PHY117 HS2024

Week 6, Lecture 1
Oct. 22nd, 2024
Prof. Ben Kilminster

Please do weekly quizzes, for your own good! You can do past weeks any time.

To avoid confusion, we have 4 ks:

K = Kelvin

k = Boltzmann constant

K = Kinetic energy

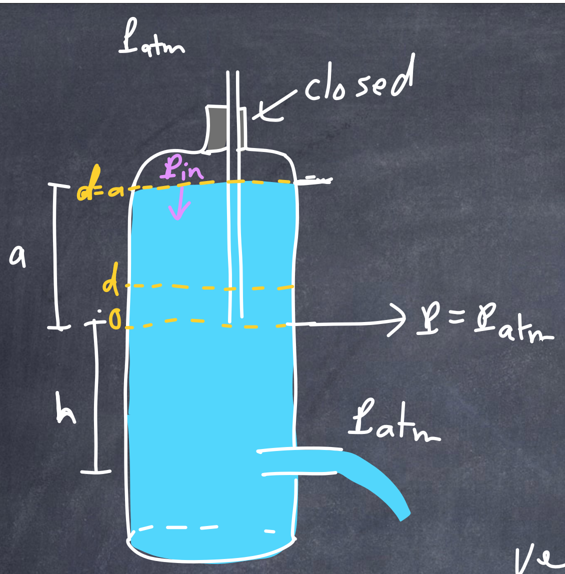
K = coefficient of thermal conductivity

Thermodynamics – study of temperature, heat, and the ~~ex~~ exchange of energy. (mechanical work)

Macroscopic scale: measurable properties:

volume, pressure, temperature.

(pressure → force/area)



Marriotte's bottle: (constant velocity of fluid)

The P_{atm} is transmitted to the depth marked as "a". P_{in} must be $P_{atm} - \rho g d$.

The velocity will be $v = \sqrt{2gh}$

The benefit here is that the velocity of water coming out does not change within the range of a.

How does this work?

P_{in}, (pressure at the top) will change.

As d changes from $d=a$ to $d=0$

$$\underbrace{P_{in}}_{\text{top, changes}} + \rho g d = \underbrace{P_{atm}}_{\text{bottom, constant}}$$

the P_{in} will be negative at the start, because this is a difference to P_{atm}

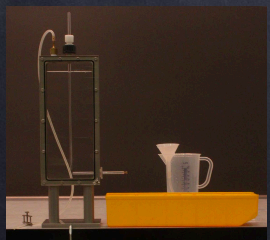
when $d=a$, $P_{in} = P_{atm} - \rho g a$

when $d=0$, $P_{in} = P_{atm}$

$$P_{in} = P_{atm} - \rho g d$$

If $d = 10\,\text{cm}$, then:

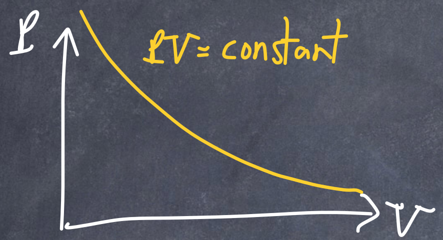
$$\rho g d = \frac{1\,\text{g}}{\text{cm}^3} \cdot \frac{1\,\text{kg}}{1000\,\text{g}} \left(\frac{100\,\text{cm}}{1\,\text{m}}\right)^3 \left(10\,\frac{\text{m}}{\text{s}^2}\right)(10\,\text{cm})\frac{1\,\text{m}}{100\,\text{cm}} = 1\,\text{kPa}$$



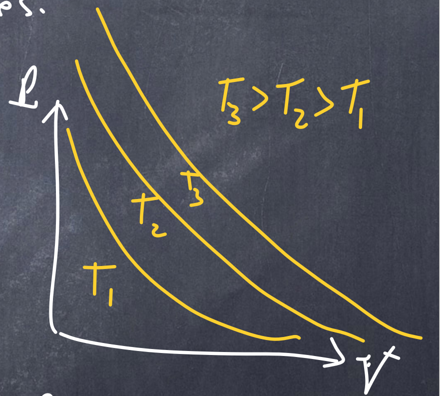
From Marriotte's bottle (fluids)
to Marriotte's Law (gases)

Ideal gases: randomly moving point particles with no inter-particle interactions

Boyle-Marriotte law: $PV =$ constant for an ideal gas at constant temperature.



