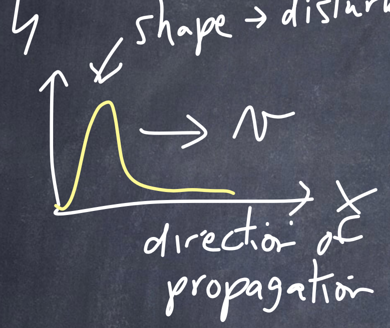
# PHY117 HS2024

Week 11, Lecture 2
Nov. 27th, 2024
Prof. Ben Kilminster

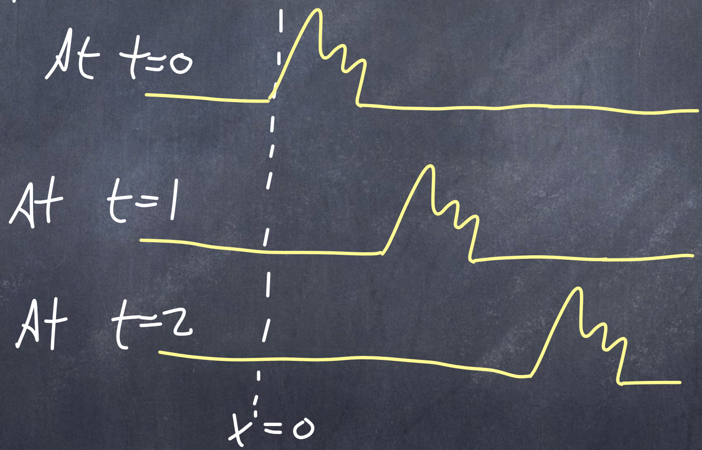
# WAVES

# WAVES:

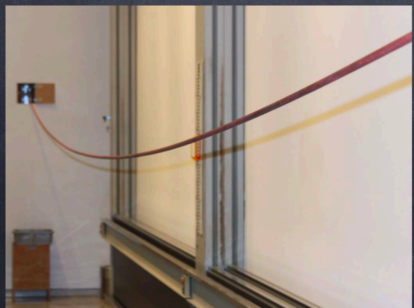
wave propagation → one makes a disturbance that propagates along some medium. ← (air, water, string, ...)




Transverse wave: disturbance $\perp$ propagation

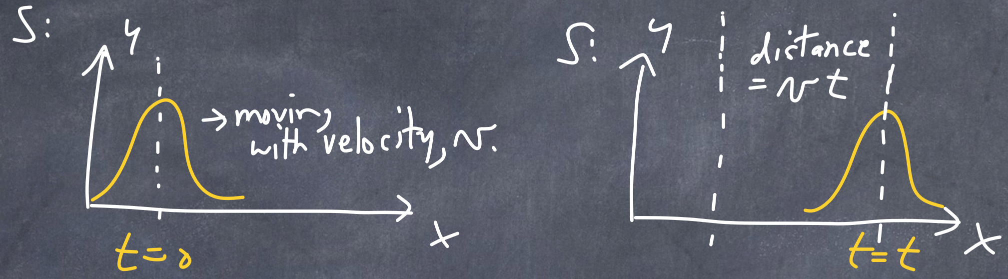



The shape of the wave $y = f(x)$ at $t=0$ is the same at $t=1,2$



we define 2 coordinate systems (or rest frames):

S: not moving




S': moving with same velocity as wave, v


$$\boxed{\begin{aligned} y' &= y \\ x' &= x - vt \end{aligned}}$$

known as Galilean transformation

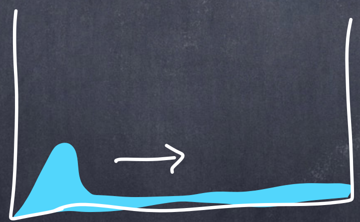
So we see that, we can describe our wave in the S' frame by transforming its position back to where it was at t=0 : $y'(x',t) = y(x - vt)$

wave function for wave moving to the right

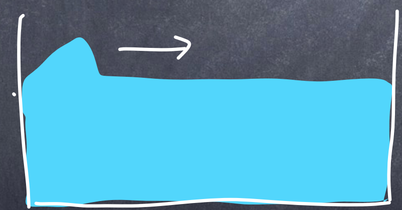
Disturbance moving in water

shallow water:

interaction with bottom



deep water: interaction mainly with surface



waves move slower in shallow water due to interaction with bottom

A wave moving to the right has a form like $y = y(x,t) = f(x - vt)$

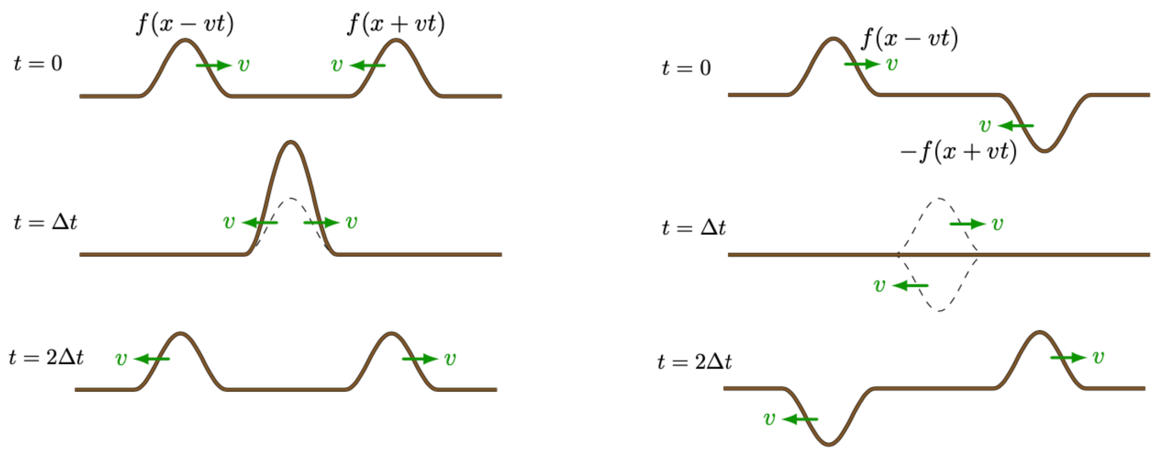
$\uparrow$ any shape

Moving to the left, $y(x,t) = f(x + vt)$

If we had 2 waves, one to the left + one to the right, we can add them linearly

$$y(x,t) = y(x - vt) + y(x + vt)$$

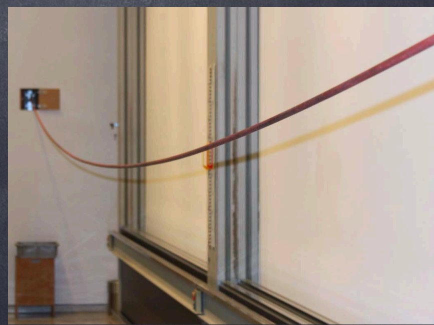
Superposition: is the addition of waves



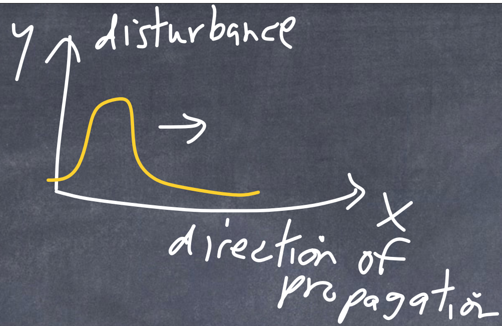

**(a)** Constructive interference happens when two oppositely waves meet on a string.

**(b)** Destructive interference. If the waves are the same but for a sign, they cancel completely.

**Figure 13.5:** Superposition between two oppositely travelling waves in the same medium is a simple linear sum.



**Transverse wave:**




disturbance ⊥ direction it moves

**Longitudinal wave**

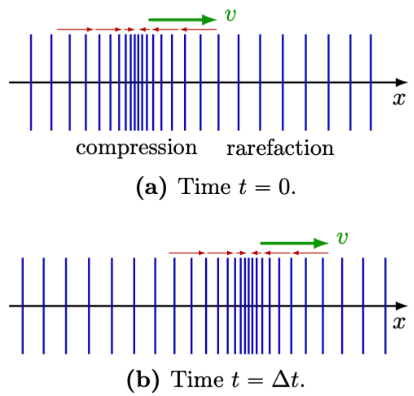
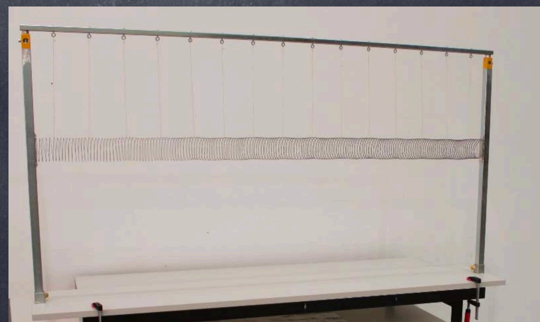



**Figure 13.10:** A traveling longitudinal wave is when the distortion happens along the direction of propagation, here shown as a local displacement.

disturbance ∥ direction it moves

