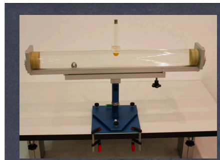
PHY 117 HS2024

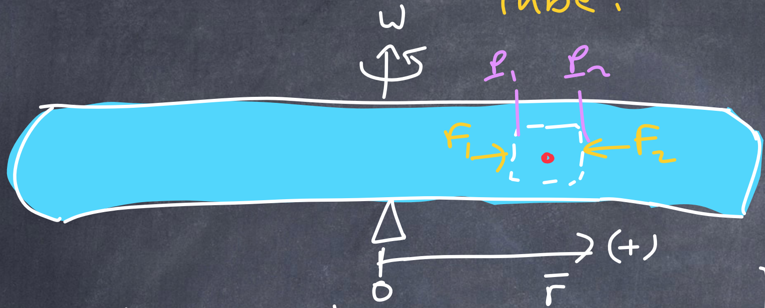
Today:

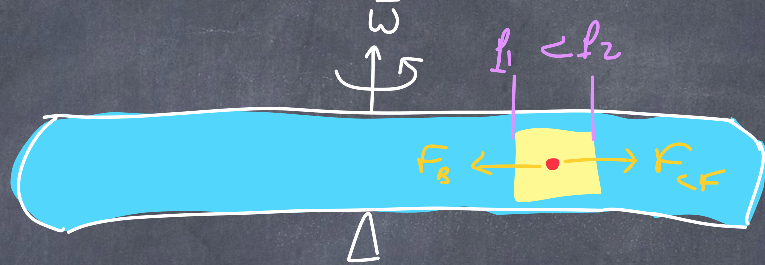
Review of centrifuge
“Pseudoforce”
Fluids in motion
Continuity equation
Bernoulli’s equation
Toricelli’s law
Venturi effect
Viscosity & resistance
Capillary action

Week 5, Lecture 2
Oct. 16th, 2024
Prof. Ben Kilminster



consider empty spinning tube:





The buoyant force comes from the centripetal force of the mass of fluid that is displaced

$$F_B = F_1 - F_2$$

$$F_B = m_e \frac{v^2}{r} = m_e \omega^2 r(-\hat{r})$$

∴ F_B is bigger at large r

An object in the centrifuge feels a buoyant force and an apparent force the "centrifugal pseudoforce" outward

$$F_{cF} = m_o \omega^2 r(\hat{r})$$

$$\Sigma F = F_c - F_B = m_o \omega^2 r - m_e \omega^2 r = V\rho_o \omega^2 r - V\rho_e \omega^2 r = V\omega^2 r(\rho_o - \rho_e)\hat{r}$$

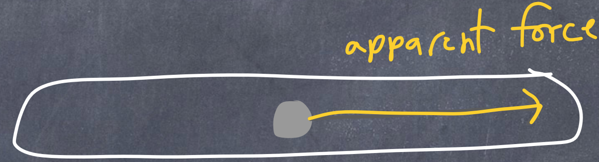
If $\rho_o > \rho_e$, the force is outward in $(+\hat{r})$ director

If $\rho_o < \rho_e$, force is inward $(-\hat{r})$

why did I call it a pseudoforce?

Because we are viewing the object in its reference frame, we see this apparent pseudoforce.

In the rest frame of the tube

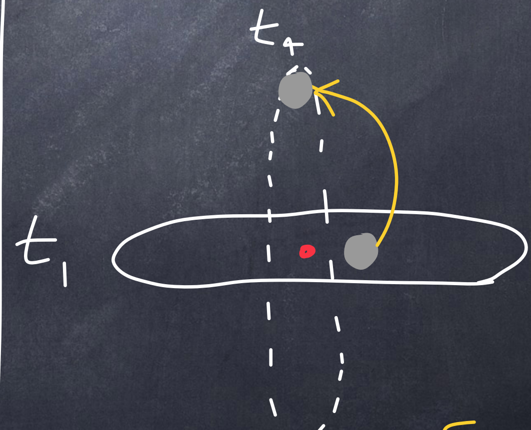


But from above (in a fixed frame), we see that:

