PHY117 HS2024

Week 7, Lecture 2
Oct. 30th, 2024
Prof. Kilminster

Adiabatic expansion: Gas volume expands without flow of heat in or out of the system.

$\Delta U = \cancel{Q}^{0} - W$ $\boxed{\Delta U = -W}$

If system expands, system does work. The work is then (+).

Then ΔU is (−) ⇒ decrease in internal energy

⇒ so the temperature decreases.

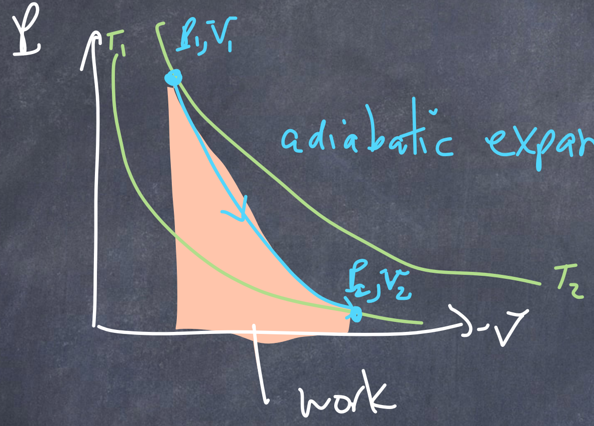
Adiabatic process can happen in 2 ways:

1) so quick that heat can't be exchanged.

2) very slowly in a well-insulated system

"quasi-static adiabatic process"

Adiabatic process $Q = 0$, $\Delta U = -W$



what is constant?
what is the work done?

we know that $dU = C_v \, dT$ for any ideal gas.

$dW = P \, dV$ work done by gas

$dU = -dW$ for an adiabatic process.

For adiabatic processes:

$$\boxed{\Delta U = -W}$$

Adiabatic processes with an ideal gas:

$$\boxed{TV^{\gamma-1} = \text{constant}}$$

$$\boxed{pV^{\gamma} = \text{constant}}$$

where

$$\gamma = \frac{c_p}{c_V}$$

$$W = \frac{1}{\gamma-1}\left(p_1V_1 - p_2V_2\right)$$

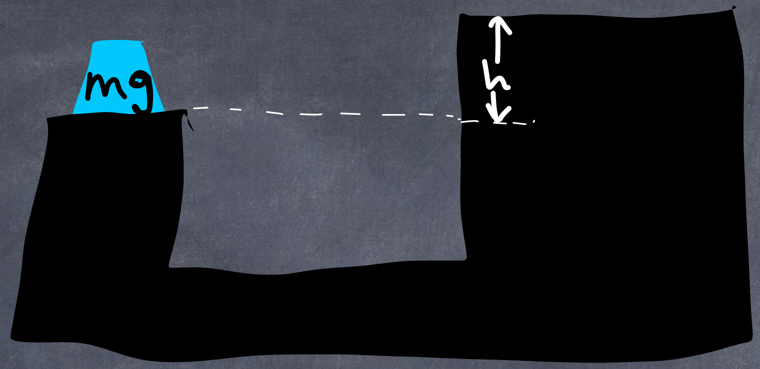
work done by a gas expanding adiabatically

Derivations in script 2

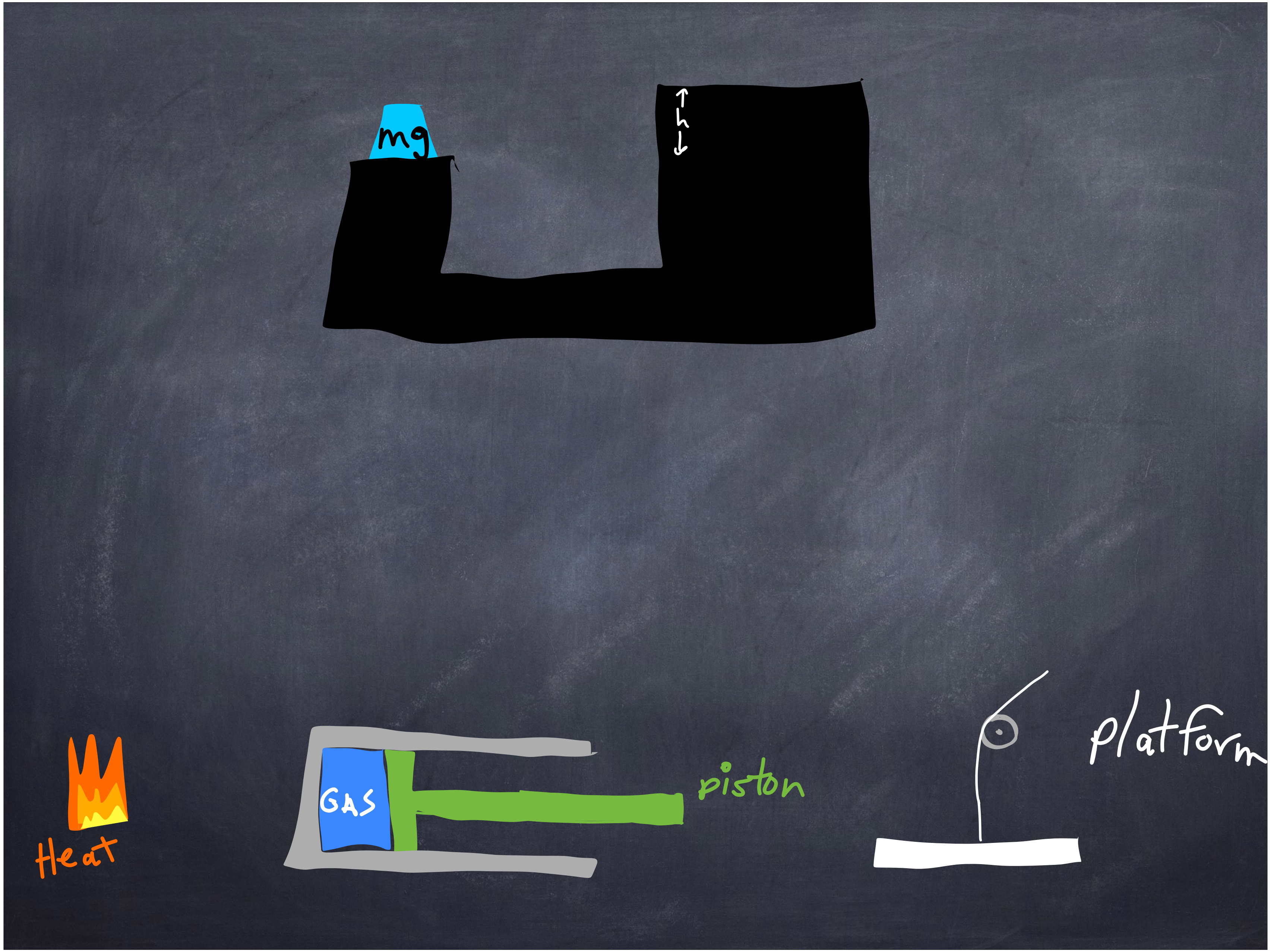
Suppose we want to lift this mass a height h.

