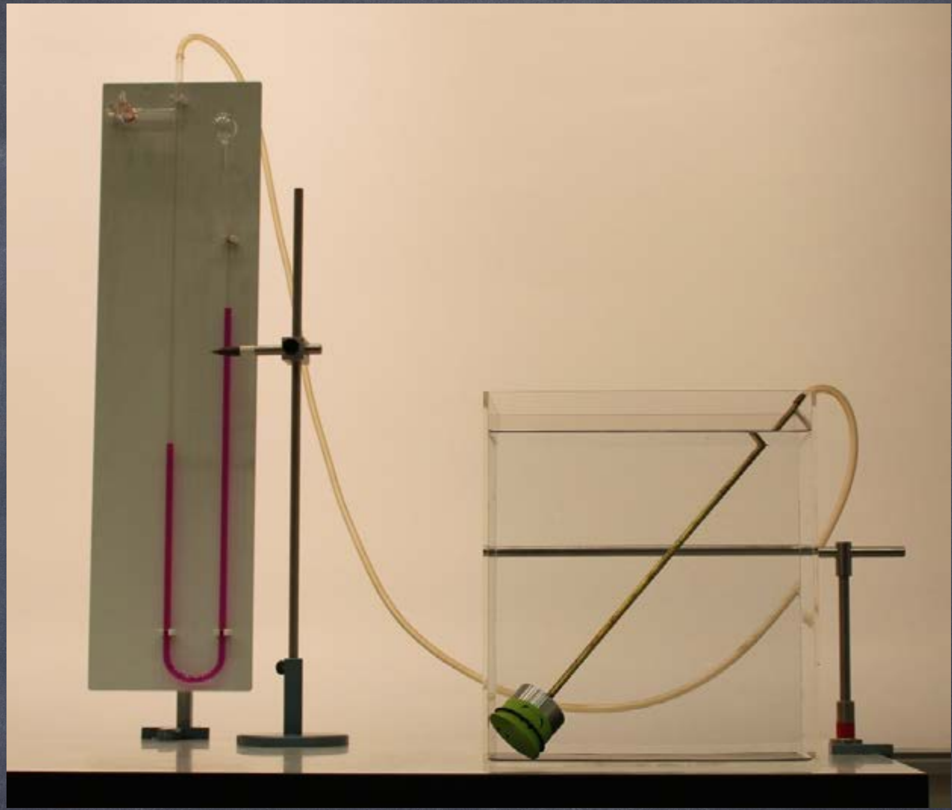
PHY 117 HS2024

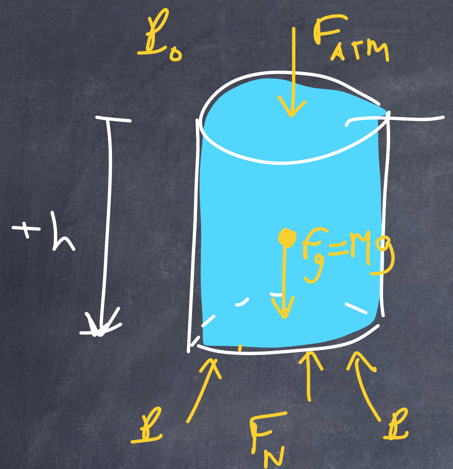
Today:

Fluids
Pressure
Pascal’s principle
Hydraulic lifts
Barometers
Manometers
Hydrometers
Archimedes’ principle
Buoyant force

Week 5, Lecture 1
Oct. 15th, 2024
Prof. Ben Kilminster



what is the pressure in a fluid?



cylinder, <u>closed</u> at bottom, open at the top.

volume $= V = \underbrace{A}_{\text{area}} \cdot \underbrace{h}_{\text{height}}$

The mass of the fluid is $M = \underset{\text{density}}{\rho}\, \underset{\text{volume}}{V}$

The weight of the fluid:

$$F_g = Mg = \rho A h g$$

The atmosphere pushes down with $F_{atm} = P_0 A$

There is normal force pushing up, F_N.