Ball slides outward

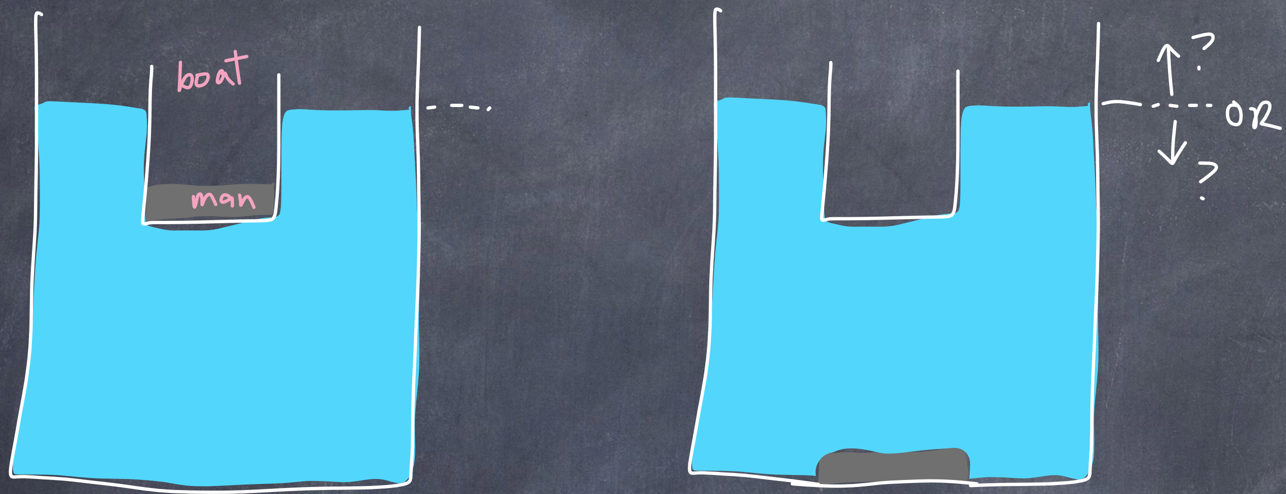
At t_4, ball now moves in a circle with wall providing $F_{centripetal}$

Movement is:



due to force of glass on ball

Man overboard!



Level goes down. Why?

On the left, the volume displaced is more than on the right. The displaced volume raises the fluid level on the left. When the boat is empty, it is less dense and displaces less fluid. So the level goes down.

Fluids. In. Motion!

pipe with fluid inside

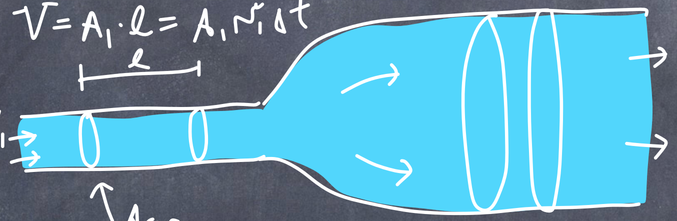
volume of water, $V_2 = A_2 v_2 \Delta t$

$l = v_1 \Delta t$

$V = A_1 \cdot l = A_1 v_1 \Delta t$

velocity of fluid, v_1 incoming

fluid flow v_1



v_2 : velocity outgoing

Area A_1

area, A_2

Consider some time Δt

Since fluids are incompressible, $V_1 = V_2$

$$A_1 v_1 \cancel{\Delta t} = A_2 v_2 \cancel{\Delta t}$$

$$A_1 v_1 = A_2 v_2 = \text{constant}$$

(tube area changes)

$$I_v \equiv vA \; : \; \left[m^2 \cdot \frac{m}{s}\right] \rightarrow \left[\frac{m^3}{s}\right]$$

