in QCD factorisation II

Decay mode	LO	NLO	NNLO	Exp.
$\bar{B}_d \to D^+\pi^-$	3.58	$3.79\,^{+0.44}_{-0.42}$	$3.93\,^{+0.43}_{-0.42}$	2.68 ± 0.13
$\bar{B}_d \to D^{*+}\pi^-$	3.15	$3.32\,^{+0.52}_{-0.49}$	$3.45\,^{+0.53}_{-0.50}$	2.76 ± 0.13
$\bar{B}_d \to D^+\rho^-$	9.51	$10.06\,^{+1.25}_{-1.19}$	$10.42\,^{+1.24}_{-1.20}$	7.5 ± 1.2
$\bar{B}_d \to D^{*+}\rho^-$	8.45	$8.91\,^{+0.74}_{-0.71}$	$9.24\,^{+0.72}_{-0.71}$	6.0 ± 0.8
$\bar{B}_d \to D^+K^-$	2.74	$2.90\,^{+0.33}_{-0.31}$	$3.01\,^{+0.32}_{-0.31}$	1.97 ± 0.21
$\bar{B}_d \to D^{*+}K^-$	2.37	$2.50\,^{+0.39}_{-0.36}$	$2.59\,^{+0.39}_{-0.37}$	2.14 ± 0.16
$\bar{B}_d \to D^+K^{*-}$	4.79	$5.07\,^{+0.65}_{-0.62}$	$5.25\,^{+0.65}_{-0.63}$	4.5 ± 0.7
$\bar{B}_d \to D^{*+}K^{*-}$	4.30	$4.54\,^{+0.41}_{-0.40}$	$4.70\,^{+0.40}_{-0.39}$	–
$\bar{B}_d \to D^+a_1^-$	10.82	$11.44\,^{+1.55}_{-1.48}$	$11.84\,^{+1.55}_{-1.50}$	6.0 ± 3.3
$\bar{B}_d \to D^{*+}a_1^-$	10.12	$10.66\,^{+1.11}_{-1.06}$	$11.06\,^{+1.10}_{-1.07}$	–

- Branching ratios in 10^{-3} for $b \to c\bar{u}d$ and 10^{-4} for $b \to c\bar{u}s$ transitions
- Have also B_s and Λ_b decays
- Non-negligible W-exchange contributions in $\bar{B}_d \to D^{(*)+}\pi^- \,/\, \rho^-$ decays? (not present in $\bar{B}_d \to D^{(*)+}K^{(*)-}$)

Tests of QCD factorisation

- Ratios of non-leptonic decay widths
  - Within error bars, no significant tension

Decay	Theory (NNLO)	Experiment
$\Gamma(\bar{B}^0 \to D^{*+}\pi^-)/\Gamma(\bar{B}^0 \to D^+\pi^-)$	$0.878^{+0.162}_{-0.150}$	1.03 ± 0.07
$\Gamma(\bar{B}^0 \to D^+\rho^-)/\Gamma(\bar{B}^0 \to D^+\pi^-)$	$2.653^{+0.163}_{-0.158}$	2.80 ± 0.47

- Extraction of $|a_1|$ from ratios of non-leptonic and semi-leptonic BRs

$$\frac{\Gamma(\bar{B}_d \to D^{(*)+}\pi^-)}{d\Gamma(\bar{B}_d \to D^{(*)+}l^-\bar{\nu})/dq^2\big|_{q^2=m_\pi^2}} = 6\pi^2|V_{ud}|^2\ f_\pi^2\ |a_1(D^{(*)}\pi)|^2$$

Decay	$\|a_1\|$ Theory (NNLO)	$\|a_1\|$ Experiment
$\bar{B}^0 \to D^+\pi^-$	1.07 ± 0.01	0.89 ± 0.05
$\bar{B}^0 \to D^{*+}\pi^-$	1.07 ± 0.01	0.96 ± 0.03
$\bar{B}^0 \to D^+\rho^-$	1.07 ± 0.01	0.91 ± 0.08
$\bar{B}^0 \to D^{*+}\rho^-$	1.07 ± 0.01	0.86 ± 0.06

  - Quasi-universal value for $|a_1| \sim 1.07$ at NNLO. Experiment favours lower $|a_1|$.
- Leaves room for (negative) power corrections to the amplitude of $\sim 10 - 15\%$

Conclusion and Outlook

- $Q_{1,2}$-contribution to penguin amplitudes α_4^u and α_4^c at NNLO ready
  - NNLO shift in amplitudes is rather sizable
  - Shift in amplitude ratios, CP asymmetries, BRs is moderate
- NNLO correction to $\bar{B} \to D\,\pi$ ready
  - NNLO BRs $\sim 20 - 30\%$ above experimental values (NNLO and FFs $\uparrow$, Exp. $\downarrow$)
  - Room for power corrections to the colour-allowed tree-amplitude of $\sim 10 - 15\%$
- Future plans
  - Other insertions to α_4^u and α_4^c at NNLO, pheno based on NNLO results
  - Connect QCDF with flavour symmetries
  - Power suppressed amplitude a_6 at NNLO
  - QED corrections