We can balance the forces:

$$F_N = F_g + F_{atm}$$

$$P \cdot \cancel{A} = \rho \cancel{A} h g + P_0 \cancel{A}$$

$$\boxed{P = P_0 + \rho g h}$$

ρ: density of fluid

g: gravity

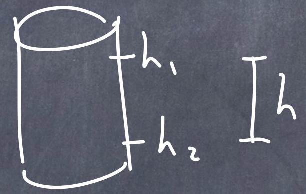
h: height of fluid

P: pressure at the bottom

P_0: pressure at the top, atm. pressure

So pressure is greater as we get deeper.
But pressure is the same at all points
with the same depth, pointing in all directions.

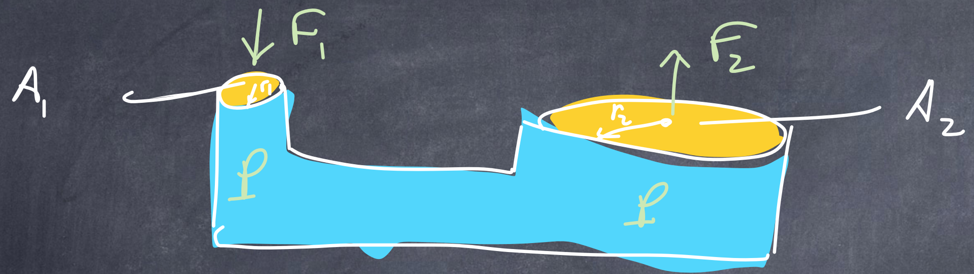
$$\boxed{\Delta P = \rho g h}$$



Pascal's principle: pressure applied to an enclosed fluid is transmitted to every point in the fluid and the walls of the container, undiminished.

same value.

This is used in a hydraulic lift.

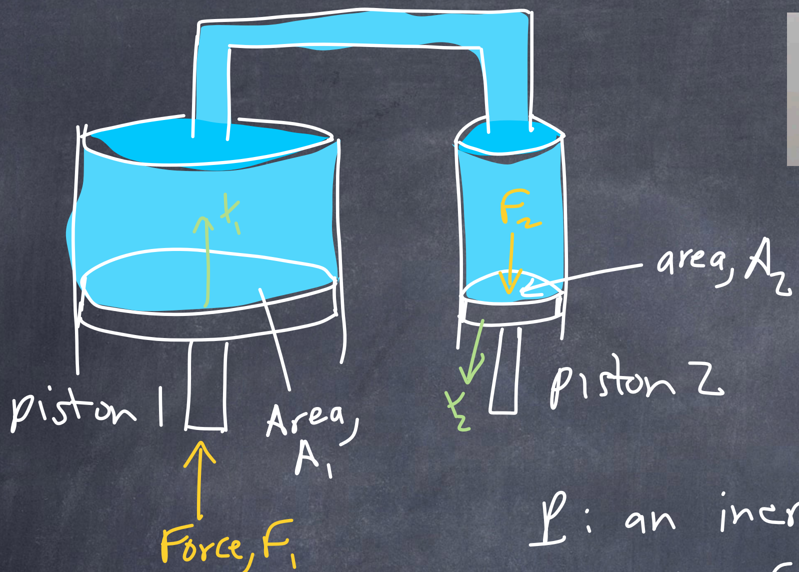


The force provides a pressure. We put a force F_1 on the small piston, and we get a different force on the large piston.

$$\rho = \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

we can lift a heavy weight with a light force.

Let's say we want to lift a car with mass 1500kg
$r_1 = 0.1\,m$ and $r_2 = 1\,m$
What force do we need to lift the car?