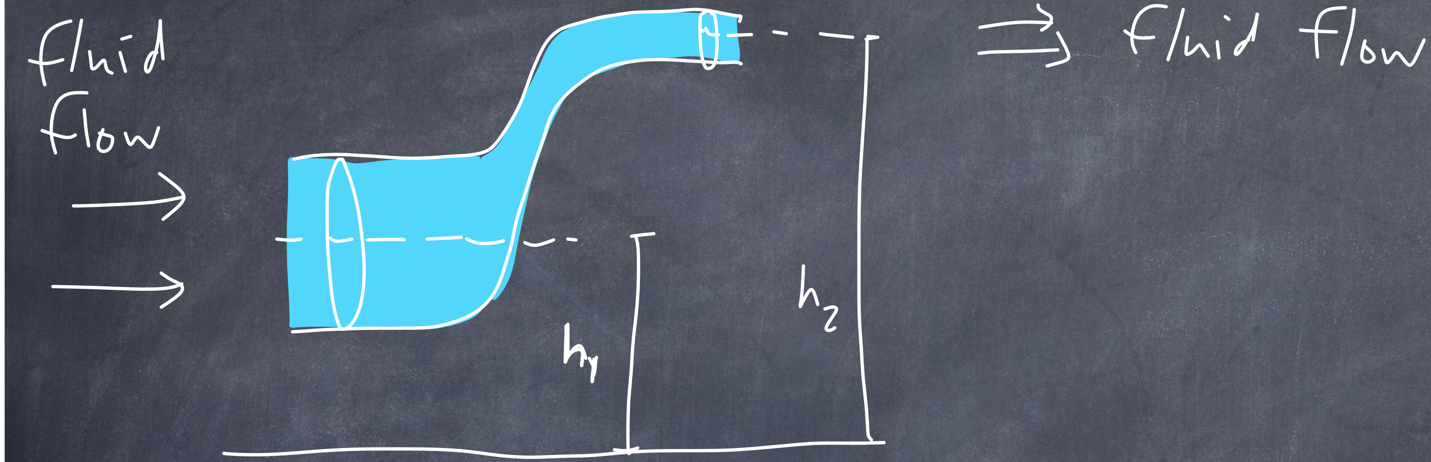
volume flow rate

$$\boxed{I_v = vA = \text{constant}}$$

continuity equation

If A gets bigger, then v gets smaller

What if it changes height?



Fluid gains potential energy, must lose kinetic energy.

In some time Δt, some amount of fluid, Δm mass, gets lifted by a height, h

change in potential energy $= \Delta U = \Delta m g h = \rho \Delta V g h$

ΔV: volume of fluid being lifted.

ΔK = change in kinetic energy $= \frac{1}{2}\Delta m v_2^2 - \frac{1}{2}\Delta m v_1^2$

$$= \frac{1}{2}\rho \Delta V (v_2^2 - v_1^2)$$

The work-energy theorem states that

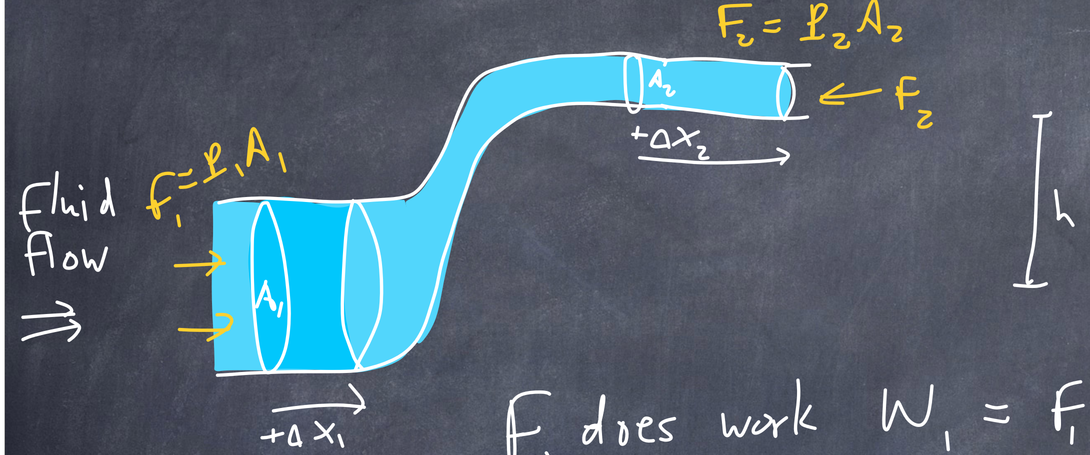
$$\boxed{W_{TOTAL} = \Delta U + \Delta K}$$

work done by fluid.

$$W_{TOTAL} = \rho \Delta V g h + \frac{1}{2}\rho \Delta V (v_2^2 - v_1^2) \quad (1)$$

we know that work comes from a force times a distance. The force, F_1, comes from the pressure P_1 at bottom and at the top, there is a force, F_2 from the pressure at the top P_2.

What if it changes height?



