PHY117 HS2024

Week 12, Lecture 1
Dec. 3rd, 2024
Prof. Ben Kilminster

Last time, standing waves in general:

$$y(x,t) = 2A\cos\omega t \sin kx$$

where $k_n = \frac{n\pi}{L}$ + $\omega_n = k_n v$ + $v = \frac{\omega}{k}$

$\omega_n = 2\pi n f_1$

$$f_1 = \frac{v}{\lambda_1} = \frac{k_1 v}{2\pi}$$

Example: standing wave on string

$n=4$

$k_4 = \frac{4\pi}{L}$, $\omega_4 = k_4 v$, $v = \sqrt{\frac{T}{\mu}}$

Vibrating systems have (in general) multiple standing waves, superimposed.

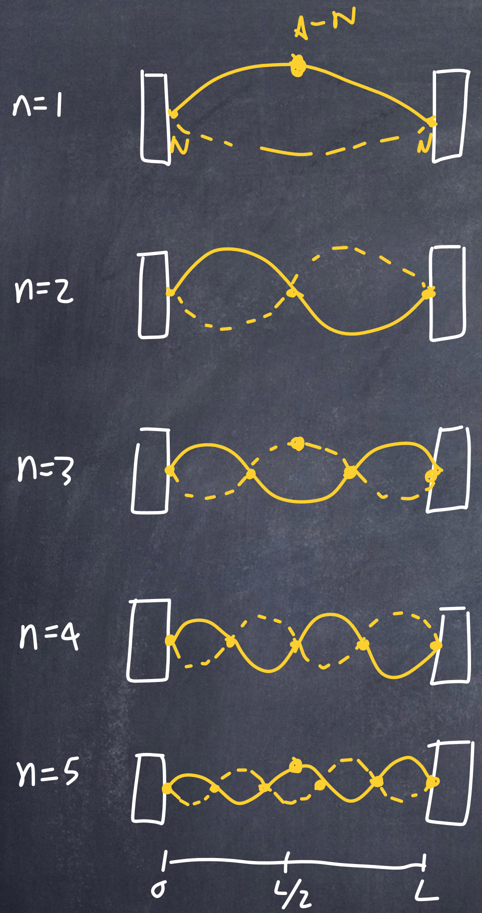
In general
$$\boxed{y(x,t) = \sum_n A_n \cos(\omega_n t + \delta_n)(\sin k_n x)}$$

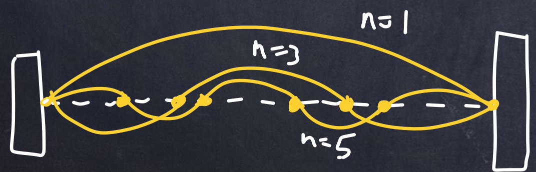
ω_n, k_n : angular frequency, wave number for some n (harmonic value, integer)

A_n, δ_n : Amplitude & phase constants

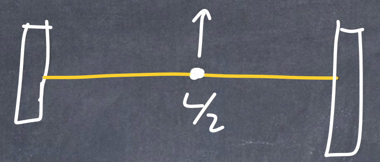
A_n^2: fraction energy of the n^{th} harmonic in our wave

δ_n : depends on initial conditions



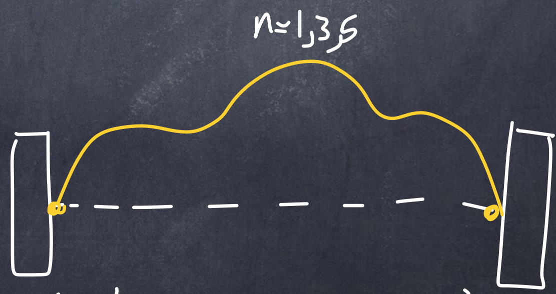


we pluck the string at $\frac{L}{2}$



we excite harmonics, depending on where we pluck the string, to get different A_n.
(relative energies of each harmonic)

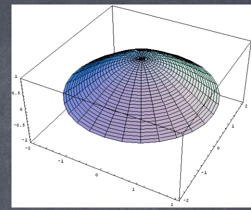
For $\frac{L}{2}$, we create an anti-node in the middle, and this excite the odd harmonics $n = 1, 3, 5, \ldots$

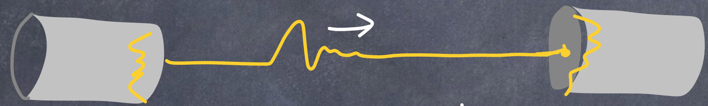


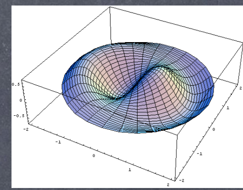
Most energy goes into the fundamental frequency, $n = 1$