Pistons: freely moving, tightly fitting

Apply a force F_1 and it pushes the other piston with F_2

P: an increase in pressure from force F_1

$$P = \frac{F_1}{A_1} = \frac{F_2}{A_2} \qquad F_2 = F_1 \frac{A_2}{A_1}$$

If we move the small piston a distance x, how far does the large piston move? (we can use work to solve this)

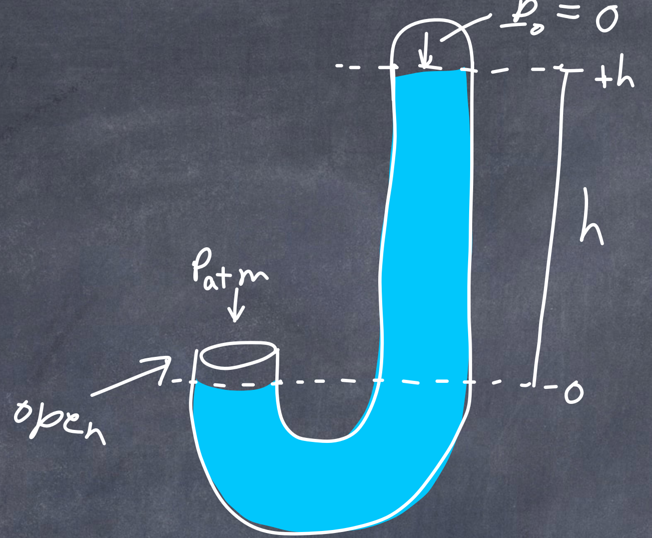
work done by piston 1 : $W_1 = F_1 x_1$

work done on piston 2 : $W_2 = -F_2 x_2$

$$W_1 = -W_2 \Rightarrow F_1 x_1 = F_2 x_2 \qquad x_2 = x_1 \frac{A_1}{A_2}$$

we must push the smaller piston a larger x to move the bigger piston a smaller x

U-tube barometer measures P_{atm}



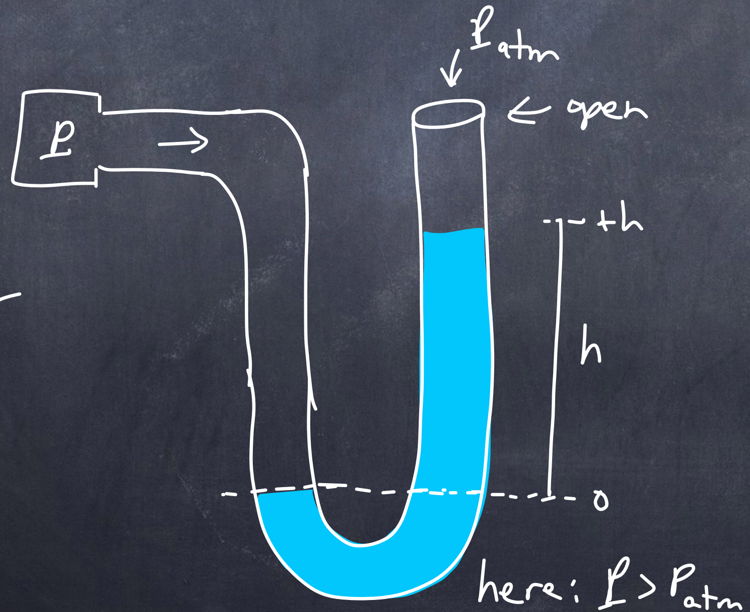
$P_{atm} = P_0 + \rho g h$

$$P_{atm} - \cancelto{0}{P_0} = \rho g h$$

$$P_{atm} = \rho g h$$

$P_{atm} \sim$ height of the fluid.

