PHY 117 HS2024

Week 3, Lecture 1
Oct. 1st, 2024
Prof. Ben Kilminster

Today:
Energy
Energy conservation
Kinetic energy
Potential energy
Work

Please do Quiz #2
on OLAT.

Types of energy:

- work
- kinetic energy
- potential energy → gravitational, → spring

Relationship of force to energy:

$[N] = \left[\frac{kg \cdot m}{s^2}\right]$ (force)

$[J] = \left[\frac{kg \cdot m^2}{s^2}\right] = [N \cdot m]$

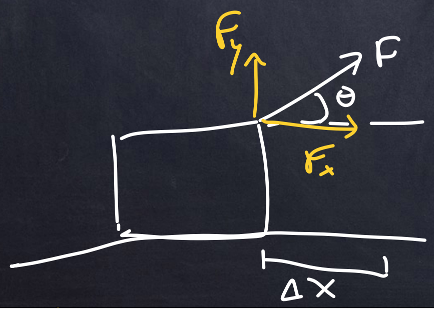
energy "Joules"

force · distance

The "work" done by a force is $W = F \Delta x$
(for the case when $\vec{F}$ is in the direction of $\vec{x}$)

ignore friction

$W = F \Delta x$



If $\vec{F}$ is not parallel to $\overline{\Delta x}$, we need to find the component of $\vec{F}$ that is parallel.

$$W = F_x \Delta x = (F \cos\theta) \Delta x$$
$$(\text{so } F_x \parallel \Delta x)$$

when the force is in the same direction as the motion, W is $(+)$

If we have a net force, then we get an acceleration.

$$\sum F_x = ma$$

Remember $v^2 = v_0^2 + 2a\Delta x \implies a = \frac{v^2 - v_0^2}{2\Delta x}$

$$\text{work} = F_x \Delta x = (ma)\Delta x = m\left(\frac{v^2 - v_0^2}{2\cancel{\Delta x}}\right)\cancel{\Delta x} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

the work-energy theorem:

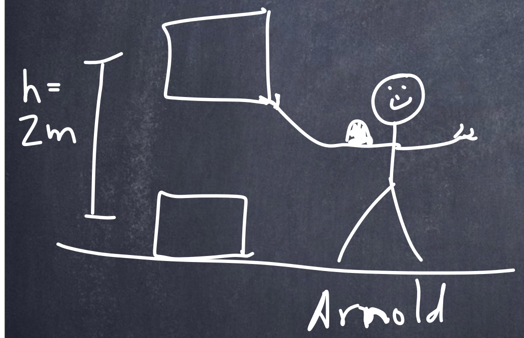
$$\boxed{W_{TOTAL} = \frac{1}{2}m\underset{\text{final}}{v_f^2} - \frac{1}{2}m\underset{\text{initial}}{v_i^2} = K_f - K_i = \Delta K}$$

$$\boxed{K = \frac{1}{2}mv^2 = \text{kinetic energy}}$$

is the kinetic energy of movement

Notes:
- K is a scalar, no direction
- K is always positive or zero
- ΔK can be negative (if object slows down)
- consider each force separately, and the work it does.

Example:



Arnold lifts a 5kg block to h=2m, let's go, using 500 N of force.

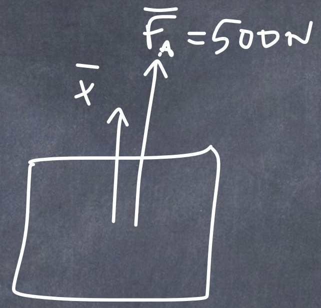
1) what is the work done by Arnold?
2) what is the work done by gravity?
3) what is the final velocity of the block when he lets go?

There are 2 forces at work, Arnold & gravity.
Initially, velocity is zero.