Fluid is pushed with F_1 by fluid pressure to the right and pushed back by the fluid pressure, at F_2

F_1 does work $W_1 = F_1 \Delta x_1 = P_1 \underbrace{A_1 \Delta x_1}_{\text{volume of cylinder}} = P_1 \Delta V$

F_2 does work $W_2 = -F_2 \Delta x_2 = P_2 \Delta V$

same volumes

The total work done is $W_{TOTAL} = W_1 + W_2$

$= P_1 \Delta V - P_2 \Delta V$

$$\boxed{W_{TOTAL} = (P_1 - P_2)\Delta V} \quad ②$$

we combine ① + ②

$$(P_1 - P_2)\cancel{\Delta V} = \rho \cancel{\Delta V} g h + \frac{1}{2}\rho \cancel{\Delta V}(v_2^2 - v_1^2)$$

$$P_1 - P_2 = \rho g (h_2 - h_1) + \frac{1}{2}\rho (v_2^2 - v_1^2)$$

we move our terms

$$\underbrace{P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2}_{\text{position 1}} = \underbrace{P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2}_{\text{position 2}}$$

In other words,

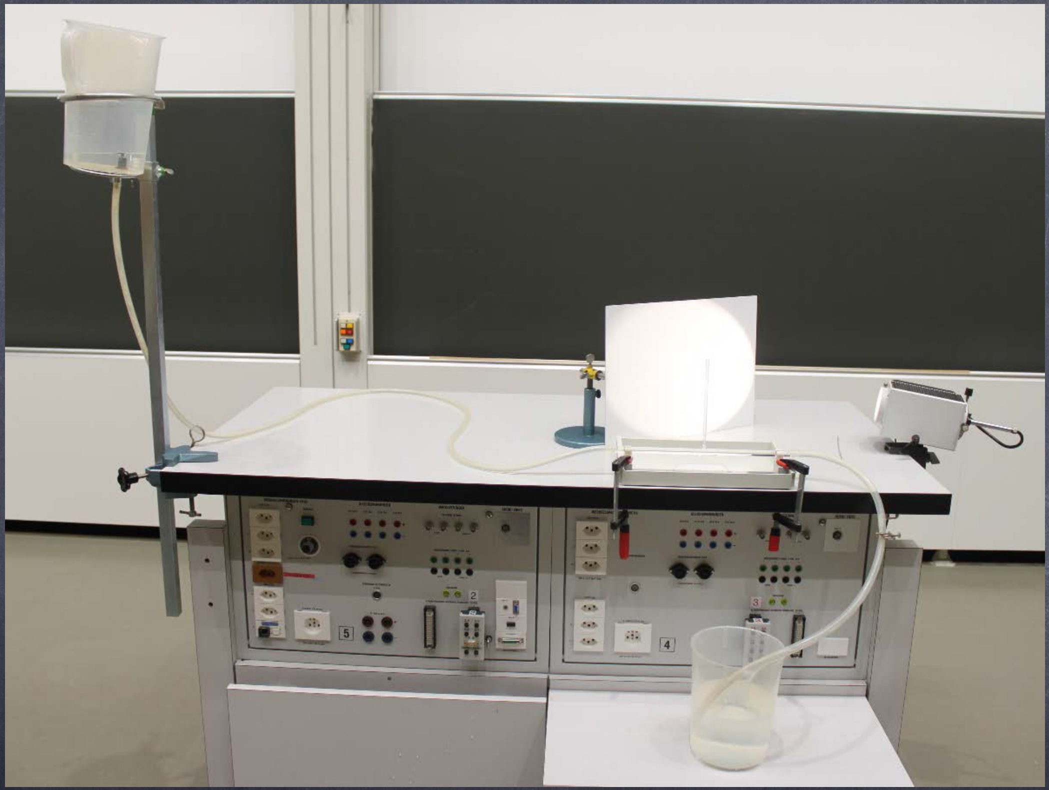
$$\boxed{P + \rho g h + \frac{1}{2}\rho v^2 = \text{constant}}$$

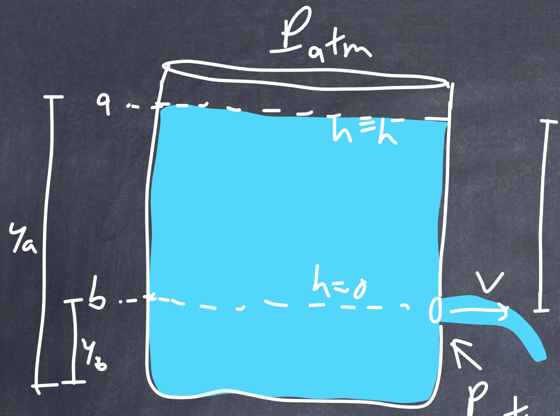
Bernoulli's equation

- every term has units of pressure.
- neglects friction

$\longrightarrow$ This combination of quantities stays constant while height, area, velocity, U, K, P changes.