Standing waves on a flat, round surface: diaphragm

T: tension

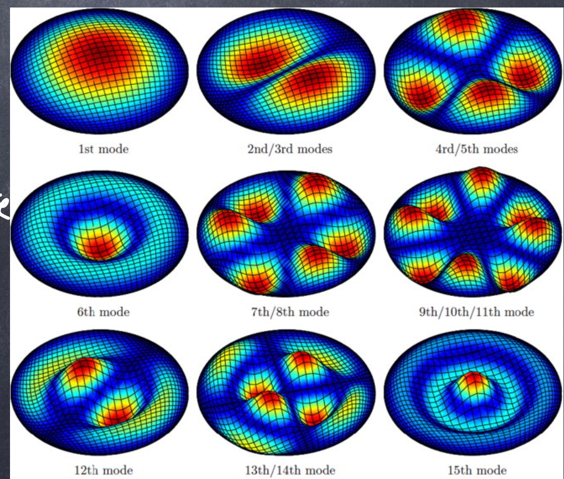


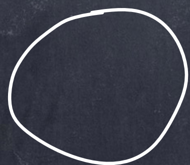




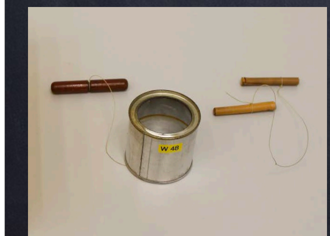
the shape of the harmonics depend on the boundary conditions, which parts are moving + which parts are fixed.

Bessel functions





the circumference is fixed as a node



Applications of standing waves (PHY 127)

Q: why do electrons in an atom only exist in some states?

A: standing waves

Wavelengths for Different States

For a hydrogen atom:

Electron wave resonance

$n=1$

$\lambda_1 = 2\pi r_1 = 6.28a_0$

$n=2$

$2\lambda_2 = 2\pi r_2$

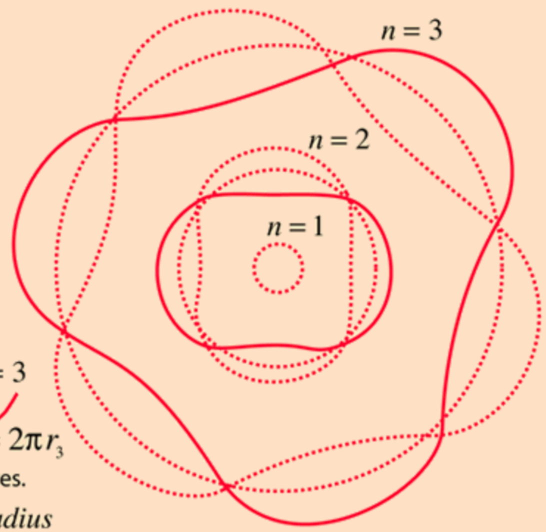
$\lambda_2 = 12.57a_0$

$n=3$

$\lambda_3 = 18.85a_0 \qquad 3\lambda_3 = 2\pi r_3$

Wavelengths for hydrogen states.

$a_0 = 0.0529nm = Bohr\ radius$



Bohr model of the atom

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Bohr model concepts

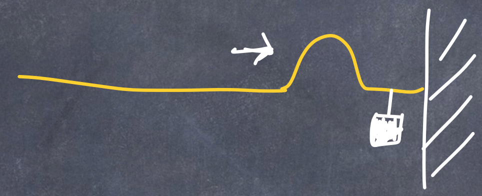
HyperPhysics***** Quantum Physics

R Nave

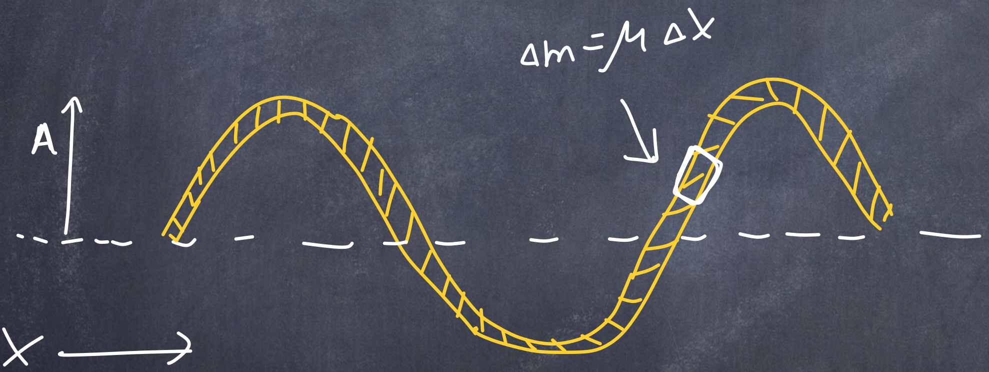
Go Back

Energy transmission in a wave (on a string)

wave can do work



pulse lifts the weight



we have a sine wave on a string, with amplitude, A, and angular frequency, ω. It has a $\frac{\text{mass}}{\text{length}} = \mu$.

