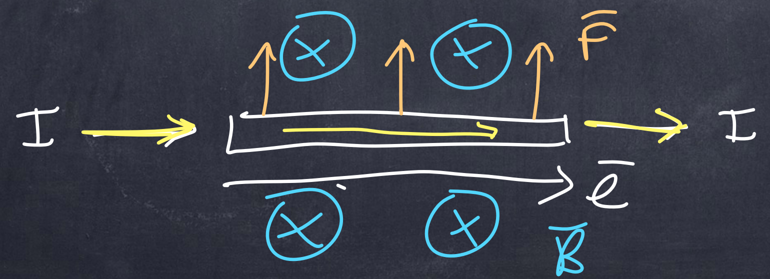
PHY117 HS2024

Week 10, Lecture 2
Nov. 20th, 2024
Prof. Ben Kilminster

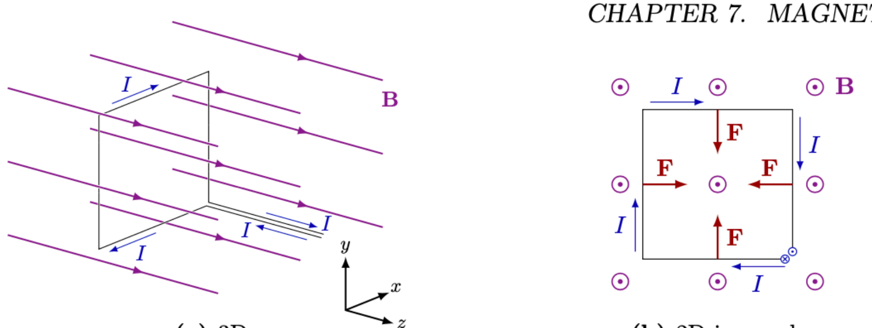
yesterday:



$F = BI\ell$

↑ length of wire

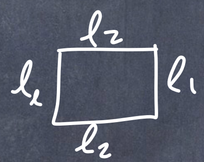
what about a loop of current?



(a) 3D. (b) 2D in xy plane.

Figure 7.9: Rectangular current loop in an external, uniform magnetic field $\mathbf{B} = B\hat{\mathbf{z}}$.

No net force, no net torque



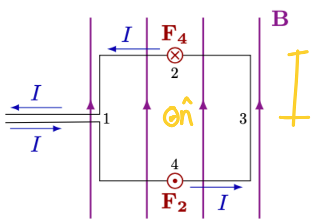


Figure 7.10: Rectangular current loop in an external magnetic field $\mathbf{B}$.

Here, there is torque $\bar{\tau} = \bar{r} \times \bar{F}$

Segments 1+3 are parallel to $\bar{B}$, so no force, no torque.

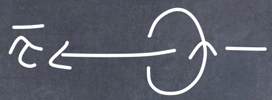
Segment 2: $F_2 = BI\ell_2$, $\tau_2 = \frac{\ell_1}{2} BI\ell_2 \hat{x}$

Segment 4: $F_4 = BI\ell_2$, $\tau_4 = \frac{\ell_1}{2} BI\ell_2 \hat{x}$

The loop will twist from the torque (Notice $\hat{n}$ of loop $\perp$ $\bar{B}$)

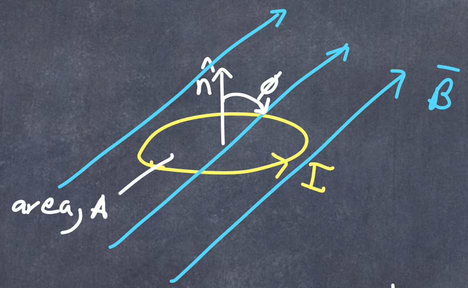
Total torque $= \tau_2 + \tau_4 = \frac{l_1}{2} B I l_2 + \frac{l_1}{2} B I l_2 = I(l_1 l_2) B = IAB\hat{x}$ (A: area)

$\hat{x} \leftarrow$



If loop $\hat{n}$ is at an angle with respect to $\bar{B}$, then in general

$$\bar{\tau} = IA\hat{n} \times \bar{B} = IAB\sin\phi$$



ϕ: is the angle from $\hat{n}$ to $\bar{B}$

$\hat{n}$: normal direction $\perp$ to the plane of the loop.

Torque is always in the direction that aligns $\hat{n}$ & $\bar{B}$

We can define the magnetic moment of the loop as $\mu = IA$

and the $\bar{\mu}$ vector

$$\bar{\mu} = (IA)\hat{n}$$

then

$$\bar{\tau} = \bar{\mu} \times \bar{B}$$

7.3. *CURRENTS IN MAGNETIC FIELDS* 81

(a) Magnetic moment of a current loop in a uniform magnetic field.

(b) Right-hand rule for the magnetic moment of a current loop.

Figure 7.11: Magnetic moment.

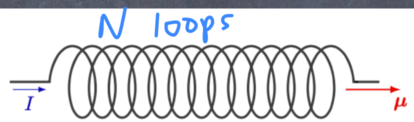


Figure 7.12: Magnetic moment of a solenoid with N windings.