This is a statement of energy conservation.

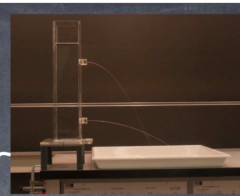




$P_a = P_b = P_{atm}$

Approximation: at the top, $v_a = 0$, valid if the top area is much larger than the hole at point b.

Tank with hole in it, open at top.



What is the velocity of water?

we apply Bernoulli's equation to solve this at heights a & b.

Level a | Level b

$$P_a + \rho g h + \tfrac{1}{2}\rho v^2 = P_b + \rho g h + \tfrac{1}{2}\rho v^2$$

$$\cancel{P_{atm}} + \rho g h + \tfrac{1}{2}\rho \cancel{v_a^2}^{\,0} = \cancel{P_{atm}} + \cancel{\rho g}\cdot 0 + \tfrac{1}{2}\rho v_b^2$$

$$\cancel{\rho} g h = \tfrac{1}{2}\cancel{\rho} v_b^2$$

v_b of water coming out of hole.

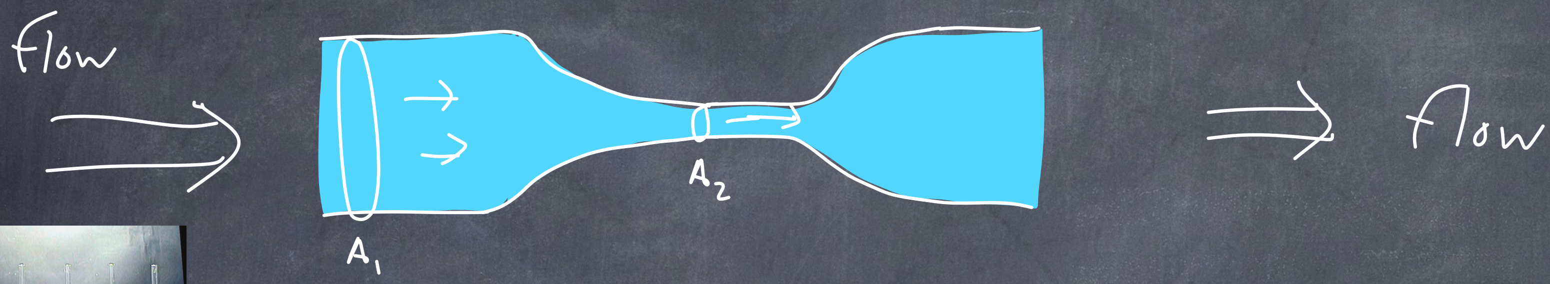
$$\boxed{v = \sqrt{2gh}}$$

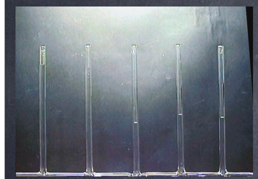
Torricelli's Law

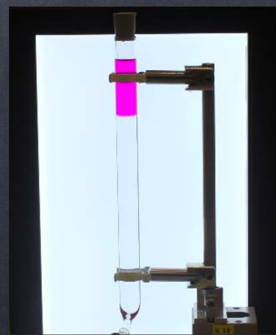
Imagine an object falling a distance x due to gravity, $v^2 = v_0^2 + 2ax$. If $a = g$, $v_0 = 0$, $x = h$

