PHY117 HS2024

Week 6, Lecture 2
Oct. 23rd, 2024
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The ideal gas law is an approximation, assuming the molecules are point particles (no size), and it neglects the force between the molecules.

$$P \quad V \quad = \quad nRT$$

More accurate:

$$\left(P + \frac{an^2}{V^2}\right)(V - bn) = nRT$$

Van der Waals equation

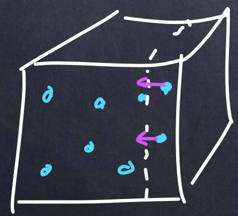
↑ correction to the volume

b: volume of 1 mole of molecules

bn: volume of n moles of molecules

a: constant that depends on the attractive molecular forces.

There is a decrease in pressure against the walls of our container due to molecular forces.

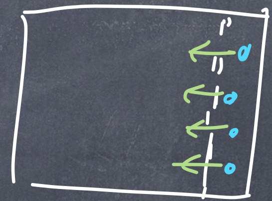


each molecule near the right wall feels a net attractive force from the molecules to the left.

The force pulling on one molecule near the wall is proportional to the density on the molecules $\sim \frac{n}{V}$

The number of molecules near the wall is also proportional to the density of molecules $\sim \frac{n}{V}$

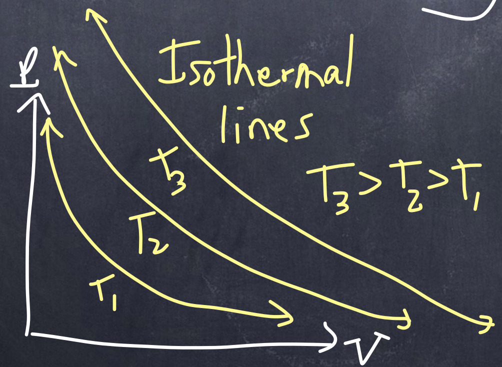
So the total force is $\sim \frac{n^2}{V^2}$



Note: Is the pressure more or less due to the attractive forces?

$$P = \frac{nRT}{V-bn} - \frac{an^2}{V^2}$$

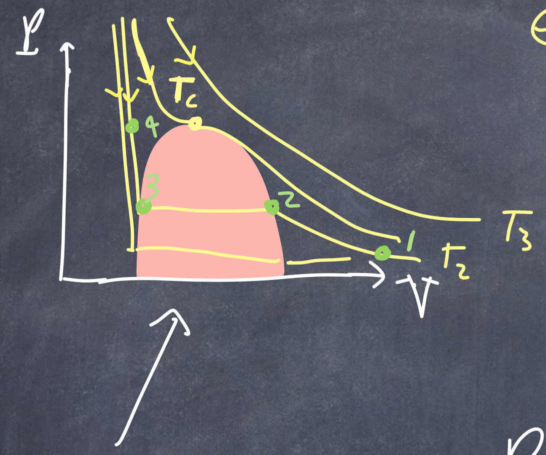
Ideal gas law



The ideal gas law is only valid above a certain temperature,

T_c : critical temperature.

Above T_c, the behavior of the gas is described by the Van der Waals equation. But below T_c, we see something different.



Each curve is for a constant temperature

Starting at low pressure & high volume at ①, we compress the gas. The pressure rises to ② as expected from the gas law.

But between ② and ③, pressure stops rising and between ③ & ④, it increases sharply. Why?

② → ③ : gas begins to liquefy (both gas & liquid exist)

③ → ④ : only liquid exists. Since liquid is nearly incompressible, pressure increases rapidly while volume changes little.

Relationship between temperature and heat

Heat : is a form of energy.

We can add heat or remove heat.

symbol, Q.

No work done → $Q = mc\,\Delta T$

- Q: heat added [J]
- m: mass of a substance [kg]
- c: specific heat of a substance
- ΔT: temperature change, $\Delta T = T_f - T_i$ [K]

$$c : \left[\frac{J}{kg \cdot K}\right]$$

In a few slides, we find: $Q = \Delta U + W$.