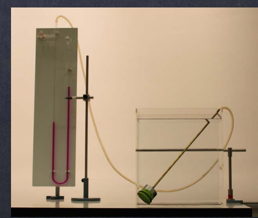
open-tube manometer measures P



$$P = P_{atm} + \rho g h$$

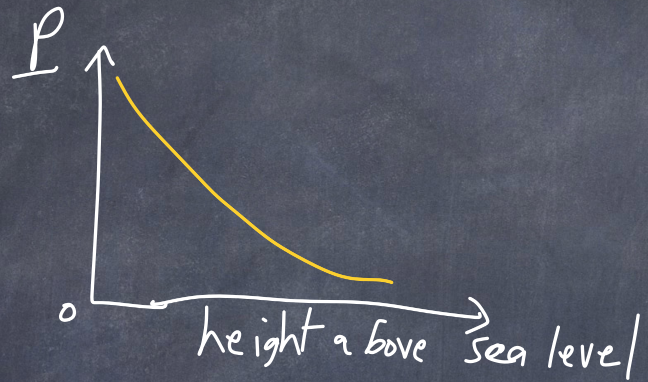
$$P - P_{atm} = \rho g h$$

$P \sim$ height of the fluid

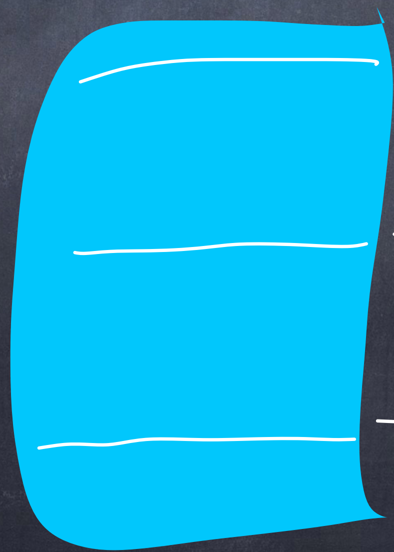


$P = 1$ atm : height of 760 mm Hg (height of mercury pushed up by 1 atm)

height of 10.3 m water



Below sea level



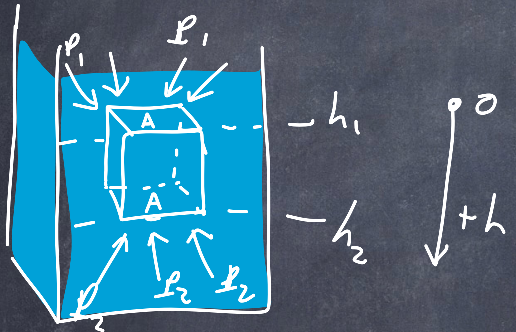
0 m, $P_{\text{sea level}} = 1$ atm

-10 m, $P_{-10\,m} = 2$ atm

-20 m, $P_{-20\,m} = 3$ atm

Archimedes principle: Body submerged in a fluid (partially or completely) is forced up by a buoyant force equal to the weight of the fluid that is displaced.

pressure of fluid:



$P_2 > P_1$ $\quad P_2 = P_1 + \rho g h$

$P_2 - P_1 = \rho_\ell g h$ ①

ρ_ℓ : density of fluid.

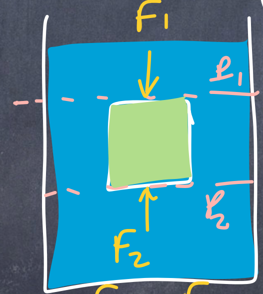
Force due to the pressure difference

$$P = \frac{F}{A} \Rightarrow F = P \cdot A$$

①$\times A$: $(P_2 - P_1)A = (\rho_\ell g h)A$

$$F_2 - F_1 = \rho_\ell g (hA) = \rho_\ell g V_\ell$$

V_ℓ : is the volume of the fluid that is displaced.



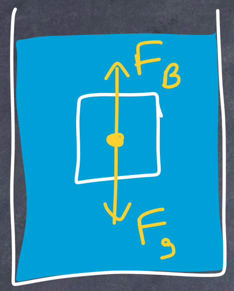
$F_2 > F_1$

If an object is fully submerged then the volume of the object is the same as the V_ℓ

The fluid pressure difference causes a force upward called the buoyant force. $F_2 - F_1 = \boxed{F_B = \rho_\ell g V_\ell}$

consider the buoyant force and the force of gravity:

$\downarrow$ (+)



$$F_B = \rho_\ell g V_\ell$$

$$F_g = m_o g = \rho_o g V_o$$

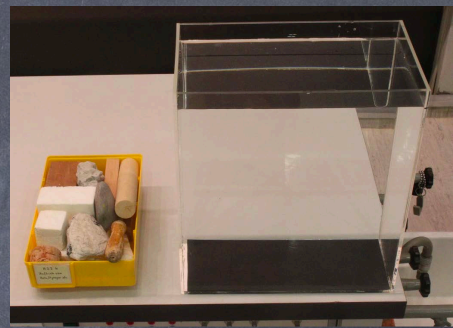
m_o: mass of object

$$\rho = \frac{m}{V} \Rightarrow m = \rho V$$

$$m_o = \rho_o V_o$$

here, $V_o = V_\ell = V$
because fully submerged.

