PHY117 HS2024

Today:
- standing sound waves
- spherical wave propagation
- interference patterns
- formula for interference

Week 12, Lecture 2
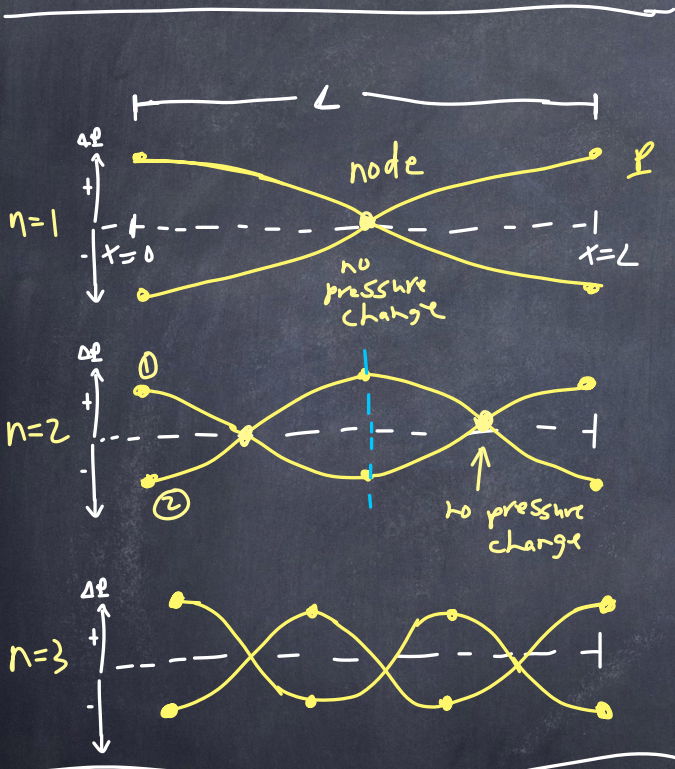Dec. 4th, 2024
Prof. Ben Kilminster

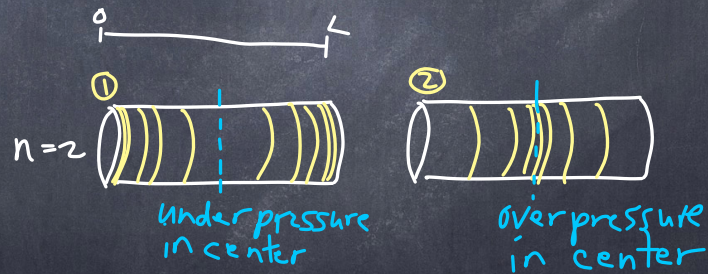# Standing sound waves in a closed tube of length L

anti-node

anti-node

The pressure is an anti-node at both ends.

$n=1$ $L = \frac{1}{2}\lambda_1$ $f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$

$n=2$ $L = \lambda_2$ $f_2 = \frac{v}{\lambda_2} = \frac{v}{L}$

$n=3$ $L = \frac{3}{2}\lambda_3$ $f_3 = \frac{v}{\lambda_3} = \frac{3}{2}\frac{v}{L}$

L

$\Delta\ell$

node

ℓ

$n=1$

$x=0$

$x=L$

no pressure change

$\Delta\ell$

①

$n=2$

②

no pressure change

$\Delta\ell$

$n=3$

← How do we understand these plots?

For $n=2$

$n=2$

①

②

underpressure in center

overpressure in center
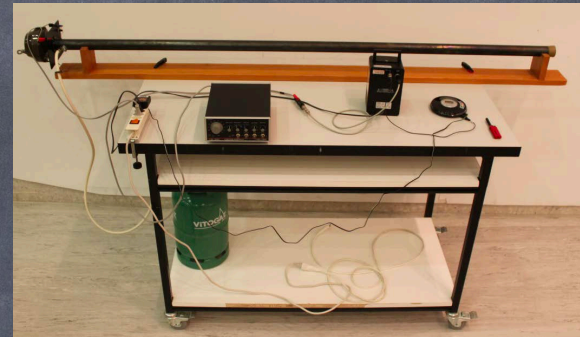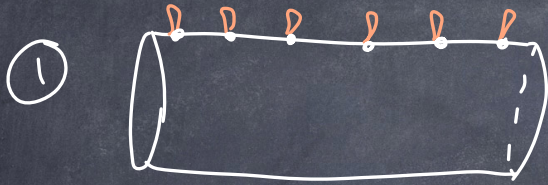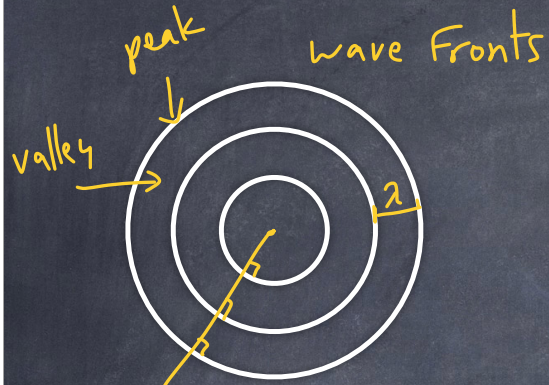
In general, $L = \frac{n}{2}\lambda_n$ $f_n = \frac{v}{\lambda_n} = \frac{nv}{2L}$

So $\lambda_n = \frac{2L}{n}$ for $n = 1, 2, 3, \ldots$

2

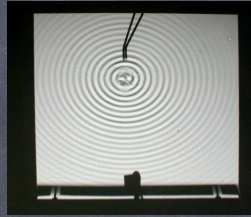# Ruben's flame tube

①

n=1

n=2

n=1 & n=2

Waves moving in 2 or 3 dimensions

peak

wave fronts

valley

λ

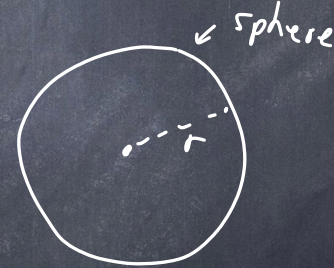lines ⊥ to wave fronts are called rays

Circular

← Consider 2-D wave from a point source (like throwing stone in water)

Energy moves in all directions uniformly

In 3-D, like sound waves, the wave moves spherically.