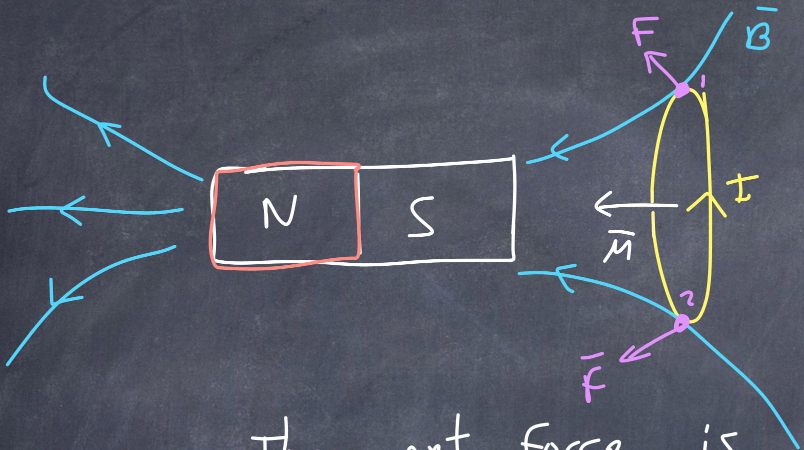
$$\bar{\mu} = N(IA)\,\hat{n}$$

The potential energy of a current loop in $\bar{B}$-field is

$$\boxed{U = -\bar{\mu}\cdot\bar{B}} + \text{constant}$$

we set the constant so that when $\bar{\mu}$ is $\parallel$ to $\bar{B}$, then $U = 0$.

What if the magnetic field is non-uniform?

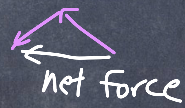


Consider 2 opposite points

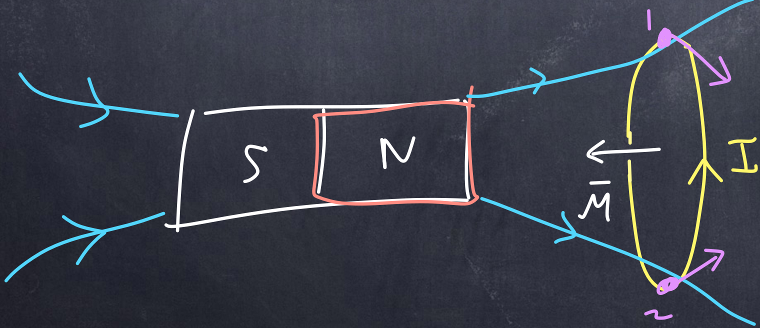
1: $\bar{F}_1 = q\bar{v} \times \bar{B}$ $\quad \left.\begin{array}{l}\bar{V} = \otimes \\ \bar{B} = \swarrow\end{array}\right\} \bar{F}: \nwarrow$

2: $\bar{F}_2$: $\quad \left.\begin{array}{l}\bar{V} = \odot \\ \bar{B} = \nwarrow\end{array}\right\} \bar{F}: \swarrow$

The net force is towards the magnet.

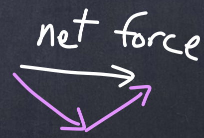


(vertical components cancel out)



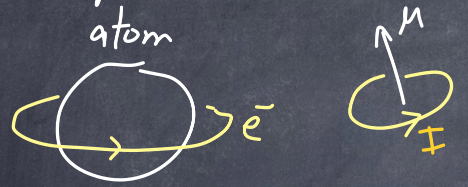
1: $\bar{F}$: $\quad \left.\begin{array}{l}\bar{V} = \otimes \\ B = \nearrow\end{array}\right\} \bar{F} = \searrow$

2: $\bar{F}$: $\quad \left.\begin{array}{l}\bar{V} = \odot \\ \bar{B} = \searrow\end{array}\right\} \bar{F} = \nearrow$