$$\sum F = F_g - F_B = (\rho_o - \rho_\ell) g V$$



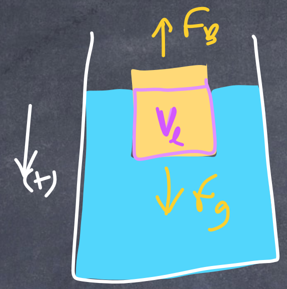
If $\rho_o > \rho_\ell$, then $\sum F$ points down.
It sinks.

If $\rho_o < \rho_\ell$, then $\sum \bar{F}$ points up.
It floats.

GAME: WILL. IT. FLOAT ?!

How much of object is submerged ?

Partially submerged object



V_ℓ : volume of liquid that is displaced.

V_o : volume of object.

$$F_B = \rho_\ell g V_\ell$$

$$F_g = \rho_o g V_o$$

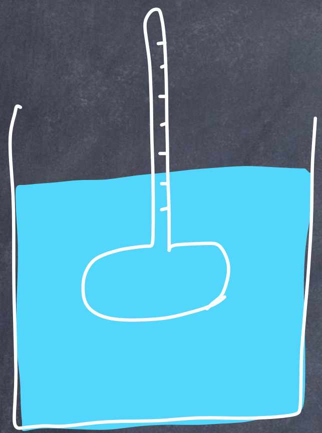
At equilibrium $\Sigma F = 0 = F_g - F_B \Rightarrow F_g = F_B$

so $\rho_\ell \cancel{g} V_\ell = \rho_o \cancel{g} V_o$

$$\frac{V_\ell}{V_o} = \frac{\rho_o}{\rho_\ell}$$

so the fraction submerged $\equiv \frac{V_\ell}{V_o}$ is just the ratio of the densities.

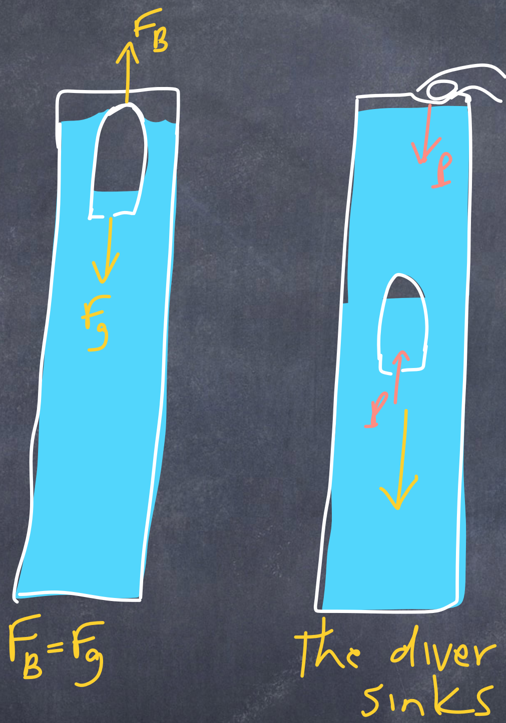
Hydrometer: measures the fraction submerged, FS
and lets us measure the density
of the fluid.



Ratio of density
to that of water
is called the
"specific gravity"

Cartesian diver

p : increase in pressure



Finger pushing down, we increase the pressure inside.

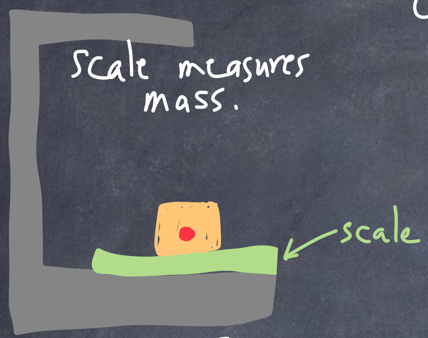
The Fluid is not compressible, but the air inside the diver is compressible.

water pushes into diver.

The diver becomes heavier, and he sinks.

