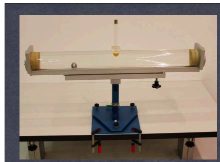
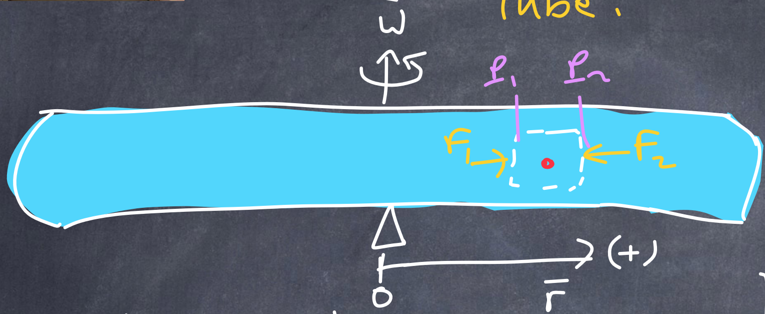
PHY 117 HS2024

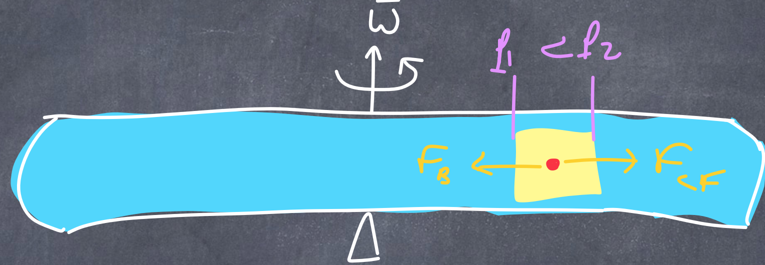
Today:

Week 5, Lecture 2
Oct. 16th, 2024
Prof. Ben Kilminster



consider empty spinning tube:





The buoyant force comes from the centripetal force of the mass of fluid that is displaced

$$F_B = F_1 - F_2$$

$$F_B = m_\ell \frac{v^2}{r} = m_\ell \omega^2 r(-\hat{r})$$

$\therefore$ F_B is bigger at large r

An object in the centrifuge feels a buoyant force and an apparent force the "centrifugal pseudoforce" outward

$$F_{cF} = m_o \omega^2 r(\hat{r})$$

$$\Sigma F = F_c - F_B = m_o \omega^2 r - m_\ell \omega^2 r = V\rho_o \omega^2 r - V\rho_\ell \omega^2 r = V\omega^2 r(\rho_o - \rho_\ell)\,\hat{r}$$

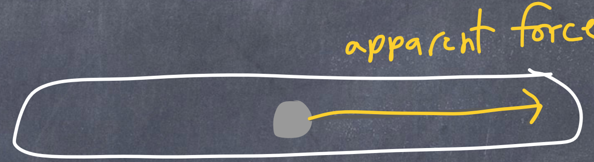
If $\rho_o > \rho_\ell$, the force is outward in $(+\hat{r})$ director

If $\rho_o < \rho_\ell$, force is inward $(-\hat{r})$

why did I call it a pseudoforce?

Because we are viewing the object in its reference frame, we see this apparent pseudoforce.

In the rest frame of the tube

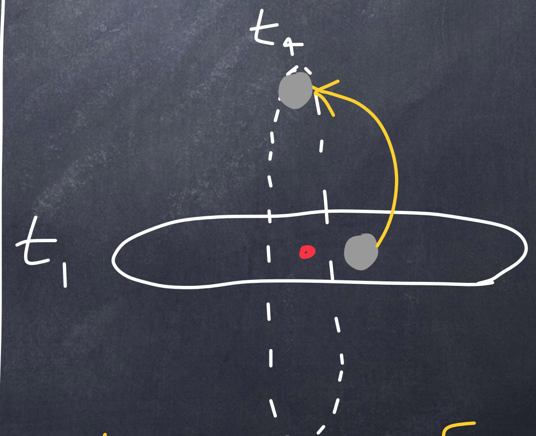


But from above (in a fixed frame), we see that:

Ball slides outward

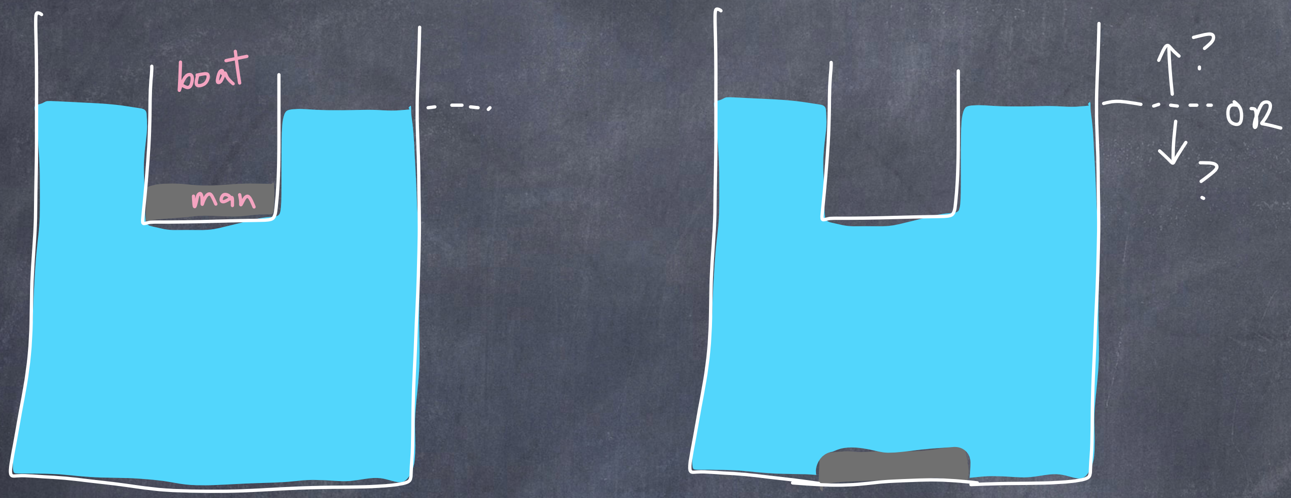
At t_4, ball now moves in a circle with wall providing $F_{centripetal}$

Movement is:



due to force of glass on ball

Man overboard !



Fluids In Motion!

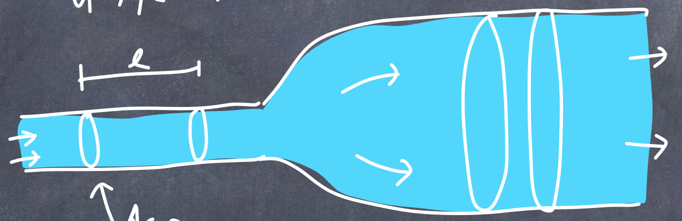
pipe with fluid inside

velocity of fluid v_1 incoming

volume V coming in time Δt

$V_1 = A_1 \ell = A_1 v_1 \Delta t$

volume $V_2 = A_2 v_2 \Delta t$



Area A_1

area, A_2

v_2: velocity outgoing

Since fluids are incompressible, $V_1 = V_2$

$$A_1 v_1 \cancel{\Delta t} = A_2 v_2 \cancel{\Delta t}$$

$$A_1 v_1 = A_2 v_2 = \text{constant}$$

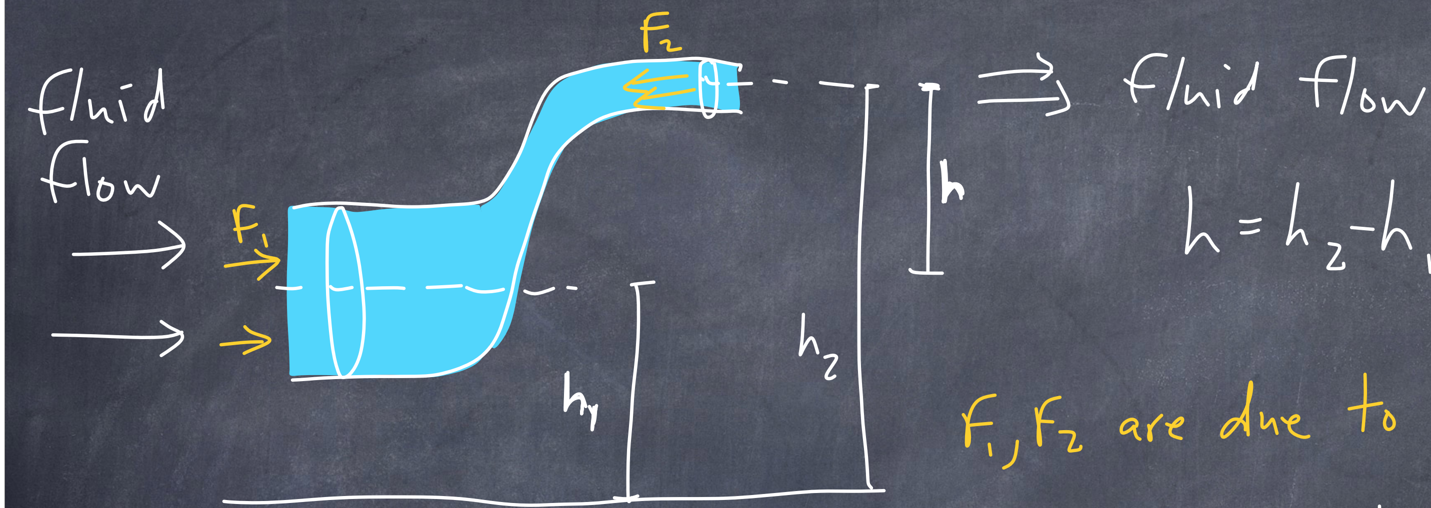
(tube changes area)