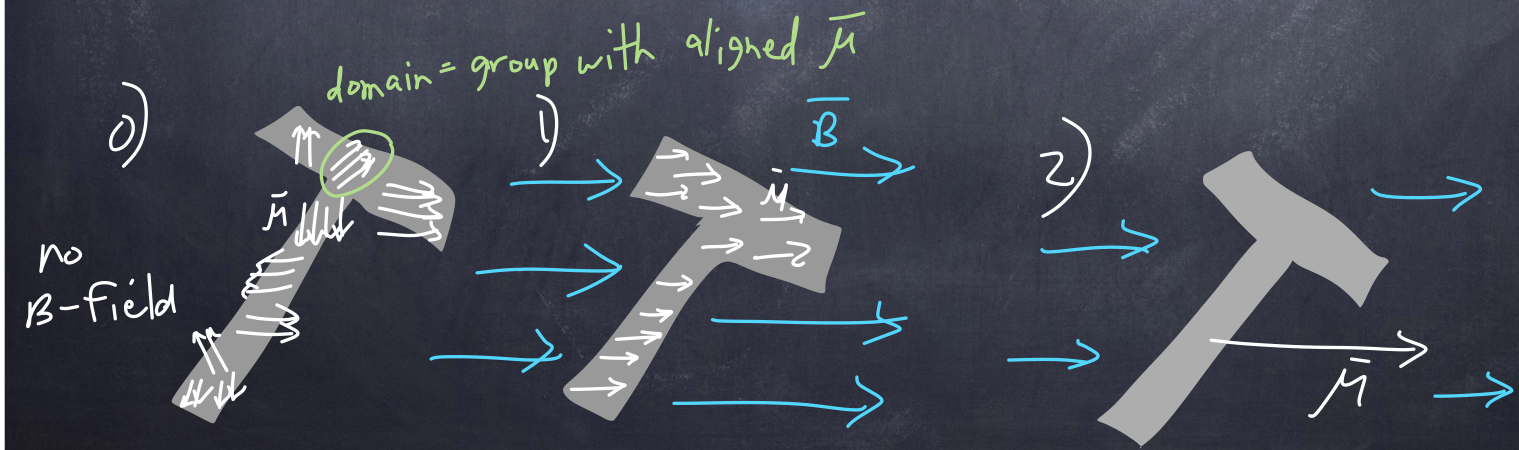
The net force is away from the magnet

Electrons and atoms can be thought of as spinning electric charges.



Each atom has a magnetic moment, $\bar{\mu}$.

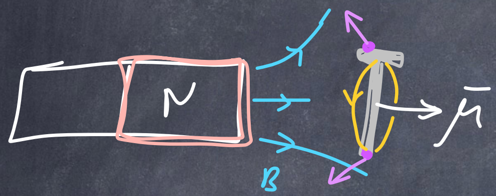
This helps us understand why an unmagnetized nail is attracted to a magnet, both the N + S side.
This happens in a few steps:



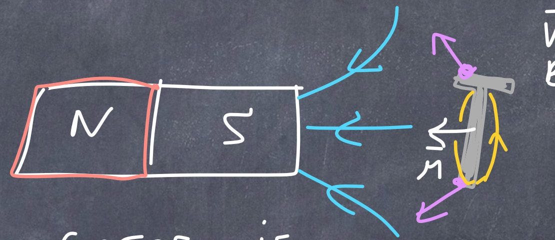
Why is nail attracted to N + S poles of magnet?

First, nail is magnetized in direction of field.

Second, divergent field causes a force of attraction



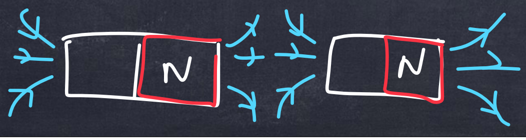
$\bar{v} = 0$, $\bar{B} = \nearrow$ } $\bar{F} = \nwarrow$



The net force in both cases is toward the magnet.

No force [In a constant magnetic field

This is also why two magnets attract

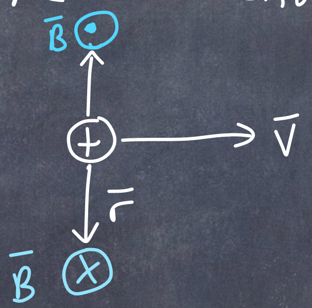


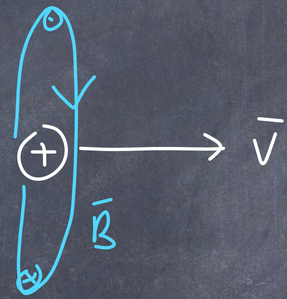
So far, we know: $\bar{F}_B = q\bar{v}\times\bar{B}$ $\quad \bar{F} \perp \bar{v}, \bar{B}$

$\bar{F}_B = I\bar{\ell}\times\bar{B}$ $\quad \bar{F} \perp \bar{\ell}, \bar{B}$

Now: A moving charge ⊕→ $\bar{v}$ generates its own magnetic field.

The direction of $\bar{B}$ is $\bar{v}\times\bar{r}$





The magnetic field loops around the direction of motion.

The magnitude of B decreases like $\frac{1}{r^2}$

$$\bar{B} = \frac{\mu_0}{4\pi} \frac{q\bar{v} \times \hat{r}}{r^2}$$

$[T]$ $\quad \left[\frac{T \cdot m}{A}\right]$ $\quad \frac{[C]\left[\frac{m}{s}\right]}{[m^2]}$

$\bar{B}$ caused by a moving charge.

μ_0: permeability of free space

↑ vacuum

$\mu_0 = 4\pi \in -7 \; \frac{T \cdot m}{A}$

$[A] = \left[\frac{C}{s}\right]$

For a current, the form is

$$d\bar{B} = \frac{\mu_0}{4\pi} \frac{I d\bar{l} \times \hat{r}}{r^2}$$

Biot - Savart law:

Integrate to solve for any shaped wire in a B-field.

we won't do any exercises with Biot - Savart law.