+x ↑

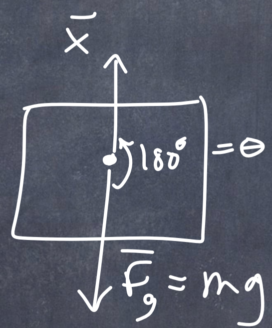
work done by Arnold

1)



$$W_A = F_A \cancelto{1}{\cos\theta}\, \Delta x = F_A \Delta x$$
$$= (500\text{ N})(2\text{ m})$$
$$= 1000\text{ J}$$

2)



$$W_g = F_g \cos(180°)\Delta x = (mg)(-1)\Delta x$$
$$= -(5\text{ kg})\left(10\frac{\text{m}}{\text{s}^2}\right)(2\text{ m})$$
$$= -100\text{ J}$$

3) $W_{TOTAL} = W_A + W_g = 1000\text{ J} - 100\text{ J} = 900\text{ J}$

$$W_{TOTAL} = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}m\cancelto{0}{v_i^2}$$

$$v_f = \sqrt{\frac{2(W_{TOTAL})}{m}} = \sqrt{\frac{2(900\text{ J})}{5\text{ kg}}} = 19\ \frac{\text{m}}{\text{s}} \quad \text{up direction}$$

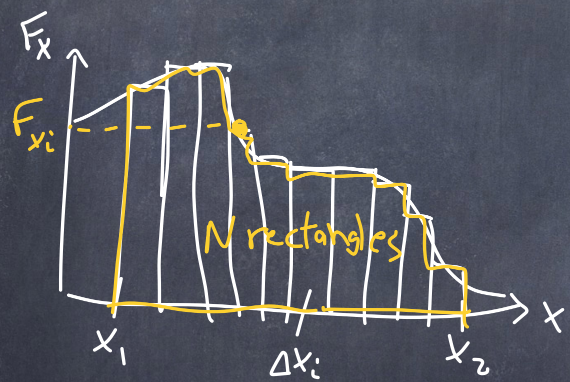
continues up!

So far, we have a constant force.

Area of curve of F vs. x

$= F \Delta x =$ work

What if our force is changing?



First, we consider force in direction of motion

The total work is

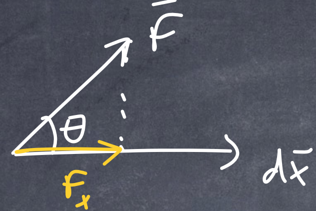
$$W = \sum^{N} F_{x_i} \Delta x_i$$] N rectangles

As $\lim_{\Delta x_i \to 0}$,

(# rectangles goes to ∞)

$$\boxed{W = \int_{x_1}^{x_2} F_x \, dx}$$

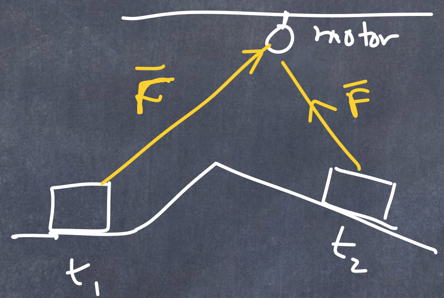
So far, we considered objects moving in x.



$$W = \int_{x_1}^{x_2} F_x \, dx = \int_{x_1}^{x_2} (F\cos\theta)\, dx$$

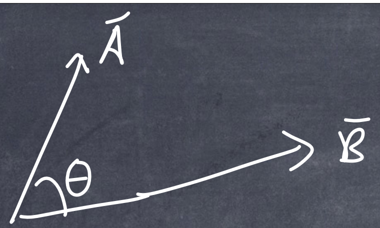
But movement direction can change:

Example:



we want to consider the force in direction of motion.

The "dot product" of force & direction gives us this.