Here, if no work done $Q = \Delta U = mc\Delta T$.

substances	$c\ [\mathrm{J/kg\cdot K}]$	$C_m\ [\mathrm{J/mol\cdot K}]$
copper	386	24.5
aluminum	900	24.2
silicon	710	42.2
water	**4186**	75.3
pine wood	1500	
oak wood	2400	

$$Q = mc\Delta T$$

$$Q = nC_m\Delta T$$

This means water is good at storing heat energy, and only changes temperature slightly.

↑ forests are also good at moderating temperature

A big lake moderates temperature changes nearby. Keeps summers cooler + winters warmer

$$Q_{cu} + Q_{water} = 0$$

$mc(T_F - T_i)$ heat gained

$1\,mol = 63.55\,g$

$T_i = -196°C$

$c = 0.386\ J/kg \cdot K$

$mc(T_i - T_F)$ heat lost

$60\,ml$ water $= 60\,g$

$T_i = 25°C$

$c = 4.186\ J/kg \cdot K$

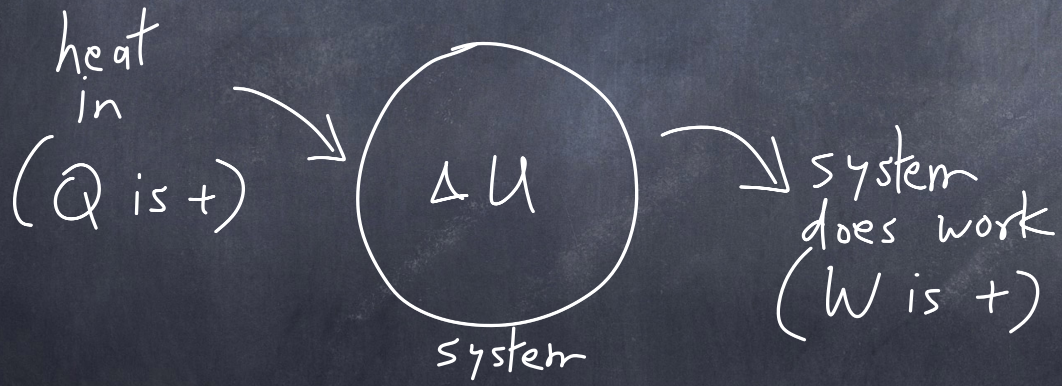
what did we omit?

Since heat is a form of energy, we can use heat to do work.

1st law of thermodynamics: a statement of energy conservation:

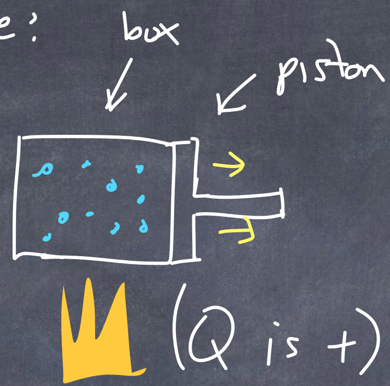
$$Q = \Delta U + W$$

- Q: heat added to a system
- ΔU: change in internal energy of the system
- W: work done by the system.



For example:

heat gas



(Q is +)

This causes the piston to move out because the gas expands.

The work is +, (the work done by the system.)

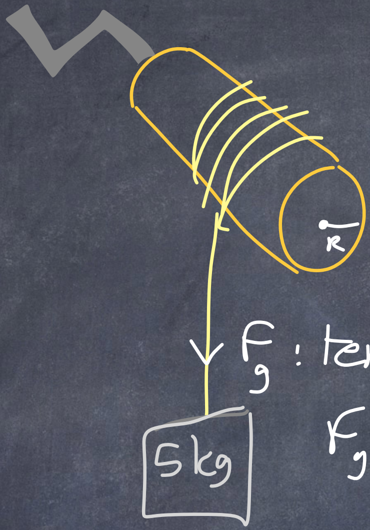
ΔU would increase (+)

Example: 3 kg of water at 80°C. We stir it with 15 kJ of energy. We also remove 50 kJ of heat. What is the final temperature?