As we change temperature, PV takes different values.



The law is written as:

$$\boxed{PV = nRT = NkT \quad \text{Ideal gas law}}$$

pressure $[Pa] = [\frac{N}{m^2}]$; volume $[m^3]$; temperature $[K]$

we see that $nR = Nk$

N: number of gas molecules

k: Boltzmann constant
$k = 1.38\,\text{e}{-23}$ J/K

n: number of moles

R: gas constant, $R = 8.314$ J/K·mol

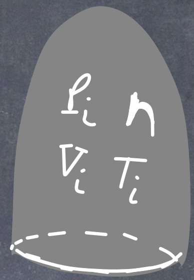
Note: $R = N_A \cdot k$
$N = n \cdot N_A$

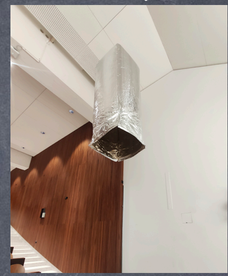
where N_A: Avogadro's number is # molecules/mole

$N_A = 6.02\,\text{e}23 \frac{\text{molecules}}{\text{mole}}$

Why does our hot air balloon rise when air inside is heated?

pressure decreases?
density decreases?
volume increases?

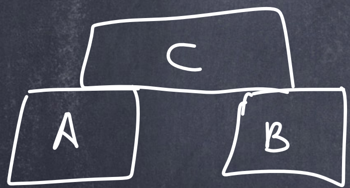




What is temperature? Intuitively, it is a measure of the hotness or coldness of something.

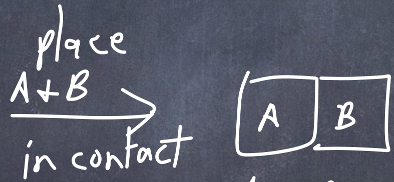
Objects have thermometric properties: gases expand, as do most solids + liquids with temperature (if allowed to). Electrical resistance changes...

0^{th} law of thermodynamics: If 2 objects are in thermal equilibrium with a third object, then they are in thermal equilibrium with each other.



C is in thermal equilibrium with A + B (no further thermometric change).

C is the same temperature as A + B.



A + B must be the same temperature.

$PV = nRT$

$P \sim T$

$-273.15°C \hat{=} 0$ Kelvin $\hat{=} 0\,K$

Since 2019, the Kelvin is defined using

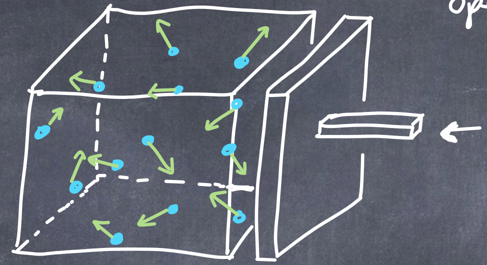
$k = 1.380649\,E-23\ J/K$

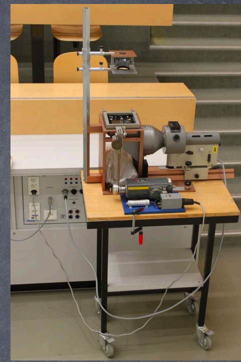
$J = \frac{kg \cdot m^2}{s^2}$ so we can get definitions of the K from m, s, kg

But what is temperature really?
At the molecular level?

box (5 closed sides)

piston pushing 6th open side

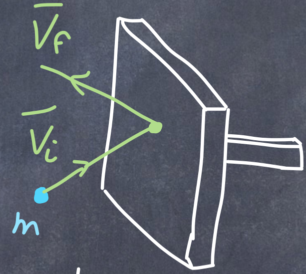




gas of molecules

we put molecules in a box, and close with a piston.

when one molecule hits the piston, its momentum changes.