← sphere

r

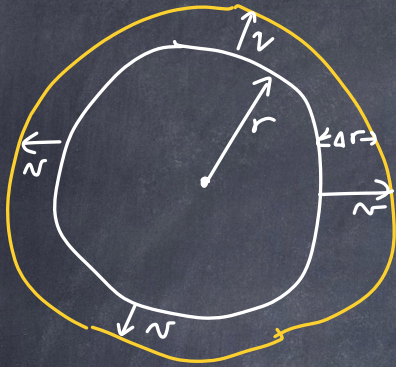surface area of a 3-D sphere is $4\pi r^2$

$$\text{Intensity} = I = \frac{\langle \text{power} \rangle}{\text{area}} \quad \leftarrow \text{average power}$$

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Relationship between I and energy per unit volume

$$\eta = \frac{\epsilon}{V}$$

$$\Delta r = v \Delta t$$

The additional energy in this spherical shell of thickness Δr

$$\Delta \epsilon = \eta \underbrace{\Delta V}_{\substack{\text{volume} \\ \text{of shell}}} = \eta \underbrace{A \Delta r}_{\Delta V} = \eta A v \Delta t$$

$$\text{Power} = \frac{\Delta \epsilon}{\Delta t} = \eta A v$$

$$\boxed{\text{Intensity} = \frac{\text{Power}}{\text{area}} = \eta v}$$

Velocity of wave

energy density

$$\text{units} = \left[\frac{W}{m^2} \right]$$

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We remember for a s.h.o., $E = \frac{1}{2} m \omega^2 A^2$

$$\Delta E = \frac{1}{2} \underbrace{\Delta m}_{\rho \Delta V} \omega^2 \underbrace{A^2}_{S_0^2 : \text{maximal displacement}}$$

$$\Delta E = \frac{1}{2} \rho \Delta V \omega^2 S_0^2$$

$$\eta = \frac{\Delta E}{\Delta V} = \frac{1}{2} \rho \omega^2 S_0^2$$

$$I = \eta \nu = \frac{1}{2} \rho \omega^2 S_0^2 \nu$$

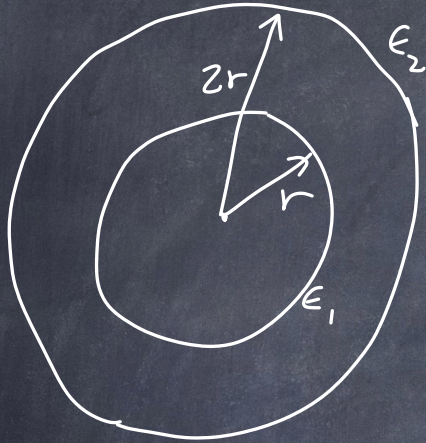for sound waves : $P_0 = \rho \omega \nu S_0$

so $S_0^2 = \dfrac{P_0^2}{\rho^2 \omega^2 \nu^2}$

$$\boxed{I = \frac{1}{2} \frac{P_0^2}{\rho \nu}}$$

Intensity of sound waves
"How loud is something"

6

Human ear can hear intensities from:

$$I = 10^{-12} \frac{W}{m^2} \rightarrow 1 \frac{W}{m^2}$$

$\underbrace{}$ breathing

$\underbrace{}$ painful
rock concert, 2m from speakers.

So we use a log scale to measure "loudness" = sound intensity

measured in decibels [dB]

Sound
intensity = $\beta = 10 \log_{10} \frac{I}{I_0}$
level

$$I_0 = 10^{-12} \frac{W}{m^2}$$

rules for $\log_{10}$:

$\log_{10} 1 = 0$

$\log_{10} 10 = 1$

$\log_{10} 100 = 2$

$\log_{10} 1000 = 3$

(# of zeroes)

breathing: $\beta = 10 \log \frac{10^{-12} \frac{W}{m^2}}{10^{-12} \frac{W}{m^2}} = 10 \log 1 = 0 \, dB$

rock concert $\beta = 10 \log \frac{1 \frac{W}{m^2}}{10^{-12} \frac{W}{m^2}} = 10 \log_{10} 10^{12} = 120 \, dB$

me talking $\beta = 10 \log 10^6 = 60 \, dB$

For 3-D waves: $\quad I = \dfrac{power}{area}$ ← for sphere: $4\pi r^2$

So Intensity decreases like $\dfrac{1}{r^2}$

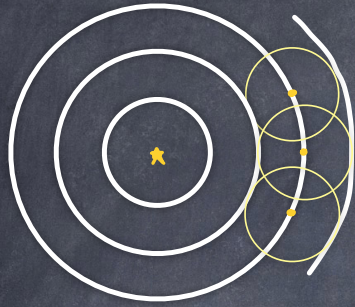If we know the intensity at r, what is intensity at $2r$?

$$r_1 = r$$
$$r_2 = 2r$$

total power at r_1 = total power at r_2

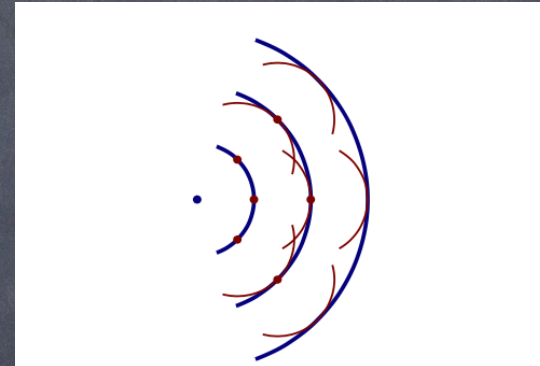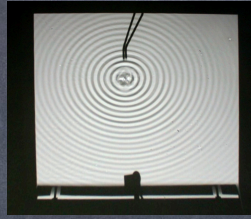$$I_1 \left(4\pi r_1^2 \right) = I_2 \left(4\pi r_2^2 \right)$$