The change in internal energy is (−).
The temperature will decrease.

For water, the volume & pressure are not changing so ΔU only changes temperature:

Can we use torque to increase temperature?



F_g : tension string

$F_g = Mg$

$F_f = F_g$

$\bar{\tau} = \bar{r} \times \bar{F} = RMg$

The work done on the cylinder to rotate it N times

$$W = \int_0^{2\pi N} \tau \, d\theta$$

$$W = \int_0^{2\pi N} RMg \, d\theta = MgR\theta \Big]_0^{2\pi N} = MgR\,2\pi N$$

work done on the system.

$$Q = \Delta U + W$$

Q : no heat added $= 0$

$$\Delta U = -W = -(-MgR\,2\pi N) = +MgR\,2\pi N$$

ΔU is the increase in internal energy of water + copper

$$(mc\Delta T)_{water} + (mc\Delta T)_{copper} = MgR\,2\pi N$$

$$\Delta U = (mc\Delta T)_{water} + (mc\Delta T)_{copper}$$

at equilibrium, $\Delta T_{water} = \Delta T_{copper}$

$c_w : 4186 \frac{J}{kg \cdot K}$ $\qquad$ $c_c : 386 \ J/kg \cdot K$

$$\Delta U = \Delta T (m_w c_w + m_c c_c) = MgR 2\pi N$$

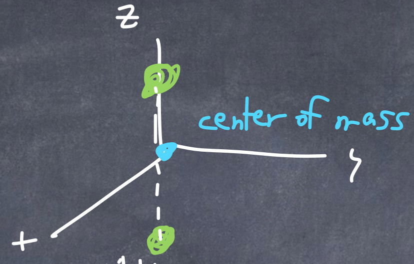
(m_c small $\rightarrow 0$)

$$\Delta T = \frac{2\pi N M g R}{m_w c_w}$$

Remember! a molecule has $\frac{1}{2}kT$ of kinetic energy per degree of freedom.
(or $\frac{1}{2}RT$ per mole)

Equipartion theorem: when a substance is in equilibrium, there is an average energy of $\frac{1}{2}kT$ per molecule or $\frac{1}{2}RT$ per mole associated with each degree of freedom. The total is called the internal energy, U.

Consider a diatomic molecule in a gas (N_2, O_2, N_2 ...)
(at constant volume)



It can rotate around the x-axis or the y-axis so it has rotational kinetic energy.

z-direction has little I_z so we ignore.

$$K_{rot} = \frac{1}{2} I_x \omega_x^2 + \frac{1}{2} I_y \omega_y^2$$

The total kinetic energy is then:

For 1 molecule

$$K = \underbrace{\tfrac{1}{2} m v_x^2}_{\frac{1}{2}kT} + \underbrace{\tfrac{1}{2} m v_y^2}_{\frac{1}{2}kT} + \underbrace{\tfrac{1}{2} m v_z^2}_{\frac{1}{2}kT} + \underbrace{\tfrac{1}{2} I_x \omega_x^2}_{\frac{1}{2}kT} + \underbrace{\tfrac{1}{2} I_y \omega_y^2}_{\frac{1}{2}kT}$$

For N molecules, with 5 degrees of freedom,

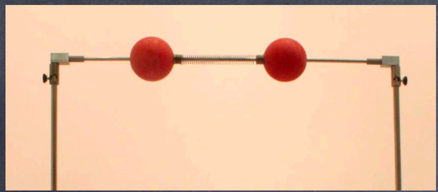
$$U = 5N\left(\tfrac{1}{2}kT\right) = \frac{5}{2} NkT = \frac{5}{2} nRT$$

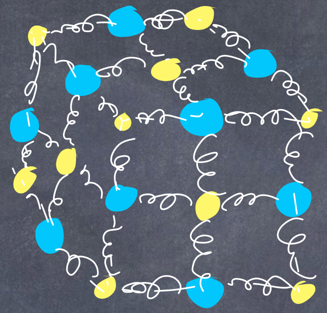
Note: if we increase U, we increase T

$$\Delta U = \frac{5}{2} nR\Delta T \Rightarrow \frac{\Delta U}{\Delta T} = \frac{5}{2} nR = C_V$$

heat capacity of diatomic gas at constant volume.