$$I_v \equiv A v : \left[m^2 \frac{m}{s}\right] \left[\frac{m^3}{s}\right]$$ volume flow rate

$$\boxed{I_v = vA = \text{constant}}$$ continuity equation

If A gets bigger, then v gets smaller

What if it changes height?



$h = h_2 - h_1$

F_1, F_2 are due to fluid pressure

Fluid gain potential energy, must lose kinetic energy.

In some time Δt, some $\underbrace{\text{amount of fluid}}_{\Delta m:\ \text{mass}}$ gets lifted by a height, h.

Change in potential energy $= \Delta U = \Delta m g h = \rho \Delta V g h$

ΔV: volume of fluid being lifted

ΔK = change in kinetic energy $= \frac{1}{2}\Delta m v_2^2 - \frac{1}{2}\Delta m v_1^2$

$$= \frac{1}{2}\rho \Delta V \left(v_2^2 - v_1^2\right)$$

The work-energy theorem states that

$$\boxed{W_{TOTAL} = \Delta U + \Delta K}$$

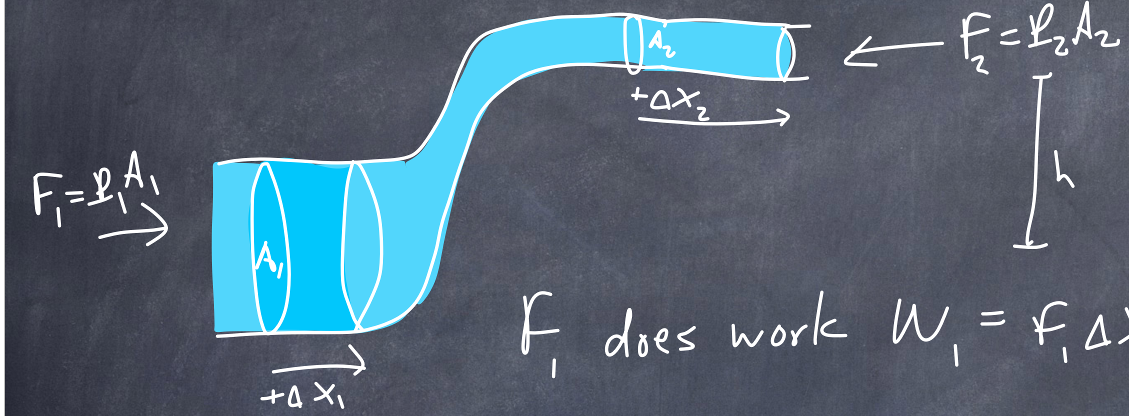
work done by fluid

$$W_{TOTAL} = \rho \Delta V g h + \frac{1}{2}\rho \Delta V \left(v_2^2 - v_1^2\right) \quad (1)$$

we know that work comes from a force times a distance. The force, F_1 comes from pressure, P_1 and F_2 comes from pressure at top, P_2.

What if it changes height?