so if $r_2 = 2r_1$,

then $\quad I_2 = \dfrac{1}{4} I_1$

Huygen's principle- a wave acts as if each point along the wave front is a spherical wave source.
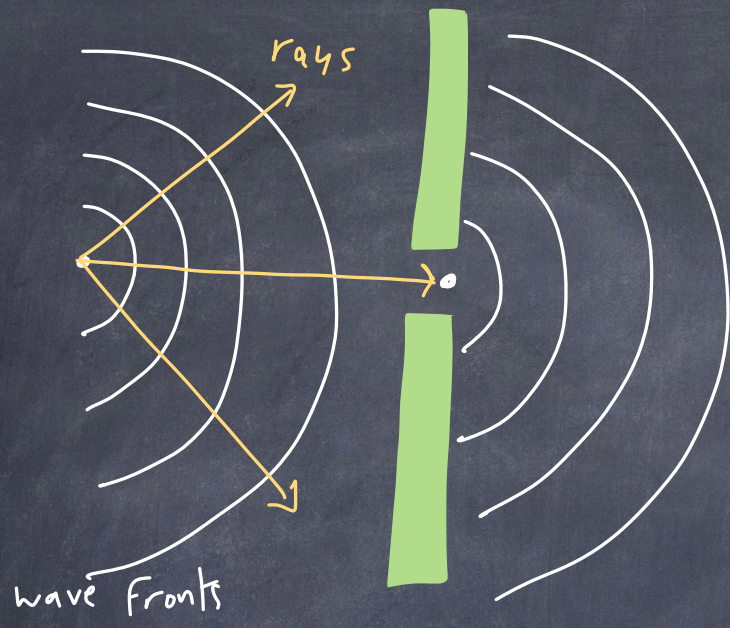
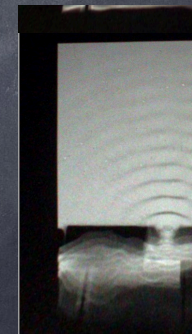← represents the wave front produced by all point sources along wave front.

(b) Each point on a wavefront is a new point source.

A plane wave is comprised of harmonic spherical waves

rays

wave fronts

wave propagates as
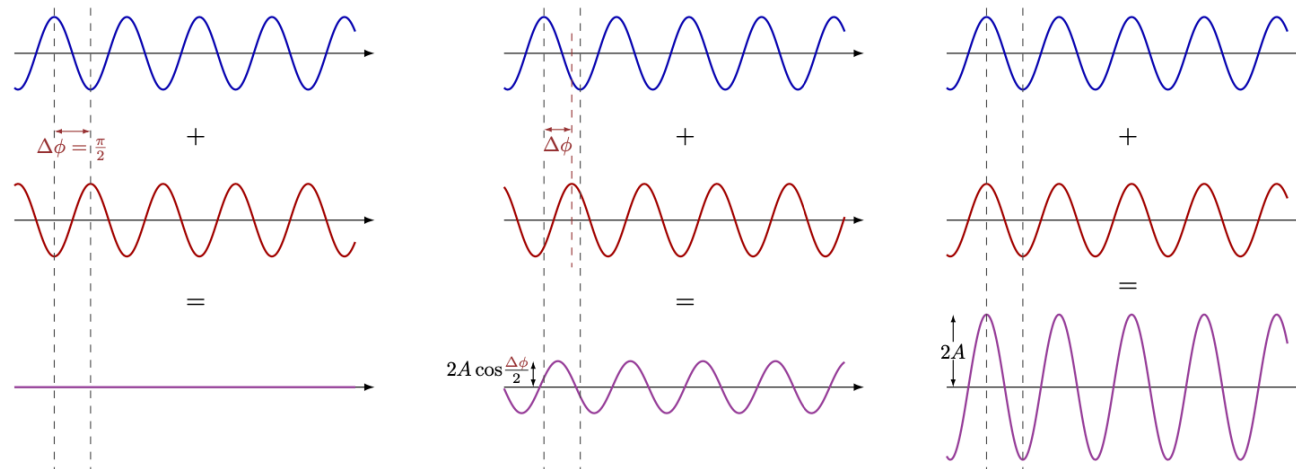if from one point

Interference: wave superposition can lead to
interference if waves of same
λ & amplitude are out of phase by
some angle Δϕ = δ

(a) If $\Delta\phi = 90°$, there is destructive interference.

(b) If $0° < \Delta\phi < 90°$, there is partial interference with amplitude $2A\cos\frac{\Delta\phi}{2}$.

(c) If $\Delta\phi = 0°$, there is constructive interference with amplitude $2A$.

Figure 14.11: Interference between two waves with the same wavelength λ and amplitude A, but phase difference $\Delta\phi$.

Adding 2 waves out of phase

$$y_1 = y_0 \sin(kx - \omega t)$$

general formula (no derivation)

$$y_2 = y_0 \sin(kx - \omega t + \delta)$$

δ = phase shift (angle)

$$y_1 + y_2 = 2y_0 \cos\left(\tfrac{1}{2}\delta\right) \sin\left(kx - \omega t + \tfrac{\delta}{2}\right)$$
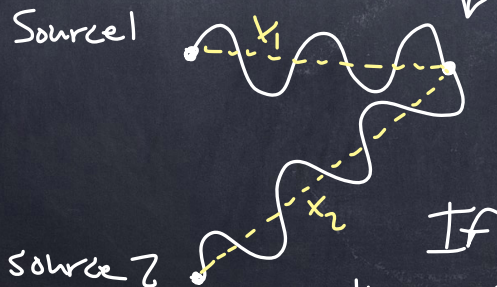
Example for pressure waves

$$\mathcal{P}_1 = \mathcal{P}_0 \sin(kx - \omega t)$$

$$\mathcal{P}_2 = \mathcal{P}_0 \sin(kx - \omega t + \delta)$$

$$\mathcal{P}_1 + \mathcal{P}_2 = 2\mathcal{P}_0 \cos\left(\tfrac{1}{2}\delta\right) \sin\left(kx - \omega t + \tfrac{\delta}{2}\right)$$

One way to get a phase difference is to have different path lengths.