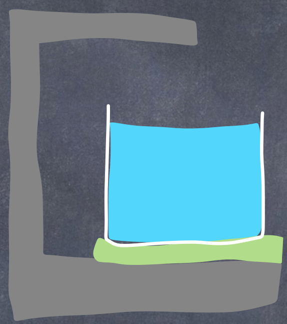
How to measure density of object, ρ_{obj}?

①



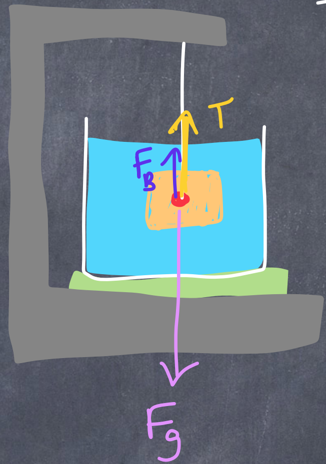
m_o = mass of object

②



zero scale

③



$T = W_A$ = apparent weight.

$$F_B = W_\ell = \underbrace{(\rho_\ell V)}_{m_\ell} g$$

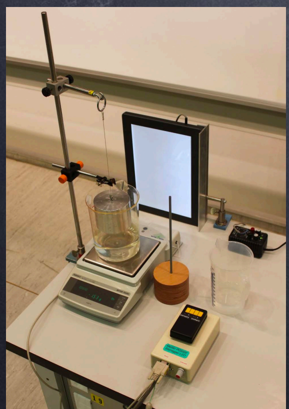
$$F_g = W_o = \underbrace{(\rho_o V)}_{m_o} g$$

At •, $\Sigma F = F_g - T - F_B = 0$

$= W_o - W_A - W_\ell = 0$ $\qquad W_\ell = W_o - W_A$

Now, we want to calculate ρ_o:

$$\frac{\rho_o}{\rho_\ell} = \frac{M_o g/\cancel{V}}{M_\ell g/\cancel{V}} = \frac{W_o}{W_\ell} \qquad \rho_o = \rho_\ell \frac{W_o}{W_\ell}$$



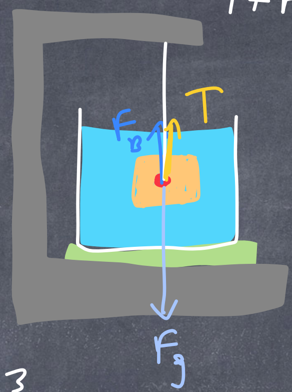
In ③, the scale feels the force $F_g - T = F_B = W_\ell$

so the scale measures m_ℓ, the mass of the fluid.

1) Case of a sinking object $F_g > F_B$

$T + F_B = F_g$

$m_o = 1663\,g$



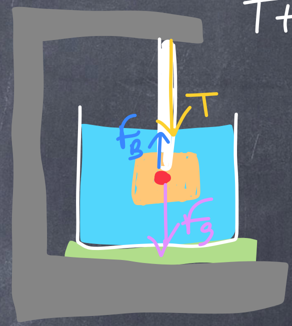
we measure $\dfrac{F_B}{g} = m_\ell = 612\,g$

So $\rho_o = \rho_{water} \cdot \dfrac{m_o}{m_\ell} = \dfrac{1.00\,g}{cm^3}\left(\dfrac{1663\,g}{612\,g}\right) = 2.72\,g/cm^3$

2) Case of Floating object: $F_B > F_g$

$T + F_g = F_B$

$m_o = 478\,g$

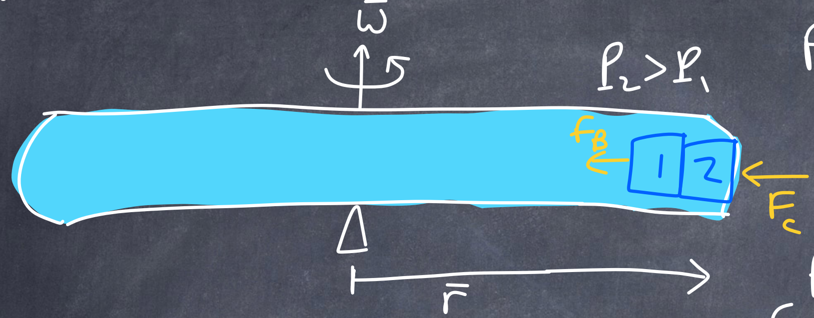


we measure $\dfrac{F_B}{g} = m_\ell = 618\,g$

But the scale still measures the mass of the fluid, F_B/g

$$\rho_o = \rho_\ell \frac{m_o}{m_\ell} = \frac{1\,g}{cm^3}\left(\frac{478\,g}{618\,g}\right) = 0.77\,g/cm^3$$