initial: $\bar{V}_i = V_{i_x}\hat{x} + V_{i_y}\hat{y} + V_{i_z}\hat{z}$

final: $\bar{V}_f = V_{f_x}\hat{x} + V_{f_y}\hat{y} + V_{f_z}\hat{z}$

considering the x-direction: $\Delta p_x = mV_{f_x} - mV_{i_x}$

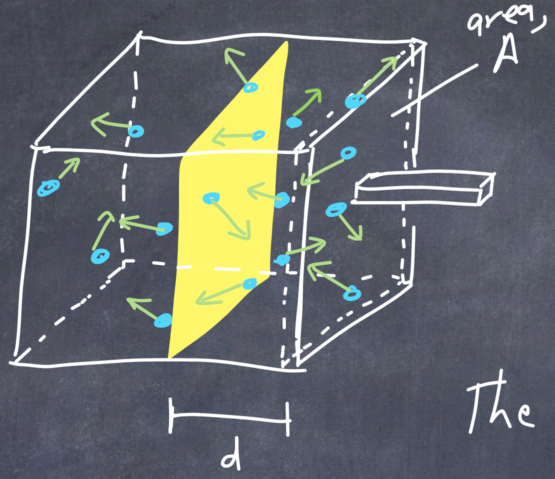
Assume it is an elastic collision, $|\bar{V}_f| = |\bar{V}_i|$

$$|V_{f_x}| = |V_{i_x}|$$

$\Delta p_x = 2mV_x$ for one molecule in gas

$(\Delta\bar{p} = \text{Impulse} = \bar{I})$

total volume V



In some Δt (time), molecules hit the piston. The ones that hit the piston must be less than a certain distance away from the piston, moving to the right, depends on velocity

$$V_x = \frac{d}{\Delta t} \qquad d = V_x \Delta t$$

The volume of this box is $A \cdot d = A \cdot V_x \Delta t$

The number of particles in this box that hit the piston:

$$\underbrace{N_R}_{\substack{\text{\# that hit the}\\ \text{piston on}\\ \text{the right}}} = \underbrace{\frac{N}{V}}_{\substack{\text{density}=\\ \text{\#/volume}}} \cdot \underbrace{V_x \Delta t A}_{\substack{\text{volume of}\\ \text{molecules}\\ \text{with correct}\\ V_x \text{ to hit}\\ \text{wall}}} \cdot \underbrace{\frac{1}{2}}_{\substack{\text{moving}\\ \text{to}\\ \text{the right}}}$$

The total momentum of ptcls hitting piston is:

$$\underbrace{\Delta p_x}_{\text{momentum of all molecules}} = \underbrace{\left(\frac{1}{2}\frac{N}{V}v_x \Delta t A\right)}_{\#\text{ molecules}} \underbrace{\left(2mv_x\right)}_{\Delta p \text{ for one molecule}}$$

$$\Delta p_x = \frac{N}{V} m v_x^2 A \Delta t$$

The total force on the wall is $F = \frac{dp}{dt} = \frac{\Delta p}{\Delta t}$

$$F_x = \frac{N}{V} m v_x^2 A \frac{\cancel{\Delta t}}{\cancel{\Delta t}}$$

Pressure is F/A

$$P = \frac{N}{V} m v_x^2 \frac{\cancel{A}}{\cancel{A}}$$

pressure due to v_x of N molecules.

so $\quad PV = N m v_x^2$

rewrite as $\quad PV = 2N\left(\frac{1}{2} m v_x^2\right) = 2N\left(\begin{array}{c}\text{Kinetic energy} \\ \text{of a molecule} \\ \text{in 1-D}\end{array}\right)$

we recognize that since $PV = NkT$,

then

$$\boxed{\begin{array}{c}\text{Kinetic} \\ \text{energy} \\ \text{of a single} \\ \text{molecule} \\ \text{in 1-D}\end{array} = \frac{1}{2} m v_x^2 = \frac{1}{2} k T}$$

Now we know that temperature is the kinetic energy of the molecules (from their velocity)

$\frac{1}{2} kT$ is the average kinetic energy in 1 dimension

$$K_{1D} = \left\langle \frac{1}{2} m v_x^2 \right\rangle = \frac{1}{2} k T$$

K : kinetic energy

k : Boltzmann constant

Note: $\langle V_x^2 \rangle > 0$ though $\langle V_x \rangle = 0$ since $\frac{1}{2}$ go in $+\hat{x}$ and $\frac{1}{2}$ go in $-\hat{x}$

Also, the molecule has equal velocities in x, y, z directions

So $K_{3D} = \frac{1}{2} m \langle V_x^2 \rangle + \frac{1}{2} m \langle V_y^2 \rangle + \frac{1}{2} m \langle V_z^2 \rangle = 3 \left(\frac{1}{2} kT \right)$