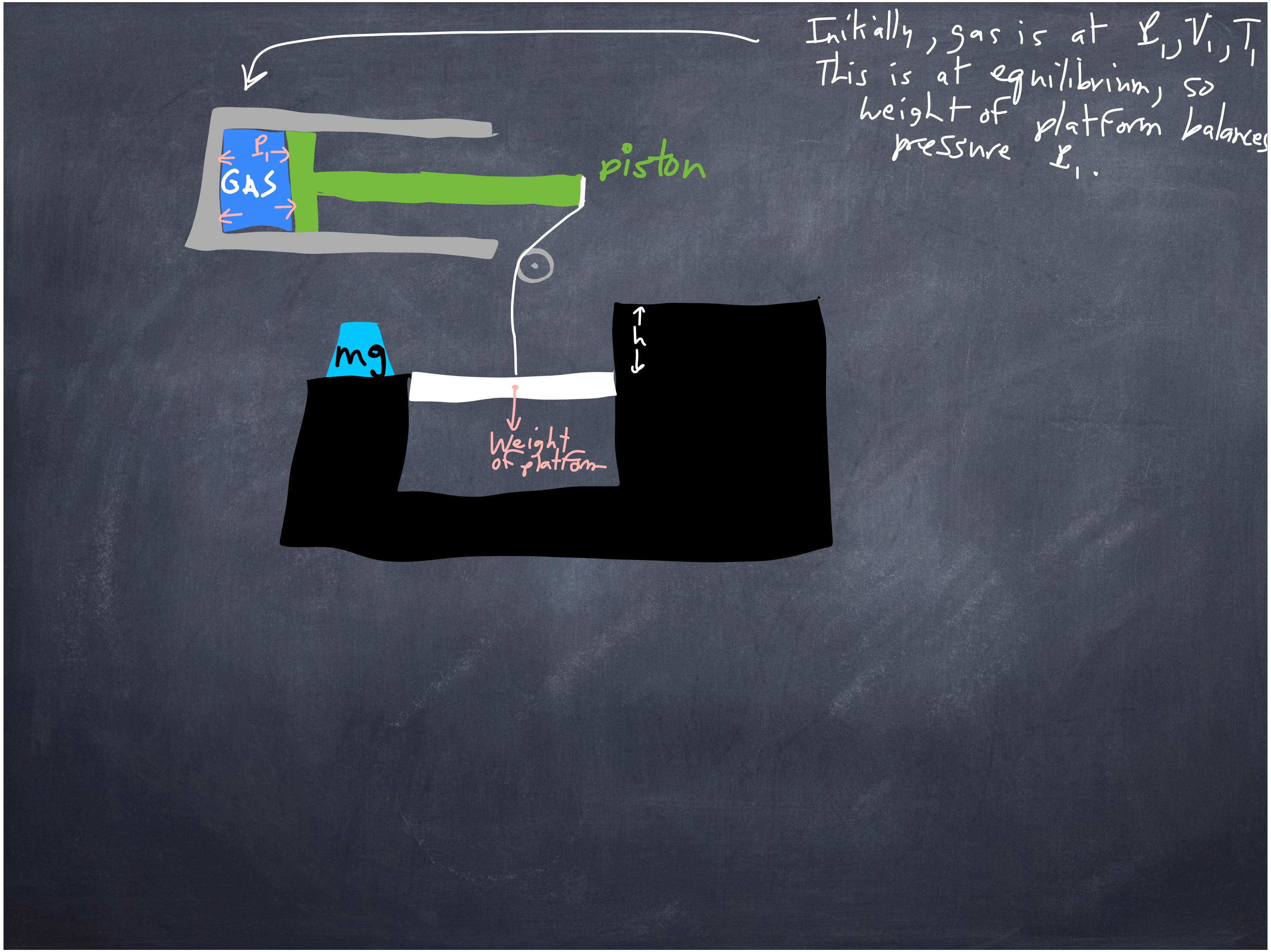
Requires work

$W = mgh$

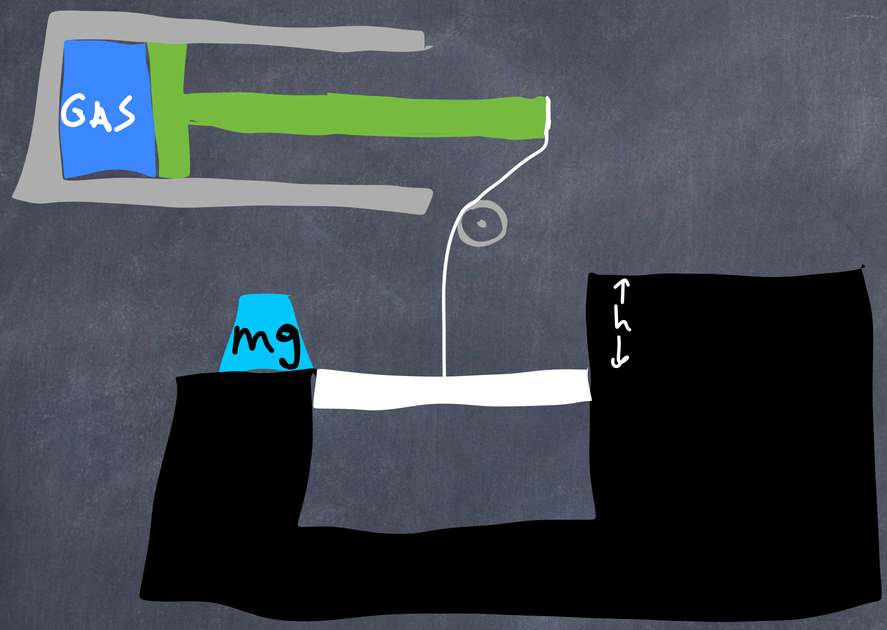


we want to use a gas to do this.