(Sound waves are longitudinal waves)



Speed of a wave on a string ] $T$: tension of string
$\mu$: mass per length

velocity of the disturbance

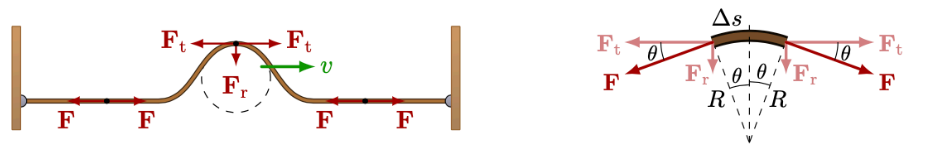
$$v = \sqrt{\frac{T}{\mu}}$$

← faster if tension is larger

← slower if mass per length is larger

Derivation of this formula in script §13.1.2

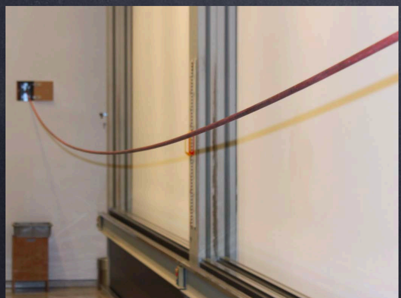





**(a)** Forces on a string. All across the string, there is a constant tension $\mathbf{F}$.

**(b)** Small segment of length $\Delta s$ experiences a tension $\mathbf{F}$ on either side.

**Figure 13.3:** The tension in a string is increased due to a disturbance.



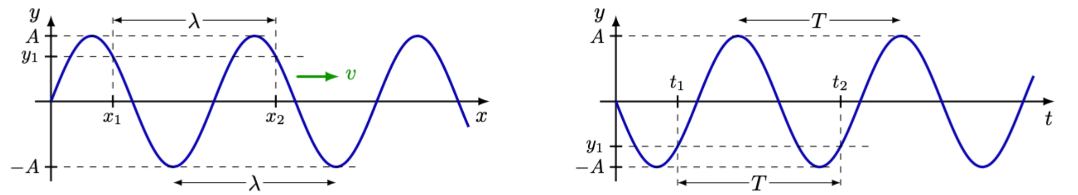
General form for a wave is $y(x,t) = f(x - vt)$

One type of wave is a sine wave:

$$\boxed{y(x,t) = A\sin(kx - \omega t)} \quad (1)$$

Inside here, we need an angle from 0 to $2\pi$ radians

$t=0$ ↓ $\qquad\qquad$ $x=0$ ↓




**(a)** Whole wave in space at time $t = 0$, given by $y(x, 0) = A\sin(kx)$.

**(b)** Local disturbance at position $x = 0$, given by $y(0, t) = -A\sin(\omega t)$.

**Figure 13.2:** A space and time slice of a travelling sine wave $y(x, t) = A\sin(kx - \omega t)$.

what is $k$? we see that the wave repeats every wavelength $\lambda$.