receiving a signal here

sources are coherent. same k, ω, λ, amplitude in phase

Source 1

Source 2

phase difference

$$\delta = (kx_1 - \omega t) - (kx_2 - \omega t)$$

$$= kx_2 - kx_1 = k \Delta x$$
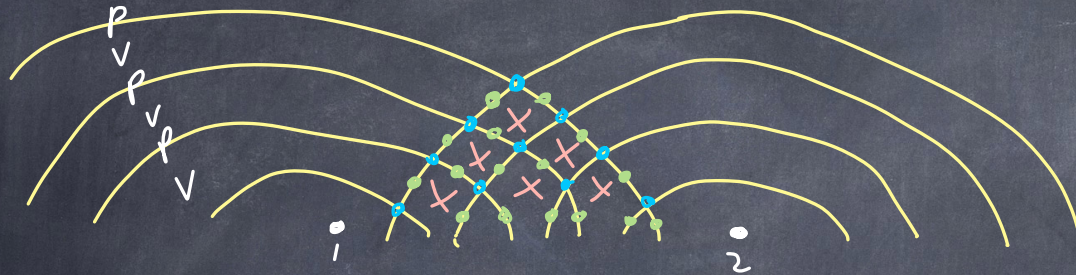
Since $k = \frac{2\pi}{\lambda}$, so $\boxed{\delta = \frac{2\pi \Delta x}{\lambda}}$ phase difference from 2 coherent sources

If $\Delta x = n\lambda$, $n = 0, 1, 2, \dots$ then $\delta = 2\pi n$, and we get constructive interference.

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2 sources coherently emitting waves
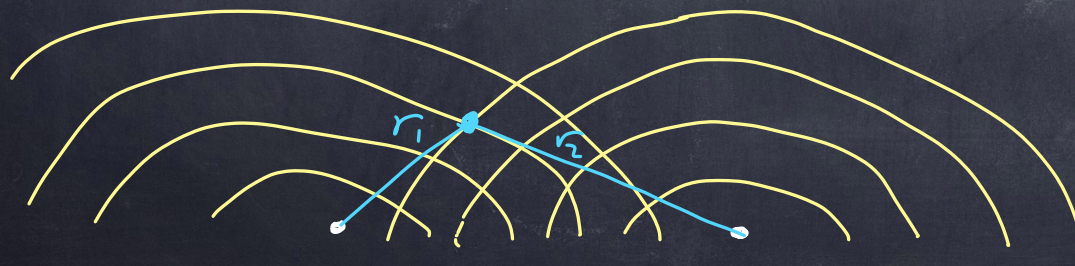
p
v
p
v
p
v

waves add together
Constructively
when peaks overlap

waves adding together
at valleys constructively

waves add destructively
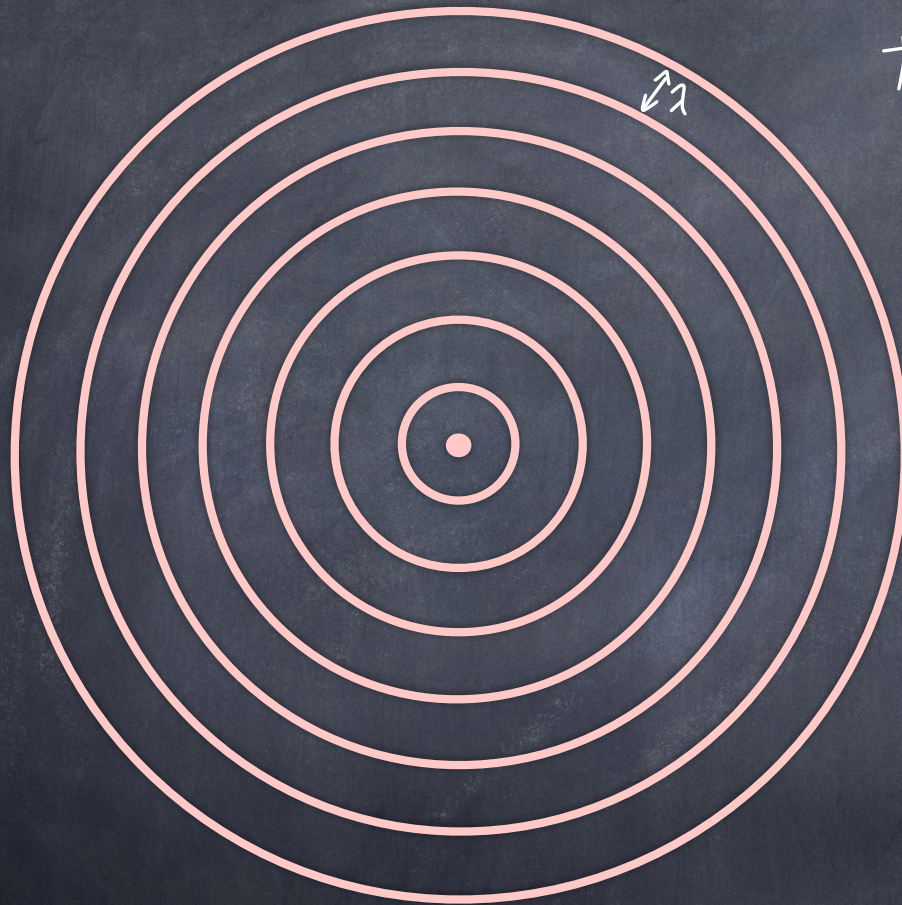(peak of one wave
+ valley of the other
wave.)