Fluid is pushed with F_1 by fluid pressure to its left and pushed back with F_2 by the fluid to the right



F_1 does work $W_1 = F_1 \Delta x_1 = P_1 \underbrace{A_1 \Delta x_1}_{\text{volume of cylinder}} = P_1 \Delta V$

same volumes

F_2 does work $W_2 = (-F_2)\Delta x_2 = -P_2 \Delta V$

The total work done is $W_{TOTAL} = W_1 + W_2$

$= P_1 \Delta V - P_2 \Delta V$

$$\boxed{W_{TOTAL} = (P_1 - P_2)\,\Delta V} \quad ②$$

we combine ① + ②

$$(P_1 - P_2)\cancel{\Delta V} = \rho \cancel{\Delta V} g h + \frac{1}{2}\rho \cancel{\Delta V}(v_2^2 - v_1^2)$$

$$P_1 - P_2 = \rho g (h_2 - h_1) + \frac{1}{2}\rho (v_2^2 - v_1^2)$$

we move our terms:

$$\underbrace{P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2}_{\text{position 1}} = \underbrace{P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2}_{\text{position 2}}$$

In other words,

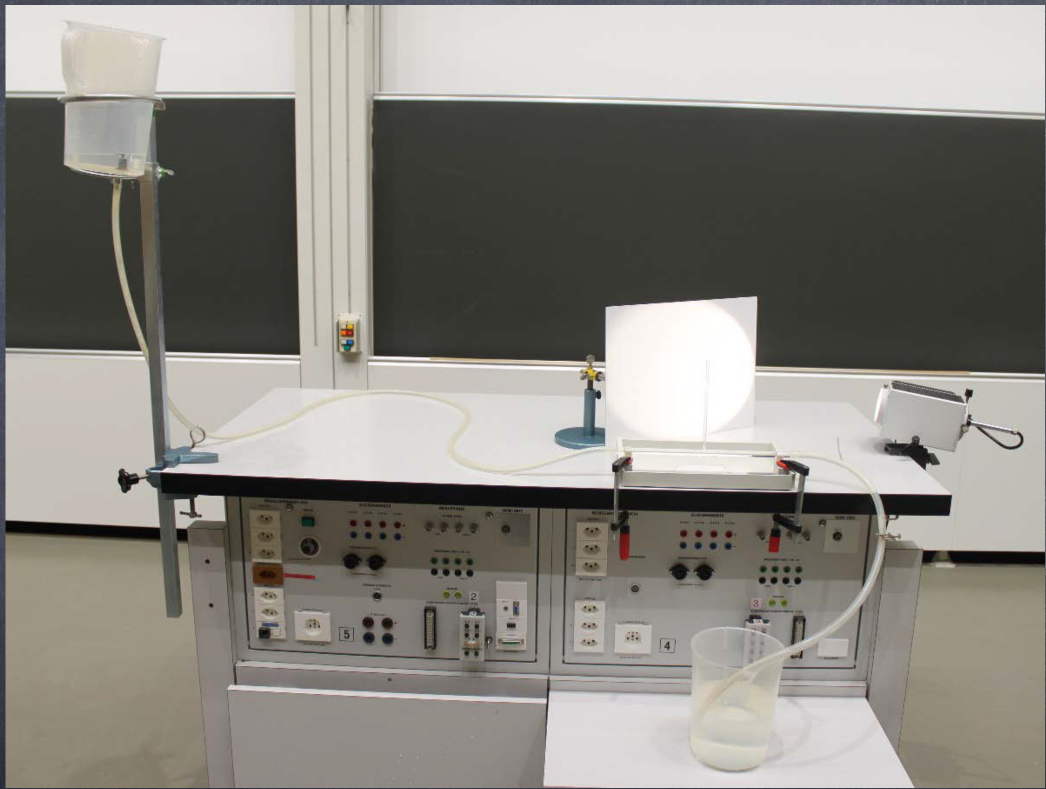
$$\boxed{P + \rho g h + \frac{1}{2}\rho v^2 = \text{constant}}$$

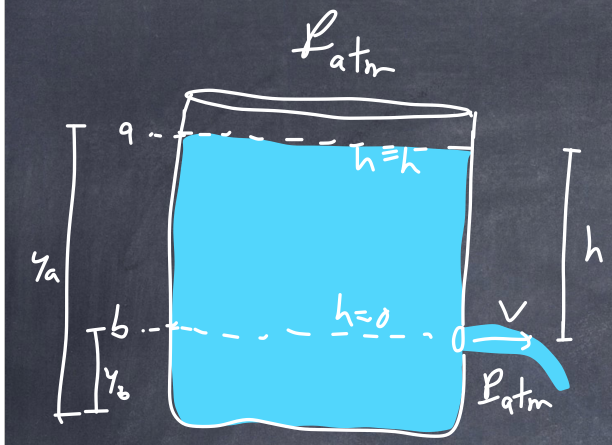
Bernoulli's equation.

- every term has units of pressure.
- neglects friction

→ This combination of quantities stays constant while height, area, velocity, U, K, P changes.