so for $x_2 = x_1 + \lambda$, $y(x_1, 0) = y(x_2, 0)$

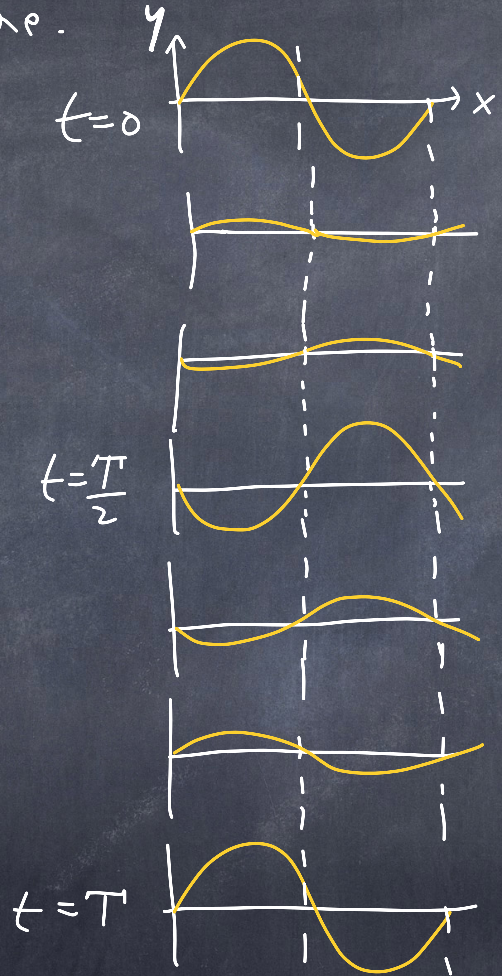
From (1) (with $t=0$), $A\sin(kx_1) = A\sin(kx_2) = A\sin(kx_1 + k\lambda)$

This happens only if $k\lambda = n(2\pi)$ where $n = 0, \pm1, \pm2, \ldots$

The smallest interval between repeating points is for $n=1$

so $k\lambda = 2\pi \implies \boxed{k = \frac{2\pi}{\lambda} \quad \text{wave number}}$

The shape of a wave changes (y vs. x) in the same place, with time.




the wave repeats every time = T

the period

when $x=0$

$\downarrow$

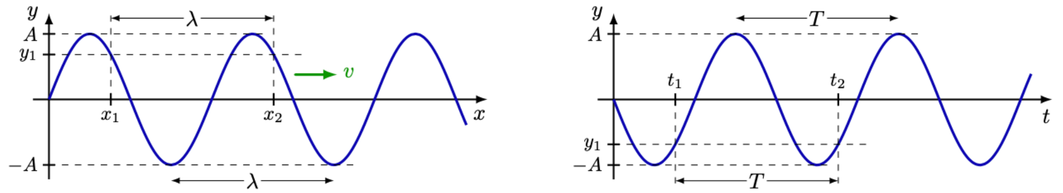



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**Figure 13.2:** A space and time slice of a travelling sine wave $y(x, t) = A\sin(kx - \omega t)$.

$$y(x,t) = A\sin(kx - \omega t)$$

when $x=0$, $y(0,t) = A\sin(-\omega t)$

Two moments in time, $t_1$ and $t_2 = t_1 + T$ must be separated by $2\pi$

so $\omega(t_1 + T) = \omega t_1 + 2\pi$

and $\boxed{\omega = \frac{2\pi}{T} \quad \text{angular velocity}}$

Summary: For our wave, $\lambda$ wavelength $[m]$

$T$ period $[s]$

$v$ velocity $\left[\frac{m}{s}\right]$

The velocity of our wave is

$$v = \frac{\lambda}{T} = f\lambda \qquad f = \frac{1}{T} \quad \left[\frac{1}{s}\right] = [Hz]$$

wave number $= k = \frac{2\pi}{\lambda} \quad \left[\frac{1}{m}\right]$

angular velocity $= \omega = 2\pi f = \frac{2\pi}{T} \quad \left[\frac{1}{s}\right]$
(angular frequency)

$$v = \frac{\lambda}{T} = \frac{\frac{2\pi}{k}}{\frac{2\pi}{\omega}} = \frac{\omega}{k} \qquad \boxed{v = \frac{\omega}{k}}$$