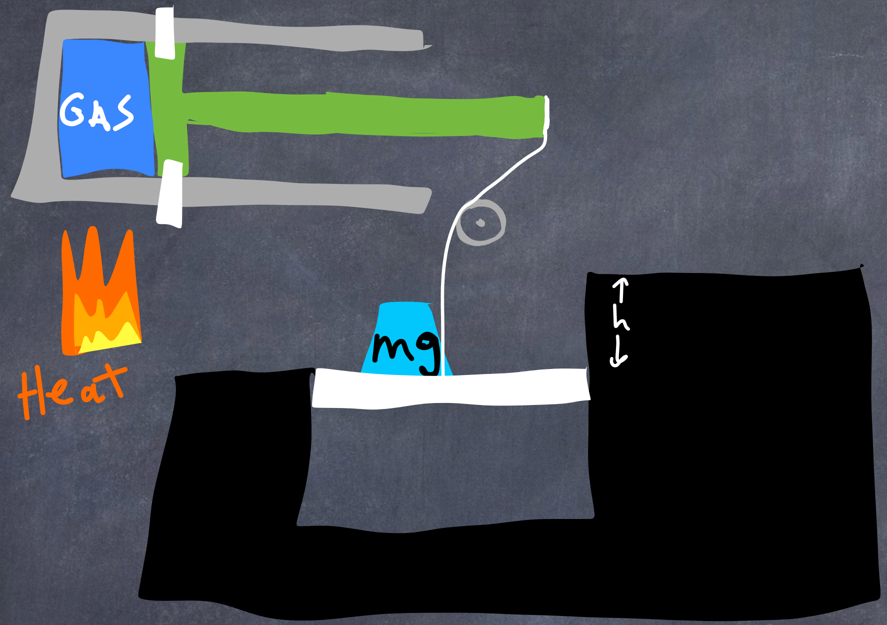

mg
h
Heat
GAS
piston
platform


Initially, gas is at P_1, V_1, T_1
This is at equilibrium, so
weight of platform balances
pressure P_1.
GAS
P_1
piston
mg
h
Weight
of platform

initial) At equilibrium, P_1, V_1, T_1



a) $P_1 \to P_2$, V_1 constant

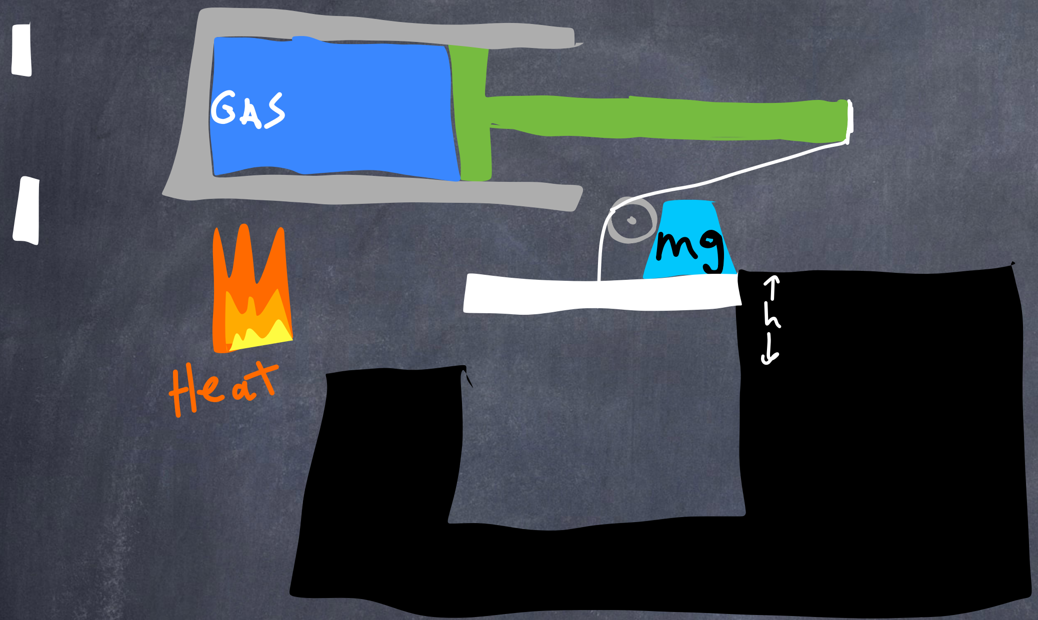


initial) At equilibrium, P_1, V_1, T_1

a) We fix volume at V_1. Then heat gas at constant volume. So pressure increases to P_2. The pressure P_2 can now hold the weight mg. We slide weight on platform.