Interference:
    constructive when $y_1 + y_2 = 2y_0$
    destructive when $y_1 + y_2 = 0$

condition for constructive interference

$$\Delta r = r_2 - r_1 = \begin{matrix} -1\lambda \\ -2\lambda, \\ 0\lambda, \\ 1\lambda \\ 2\lambda \end{matrix}$$

then we get
Constructive interference

Here is a source emitting a spherical wave.
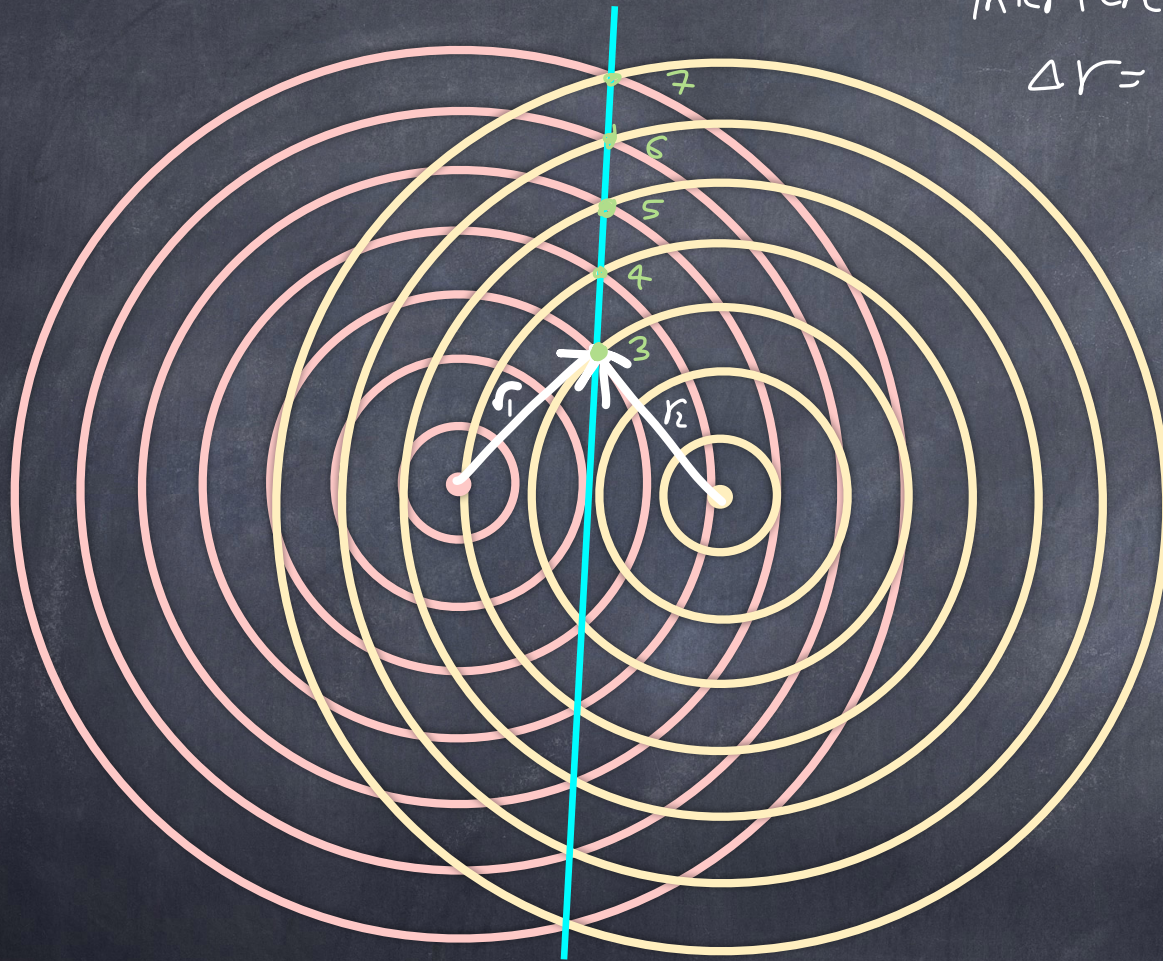The distance between peaks is λ.

This is a s.h.o.

λ

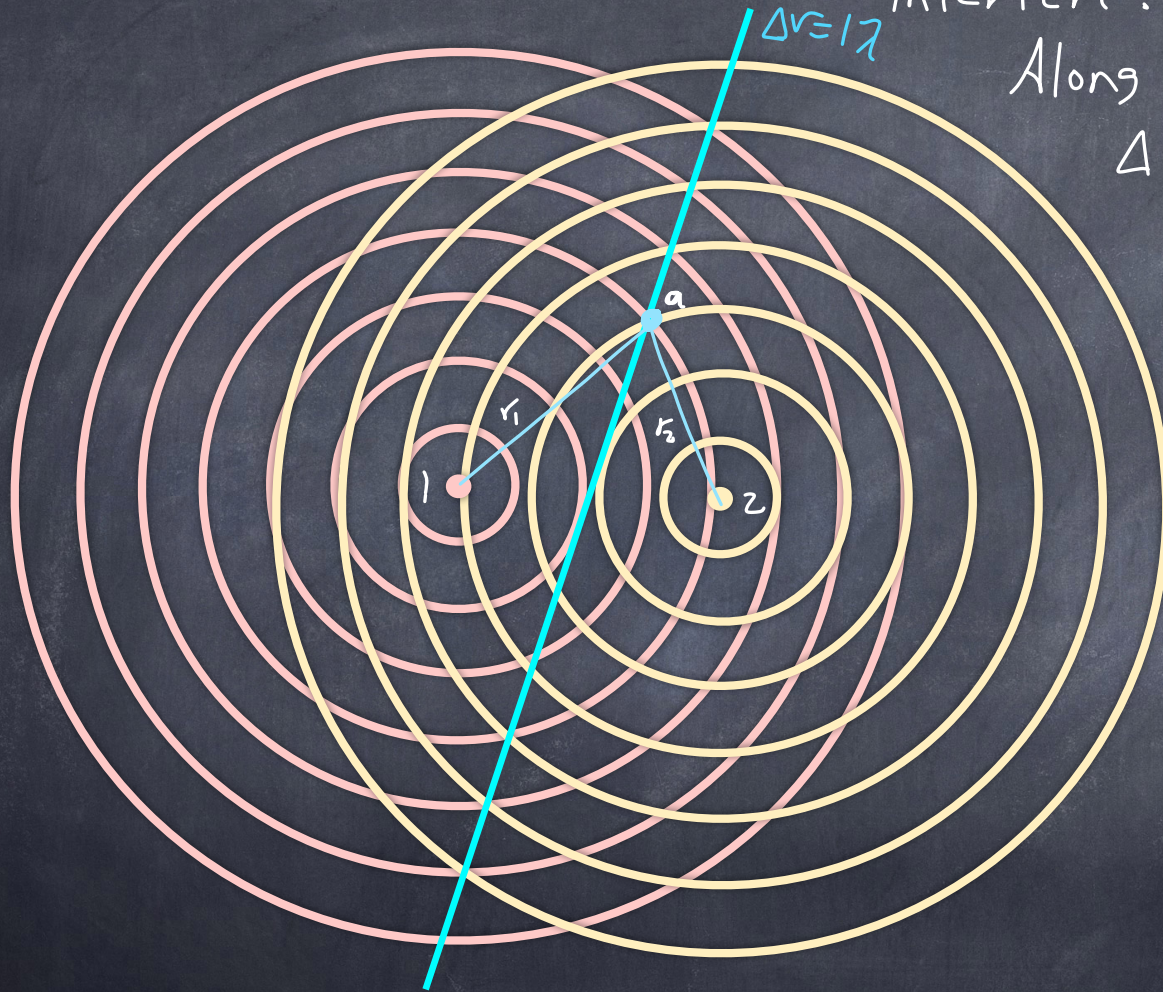Here are 2 coherent sources: Same ω, k, in phase at source.