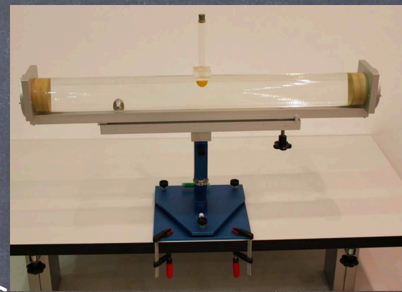
We get a buoyant force from a difference in pressure. This also works with a centrifuge.



Fluid feels a from the wall of the glass container

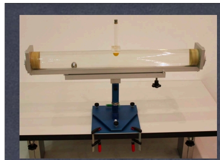
$$F_c = \frac{mv^2}{r}$$

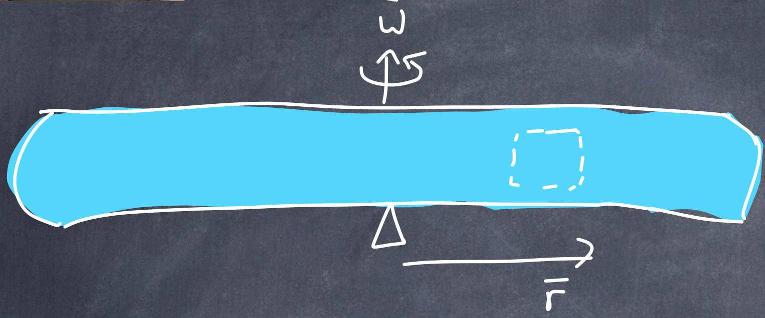
Fluid in $\boxed{2}$ feels force from $\boxed{1}$ but it is less.

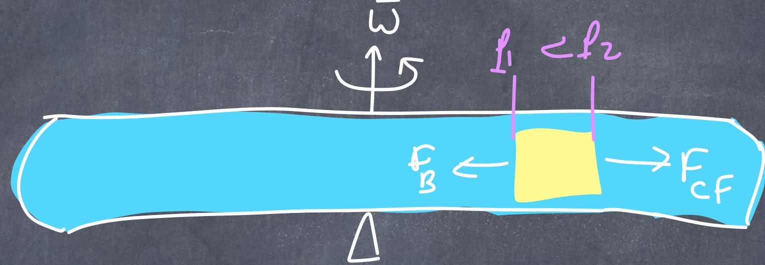


Pressure of fluid varies as a function of r. This provides a buoyant force.

An object in the centrifuge feels a buoyant force towards $r=0$, and a "centrifugal force" towards the wall.







The buoyant force comes from the centripetal force of the fluid:

$$F_B = m_\ell \frac{v^2}{r} = m_\ell \omega^2 r(-\hat{r})$$

F_B is bigger at large r

An object in the centrifuge feels a buoyant force and an apparent force called the "centrifugal pseudoforce" outward.

$$F_{cF} = m_o \omega^2 r(\hat{r})$$

$$\Sigma F = F_c - F_B = m_o \omega^2 r - m_\ell \omega^2 r = V\rho_o \omega^2 r - V\rho_\ell \omega^2 r = V\omega^2 r(\rho_o - \rho_\ell)\hat{r}$$