The dot product of $\bar{A}$ & $\bar{B}$ is

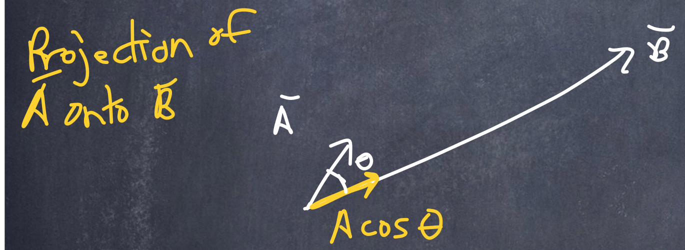
$$\bar{A}\cdot\bar{B} = |\bar{A}|\,|\bar{B}|\cos\theta$$

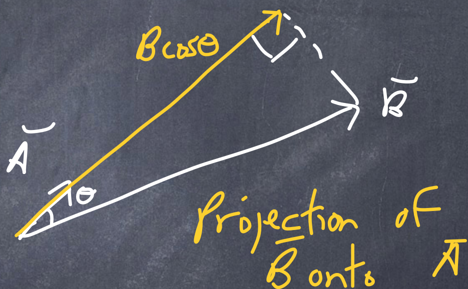
(It is a scalar quantity)

A dot product multiplies the parallel components of 2 vectors. $|\bar{A}|$ = magnitude of $\bar{A} = \sqrt{A_x^2 + A_y^2 + A_z^2}$

we are projecting one vector onto another

Two ways to think about it:





$$\bar{A}\cdot\bar{B} = (A\cos\theta)\,B = AB\cos\theta$$

magnitude

$$\bar{A}\cdot\bar{B} = (B\cos\theta)(A) = AB\cos\theta$$

Both give the same answer

(Also $\bar{A}\cdot\bar{B} = \bar{B}\cdot\bar{A}$)

In 3-D coordinates (x, y, z)

$$\bar{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z}$$
$$\bar{B} = B_x \hat{x} + B_y \hat{y} + B_z \hat{z}$$

$$\bar{A} \cdot \bar{B} = A_x B_x + A_y B_y + A_z B_z$$

consider ①

$\bar{A} \cdot \bar{B} = AB$
maximal

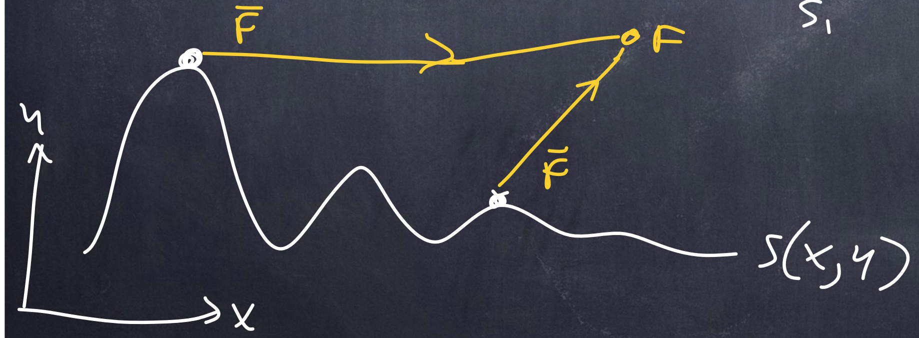
$\bar{A} \cdot \bar{B} = \emptyset$

So our formula in general is

$$W = \int_{S_1}^{S_2} \bar{F} \cdot d\bar{s}$$

$\bar{s}$ is the path of the object.

$d\bar{s}$ could be $d\bar{x}$ if object moves only in $\hat{x}$ direction



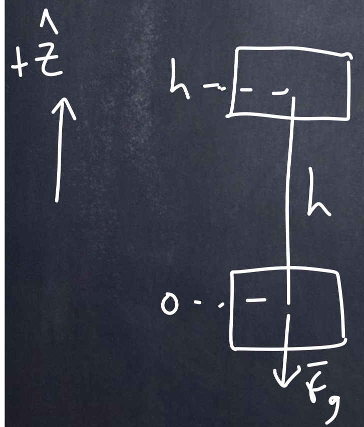
The work done on a system can be <u>stored</u> as potential energy, U.

Potential energy can change $\Delta U = U_f - U_i = U_2 - U_1$

$$\boxed{-\Delta U = W = \int_{s_1}^{s_2} \bar{F} \cdot d\bar{s}}$$

Notice that $\Delta U = -W$

Example: Assume we lift an object of mass, m, to a height, h.