Likewise, for a solid, such as NaCl





Atoms are held together bound like springs.

$$K = \underbrace{\tfrac{1}{2}mv_x^2 + \tfrac{1}{2}mv_y^2 + \tfrac{1}{2}mv_z^2}_{\text{translational}} + \underbrace{\tfrac{1}{2}k_s x^2 + \tfrac{1}{2}k_s y^2 + \tfrac{1}{2}k_s z^2}_{\text{Springs in 3-D}}$$

For 6 degrees of freedom:

$$U = 6 \cdot \tfrac{1}{2} nRT = 3nRT$$

$$U = 6 \cdot \tfrac{1}{2} NkT = 3NkT$$

Note: if we increase U, T increases:

$$\Delta U = 3nR\Delta T$$

So $\frac{\Delta U}{\Delta T} = 3nR$ for solids with 6 d.o.f.

$\frac{\Delta U}{\Delta T} = 3R$ per mole $\qquad R = 8.314\ J/mol \cdot K$

$= 24.9\ J/mol \cdot K$

This is very close to the value of C_m from our previous table

$C_m \cong 3R$ and $\frac{\Delta U}{\Delta T} = n\,C_m$

Be careful with units on C

C can be given in several units:

$\left[\frac{J}{mol \cdot K}\right] \quad \left[\frac{J}{kg \cdot K}\right] \quad \left[\frac{kcal}{kg \cdot {}^\circ C}\right]$

check $U: [U]$

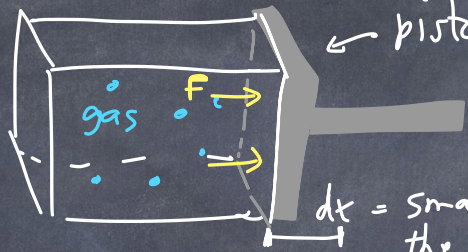
$T: [K]$

Correction to N_2 velocity we calculated yesterday:

N_2 has 2 rotational degrees of freedom we did not consider.