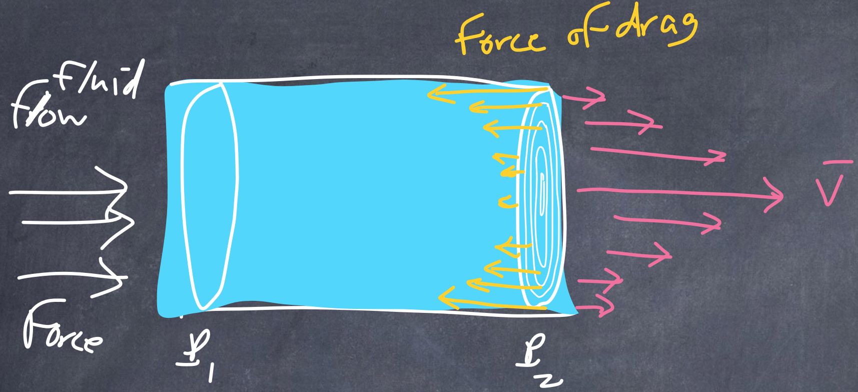
$$v^2 = 2gh \leftarrow v = \sqrt{2gh}$$

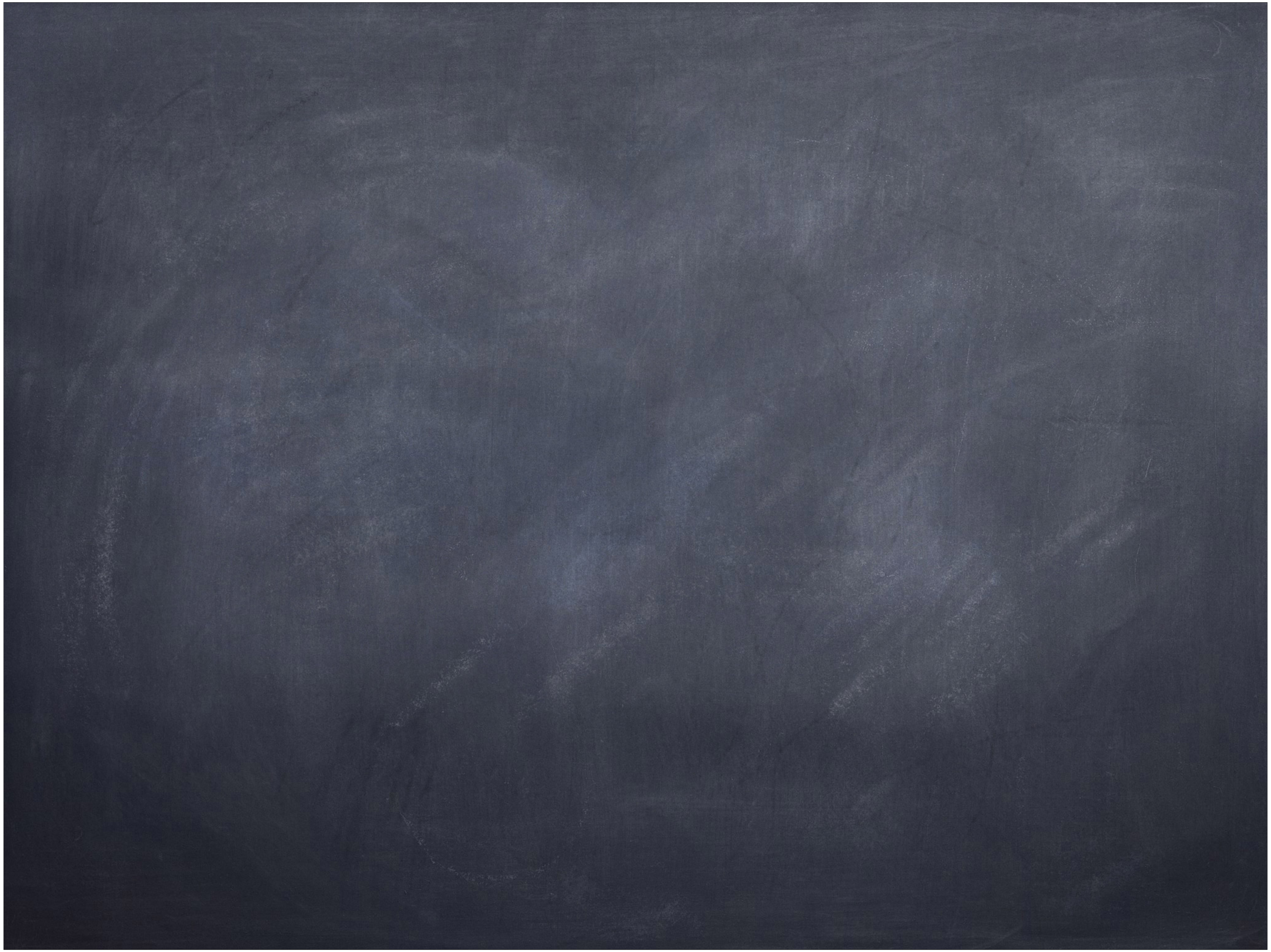
Fluid moving in a pipe with changing area