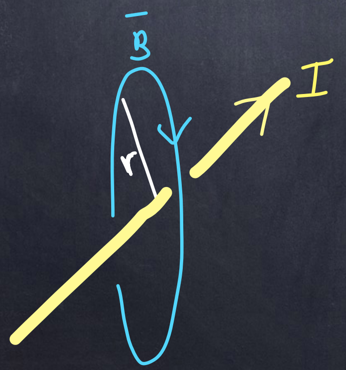
$$\bar{B} = \int_l \frac{\mu_0}{4\pi} \frac{I d\bar{l} \times \hat{r}}{r^2}$$

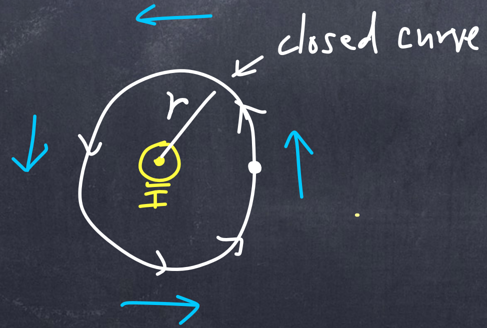
However, for simple configurations of current, there is an easier way. (like Gauss' Law)

Ampere's Law

$$\oint_{\text{closed curve}} \bar{B} \cdot d\bar{\ell} = \mu_0 I_c$$

I_c : current passing through the closed curve.





we pick a curve where $\bar{B} \parallel \bar{\ell}$

$$\oint_c \vec{B}\cdot d\vec{l} = \mu_0 I_c$$

$$\downarrow$$

$$B\oint_c dl = \mu_0 I$$

$$\oint_c dl = 2\pi r \text{ , the circumference of a circle}$$

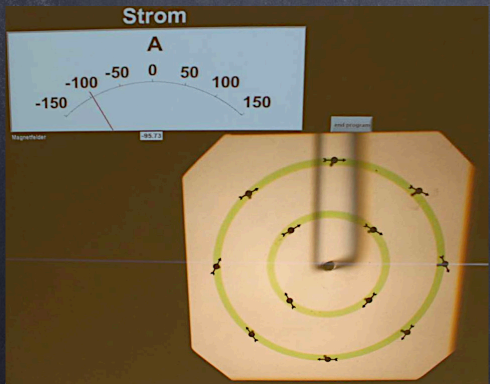
$$\downarrow$$

$$B(2\pi r) = \mu_0 I$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

we see here:

$$B \propto \frac{1}{r} \qquad B \propto I$$



Using Ampere's law on a solenoid:

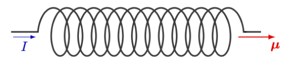


Figure 7.12: Magnetic moment of a solenoid with N windings.

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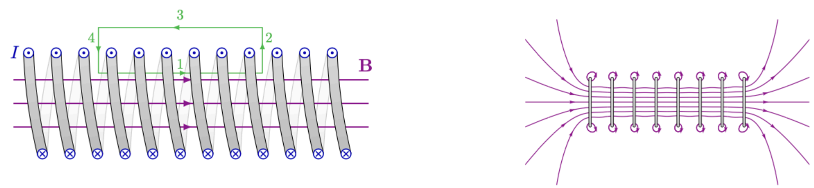


(a) Using Ampère's law on a rectangular loop. **(b)** Realistic field of a solenoid.

Figure 8.6: Magnetic field due to a solenoid.

sides 1 & 3 have length, x

$$n = \frac{N \text{ loops}}{\text{length}}$$

$$I_c = (nx) I$$

Then $\oint_c \bar{B} \cdot d\bar{\ell} = \mu_0 I_c$

$$\underbrace{Bx}_{1} + \underbrace{0}_{2} + \underbrace{0}_{3} + \underbrace{0}_{4} = \mu_0 n x I$$

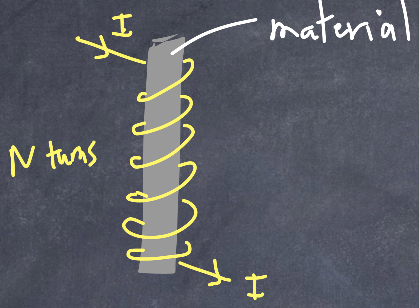
1: $\bar{B} \parallel d\bar{\ell}$; 2: $\bar{B} \perp d\bar{\ell}$; 3: $\bar{B} \cong 0$; 4: $\bar{B} \perp d\bar{\ell}$

$$B\cancel{x} = \mu_0 n \cancel{x} I$$

$$\boxed{B = \mu_0 n I = \mu_0 \frac{N}{\ell} I}$$

magnetic field in a hollow Solenoid.

If there is a material inside,



$$B = \mu n I \quad \text{where } \mu = \mu_0 K$$

Here, K is the relative permeability

material	K $\left(\frac{\mu}{\mu_0}\right)$
air	1.000 000 37
water	0.99999 2
copper	0.999 994
pure iron (99.95%)	200 000
iron 99.8%	5000

A current I_1 produce a magnetic field $B_1 = \frac{\mu_0 I_1}{2\pi r}$

Another current I_2 will feel a force from B_1: $F = B_1 I_2 \ell$

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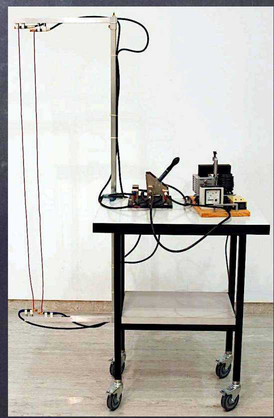
(a) Parallel current. **(b)** Anti-parallel current.