This is a statement of energy conservation.





$P_a = P_b = P_{atm}$

Tank with a hole in it, open at top.

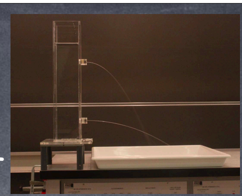


Figure out the velocity of water

we apply Bernoulli's equation at heights a + b.

Level a = Level b

$$P_a + \rho g h + \tfrac{1}{2}\rho v^2 = P_b + \rho g h + \tfrac{1}{2}\rho v^2$$

$$\cancel{P_{atm}} + \rho g h + \tfrac{1}{2}\rho \cancel{v_a^2}^{\,0} = \cancel{P_{atm}} + \rho g \cancel{h}^{\,0} + \tfrac{1}{2}\rho v_b^2$$

$$\cancel{\rho} g h = \tfrac{1}{2}\cancel{\rho} v_b^2$$

$v_b = v$ of water exiting the hole.

Approximation: $v_a = 0$. Valid if the opening at the top is much larger than the hole at a.

$$\boxed{v = \sqrt{2gh}}$$

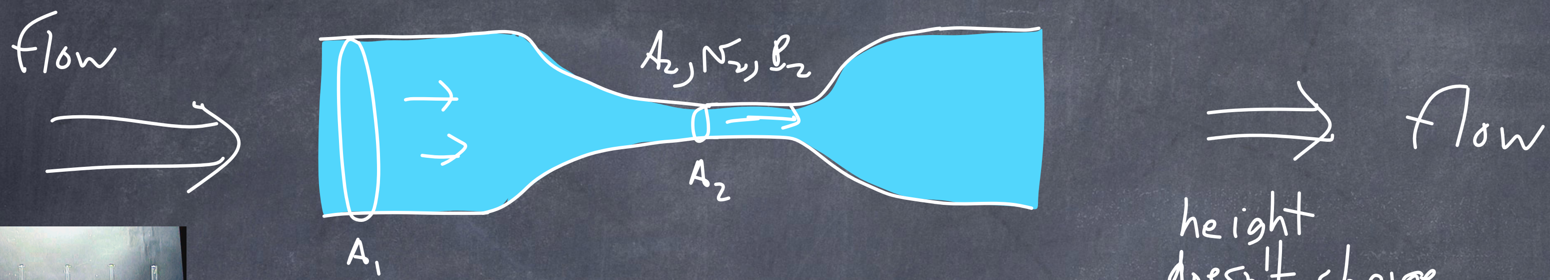
Torricelli's Law

Imagine an object falling a distance x.

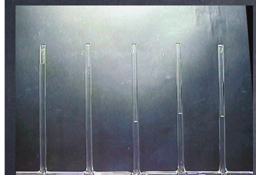
$$v^2 = v_0^2 + 2ax \Rightarrow \quad \text{If } a = g,\ v_0 = 0,\ x = h$$

$$v^2 = 2gh \Rightarrow v = \sqrt{2gh}$$

Fluid moving in a pipe with changing area



height doesn't change



A_1, v_1, P_1

we know that $A_1 v_1 = A_2 v_2$ so $v_2 = v_1\left(\frac{A_1}{A_2}\right)$

Bernoulli's equation becomes: $P + \frac{1}{2}\rho v^2 = \text{constant}$
(constant height)

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

since $v_2 > v_1$, then it must be that

$$P_2 < P_1$$

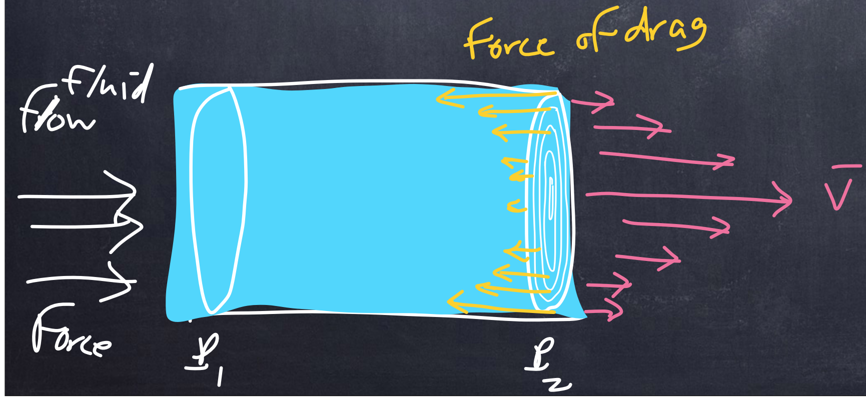
Venturi effect: when the speed of a fluid increases, then the pressure gets smaller.