Force of gravity is $F_g = mg$, $\bar{F}_g = -mg\,\hat{z}$

$d\bar{s} = dz\,\hat{z}$

$W = \int \bar{F} \cdot d\bar{s}$

work done by gravity:

$$W = \int_0^h (-mg\,\hat{z}) \cdot dz\,\hat{z} = \int_0^h -mg\,dz$$

$\hat{z} \cdot \hat{z} = 1$

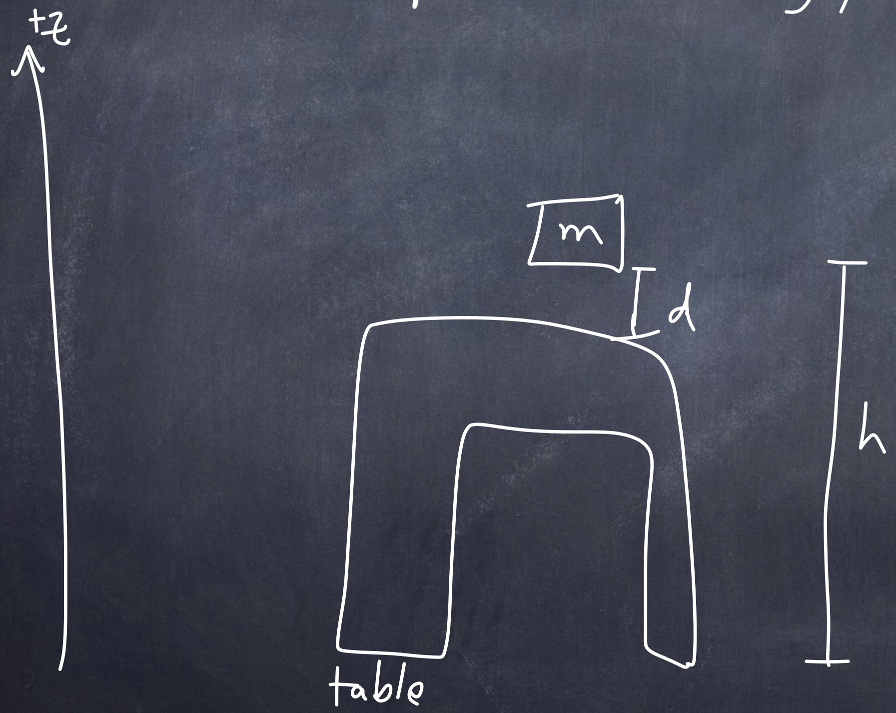
$$W = -mgz\Big]_0^h = -mgh$$

change in potential energy $\Delta U = -W = -(-mgh) = mgh$

In general, gravitational potential energy is $U = mgz$

$\uparrow$ height

But potential energy is relative.



$U = mgz$

U relative to the floor:

$U = mgh$

U relative to the table:

$U = mgd$

conservation of energy

$$\boxed{E_{before} = E_{after}}$$

always true.
If we consider all sources of energy, then the sum of energies is conserved, before & after anything happens.

For conservative forces, potential energy plus kinetic energy is conserved.

$$\boxed{(K + U)_{before} = (K + U)_{after}}$$

only true for conservative forces

summary so far:

Energy is conserved

$$\boxed{E_{before} = E_{after}}$$

$$\boxed{\text{Potential energy from gravity} = mgh = U}$$

$$\boxed{\text{Kinetic energy} = \tfrac{1}{2}mv^2 = K}$$

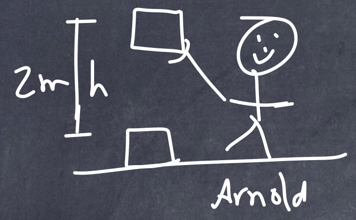
work-energy theorem

$$\boxed{W_{TOT} = \Delta K = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}m_i v_i^2}$$

potential energy + work relation

$$\boxed{W = -\Delta U}$$

$$\boxed{W = \int_{s_1}^{s_2} \bar{F} \cdot d\bar{s}}$$

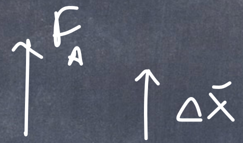


$m = 5\,kg$

Consider Arnold again. If he lifts the block with 50 N of force, to a height of 2m, and then let go. How fast will it travel when it hits the ground