Our piece of string (Δx) oscillates up & down as a simple harmonic oscillator (s.h.o.)

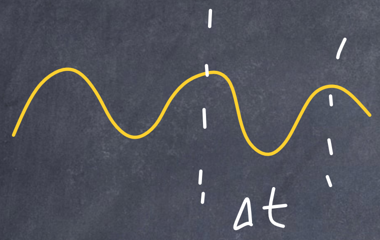
Previously, we determined that the energy of a s.h.o.

k: spring constant

is $E = \frac{1}{2} k A^2$ $\quad \omega = \sqrt{\frac{k}{m}}$ $\quad k = \omega^2 m$

$$\Delta E = \frac{1}{2} k A^2 = \frac{1}{2}\omega^2 \Delta m \, A^2 = \frac{1}{2}\omega^2 \underbrace{(\mu \Delta x)}_{(\Delta m)} A^2$$

$$\text{Energy} = \Delta E = \frac{1}{2}\omega^2 \mu \Delta x \, A^2$$



In Δt, $\Delta x = v \Delta t$

$$\boxed{\frac{\Delta E}{\Delta t} = \frac{1}{2}\omega^2 \mu \frac{\Delta x}{\Delta t} A^2 = \frac{1}{2}\omega^2 \mu v A^2 = \text{Power}}$$

Note that energy, $E \propto A^2$, $E \propto \omega^2$

$E \propto \mu$

$\text{Power} \propto \frac{1}{\Delta t}$

Longitudinal waves

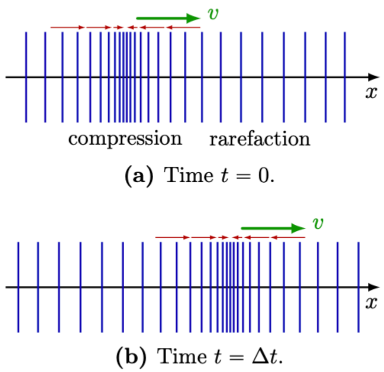


Figure 13.10: A traveling longitudinal wave is when the distortion happens along the direction of propagation, here shown as a local displacement.

Sound waves are longitudinal waves

in air or a fluid.

air molecules

$t=0$

$t=1$

$t=2$

$t=3$

$\longrightarrow$ disturbance propagates

What is sound? A pressure increase ΔP that moves with a velocity that depends on medium.

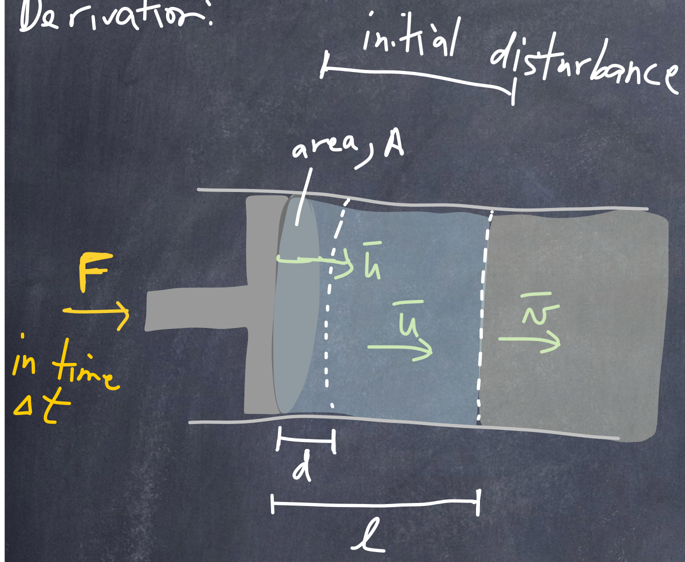
How fast is sound in a fluid?

Derivation:

ρ: fluid density

B = Bulk modulus

remember
$$\boxed{B = \frac{\Delta P}{-\left(\frac{\Delta V}{V}\right)}} \quad (1)$$



we push the piston with velocity u for some time Δt, so a distance $d = u\Delta t$. This makes a pulse of ΔP.

we assume that the speed of the wave, v, is much larger than u. We further assume that the effect of the piston is to cause all the fluid up to the distance ℓ to get pushed by a velocity, u.

Now, this initial disturbance propagates with velocity, v.

i) compress our piston quickly in time Δt, with velocity $u \Rightarrow d = u\Delta t$

This causes a pressure increase $\Delta P = \frac{F}{A}$

$$\boxed{F = A\Delta P} \quad (2)$$

ii) This creates a pulse of pressure, which is a sound wave $\equiv$ pressure wave.