At room temperature, no spring degrees of freedom

work done by a gas to move a piston



← piston has area, A

dx = small distance the piston moves

$$A\Delta x = \Delta V \;\rightarrow\; A\,dx = dV$$

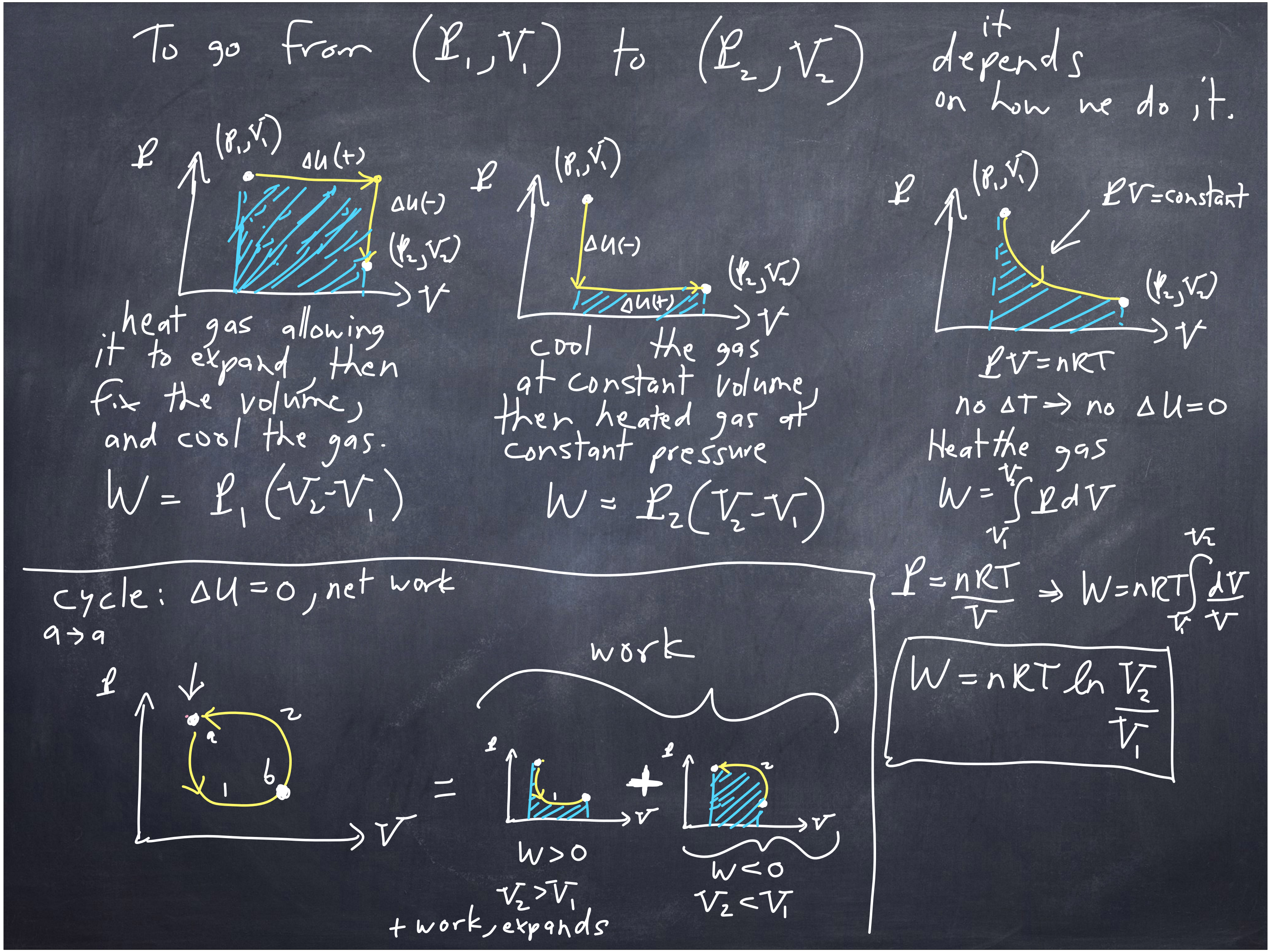
$$F = pA$$

$$dW = F\,dx = pA\,dx = p\,dV$$

$$\int dW = \int p\,dV \implies W = \int p\,dV$$

$$W = \int_{V_1}^{V_2} p\,dV$$

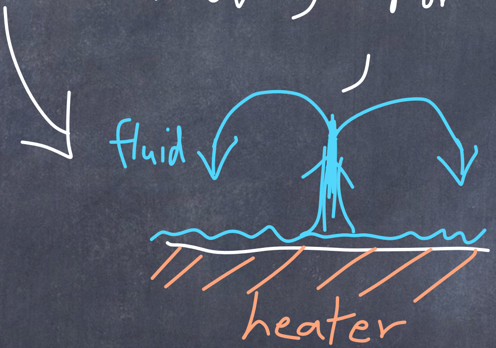
work done by a gas is the area under a p vs. V curve.

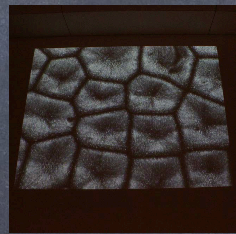

To go from (P1, V1) to (P2, V2)
it depends on how we do it.
(P1,V1)
ΔU(+)
ΔU(-)
(P2,V2)
heat gas allowing it to expand then fix the volume, and cool the gas.
W = P1 (V2 - V1)
cool the gas at constant volume, then heated gas at constant pressure
W = P2 (V2 - V1)
PV = constant
PV = nRT
no ΔT ⇒ no ΔU = 0
Heat the gas
W = ∫ from V1 to V2 P dV
P = nRT/V ⇒ W = nRT ∫ from V1 to V2 dV/V
W = nRT ln V2/V1
cycle: ΔU = 0, net work
a → a
a
b
1
2
work
W > 0
V2 > V1
+ work, expands
W < 0
V2 < V1