Current of Fluid, I_V moves with "viscous" flow.

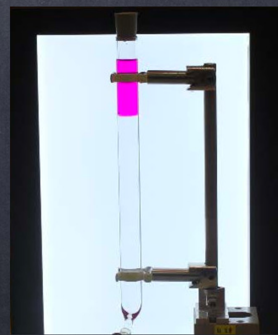
Bernoulli's equation states that the pressure is the same anywhere in a pipe at constant height and area.

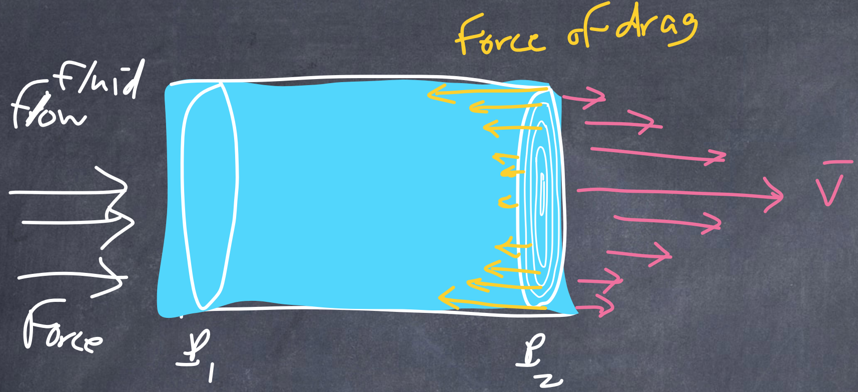
$P_1 = P_2$

In practice, we see a pressure drop. The pressure drop comes from a drag force from the surface of the pipe on the fluid, but also from each layer of fluid on the next layer.



velocity is faster in center of tube.





velocity faster in center

Pressure drop from P_1 to P_2

$$P_1 - P_2 = I_v R = (v A) R$$

velocity of the fluid

area of the pipe

constant of resistance

Pressure difference $P_1 - P_2 = \underbrace{I_v}_{I_v = vA} \underbrace{R}_{\text{constant of resistance}}$

The resistance for steady flow in a cylindrical pipe is

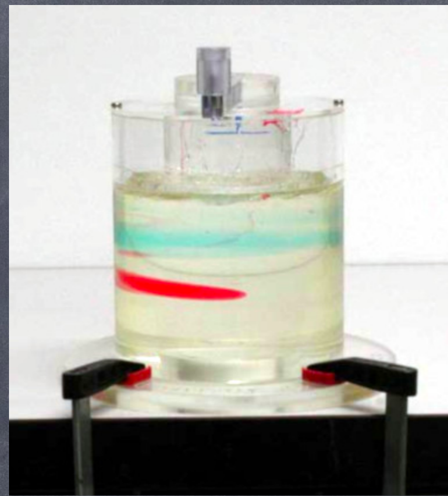
$$R = \frac{8 \eta L}{\pi r^4}$$

L: length of the pipe
r: radius of the pipe
η: coefficient of viscosity.

η: has units of $\left[\frac{N \cdot s}{m^2}\right] = [Pa \cdot s]$

$1\, Pa \cdot s = 10$ Poise

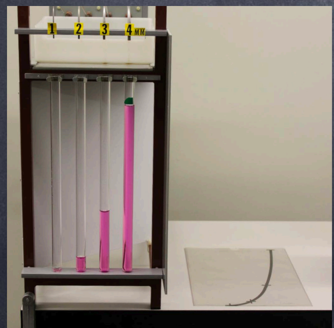

a



For our pipe,

$$\Delta P = I_V R = I_V\left(\frac{8\eta L}{\pi r^4}\right)$$

so $$\underbrace{I_V}_{vA} = \Delta P\left(\frac{\pi r^4}{8\eta L}\right)$$



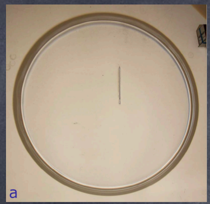
$I_V \sim r^4$

$I_V \sim \frac{1}{L}$

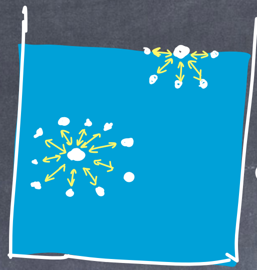
For constant A,

$v \sim \frac{1}{L}$

surface tension



molecules are attracted to each other, a cohesive force.