The wave has velocity v

it moves a distance $l = v\Delta t$

The mass of the fluid that moves in Δt is $m = \rho V$

$$\boxed{m = \rho A \cdot l = \rho A v \Delta t} \quad (3)$$

iii) we want to know v. we can do this with momentum conservation.

we can go from $F \to$ impulse, $\Delta p \to$ momentum, $p \to$ velocity

using (2) $\quad$ Impulse $= F\Delta t = A\Delta P\Delta t$ (4)

p = momentum $\neq P$ = pressure

Momentum conservation tells us that:

Δp of piston $= \Delta p$ of fluid that moves

from (4) $\quad A\Delta P\Delta t = m\bar{u} = \rho A v \Delta t\, \bar{u}$

$\nwarrow$ from (3)

$$\cancel{A}\,\Delta P\,\cancel{\Delta t} = \rho\,\cancel{A}\,v\,\cancel{\Delta t}\,u$$

$$\Delta P = \rho v u$$

using ①

$$-B\frac{\Delta V}{V} = \rho v u$$

V: is volume of fluid experiencing wave

ΔV: is volume of fluid moved by piston, $\Delta V = -A\underbrace{u\Delta t}_{d}$ (−) gets compressed.

$$\frac{\Delta V}{V} = \frac{-\cancel{A}\,u\,\cancel{\Delta t}}{\cancel{A}\,v\,\cancel{\Delta t}} = -\frac{u}{v}$$

$$-B\left(\frac{-u}{v}\right) = \rho v u \qquad \frac{B}{v} = \rho v$$

$$\boxed{v = \sqrt{\frac{B}{\rho}}}$$

This is the speed of sound in a fluid. Related to the compressibility & density of the fluid.

For air, $B = 142\ \text{kPa}$, $\rho = 1.2\ \text{kg/m}^3$

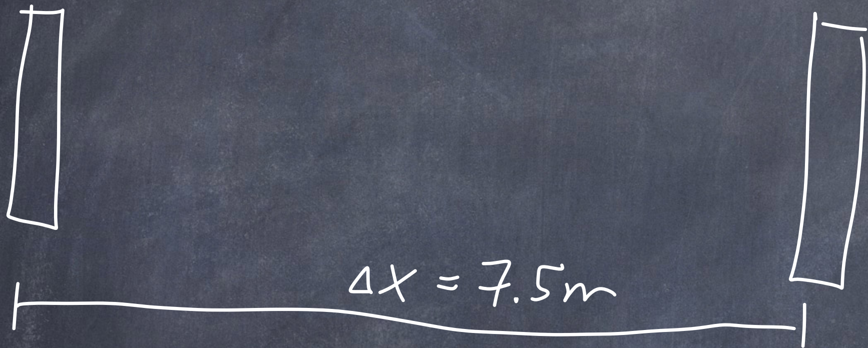
$$v_{\substack{\text{sound}\\ \text{in air}}} = \sqrt{\frac{B}{\rho}} = 343\ \frac{m}{s}$$

prediction

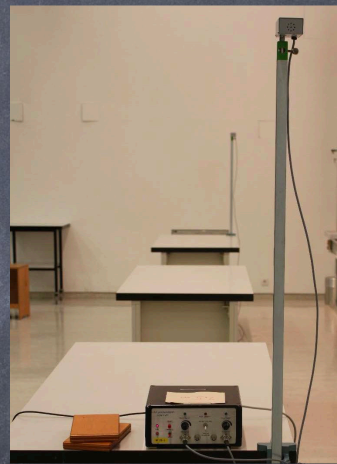
Note on temperature, T for molecules

$T \propto \frac{1}{2} m v^2$

$v \propto \sqrt{T}$



$\Delta x = 7.5\text{m}$



time professor:

time student:

time sound:

$$v_{\substack{\text{air}\\ \text{sound}}} = \frac{\Delta x}{\Delta t} =$$

Speed of sound in a solid?