Example for a sine wave

$$y(x,t) = A\sin(kx - \omega t)$$

wave $\boxed{\text{function}}$ for a moving sine wave

$$y(x,t) = A\sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]$$

same formula with $\lambda$ + $T$

wave equation $\dfrac{\partial^2 y}{\partial x^2} = \dfrac{1}{v^2}\dfrac{\partial^2 y}{\partial t^2}$ $\quad y(x,t)$

see derivation $\longrightarrow$ **13.2 Wave equation**

all wave functions satisfy the wave equation:

check: consider $F(x,t) = A\sin(kx-\omega t)$

$$\frac{\partial F}{\partial x} = Ak\cos(kx-\omega t), \quad \frac{\partial^2 F}{\partial x^2} = -Ak^2\sin(kx-\omega t)$$

$$\frac{\partial F}{\partial t} = -A\omega\cos(kx-\omega t), \quad \frac{\partial^2 F}{\partial t^2} = -A\omega^2\sin(kx-\omega t)$$

$$\frac{\partial^2 F}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 F}{\partial t^2}$$

$$-\cancel{A}k^2\cancel{\sin(kx-\omega t)} = \frac{1}{v^2}\left(-\cancel{A}\omega^2\cancel{\sin(kx-\omega t)}\right)$$

$$\frac{k^2}{\omega^2} = \frac{1}{v^2} \Rightarrow v = \frac{\omega}{k}$$ , which is true.

so our check works.

Standing waves — when we confine waves, the waves will reflect and combine by the superposition principle.

There are resonant frequencies and wavelengths in standing waves.

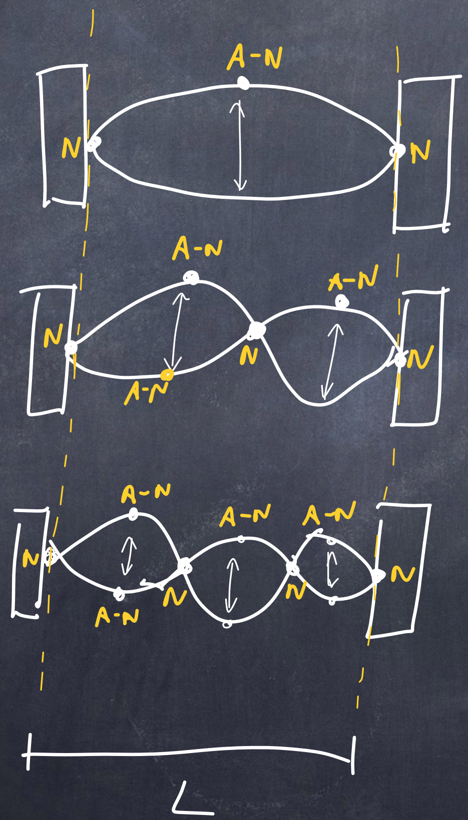
N : Nodes (points that don't move)

A-N : anti-nodes (points that move maximally)

Fundamental harmonic (first harmonic) $n=1$

2nd harmonic $n=2$

3rd harmonic $n=3$



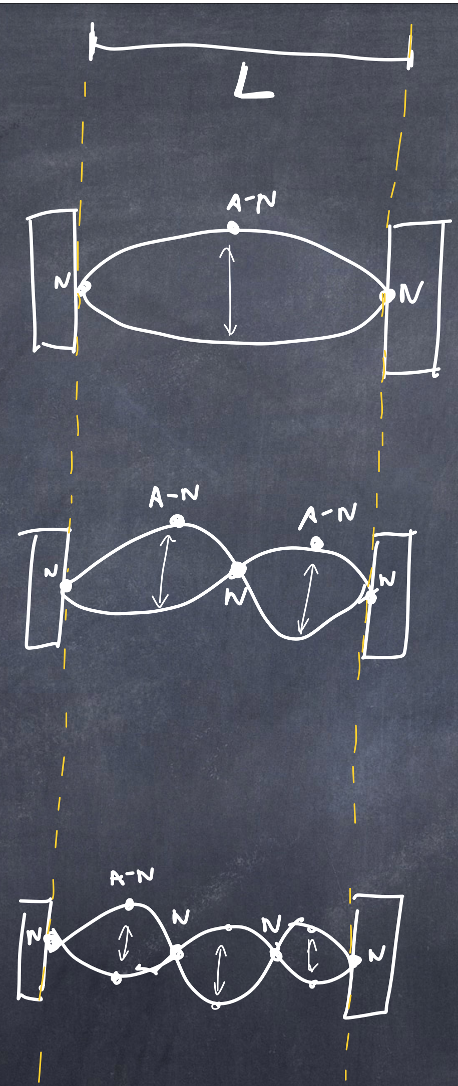

L: size of the space that the wave is confined

How many waves per length?