Along this line, where $r_1 = r_2$, the waves constructively interfere.

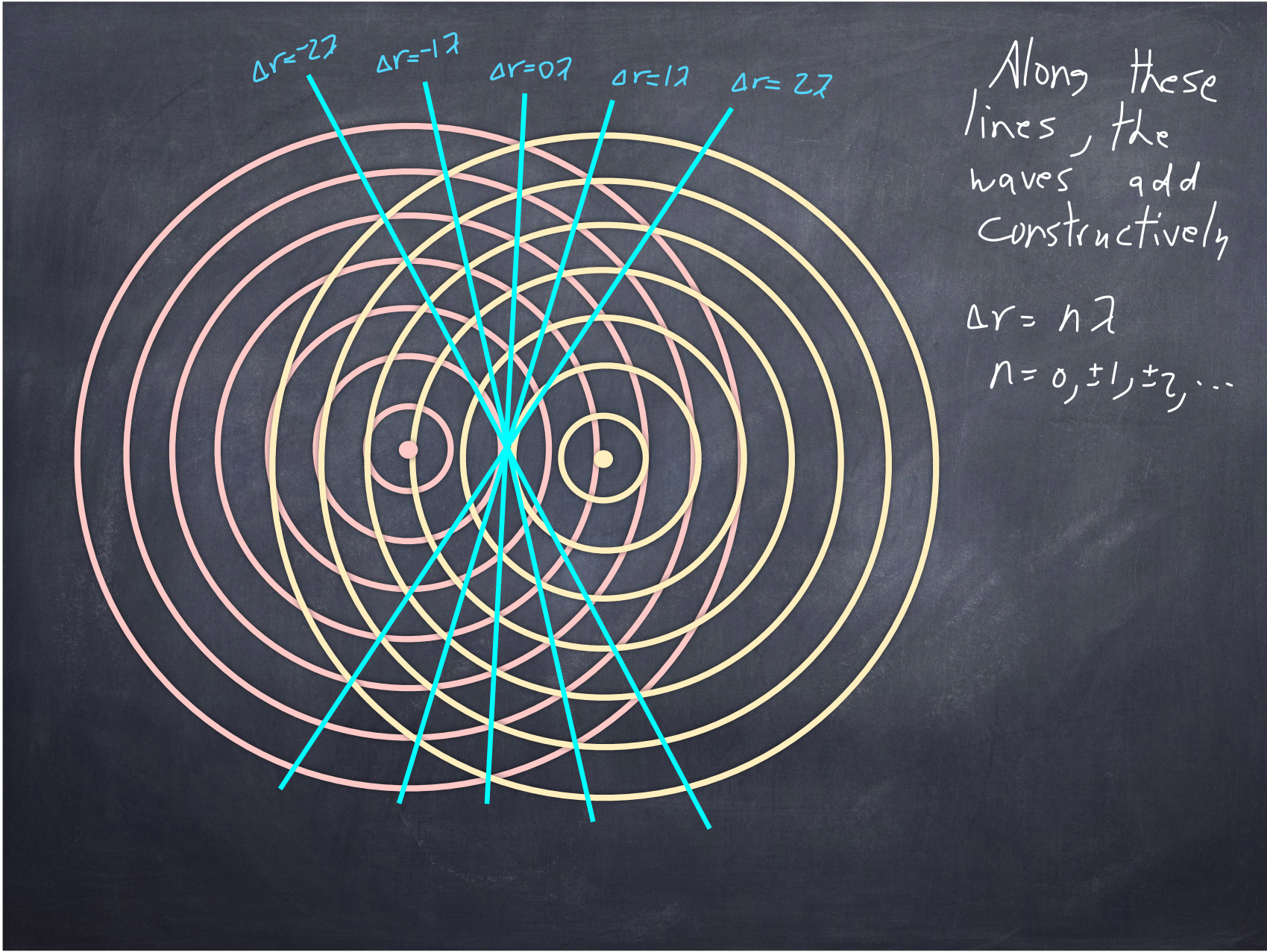$$\Delta r = r_2 - r_1 = 0\lambda = 0$$

At point 3, the 3rd peak of one wave adds with the 3rd peak of other wave.