Figure 8.7: Magnetic force between current-carrying wires.

$$F_{12} = \frac{\mu_0 I_1 I_2 \ell}{2\pi r}$$

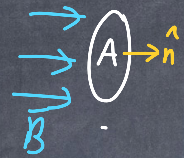
also $\vec{F}_{12} = -\vec{F}_{21}$



attractive or replsive!

Magnetic flux :

For a loop ⊥ to $\bar{B}$-field



we can quantify the $\bar{B}$-field by

$$\boxed{\phi_m = BA}$$

A : area

where ϕ_m is known as the magnetic flux

If $\hat{n}$ is not ∥ to $\bar{B}$, then

$$\boxed{\phi_m = \bar{B} \cdot \hat{n} A = BA\cos\theta}$$

units are Weber :

$$1\,[Wb] = 1\,[T \cdot m^2]$$

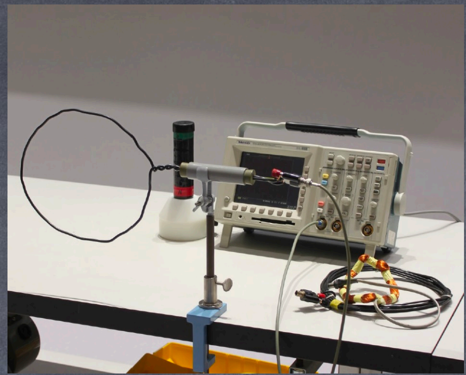
If magnetic flux changes, an electric field will be produced. The electric field produces an Emf

$$\boxed{\mathcal{E} = \oint_C \bar{E} \cdot d\bar{l} = -\frac{d\phi_m}{dt}}$$

↑ voltage supplied

Known as Faraday's Law.

Notice $[V] = \left[\frac{Wb}{s}\right]$



$\phi_m = \bar{B} \cdot \hat{n} A$

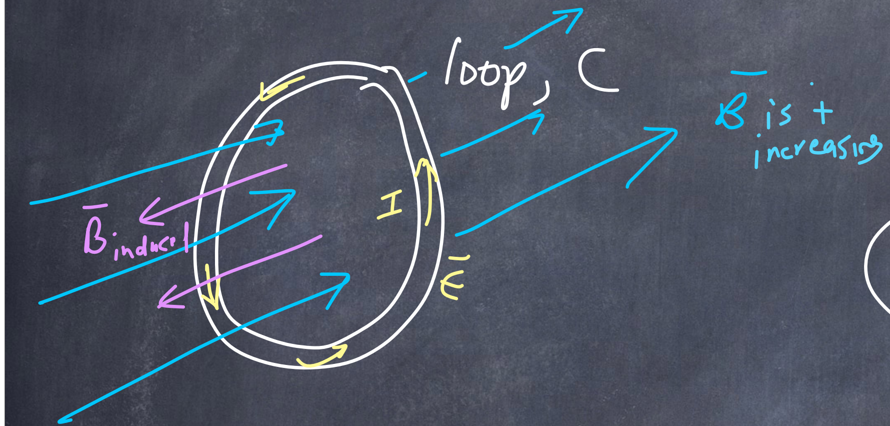
can be changed by changing B or A or $\hat{n}$!

If magnetic flux changes, an electric field will be produced. The electric field produces an Emf. This electric field means that a current is produced. But a current produces a magnetic field! what?

Lenz's Law: "The induced Emf and induced current are in such a diretion so as to oppose the change that produces them."

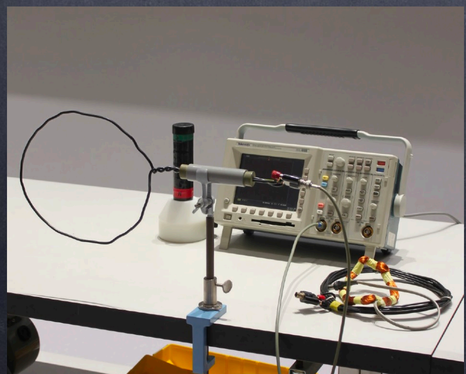
This means:

1) A moving magnet induces magnets in the opposite direction.

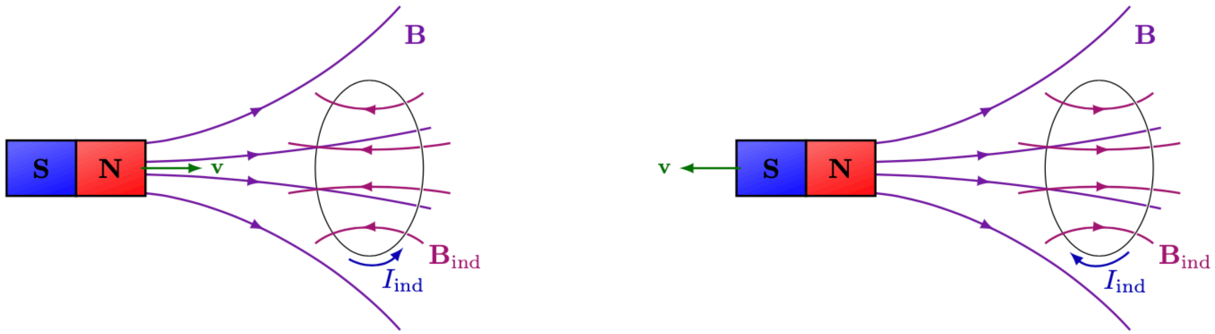


If we increase $\bar{B}$, I is produced.