a) $P_1 \to P_2$, V_1 constant

b) $V_1 \to V_2$, P_2 constant



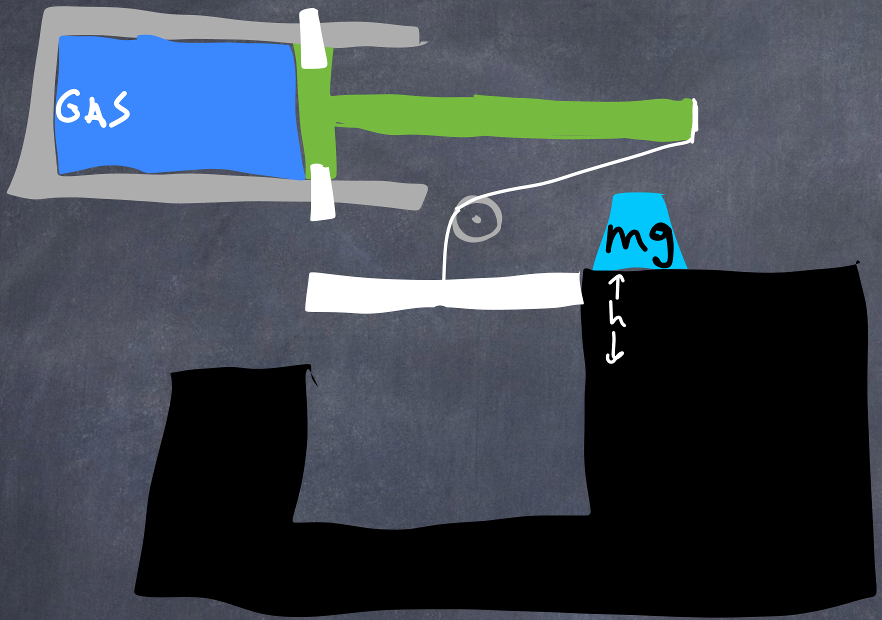
initial) At equilibrium, P_1, V_1, T_1

a) We fix volume at V_1. Then heat gas at constant volume. So pressure increases to P_2. We slide weight on platform. The pressure P_2 can now hold the weight mg.

b) Unfix volume. Heat the gas until volume increases to V_2. This raises the weight a height h.

a) $P_1 \to P_2$, V_1 constant
b) $V_1 \to V_2$, P_2 constant

c) $P_2 \to P_1$, V_2 constant



initial) At equilibrium, P_1, V_1, T_1

a) We fix volume at V_1. Then heat gas at constant volume. So pressure increases to P_2. We slide weight on platform. The pressure P_2 can now hold the weight mg.

b) Unfix volume. Heat the gas unti volume increases to V_2. This raises the weight a height h.

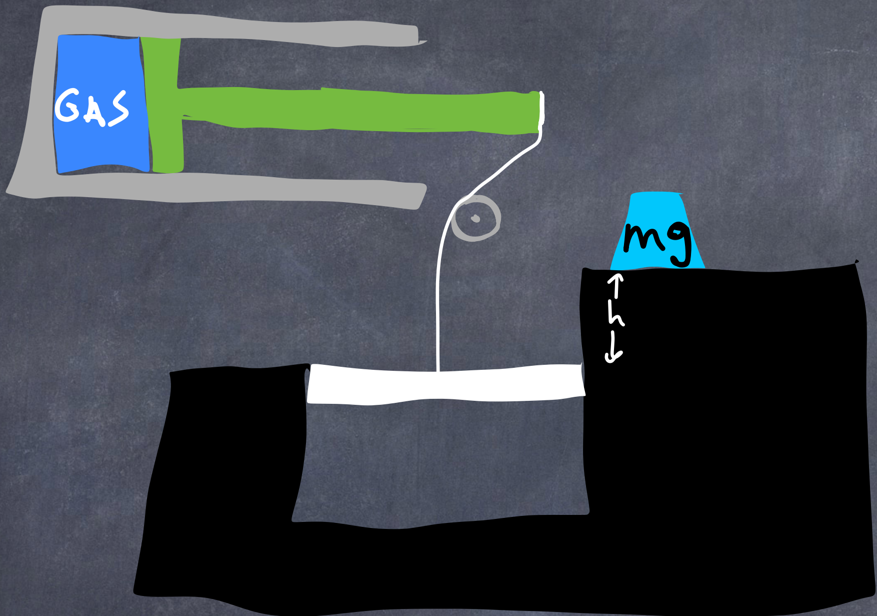
c) we fix the volume at V_2. slide over the weight. Remove the heat. Pressure will decrease at constant volume, V_2 to P_1.

a) $P_1 \to P_2$, V_1 constant
b) $V_1 \to V_2$, P_2 constant
c) $P_2 \to P_1$, V_2 constant
d) $V_2 \to V_1$, P_1 constant

initial) At equilibrium, P_1, V_1, T_1

a) We fix volume at V_1. Then heat gas at constant volume. So pressure increases to P_2. We slide weight on platform. The pressure P_2 can now hold the weight mg.

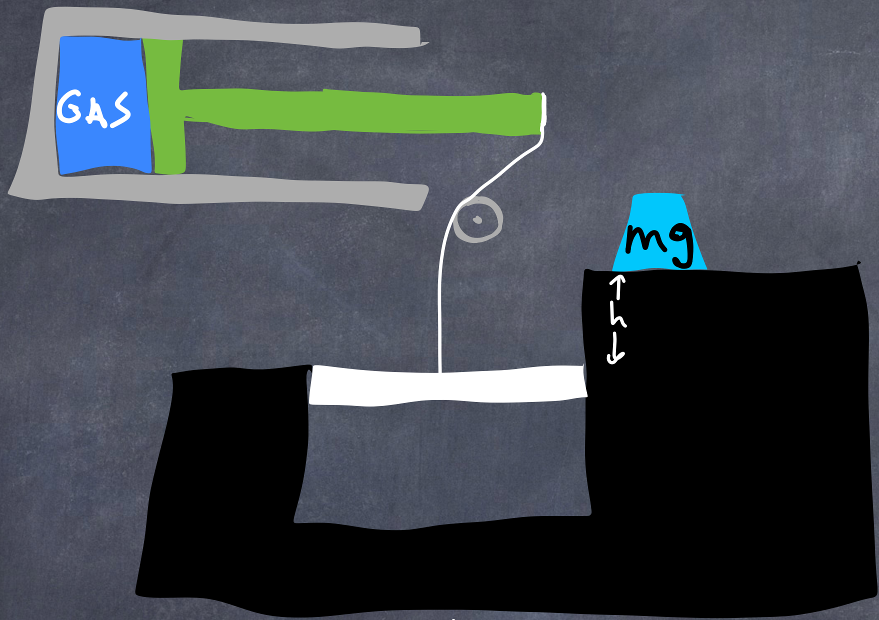
b) Unfix volume. Heat the gas until volume increases to V_2. This raises the weight a height h.