Backup slides

Some definitions

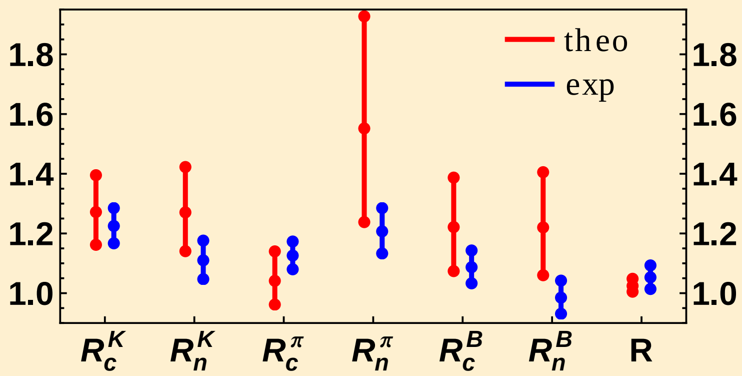
$$A_{\pi\pi} = i\,\frac{G_F}{\sqrt{2}}\,m_B^2 F_+^{B\to\pi}(0) f_\pi$$

$$r_{\rm sp} = \frac{9 f_\pi \hat{f}_B}{m_b \lambda_B F_+^{B\to\pi}(0)}$$

$$\lambda_B^{-1} = \int_0^\infty \frac{d\omega}{\omega}\,\phi_B(\omega,\mu)$$

$$\Delta A_{\rm CP}^-(\pi K) = A_{\rm CP}(B^- \to \pi^0 K^-) - A_{\rm CP}(\bar{B}^0 \to \pi^+ K^-) = \Delta A_{\rm CP}(\pi K)$$

$$\Delta A_{\rm CP}^0(\pi K) = A_{\rm CP}(B^- \to \pi^- \bar{K}^0) - A_{\rm CP}(\bar{B}^0 \to \pi^0 \bar{K}^0)$$



Results: Direct CP asymmetries III

- Direct CP asymmetries in percent

f	NLO	NNLO	NNLO + LD	Exp
$\rho^-\bar{K}^0$	$0.38\,^{+0.07}_{-0.07}\,^{+0.16}_{-0.27}$	$0.22\,^{+0.04}_{-0.04}\,^{+0.19}_{-0.17}$	$0.30\,^{+0.06}_{-0.06}\,^{+2.28}_{-2.39}$	-12 ± 17
$\rho^0 K^-$	$-19.31\,^{+3.42}_{-3.61}\,^{+13.95}_{-\ 8.96}$	$-4.17\,^{+0.75}_{-0.80}\,^{+19.26}_{-19.52}$	$43.73\,^{+7.07}_{-7.62}\,^{+\ 44.00}_{-137.77}$	37 ± 11
$\rho^+ K^-$	$-5.13\,^{+0.95}_{-0.97}\,^{+6.38}_{-4.02}$	$1.50\,^{+0.29}_{-0.27}\,^{+\ 8.69}_{-10.36}$	$25.93\,^{+4.43}_{-4.90}\,^{+25.40}_{-75.63}$	20 ± 11
$\rho^0\bar{K}^0$	$8.63\,^{+1.59}_{-1.65}\,^{+2.31}_{-1.69}$	$8.99\,^{+1.66}_{-1.71}\,^{+3.60}_{-7.44}$	$-\ 0.42\,^{+0.08}_{-0.08}\,^{+19.49}_{-\ 8.78}$	6 ± 20
$\delta(\rho\bar{K})$	$-14.17\,^{+2.80}_{-2.96}\,^{+7.98}_{-5.39}$	$-5.67\,^{+0.96}_{-1.01}\,^{+10.86}_{\ \ -9.79}$	$17.80\,^{+3.15}_{-3.01}\,^{+19.51}_{-62.44}$	17 ± 16
$\Delta(\rho\bar{K})$	$-8.75\,^{+1.62}_{-1.66}\,^{+4.78}_{-6.48}$	$-10.84\,^{+1.98}_{-2.09}\,^{+11.67}_{-\ 9.09}$	$-\ 2.43\,^{+0.46}_{-0.42}\,^{+\ 4.60}_{-19.43}$	-37 ± 37

More on theory approaches to nonleptonic B-decays

- Perturbative QCD (PQCD) approach based on k_T-factorisation *[see e.g. Keum,Li,Sanda'01]*
  - Factorises amplitudes according to

$$A(B \to M_1 M_2) = \phi_B \otimes H \otimes J \otimes S \otimes \phi_{M_1} \otimes \phi_{M_2}$$

  - Generates larger strong phases. Avoids endpoint divergences.
  - However: Organises amplitude differently
  - Introduces additional infrared prescriptions, e.g. exponentiation of Sudakov logarithms, phenomenological model for transverse momentum effects
  - Discussion of theoretical uncertainties difficult, since no complete NLO ($\mathcal{O}(\alpha_s^2)$) analysis available
  - Independent information on hadronic input functions not available