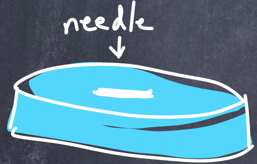
← Here we see an unbalanced force at top

← Here, equal force from all directions.

surface tension is the unbalanced force from the cohesive force of molecules below, which makes the surface like a stretchable membrane.

side view

surface is like a stretched membrane.



There is a restoring force F_{ST}, proportional to the total length of object

$$F_{ST} = \gamma L$$

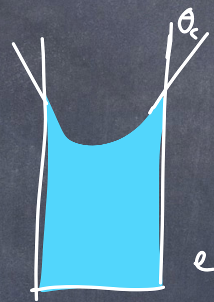
where γ is a constant that depends on the fluid & its temperature and gas in contact.

$\gamma = 0.073\ \frac{N}{m}$ for water at room temperature.

One consequence of this is capillary action.

This comes from the adhesive & cohesive forces. The adhesive force is between the fluid and the walls of the container.

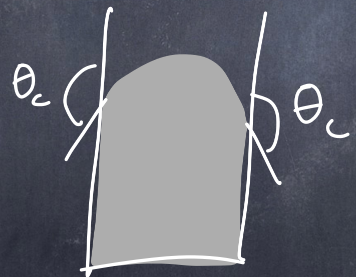
θ_c : contact angle



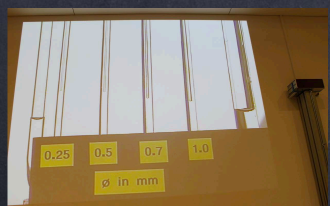
If adhesive forces > cohesive forces.
This shape

is known as the meniscus.

e.g. water + glass
$\theta_c = 0°$



If adhesive forces < cohesive forces,
e.g. mercury & glass, ($\theta_c = 140°$)

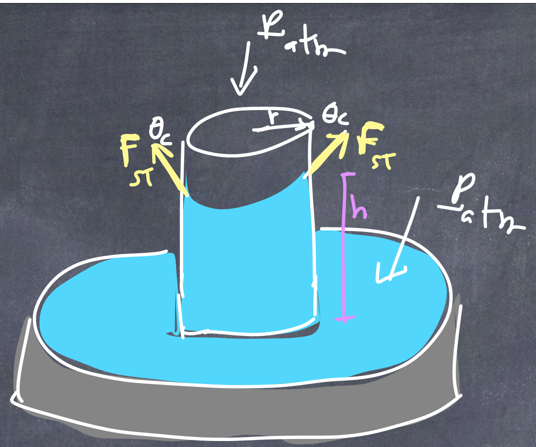


θ_c : measure, depends on the fluid and the container

Consider cylinder, radius r

open on top + bottom

(+)



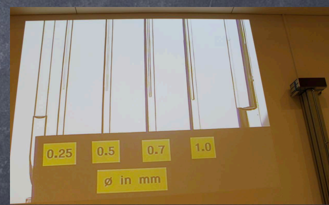
Adhesive force pulls fluid upward

F_{ST} : adhesive force pulling upward.

$$h = \frac{2\gamma\cos\theta_c}{\rho r g}$$: the height that the adhesive force raises a fluid in a container.

when $\theta_c < 90^\circ$, $\cos\theta_c > 0 \Rightarrow h$ is (+)

when $\theta_c > 90^\circ$, $\cos\theta_c < 0 \Rightarrow h$ is (−)



Derivation of height formula in the backup slides.

end

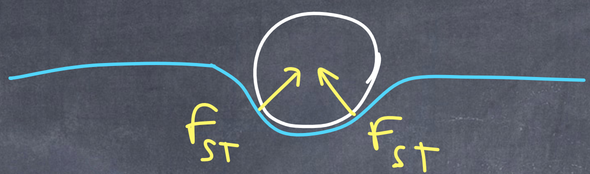
After this, there are a few derivations for your information.