Note: $\langle V^2 \rangle = \langle V_x^2 \rangle + \langle V_y^2 \rangle + \langle V_z^2 \rangle = 3 \langle V_x^2 \rangle$

So $\underbrace{K_{3D}}_{\text{Kinetic energy in 3-D for 1 molecule}} = 3 \left(\frac{1}{2} m \langle V_x^2 \rangle \right) = \frac{1}{2} m \langle V^2 \rangle = 3 \underbrace{\left(\frac{1}{2} kT \right)}_{\text{Kinetic energy per degree of freedom}}$

3 degrees of freedom (x, y, z)

we call $\sqrt{\langle V^2 \rangle} \equiv V_{rms}$

V_{rms} = "root mean square" speed

For N particles, the translational kinetic energy (in 3 dimensions) is:

$$K_{3D} = N\left(\frac{3}{2}kT\right) = \frac{3N}{2}m\langle v_x^2\rangle = \frac{N}{2}m\langle v^2\rangle$$

$$\cancel{N}\frac{3}{\cancel{2}}kT = \frac{\cancel{N}}{\cancel{2}}m\langle v^2\rangle \Rightarrow T = \frac{m\langle v^2\rangle}{3k}$$

for 3 degrees of freedom (independent of N)

$$\langle v^2\rangle = \frac{3kT}{m}$$

Q: What is the rms speed of Nitrogen (N_2) molecules at room temperature?

$T = 293\,K$, molar mass $M = \frac{28\,g}{mol}$

m = mass of 1 molecule $= \frac{M}{N_A}$ $\qquad M = mN_A$

A: $$V_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3N_AkT}{mN_A}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3\left(8.314\frac{J}{\cancel{K\cdot mol}}\right)(293\,\cancel{K})}{0.028\,kg/\cancel{mol}}}$$

$$V_{rms} = 510\,\frac{m}{s}$$

Note: velocity depends on T + mass

velocity of particles in a gas:

$$\frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} kT$$

$$V_{rms} = \sqrt{\langle V^2 \rangle} = \sqrt{\frac{3kT}{m}}$$

The distribution of velocities is

$$\boxed{f(v) = \frac{4}{\sqrt{\pi}} \left(\frac{m}{2kT} \right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}}}$$

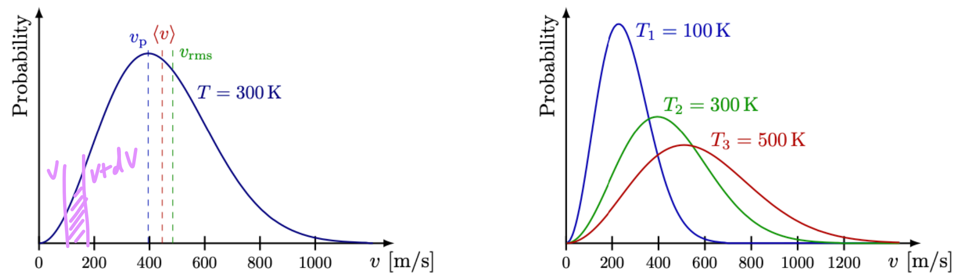
Maxwell-Boltzmann distribution for velocities.

m: mass of a molecule
V: velocity of a molecule
T: temperature

If we had N total molecules, how many in a range of velocities. $v \to v + dv$?

$dN = N f(v) dV$: dN is the number of molecules within a velocity range of $V \to V + dV$

Integrate: $\int dN = \int_{V_1}^{V_2} N f(v) dV$



(a) The distribution is asymmetric. **(b)** For different temperatures.

Figure 17.4: Maxwell-Boltzmann distribution for oxygen gas O_2 with atomic weight ~ 16 per atom.

$$N_{v_1 \to v_2} = \int_{v_1}^{v_2} N f(v)\, dv$$

$V_p = V_{max}$: speed for which $f(v)$ is maximum $= \sqrt{\frac{2kT}{m}}$

V_{av} : mean of $f(v)$

V_{rms} : $\sqrt{\langle v^2 \rangle}$

we could rewrite $f(v)$ as $f(E)$ where $E = \frac{1}{2}mv^2$ is kinetic energy

$$\boxed{f(E) = \frac{2}{\sqrt{\pi}} \left(\frac{1}{kT}\right)^{3/2} E^{1/2} e^{-E/kT}}$$

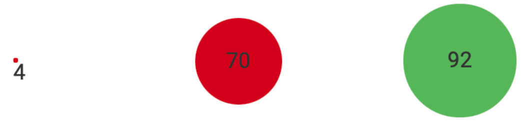
Maxwell-Boltzmann energy distribution.

$$dN = N f(E)\, dE$$

is the # of particles with an energy between E and $E + dE$

Quiz 3

The area under the force vs. time curve represents work.