c) We fix the volume at V_2. Slide over the weight. Remove the heat. Pressure will decrease at constant volume V_2 to P_1

d) Unfix the volume. We continue to allow heat to be removed. The volume will decrease at constant pressure P_1 to V_1, lowering platform.

a) $P_1 \to P_2$, V_1 constant
b) $V_1 \to V_2$, P_2 constant
c) $P_2 \to P_1$, V_2 constant
d) $V_2 \to V_1$, P_1 constant



initial) At equilibrium, P_1, V_1, T_1

a) We fix volume at V_1. Then heat gas at constant volume. So pressure increases to P_2. We slide weight on platform. The pressure P_2 can now hold the weight mg.

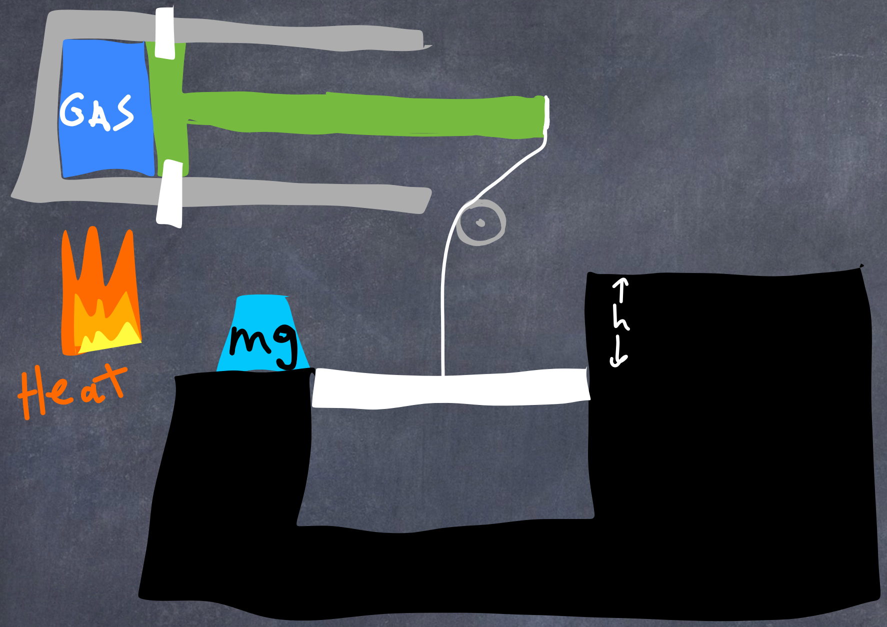
b) Unfix volume. Heat the gas until volume increases to V_2. This raises the weight a height h.

c) we fix the volume at V_2. Slide over the weight. Remove the heat. Pressure will decrease at constant volume V_2 to P_1

d) Unfix the volume. We continue to allow heat to be removed. The volume will decrease at constant pressure P_1 to V_1, lowering platform.

Final = initial) P_1, V_1, T_1, platform at original position, but we've done work $W = mgh$

Summary:



Cycle:

a: heat at fixed volume, pressure increases

b: heat at fixed pressure, volume increases. work(+).

c: cool at fixed volume, pressure decreases

d: cool at fixed pressure, volume decreases. work(−).

Total work done by system = $\bar{F}\cdot\bar{x} = F_g h = mgh \Rightarrow W = mgh$

Cycle:

a: heat at fixed V, P increases

b: heat at fixed P, V increases

c: cool at fixed V, P decreases

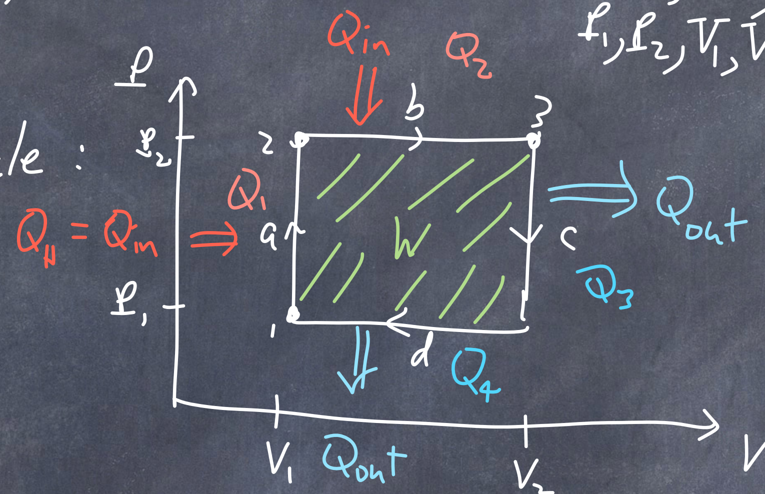
d: cool at fixed P, V decreases

Draw: P vs. V cycle, showing heat coming in and out, show the work.

Calculate: ΔU, W, Q_{in}, Q_{out}. How do $P_1, P_2, V_1, V_2, Q_{in}, Q_{out}$ relate to h.