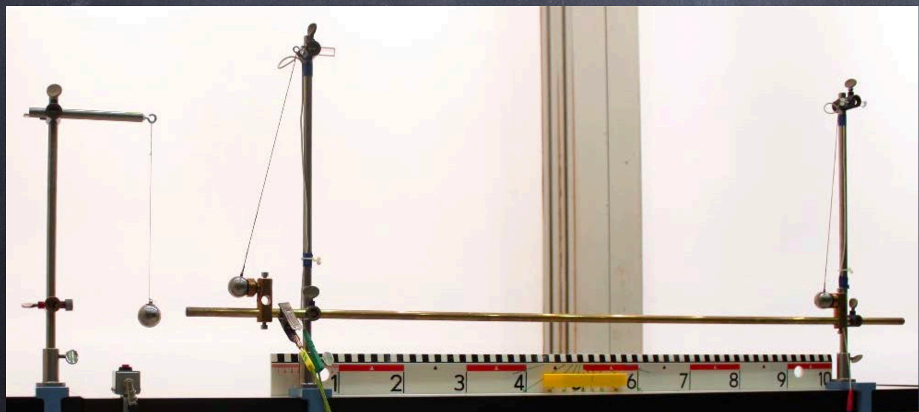
$$v = \sqrt{\frac{Y}{\rho}}$$

Y = young's modulus

brass: $Y = 10 \times 10^{10}\ \frac{N}{m^2}$

$\rho = 8.73\ \frac{g}{cm^3} = 8730\ \frac{kg}{m^3}$

$$v = \sqrt{\frac{10e10\ \frac{N}{m^2}}{8730\ \frac{kg}{m^3}}} = 3384\ \frac{m}{s}$$ speed of sound in brass



$x = 1\ m$

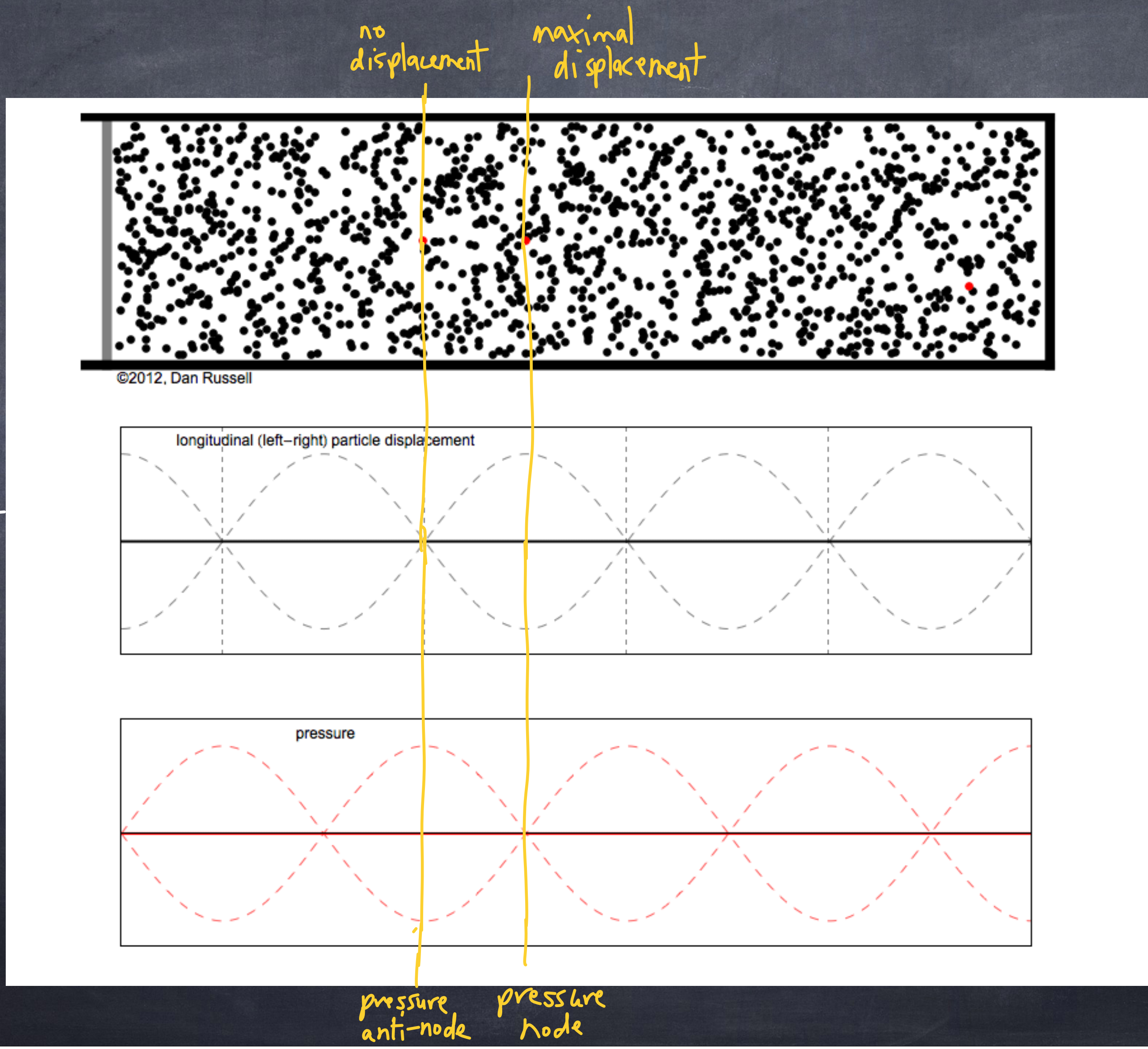
$t =$

$v =$

$v_{\text{sound in brass}} =$

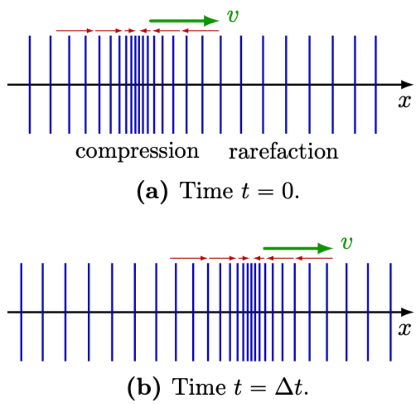
we can create standing waves of sound in a tube