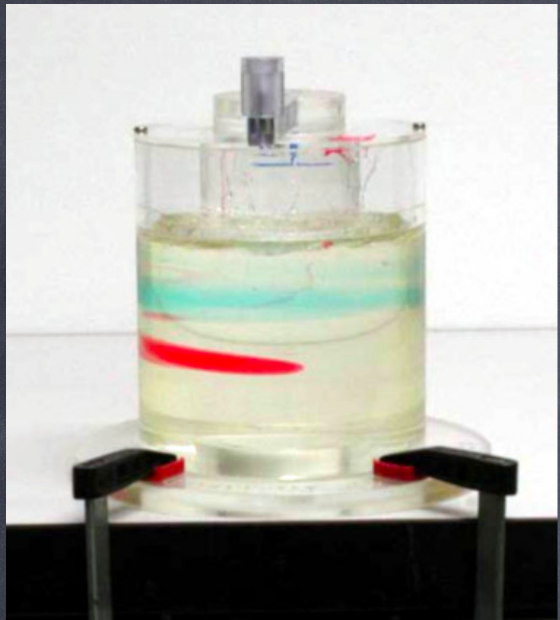


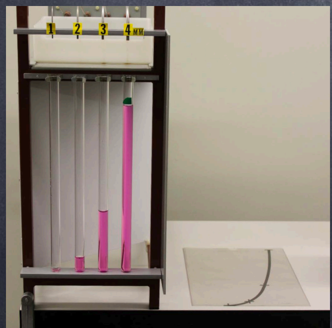




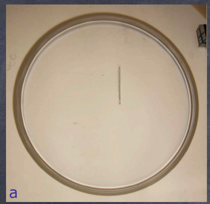

Force of drag
fluid flow
Force
P_1
P_2
$\bar{V}$

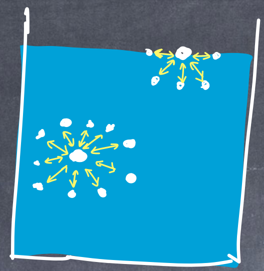


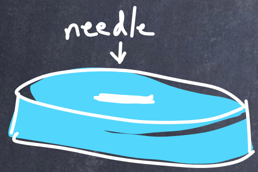




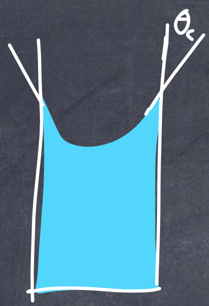
surface tension

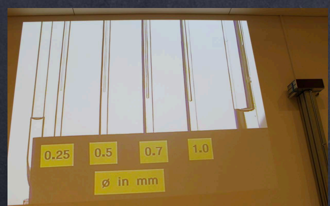


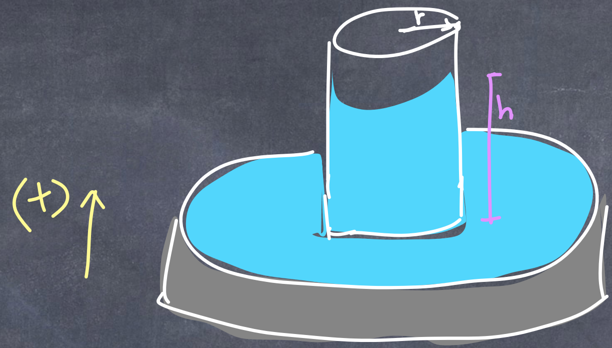


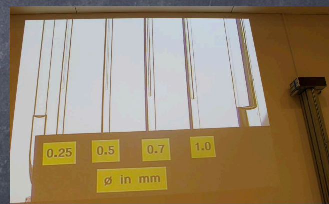


side view


θ_c


0.25
0.5
0.7
1.0
ø in mm


(+)
r
h


0.25
0.5
0.7
1.0
ø in mm

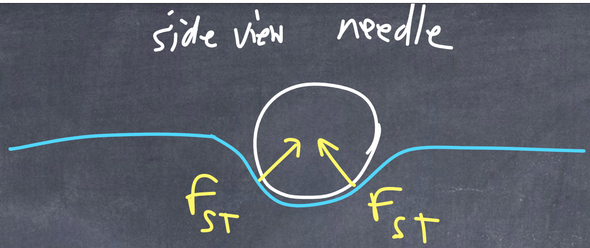
end

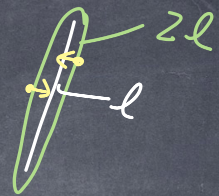
After this, there are a few derivations for your information.