If $\rho_o > \rho_\ell$, the force is outward in $+\hat{r}$ director

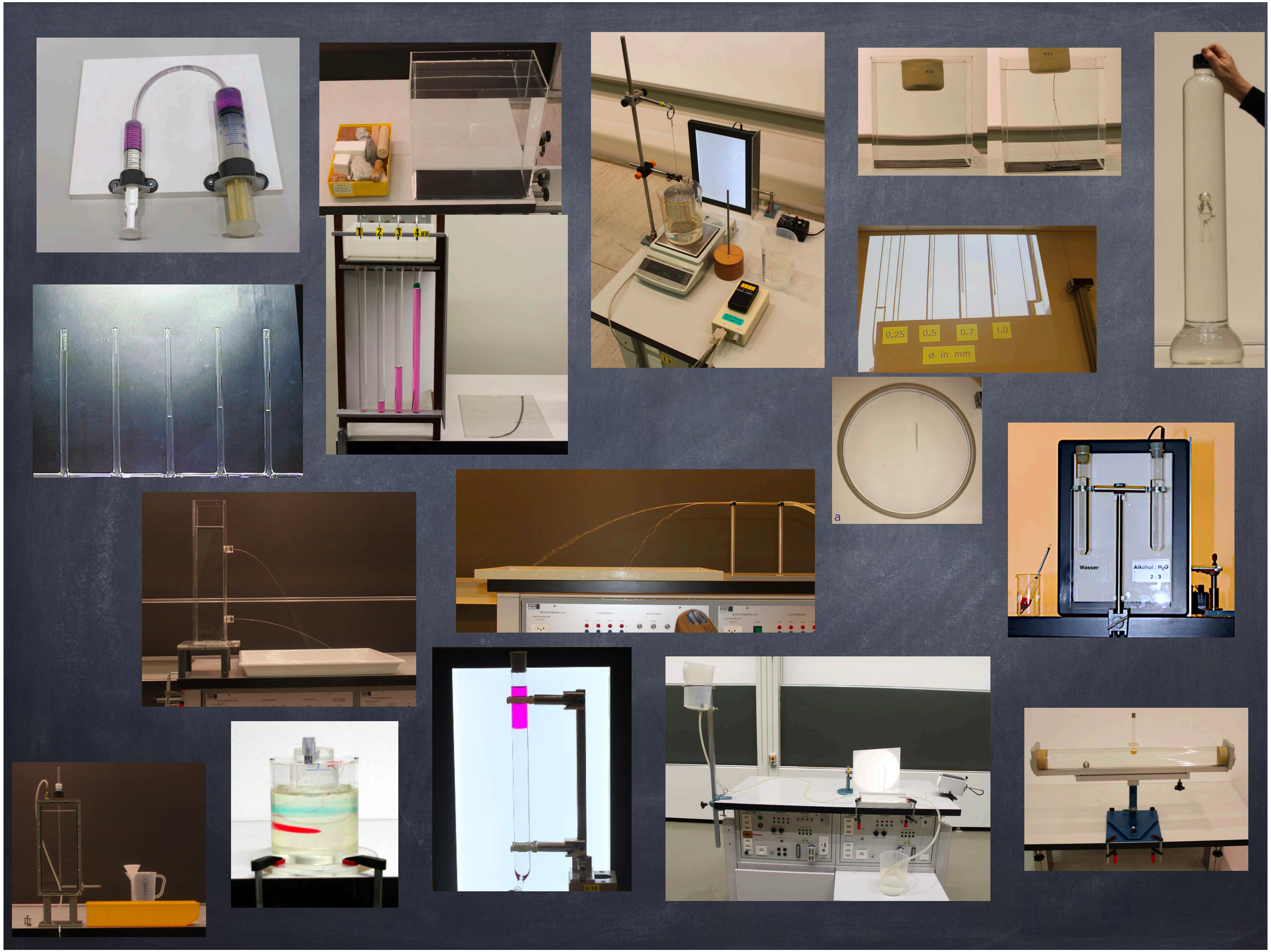
If $\rho_o < \rho_\ell$, force is inward $(-\hat{r})$

why did I call it a pseudoforce?

The actual force is the centripetal force, which causes the object to move in a circle because it is being accelerated towards the center.

But because we are viewing the object in its reference frame (ignoring that it is spinning), we see this apparent pseudoforce.

End


0.25
0.5
0.7
1.0
ø in mm
Wasser
Alkohol : H_2O
2 : 3