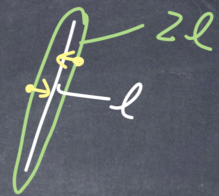
There is a surface force on both sides of the needle, so

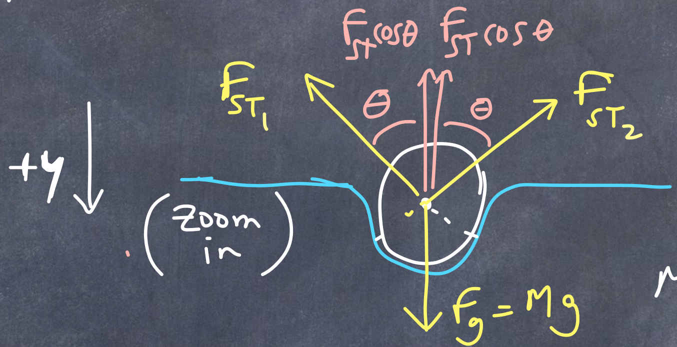
$$F_{ST} = \gamma L = \gamma 2\ell$$

ℓ, length of the needle

side view needle







$\longleftrightarrow$ in horizontal direction, these forces cancel out.

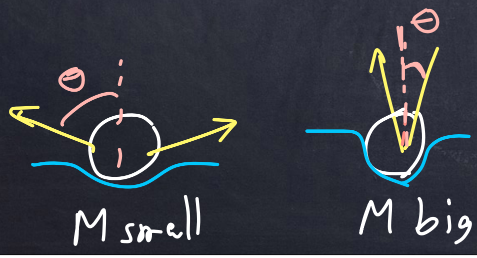
$F_{ST} \rightarrow$

M: total mass of needle

In the y-direction, the total surface tension is

$$F_{ST_y} = \overset{(\gamma\ell\cos\theta)}{F_{ST_1}\cos\theta} + \overset{(\gamma\ell\cos\theta)}{F_{ST_2}\cos\theta} = 2F_{ST}\cos\theta = 2\gamma\ell\cos\theta$$

The needle floats as long as $F_{ST_y} > F_g$



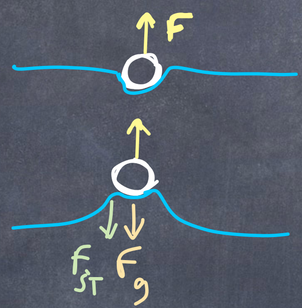
As M gets larger, θ decreases.

When $\theta = 0°$, $\cos\theta = 1 \Rightarrow F_{ST} = 2\gamma\ell$

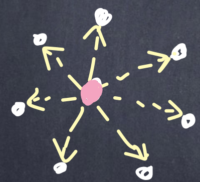
The maximum mass allowed is when

$Mg = 2\gamma\ell\cos 0°$ $\quad m_{max} = 2\gamma\ell/g$

The force to lift the needle off the surface is $F = mg + \gamma 2L$



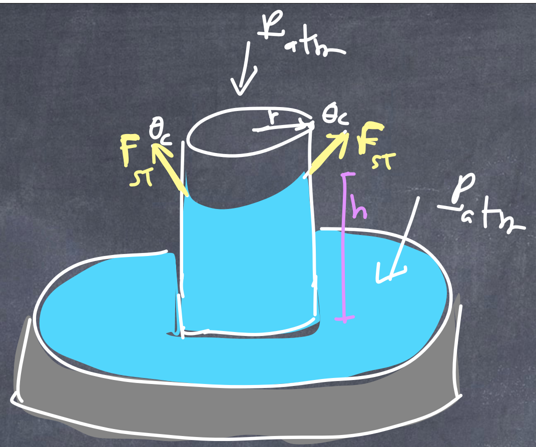
In this case, the surface tension resists us pulling the needle up, because we are stretching the fluid membrane upward.



cohesive force on one molecule is coming from the surrounding molecules.

Consider cylinder, radius r

open on top + bottom



$(+)\uparrow$

Adhesive force pulls fluid upward

F_{ST} : adhesive force pulling upward

vertical direction $\left\{ \sum F = F_{ST}\cos\theta_c - mg = 0 \right.$

$$\gamma L \cos\theta_c = mg$$

$$\gamma 2\pi r \cos\theta_c = \rho V g$$

$$\gamma 2\cancel{\pi}\cancel{r}\cos\theta_c = \rho(\cancel{\pi} r^{\cancel{2}} h)g$$

what is L? It's the length of contact between fluid + container

$L = 2\pi r$

$$h = \frac{2\gamma\cos\theta_c}{\rho r g}$$

: The height that the adhesive force raises a fluid in a container

when $\theta_c < 90^\circ$, $\cos\theta_c > 0 \Rightarrow h$ is $(+)$

when $\theta_c > 90^\circ$, $\cos\theta_c < 0 \Rightarrow h$ is $(-)$