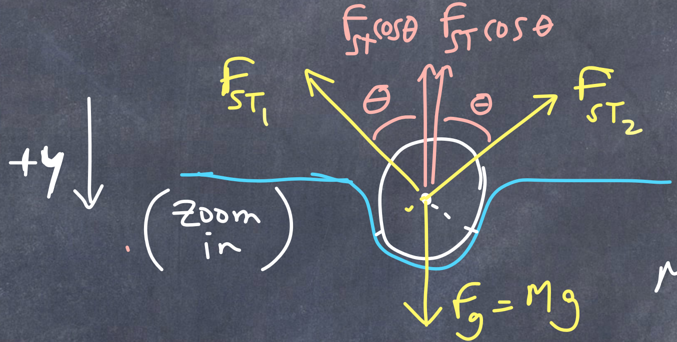
There is a surface force on both sides of the needle, so

$$F_{ST} = \gamma L = \gamma 2l$$

l, length of the needle







$\longleftrightarrow$ in horizontal direction, these forces F_{ST} cancel out.

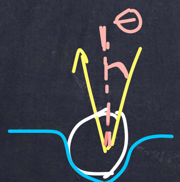
M: total mass of needle

In the y-direction, the total surface tension is

$$F_{ST_y} = \overbrace{F_{ST_1}\cos\theta}^{(\gamma l\cos\theta)} + \overbrace{F_{ST_2}\cos\theta}^{(\gamma l\cos\theta)} = 2F_{ST}\cos\theta = 2\gamma l\cos\theta$$

The needle floats as long as $F_{ST_y} > F_g$

M small



M big

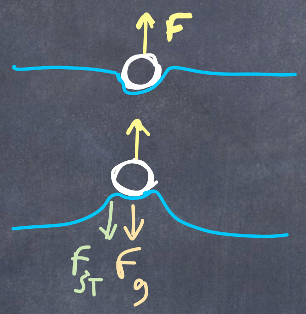
As M gets larger, θ decreases.

When $\theta = 0^\circ$, $\cos\theta = 1 \Rightarrow F_{ST} = 2\gamma l$

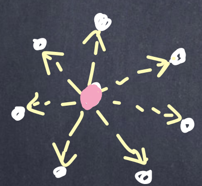
The maximum mass allowed is when

$Mg = 2\gamma l\cos 0^\circ$ $\quad m_{max} = 2\gamma l/g$

The force to lift the needle off the surface is $F = mg + \gamma 2L$



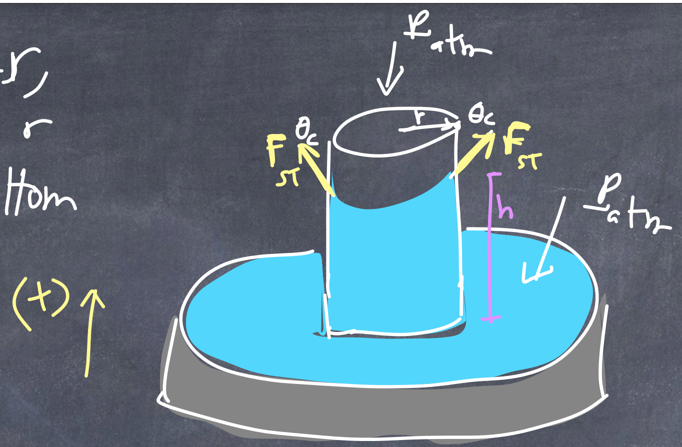
In this case, the surface tension resists us pulling the needle up, because we are stretching the fluid membrane upward.



cohesive force on one molecule is coming from the surrounding molecules.

Consider cylinder, radius r

open on top & bottom



Adhesive force pulls fluid upward

F_{ST} : adhesive force pulling upward

vertical direction $\Big\{$ $\Sigma F = F_{ST}\cos\theta_c - mg = 0$

$$\gamma L \cos\theta_c = mg$$

$$\gamma\, 2\pi r \cos\theta_c = \rho V g$$

$$\gamma\, 2\cancel{\pi}\cancel{r} \cos\theta_c = \rho(\cancel{\pi} r^{\cancel{2}} h) g$$

what is L? It's the length of contact between fluid & container

$L = 2\pi r$

$h = \dfrac{2\gamma\cos\theta_c}{\rho r g}$: The height that the adhesive force raises a fluid in a container

when $\theta_c < 90°$, $\cos\theta_c > 0 \Rightarrow h$ is (+)

when $\theta_c > 90°$, $\cos\theta_c < 0 \Rightarrow h$ is (−)