Here, with a piston:



Displacement

Pressure 90° out of phase



(a) Time $t = 0$.

(b) Time $t = \Delta t$.

Figure 13.10: A traveling longitudinal wave is when the distortion happens along the direction of propagation, here shown as a local displacement.

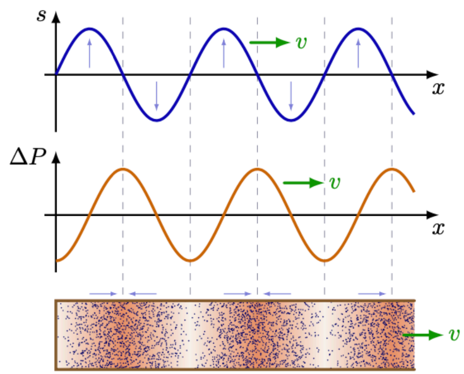


Figure 13.11: Sound wave traveling in a tube of air, shown as a local, average displacement s of air molecules in the longitudinal (x) direction (blue), and a local pressure variation ΔP (orange), 90° out of phase with s.

Displacement $S = S_0 \sin(kx - \omega t)$

maximal displacement

90° phase difference

ΔP = pressure change $\equiv P = P_0 \sin\left(kx - \omega t - \frac{\pi}{2}\right)$

$P_0 = \rho \omega v S_0$

ρ: density of medium

ω: angular frequency

v: speed of sound

Important for today

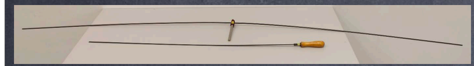
node: position where pressure does not change

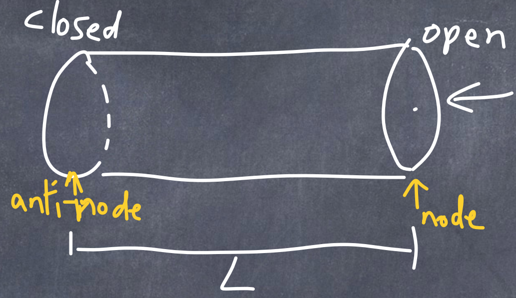
anti-node: position where pressure changes maximally

oscillates between $p + \Delta p$ to $p - \Delta p$

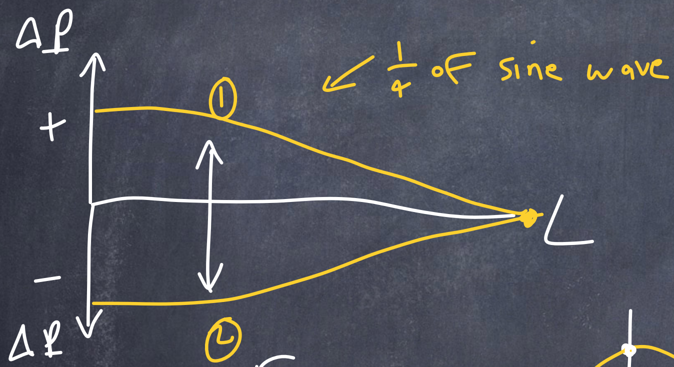
Standing waves in a tube closed on one end, open on the other end.

similar to this:





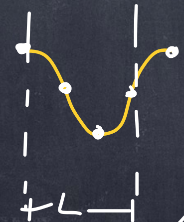
P_{atm} = boundary condition that forces the open end to always have P_{atm} (this is a pressure node)



For $n=1$

$\frac{1}{4}\lambda_1 = L \quad 4L = \lambda_1 \Rightarrow f_1 = \frac{v}{\lambda_1} = \frac{v}{4L}$

For $n=3$



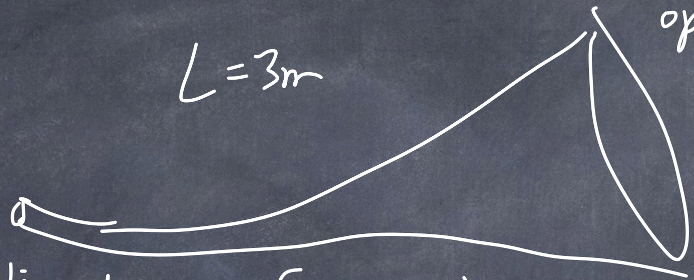
$\frac{3}{4}\lambda_3 = L \quad \frac{4L}{3} = \lambda_3 \Rightarrow f_3 = \frac{v}{\lambda_3} = \frac{3}{4}\frac{v}{L}$