↑ height

Ideal cycle:



$\Delta U = 0$

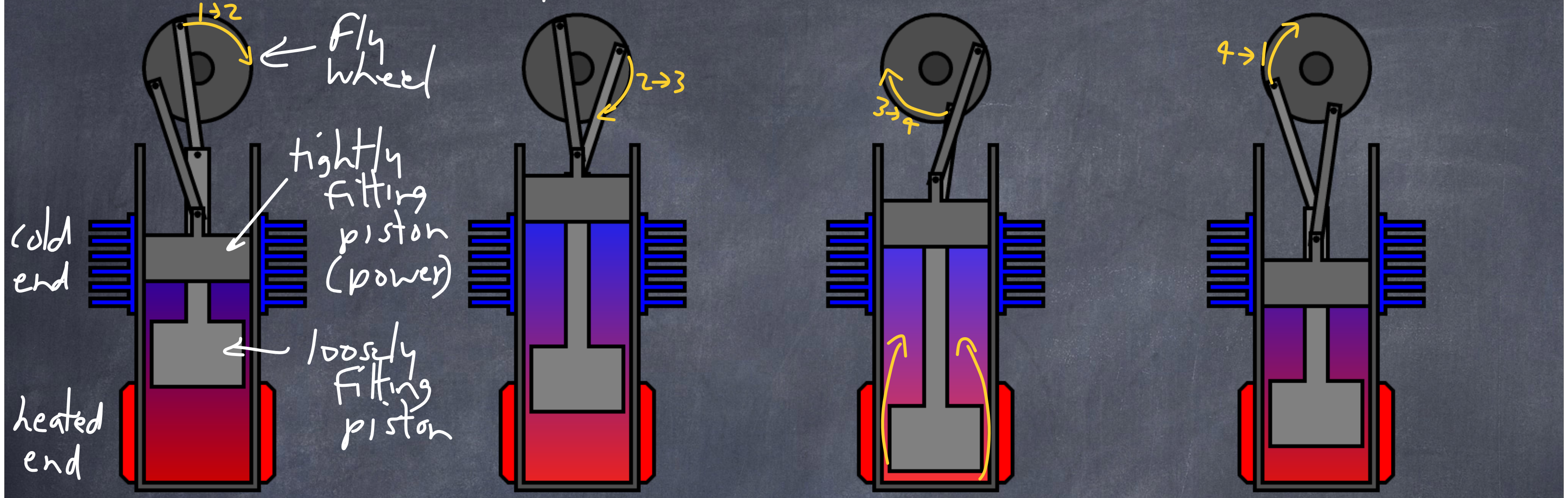
work done: $W = \overbrace{P_2(V_2 - V_1)}^{b(+)} - \overbrace{P_1(V_2 - V_1)}^{d(-)} = (P_2 - P_1)(V_2 - V_1)$

$Q_{in} = Q_1 + Q_2$ heat into system $(+)$

$Q_{out} = Q_3 + Q_4$ heat out system $(-)$

$W = mgh \simeq$ area on PV diagram $\simeq (P_2 - P_1)(V_2 - V_1) = Q_{in} - Q_{out}$

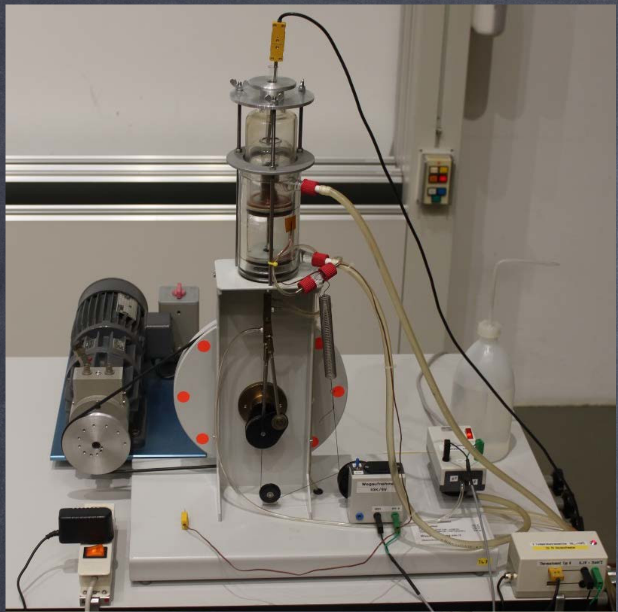
Stirling "Beta type" motor : 1 chamber + 2 pistons.

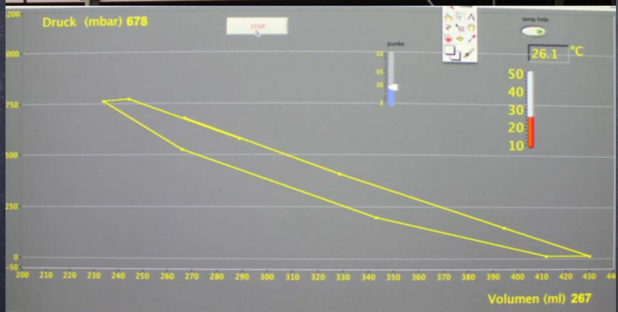


(1) (2) (3) (4)

(1) Most gas is in the hot end. So the gas increases in pressure from heat and expands into the cold area the tight piston is pushed up (power stroke).
(2) The wheel is spinning, and pushes loose piston down (3). This moves the hot gas to the cold end. The hot gas is cooled, causing gas to contract, so tight piston is pulled in (4). Wheel continues spinning pulling loose piston up. Back to (1).

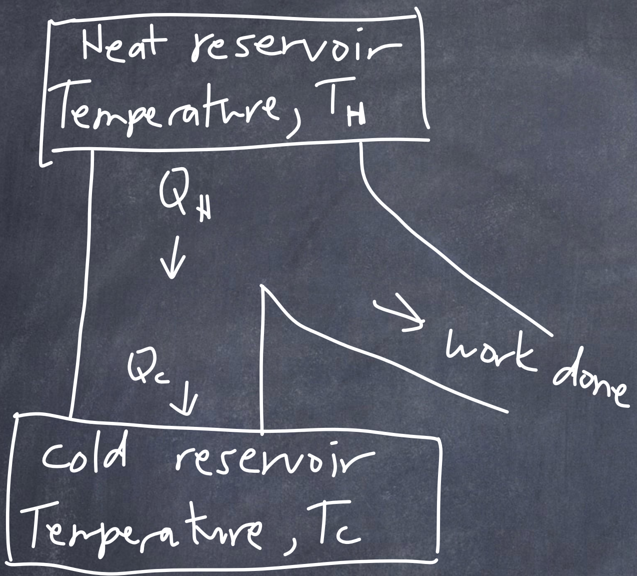
stirling "Beta" engine





← actual cycle

Heat engine (heat converted to work)