$L = \lambda \cdot ?$

one $\lambda$ = 

key




$n=1$ ← $\frac{1}{2}$ of a wave $\qquad L = \frac{\lambda}{2} \Rightarrow \lambda = 2L$

$n=2$ $\qquad L = \lambda$

$n=3$ $\qquad 1 + \frac{1}{2}$ waves $\qquad L = \frac{3}{2}\lambda$

$\lambda = \frac{2}{3} L$

In general, the wavelengths of the harmonics are $\lambda_n = \frac{L}{n} 2$

$n = 1, 2, 3, \ldots$

The frequencies of the $n^{th}$ harmonic $\quad v = f\lambda$

$$f_n = \frac{v}{\lambda_n} = \frac{v}{\frac{2L}{n}} = \frac{nv}{2L} \quad \text{for } n = 1, 2, 3, \ldots$$

For $n = 1$,
first harmonic

$$f_1 = \frac{1 \cdot v}{2L} = \frac{v}{2L}$$

for a string, $v = \sqrt{\frac{T}{\mu}}$

$$f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

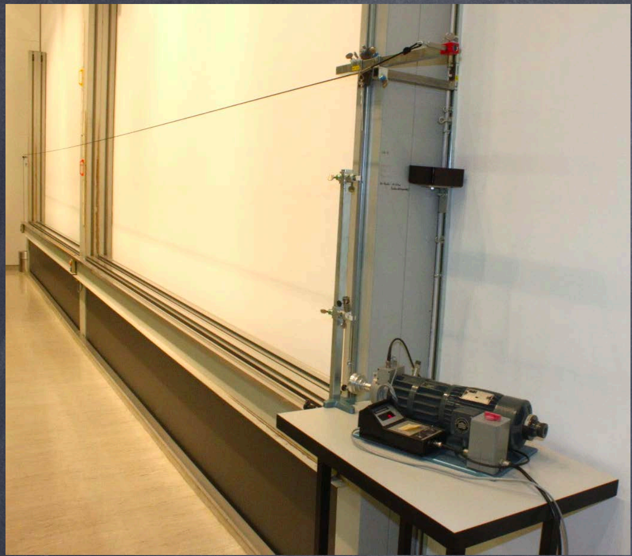
tension on string

mass per length of string

If we know $f_1$, $f_n = n \cdot f_1$

$f_1 = \frac{v}{2L} \qquad f_n = \frac{nv}{2L}$

All standing waves have a frequency that is a multiple of the first harmonic, $f_1$, depends on tension, length, $\frac{\text{mass}}{\text{length}}$



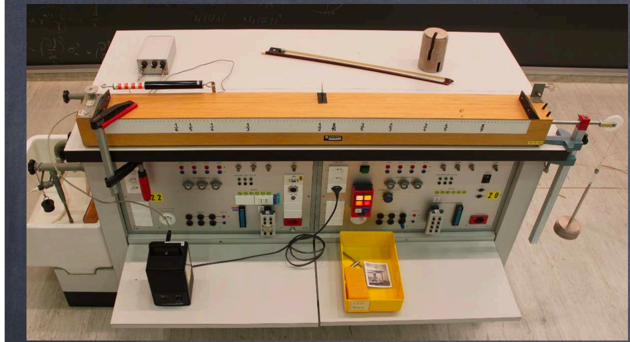
$$f_n = n f_1$$

first, we find $f_1$, the frequency of the first harmonic.

Then, $f_2 = 2 f_1$

$f_3 = 3 f_1$

...