Along this line, the waves constructively interfere.
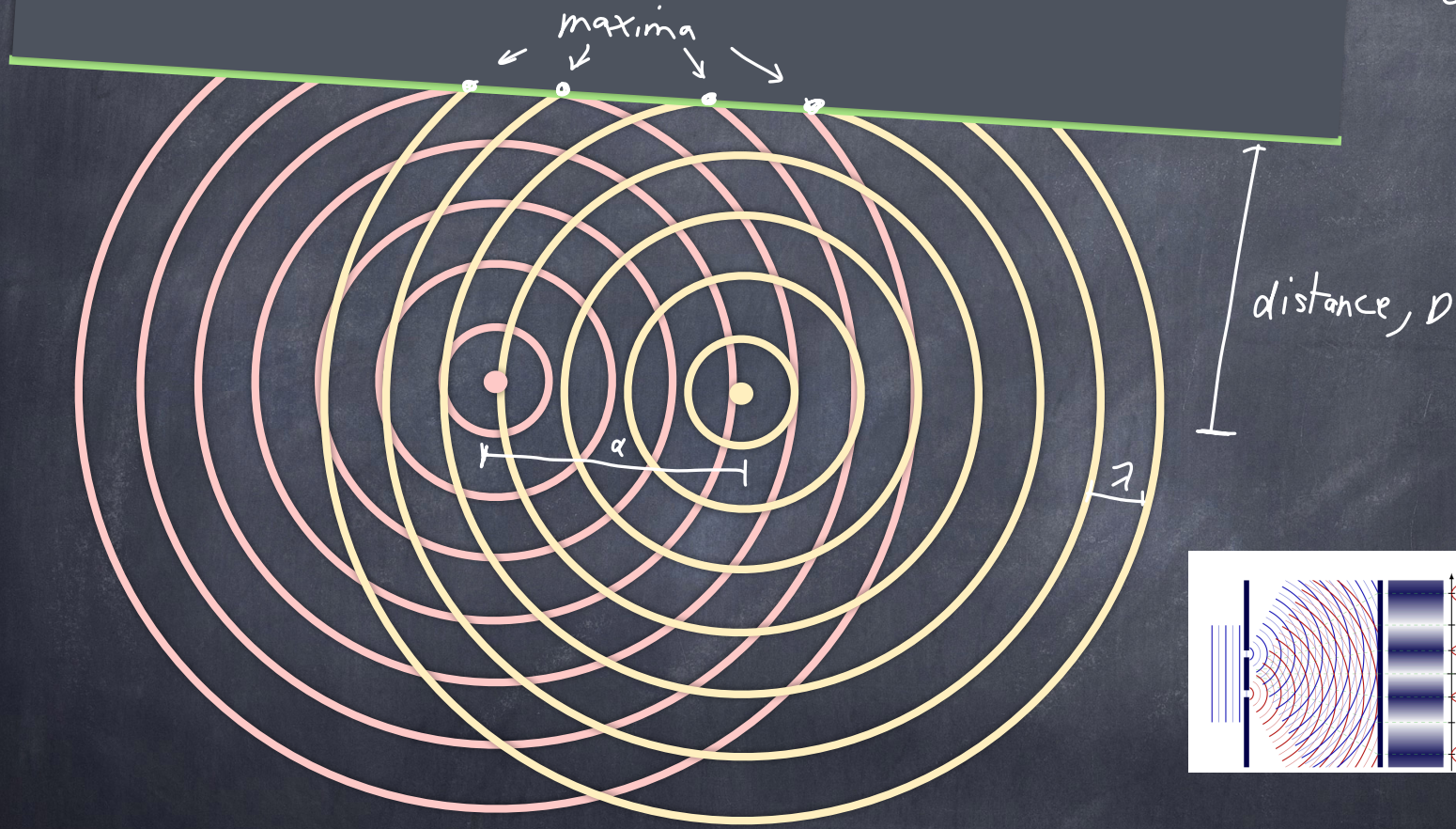
Along this line, $\Delta r = 1\lambda$

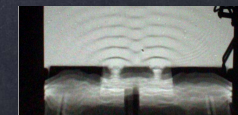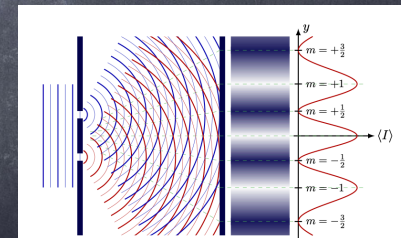At point a, the 4th peak of source 1 adds with the 3rd peak of source 2.

$\Delta r = 1\lambda$

r_1

r_2

1

2

a

Δr=-2λ Δr=-1λ Δr=0λ Δr=1λ Δr=2λ

Along these
lines, the
waves add
constructively

$\Delta r = n\lambda$
$n = 0, \pm 1, \pm 2, \ldots$

If we measure the sum of the waves along the green line, there will be constructive maxima at the intersection of the peaks from the 2 sources

maxima

distance, D

λ

α
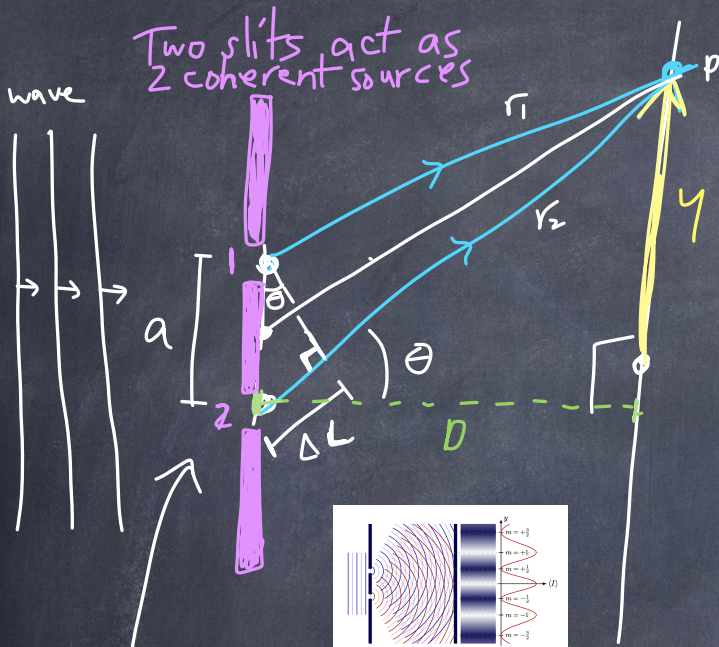

sources are a distance, a, apart

Calculate the location of the maxima. We measure at point P, at a distance, D. 2 coherent sources, a, apart.