Lifting part:

1)



The work of Arnold:

$$W_A = \bar{F}_A \cdot \Delta\bar{x} = F_A \Delta x = F_A h$$
$$= (50N)(2m) = 100J$$

2) The work of gravity

$$W_g = \bar{F}_g \cdot \Delta\bar{x} = -F_g \Delta x = -mgh$$
$$= (5kg)\left(10\,\frac{m}{s^2}\right)(2m) = -100J$$

3) the total work = $W_A + W_g = 100J + (-100J)$
$$= 0\,J$$

The total work is zero, then velocity is zero at top

Now Arnold drops the weight, how fast is it when (just before) it hits the ground?

At top, the block has $U = mgh$

$K = 0 \quad (v=0)$

when he drops it, the potential transforms into kinetic energy.

$$(K+U)_{\substack{\text{before} \\ =\text{top}}} = (K+U)_{\substack{\text{after} \\ =\text{bottom}}}$$

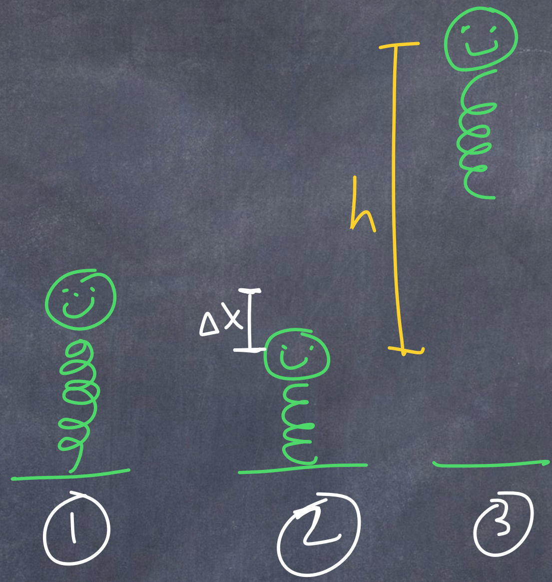
$$0 + mgh = \frac{1}{2}mv^2 + 0$$

$$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2$$

$$v = \sqrt{2gh} = \sqrt{2\left(10\,\frac{m}{s^2}\right)2m} = \sqrt{40}\ \frac{m}{s}$$

$$v = 6.3\ \frac{m}{s}$$

How high does the grasshopper jump?



1. grasshopper just sitting there.
2. grasshopper compresses spring.
3. grasshopper releases spring, jumps a height, h.

Three types of energy here:

U_s: potential energy of spring

U_g: potential energy of gravity

K: kinetic energy

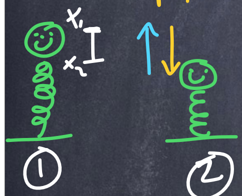
Focus on spring energy, U_s.

we know the force on a spring:

$$F_s = -k\Delta x$$

(−) force points opposite the stretching of the spring.

F: me pushing down



Force of grasshopper spring pushing back at me

we can measure k:

$$k = \frac{F_s}{\Delta x} = \frac{(2.5\,\text{kg})(9.81\,\frac{\text{m}}{\text{s}^2})}{0.04\,\text{m}} = 612.5\ \frac{\text{N}}{\text{m}}$$

Now we calculate work done by the spring when we compress it.

$$W = \int_{x_1}^{x_2} \bar{F} \cdot d\bar{x} = \int_{x_1}^{x_2} (-kx)\,dx$$

$$W = -\frac{1}{2}kx^2\Big]_{x_1}^{x_2} = -\frac{1}{2}k(\Delta x)^2$$

spring does negative work.

$$\Delta U_s = -W = \frac{1}{2}k(\Delta x)^2$$

(+) means potential spring has increased.