(But in opposite direction)



$\bar{B}_{induced}$ opposes changes in $\bar{B}$



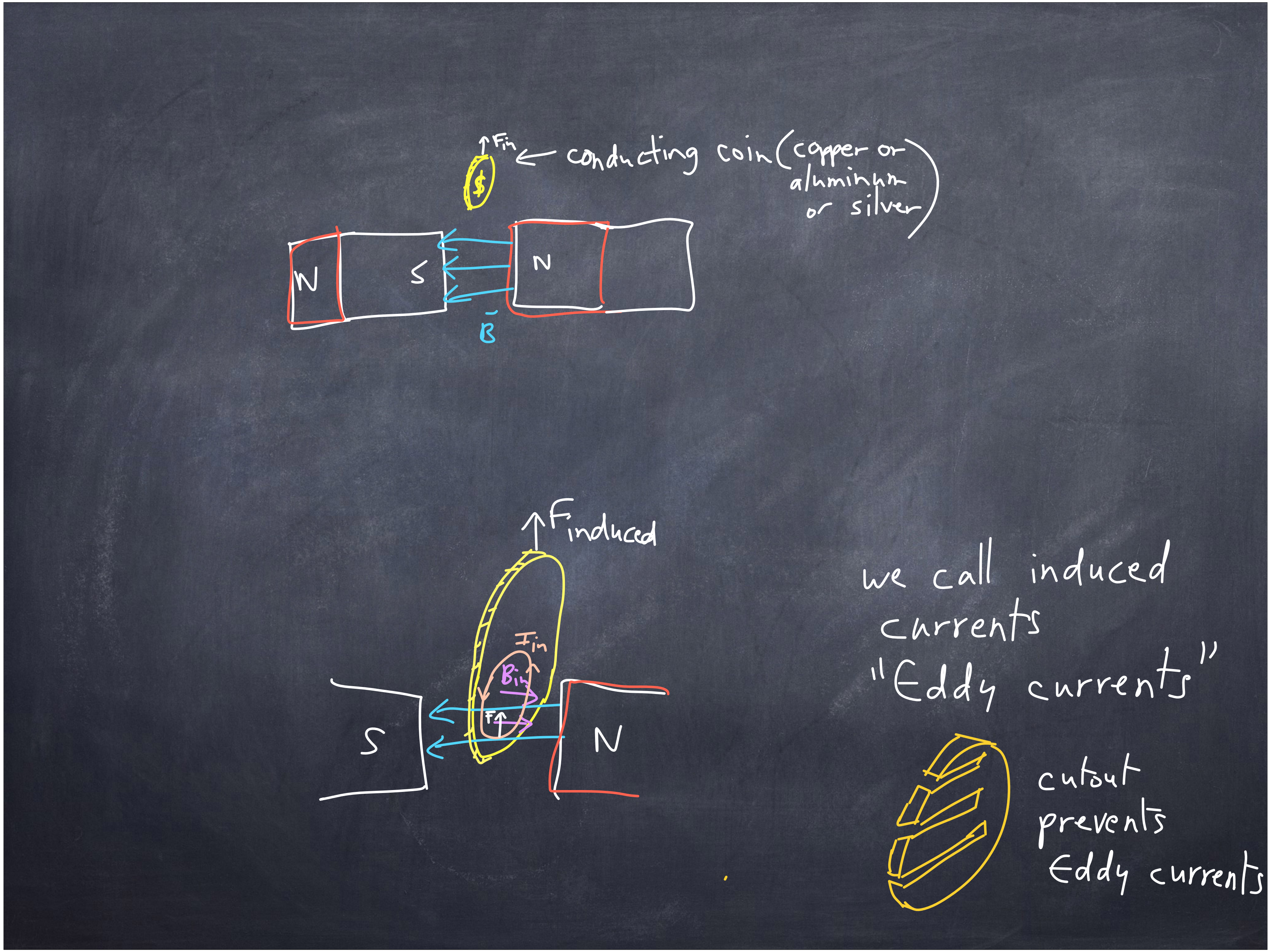
(a) Field moving toward the loop. **(b)** Field moving away from the loop.

Figure 8.8: The magnetic field **B** of a moving bar magnet will induce a current I_{ind} in a conducting loop and therefore a magnetic field $\mathbf{B}_{ind}$.

If $\bar{B}$ increases, $\bar{B}_{ind}$ is opposite $\bar{B}$

If B decreases, $\bar{B}_{ind}$ is in the same direction as $\bar{B}$

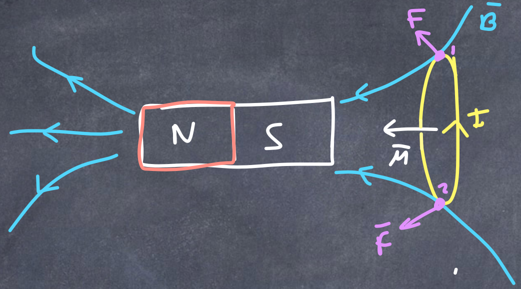
Dropping a conductor through a magnet.