1) $v = f_n \lambda_n$ → If we half the wavelength, frequency doubles (octave)

2) $f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

tension on string

mass per length of string

→ If we increase $T$ by ×4, $f_1$ doubles

## standing wave functions

– consider the superposition of a wave moving to the left and to the right.

$$y_R = A\sin(kx - \omega t)$$

$$y_L = A\sin(kx + \omega t)$$

k + w are both the same

$$y(x,t) = y_R + y_L = A\left[\sin(kx-\omega t) + \sin(kx+\omega t)\right]$$

substitution

$$\sin\theta_1 + \sin\theta_2 = 2\cos\tfrac{1}{2}(\theta_1-\theta_2)\sin\tfrac{1}{2}(\theta_1+\theta_2)$$

we set $\theta_1 = kx + \omega t$

and $\theta_2 = kx - \omega t$

add these: $\theta_1 + \theta_2 = 2kx$

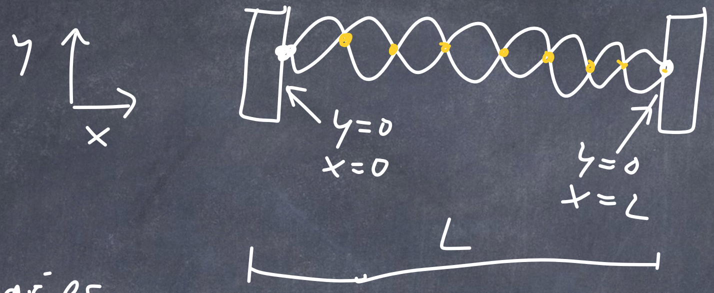
$\theta_1 - \theta_2 = 2\omega t$

$$\boxed{y(x,t) = 2A(\cos\omega t)(\sin kx)}$$

formula for a standing wave with 2 fixed ends (nodes)

Now only some $\omega$ + $k$ work for standing waves.
we consider the boundary conditions

$y = 0$ when $x = 0$, or $x = L$




At boundaries, when does $y = 0$? Happens when $\sin kL = 0$

so $k_n L = n\pi$ $\quad n = 1, 2, 3, \ldots$

since $\lambda = \frac{2\pi}{k}$

$$\boxed{k_n = \frac{n\pi}{L} \quad \text{for } n = 1, 2, \ldots}$$

allowed wave numbers

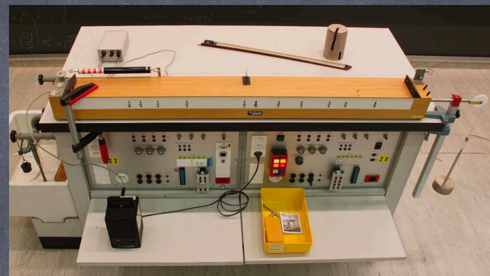
we know $f_n = \frac{v}{\lambda_n} = \frac{v n}{2L}$ $\quad n = 1, 2, 3, \ldots$

$$\boxed{f_1 = \frac{1}{2L}\sqrt{\frac{F}{\mu}}}$$

$f_n = n \cdot f_1$

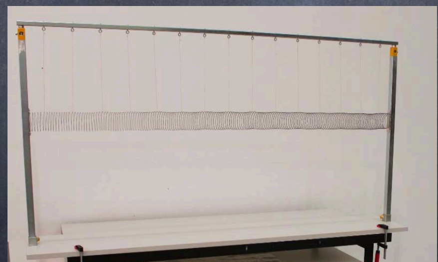
since $\omega_n = 2\pi f_n$

$$\boxed{\omega_n = 2\pi n f_1 \quad \text{for } n = 1, 2, 3,}$$



W14

W18



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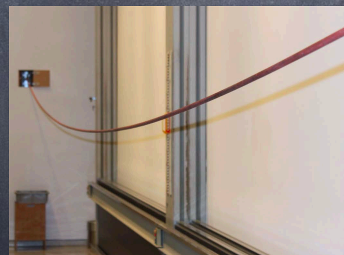

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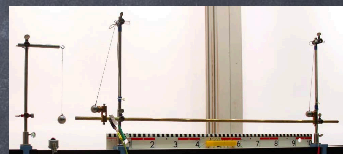

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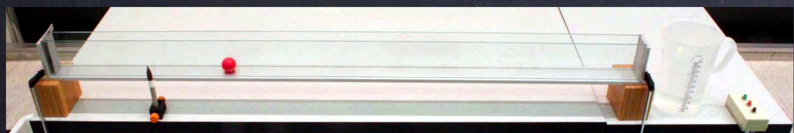

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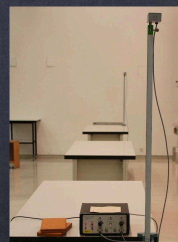

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