In a cycle, initial + final state are the same, no change in internal energy (no change in U)

From 1st law of thermodynamics

$$Q = \cancel{\Delta U} + W$$

$$W = Q_H - |Q_c|$$

(written like this to avoid confusion)

The efficiency is defined as the work divided by heat from hot reservoir:

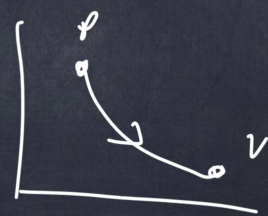
$$\epsilon = \frac{W}{Q_H} = \frac{Q_H - |Q_c|}{Q_H} = \boxed{1 - \frac{|Q_c|}{Q_H} = \text{This is the maximum possible efficiency}}$$

The 2nd law of thermodynamics for heat engines:
It is impossible for a heat engine to convert 100% of heat from a heat source (at constant temp.) into work energy. * caveat

Typical efficiencies:

steam engine ~ 40%
internal combustion engine ~ 25%
Formula one engine ~ 47%
rocket engine ~ 70%

* It is possible during a thermal expansion step, but not in a cycle.



What is the maximum possible efficiency of a heat engine cycle? We can calculate this for a reversible process, no energy lost to friction, no heat conduction, no radiation ...

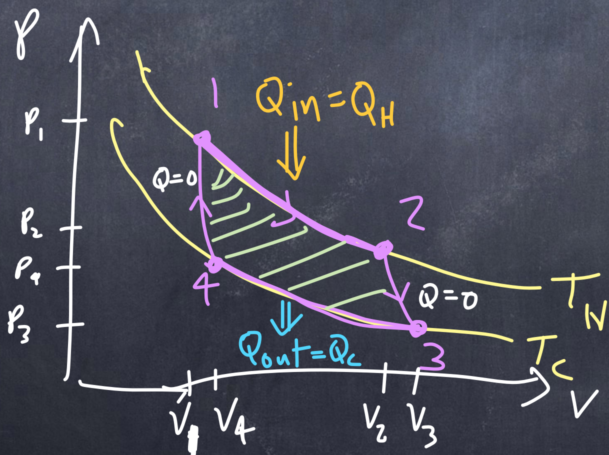
All reversible engines have the same efficiency. So we just need one case to calculate. Solved by Carnot in 1824, he used an ideal gas cycle.

1: P_1, V_1, T_H

2: P_2, V_2, T_H

3: P_3, V_3, T_C

4: P_4, V_4, T_C



$1 \to 2$: isothermal expansion $\Delta U = 0$

$2 \to 3$: adiabatic expansion $Q = 0$

$3 \to 4$: isothermal compression $\Delta U = 0$

$4 \to 1$: adiabatic compression $Q = 0$

efficiency of cycle: $\boxed{\epsilon = 1 - \frac{|Q_C|}{Q_H}}$

$1 \to 2$: $Q_H = W = \int_{V_1}^{V_2} P\,dV = nRT_H \int_{V_1}^{V_2} \frac{dV}{V} = nRT_H \ln \frac{V_2}{V_1}$

$(\Delta U = 0)$

$3 \to 4$: $|Q_c| = |W| = nRT_c \ln \frac{V_3}{V_4}$

Look at $2 \to 3$ & $4 \to 1$. These are adiabatic processes,

$TV^{\gamma-1} = \text{constant}$ & $PV^{\gamma} = \text{constant}$

$2 \to 3$: $T_H V_2^{\gamma-1} = T_c V_3^{\gamma-1}$ $\qquad \frac{T_H}{T_c} = \frac{V_3^{\gamma-1}}{V_2^{\gamma-1}}$

$4 \to 1$: $T_c V_4^{\gamma-1} = T_H V_1^{\gamma-1}$ $\qquad \frac{T_H}{T_c} = \frac{V_4^{\gamma-1}}{V_1^{\gamma-1}}$

It must be that $\frac{V_3}{V_2} = \frac{V_4}{V_1} \Rightarrow \frac{V_3}{V_4} = \frac{V_2}{V_1}$

$$\epsilon = 1 - \frac{|Q_c|}{Q_H} = 1 - \frac{T_c \cancel{\ln\left(\frac{V_3}{V_4}\right)}}{T_H \cancel{\ln\left(\frac{V_2}{V_1}\right)}} = 1 - \frac{T_c}{T_H}$$

same

ϵ_c: The Carnot efficiency

$$\boxed{\epsilon_c = 1 - \frac{T_c}{T_H} \quad \text{where} \quad \frac{T_c}{T_H} = \frac{|Q_c|}{Q_H}}$$

This is the efficiency for a <u>perfect</u> reversible engine. These are the conditions relating T_c, T_H, Q_c, Q_H, ϵ_c

The Carnot efficiency cannot be beat!
An efficiency higher than this violates the 2nd law of thermodynamics.

The maximum efficiency only depends on the temperature difference.

Engines are often defined by the ratio

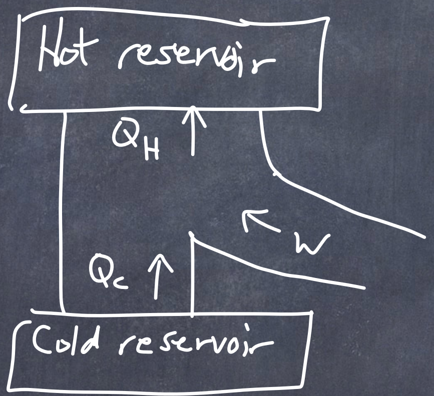
$$\epsilon_{SL} = \frac{\text{actual efficiency}}{\text{Carnot efficiency}} = \frac{\epsilon}{\epsilon_c}$$

efficiency with respect to best possible efficiency from second law (SL)

Example: An engine has 600 K high temperature, and a 300 K low temperature and is 30% efficient. What is ϵ_{SL}?

It's $\epsilon_{SL} = \frac{\epsilon}{1 - \frac{T_C}{T_H}} = \frac{30\%}{1 - \frac{300\,K}{600\,K}} = \frac{30\%}{0.5} = 60\%$

A refrigerator is like a heat engine, but running backwards. Work is done (put into system) to extract heat from a cold reservoir, and put it into a hot reservoir. The piston is used to lower the pressure in one cylinder → lowers temperature.