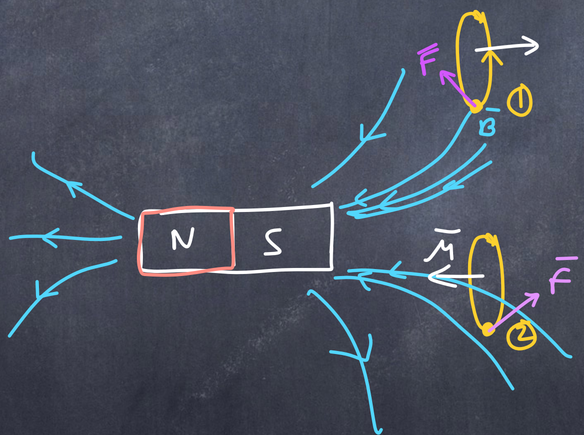
F_{in}
conducting coin (copper or aluminum or silver)
N
S
N
B
$F_{induced}$
I_{in}
B_{in}
F
S
N
we call induced currents "Eddy currents"
cutout prevents Eddy currents

What if the magnetic field is non-uniform?

Remember:



In this case, the loop falls through the non-uniform field.



As it falls, the B-field gets bigger, so the induced current opposes the external B-field

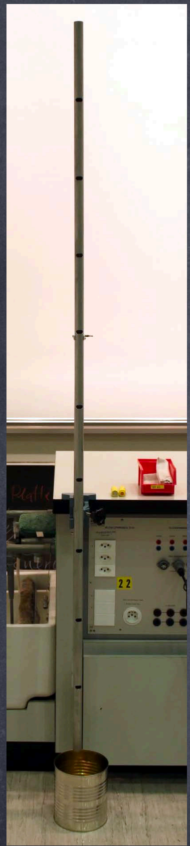
• ①: $\bar{V}$: ⊗ $\bar{B}$: ↙ $\bar{F}$ = ↖

Below, the induced magnetic field is in the same direction, resulting in an upward force

• ②: $\bar{V}$ = ⊙ $\bar{B}$ = ↖ : $\bar{F}$ = ↗

Force has up component in both cases

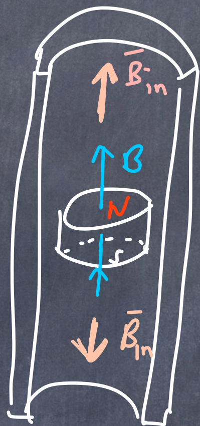
(keep in mind there are two magnets so horizontal force cancels out)

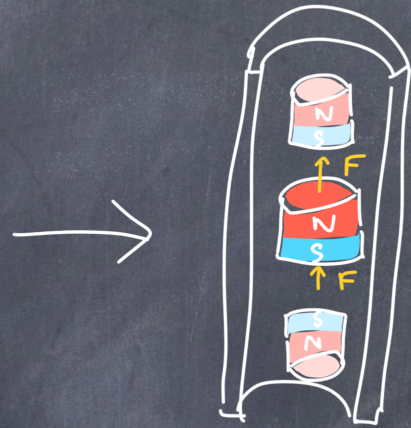


As magnet falls:

$\bar{B}$ decreases above magnet

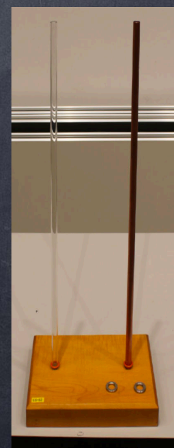
$\bar{B}_{in}$ is same direction as $\bar{B}$

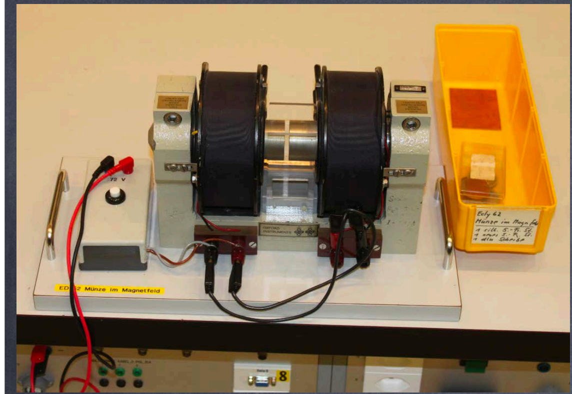




$\bar{B}$ increases below the magnet

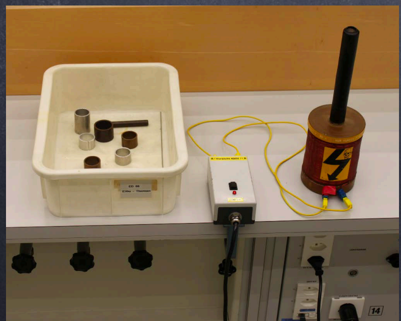
$\bar{B}_{induced}$ opposite of $\bar{B}$