In general $\lambda_n = \frac{4L}{n} \quad f_n = \frac{v}{\lambda_n} \quad n = 1, 3, 5, \ldots$

Alphorn

closed end
P changes
(vibrates on lips)
anti-node

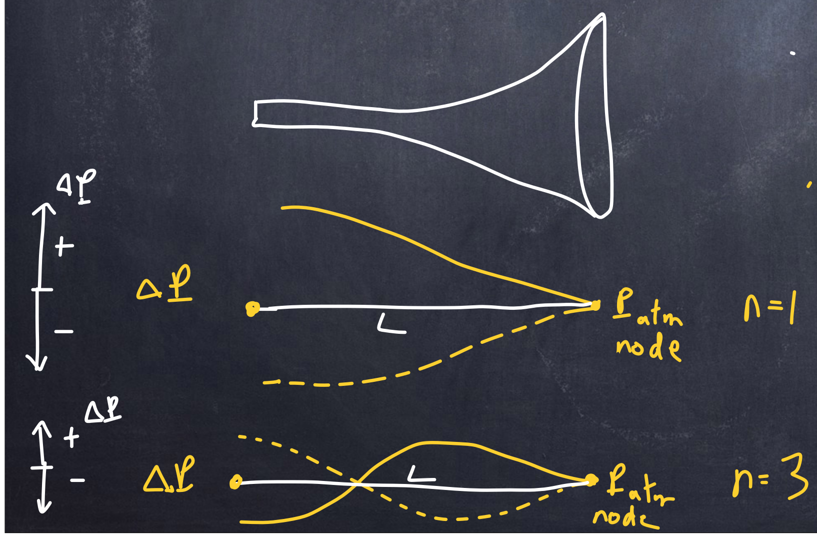
$L = 3\,m$



open end = node
$P = P_{atm}$

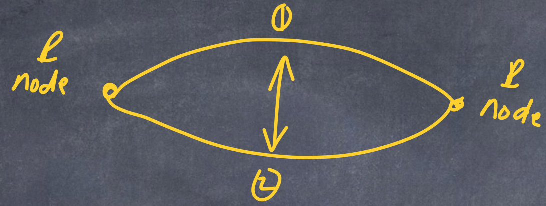
Fundamental frequency for Alphorn $= f_1 = \frac{v}{\lambda_1} = \frac{v}{4L} = \frac{340\,m/s}{4(3m)} = 28\ Hz$

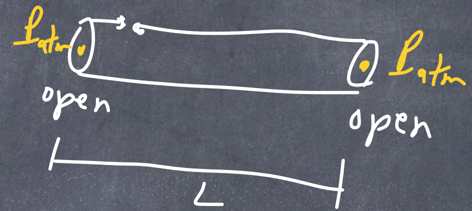
Next, $f_3 = \frac{3v}{4L} = 84\ Hz$



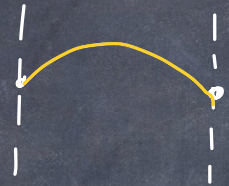
Standing sound waves, tube open on both ends (flute)

For $n=1$, length, L



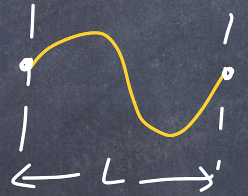


$n=1$



$$L = \frac{\lambda_1}{2}$$

$n=2$



$$L = \lambda_2$$

In general,

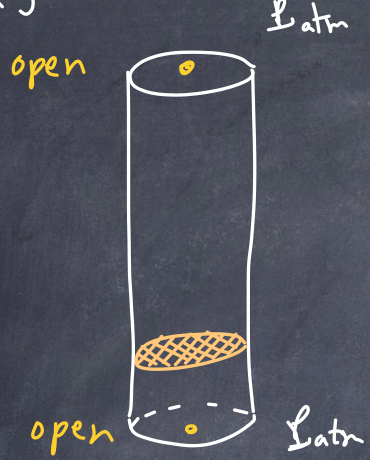
$$\lambda_n = \frac{2L}{n} \qquad n = 1, 2, \ldots$$

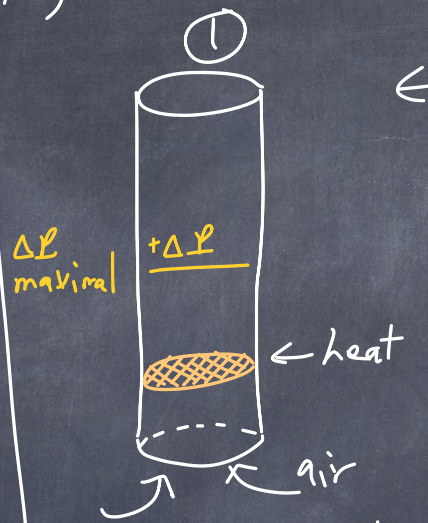
$$f_n = \frac{v}{\lambda_n} \qquad v = v_{\text{sound in air}}$$

flute

Finger positions excite different harmonics (fingers put anti-nodes)

Rijke tube – self-amplifying standing sound waves.