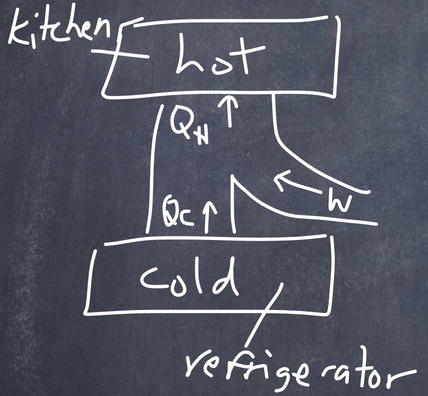
$$|Q_H| = |Q_c| + W$$

2nd law of thermodynamics for a refrigerator: It is impossible for a refrigeration cycle to only transfer heat from a cold object to a hot object (one needs work to do this)

The measure of performance of a refrigerator is C.O.P. = coefficient of performance $= \dfrac{Q_c}{W}$

2nd law says: C.O.P. must <u>not</u> be ∞

For a typical refrigerator has C.O.P. ~ 5.5 how much work + power is needed to make ice cubes from 1 liter of water at 10°C if we do it in 10 seconds? How much heat do we put into our kitchen doing this?



How much heat needs to be removed to make ice?

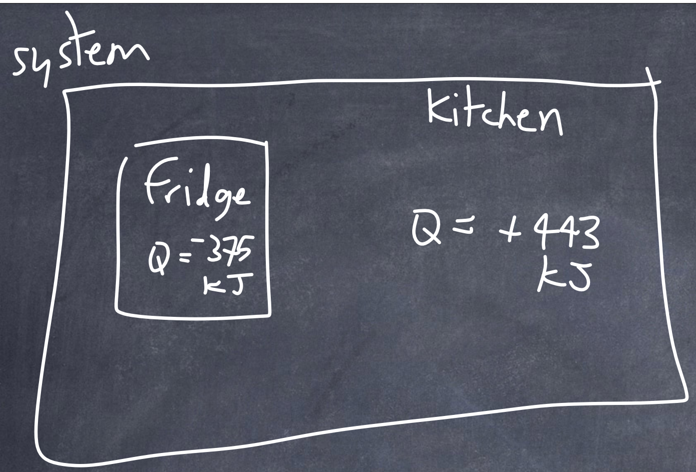
$$Q_C = \underset{\substack{\uparrow \\ mc\Delta T}}{Q_1} + \underset{\substack{\uparrow \\ mL_f}}{Q_2} = 375\text{ kJ}$$

$$W = \frac{|Q_c|}{\text{C.O.P.}} = \frac{375\text{ kJ}}{5.5} = 68\text{ kJ}$$

$$|Q_H| = \text{heat into kitchen} = |Q_c| + W = 375\text{ kJ} + 68\text{ kJ} = 443\text{ kJ}$$

$$\text{Power} = \frac{\text{work}}{\text{time}} = \frac{68\text{ kJ}}{10\text{ seconds}} = 6.8\text{ kWatts}$$

typical fridge uses ~ 25 watts



Kitchen + fridge is our system

$$Q_{total} = Q_{fridge} + Q_{kitchen}$$
$$= -375 + 443 \text{ kJ}$$
$$= +68 \text{ kJ}$$

⇒ You can't cool a room down by opening a fridge!

Previously, we found that the efficiency for a perfect reversible engine is related to the temp. difference $\varepsilon_c = 1 - \frac{T_c}{T_H}$ + $\frac{T_c}{T_H} = \frac{|Q_c|}{|Q_H|}$

Carnot efficiency

Power plant generating electricity (fossil fuels): $\varepsilon_c \sim 50\%$

Actual efficiency is 40% $\left(\varepsilon_{SL} = \frac{\varepsilon}{\varepsilon_c} = 80\%\right)$

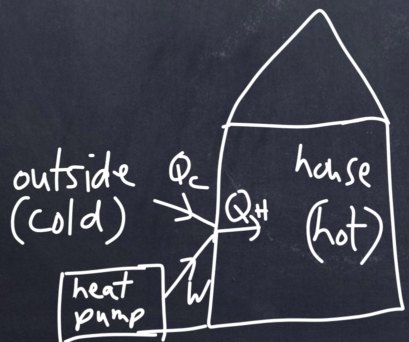
Electrical house heater efficiency =

power plant efficiency * transmission efficiency * electrical heater efficiency

= (67%) * (95%) * (100%) = 64%

But, if goal is to heat house, we can do this with a much better efficiency.

A heat pump is a refrigerator that removes Q_C of heat from a cold reservoir and puts Q_H of heat into a hot reservoir, using W amount of work.



What is the efficiency of a heat pump?

$$|Q_H| = W + |Q_c|$$

$$\text{C.O.P.} = \frac{|Q_c|}{W} = \frac{|Q_c|}{|Q_H| - |Q_c|}$$ divide top + bottom by $|Q_H|$ $\Rightarrow$ $$\text{C.O.P.} = \frac{\frac{|Q_c|}{|Q_H|}}{1 - \frac{|Q_c|}{|Q_H|}}$$

using our Carnot conditions (replace $Q_c \to T_c$, $Q_H \to T_H$)

$$\text{C.O.P.} = \frac{\frac{T_c}{T_H}}{1 - \frac{T_c}{T_H}} = \frac{T_c}{T_H - T_c} = \boxed{\frac{T_c}{\Delta T} = COP_{max}}$$

As an efficiency

$$\varepsilon = \frac{Q_H}{W} = \frac{\cancel{W}(1 + \text{C.O.P.})}{\cancel{W}} = 1 + \text{C.O.P.}$$

Compare to 50% for power plant

Typical C.O.P. ~ 3.5, then $\varepsilon = 1 + 3.5 = \boxed{450\%}$

very efficient

or 80% for a gas furnace

Examples done in class of refrigerators + heat pumps