Dropping a conductor
in a magnet

cutout
prevents
Eddy currents



turning a conductor into
an opposing magnet

dropping magnet in
a conductor

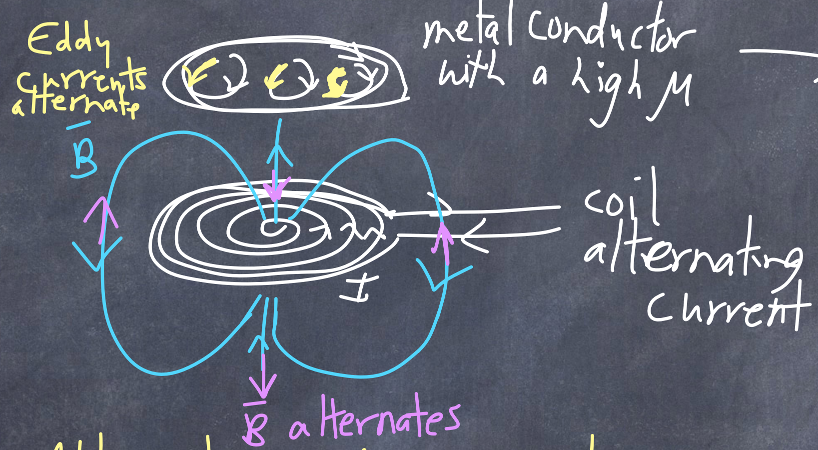
Summary of magnetic field concepts:

1) A moving electric charge may feel a force from a magnetic field.

2) A moving electric charge generates its own magnetic field. (A changing electric field produces a magnetic field.)

3) A changing magnetic field generates electric currents that produce an opposing magnetic field.

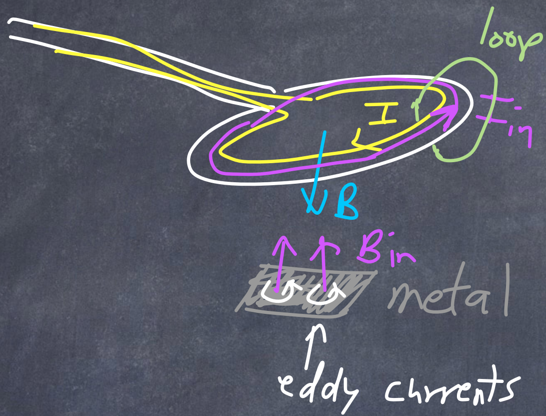
Induction stove uses Eddy currents:



→ movement of I generates heat.

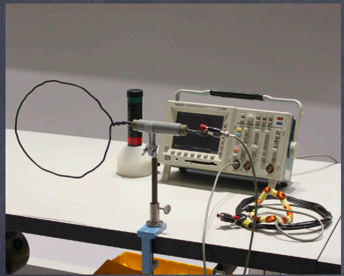
Alternating eddy currents generate heat in a conductor (Joule heating)

Metal detector uses Eddy currents



I_{in} is generated in opposite direction, tends to decrease current in metal detector.

Metal detector searches for currents in opposite directions.



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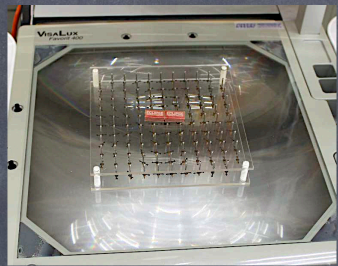

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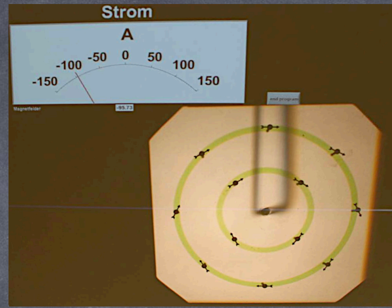

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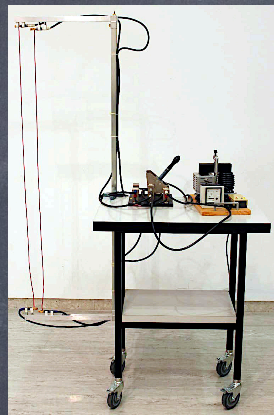
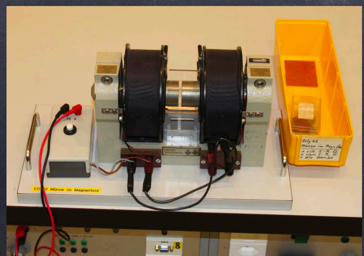
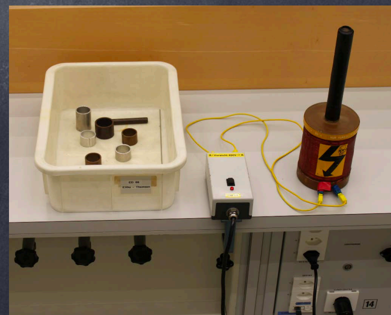


ED14

ED62



ED61



ED66