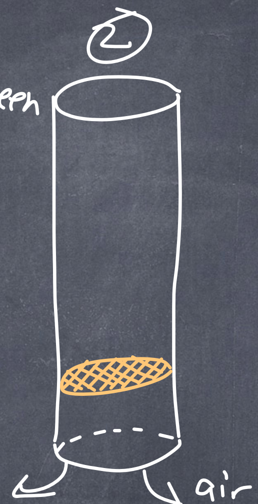
$\frac{1}{2}$ of the cycle,

air comes in and create a high pressure region in center

oscillation between ① + ②

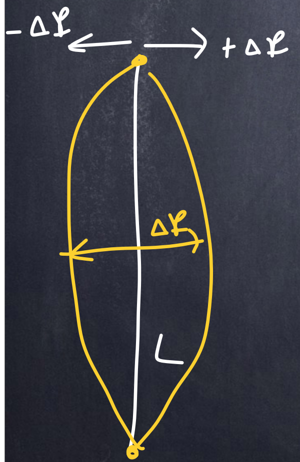
$pV = nRT$

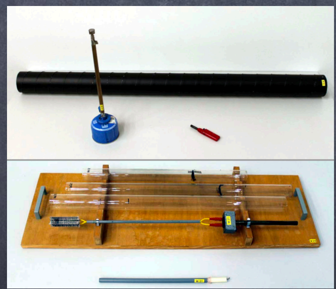
$\Delta p \sim \Delta T$

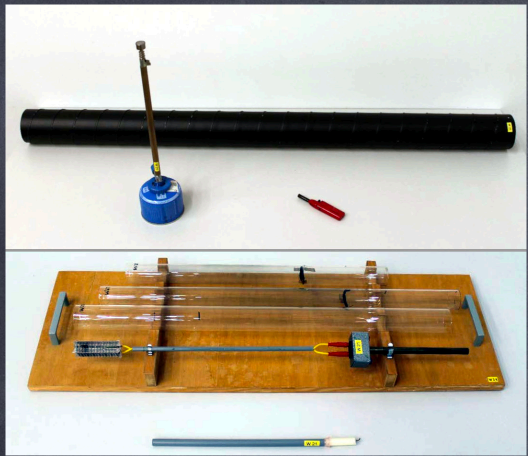


$\frac{1}{2}$ of cycle,

air flows away from high pressure region, away from center, low pressure in center







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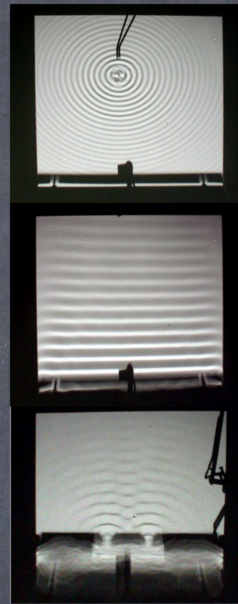

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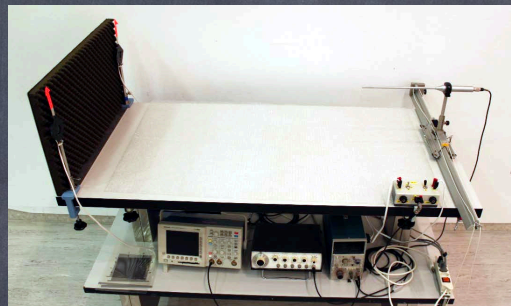

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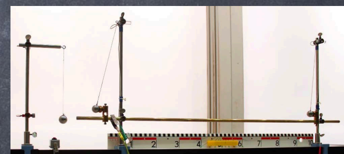

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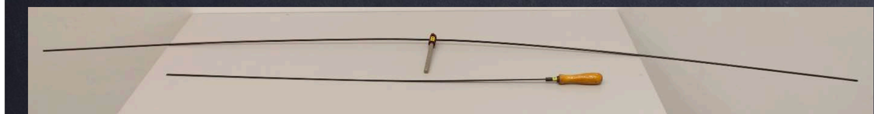

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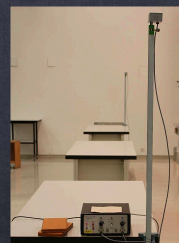

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