ΔL = path difference from sources to point y,

$$\Delta L = r_2 - r_1$$

$$\tan \theta = \frac{y}{D}$$

IF $D \gg a$, then we can use the small angle approximation:

$$\sin \theta \sim \tan \theta \sim \theta \qquad \text{(radians)}$$

$$\uparrow \frac{y}{D}$$

Two slits act as 2 coherent sources

wave

r_1

r_2

y

a

θ

θ

ΔL

D

P

from our small triangle

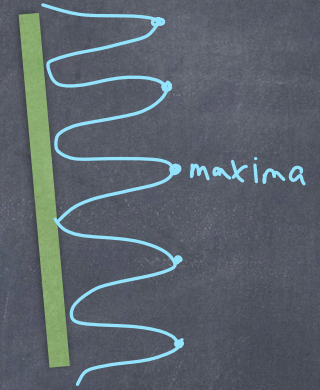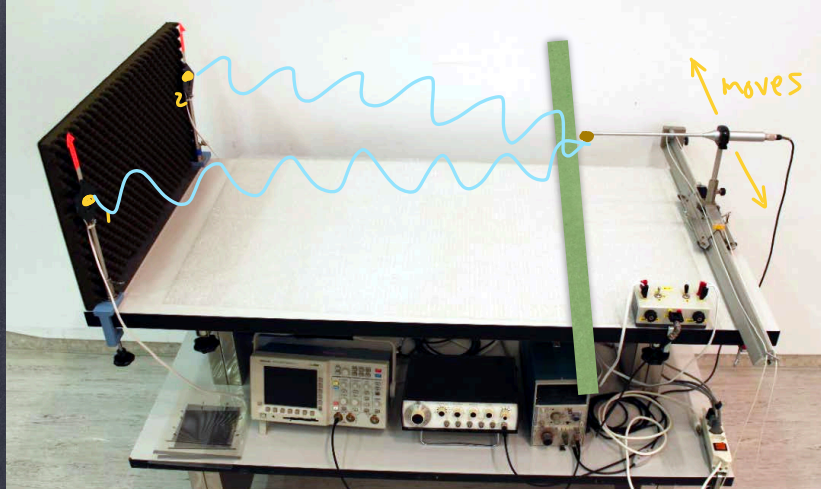$$\sin \theta = \frac{\Delta L}{a}$$

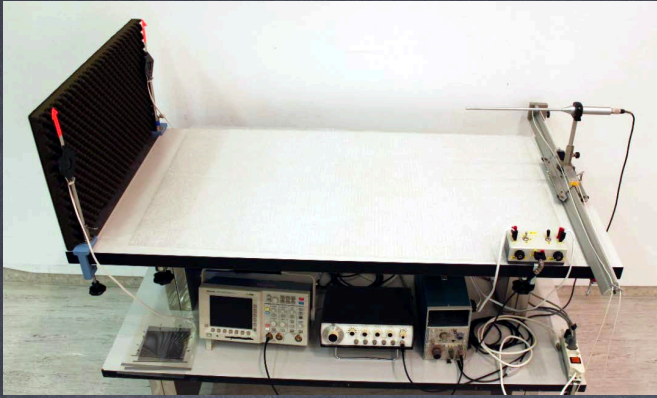$$\frac{\Delta L}{a} = \frac{y}{D} \Rightarrow \Delta L = \frac{ya}{D}$$

There are interference maxima when $\Delta L = n\lambda$ $n = 0, \pm 1, \pm 2, \ldots$

So we can set $\Delta L = n\lambda = \frac{ya}{D} \Rightarrow \boxed{y_n = \frac{n\lambda D}{a} \quad \text{location of maxima}}$
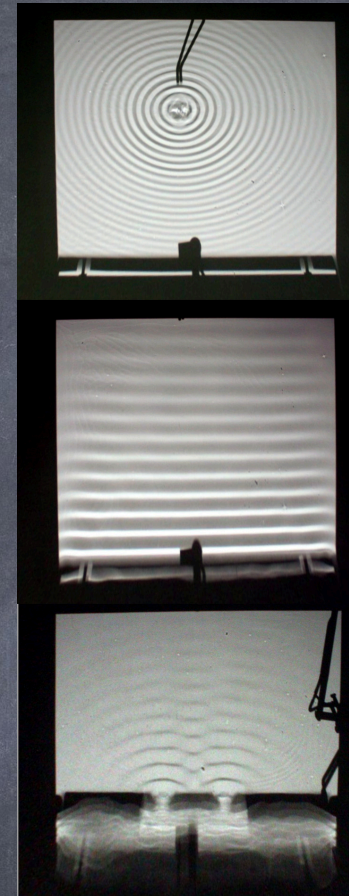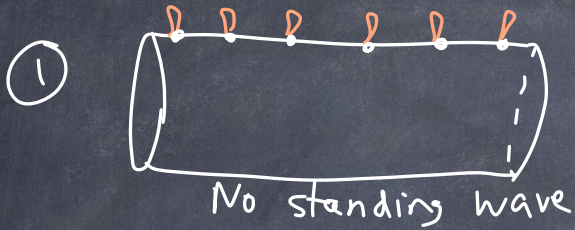
20

2
Sources

1
receiver

moves

maxima

W110

W108

Ruben's flame tube — more understanding

①

No standing wave

n = 1

$\Delta \ell$

oscillita

① $\Delta \ell$ is +

node

② $\Delta \ell$

①→②→① ...

highest flame

n = 2

① $\Delta \ell$

node   node

② $\Delta \ell$

①→②→①

why is
pressure
higher
at pressure
nodes?

For Ruben's flame tube!

For quiet sounds, ΔP of the gas $<$ P of the gas

from Bernoulli's equation, gas flow is proportional to square root of the pressure difference between inside + outside tube.

$$\text{flow} \sim \sqrt{P_{inside} - P_{outside}}$$

(The flow of gas out of the pipe)

When $\Delta P_{maximal}$, anti-nodes produce lower flames
(flow rate is lower)

When $\Delta P = 0$, nodes, flow rate is higher

Part of the cycle, pressure is higher than average but part is lower. On average

This is why pressure is higher at nodes:

$$\sqrt{\underset{\text{at anti-nodes}}{P + \Delta P}} \quad < \quad \sqrt{\underset{\text{at nodes}}{P + \Delta P}^{\to 0}}$$