Transfer of thermal energy is done by 3 main processes:

conduction
convection
radiation

convection: heat transported by a mass of material moving. For instance, hot air is less dense and it rises.





radiation: energy absorbed + emitted in electromagnetic radiation (visible light, infrared light, X-rays)

Quiz 3

If the objects reaches a constant velocity (terminal velocity), gravity is still doing work on the object.	2	105	59
If the objects reaches a constant velocity (terminal velocity), there is no net work on the object.	5	77	84

Question

Which is true about inelastic collisions?

Total energy is conserved	2	89	75
Momentum of the whole system is conserved	2	100	64

Quiz 4

When torque is zero, angular momentum is zero.

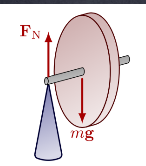
1 41 38

Question

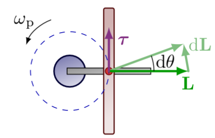
Which direction does a spinning object precess.

In the direction of the angular momentum of the spinning object.

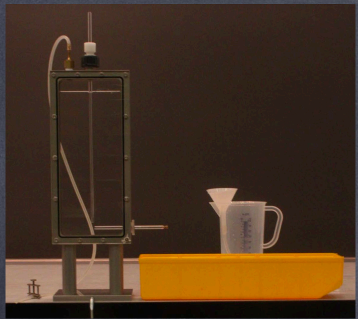
7 31 42



(a) The handle allows the disk to spin around its axis and around the pivot.

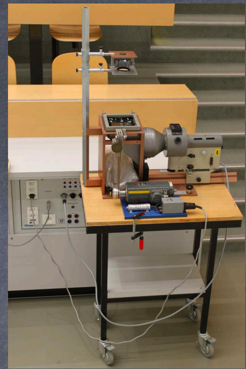


(d) Torque $\boldsymbol{\tau}$ perpendicular to angular momentum $\mathbf{L}$, will only change its direction.



H21

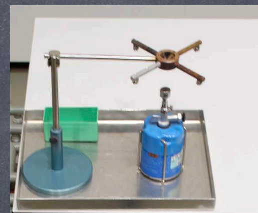
Th57



Th36

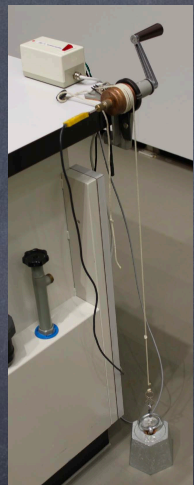
Th58

Th12



Th20

Th63



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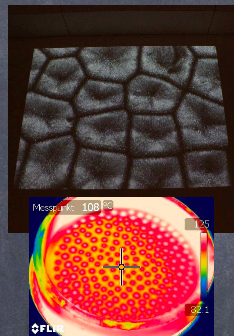
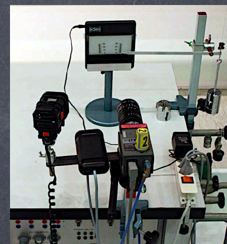
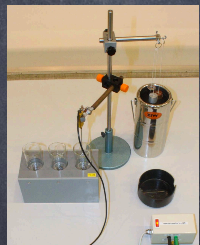


Th35

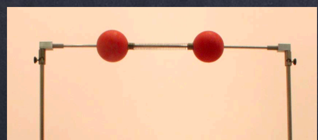


E12

Th19

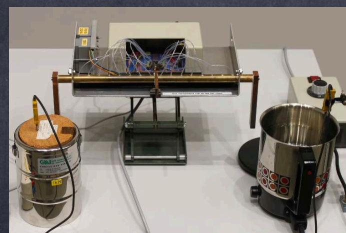


Th28

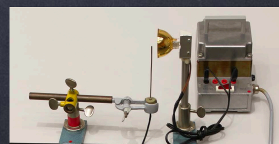


Th27

Th2



Th22



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