The area under the velocity vs. time curve represents acceleration.

The dot product of a constant force and the 1-D distance the force pushes an object is the work.

Quiz 3

If the objects reaches a constant velocity (terminal velocity), gravity is still doing work on the object.

2 105 59

If the objects reaches a constant velocity (terminal velocity), there is no net work on the object.

5 77 84

Question

Which is true about inelastic collisions?

Total energy is conserved

2 89 75

Momentum of the whole system is conserved

2 100 64

Quiz 4

When torque is zero, angular momentum is zero.

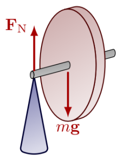
1 41 38

Question

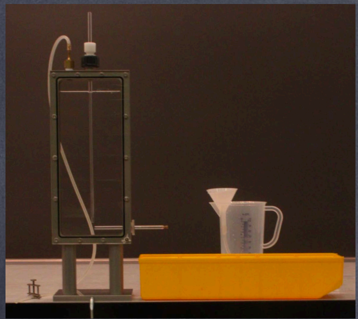
Which direction does a spinning object precess.

In the direction of the angular momentum of the spinning object.

7 31 42

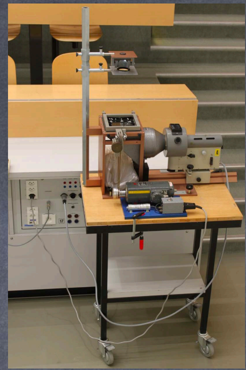


(a) The handle allows the disk to spin around its axis and around the pivot.



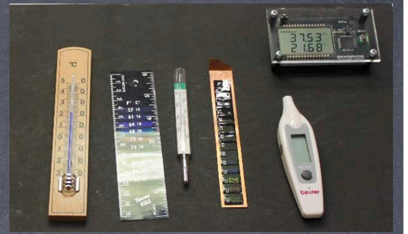
H21

Th57



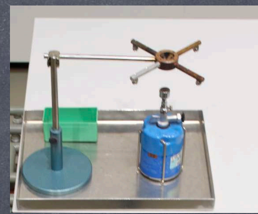
Th36

Th58

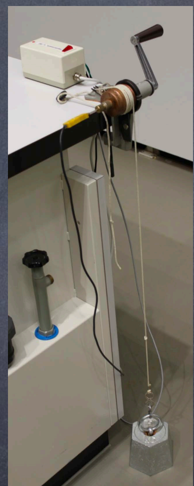


Th12

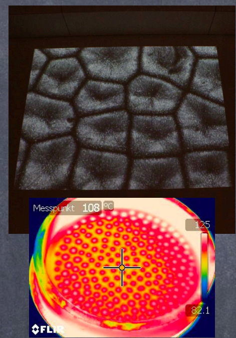
Th63



Th20



Th54



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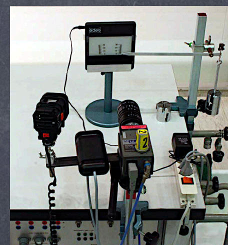
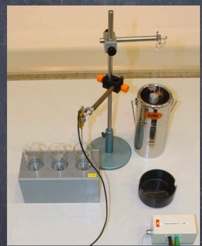


E12

Th19



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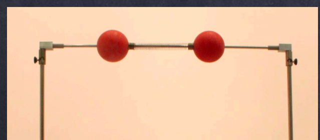
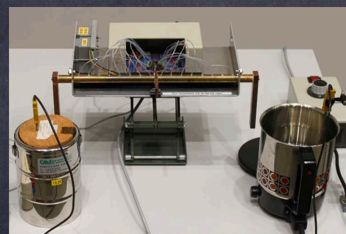
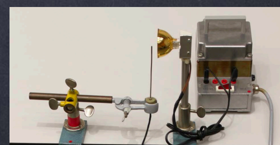


Th27

Th2



Th22



Th48