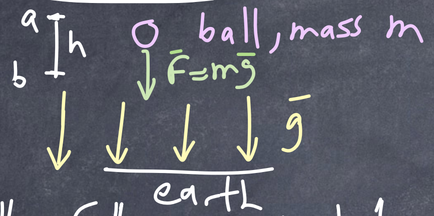
PHY 117 HS2024

Week 9, Lecture 1
Nov. 12th, 2024
Prof. Ben Kilminster

Potential Energy

Gravitational

a: initial
b: final



As ball falls, potential energy decreases

$$U_a > U_b$$

$$mga > mgb$$

$$\Delta U = U_b - U_a = -mgh$$

The work done by gravity is $-\Delta U$

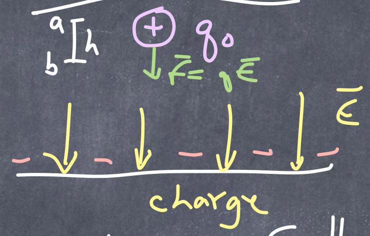
$$W_{a\to b} = -\Delta U = mgh$$

Remember

$$W_{a\to b} = \int_a^b \bar F \cdot d\bar l = +mgh$$

(+)

Electrical



$$\bar F = q_0 \bar E$$

As (+) charge falls, potential energy decreases.

$$U_a > U_b$$

$$q_0Ea > q_0Eb$$

$$\Delta U = U_b - U_a = -q_0Eh$$

The work done by $\bar E$-field is $-\Delta U$

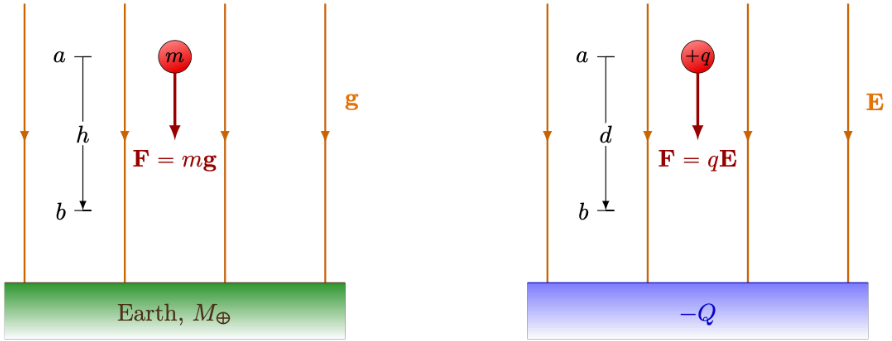
$$W_{a\to b} = -\Delta U = +q_0Eh$$

$$W_{a\to b} = \int_a^b F\cdot d\bar l = Fl\Big]_a^b = Fb - Fa$$

$$F = q_0E$$

$$W_{a\to b} = q_0Eh$$

3.1 Electric potential energy



(a) Gravitational: $\Delta U = -mgh$. **(b)** Electric: $\Delta U = -qEd$.

Figure 3.1: Comparison of potential energy difference $\Delta U = U_b - U_a$ in a force field.

when the movement is in the same direction as the force, there is a decrease in U.

we often use the electric potential, V, or the electric potential difference, ΔV.

$$\Delta V = V_b - V_a = \frac{U_b - U_a}{q_0} = \frac{\Delta U}{q_0} \left(= \frac{-\cancel{q_0} E h}{\cancel{q_0}} \right)$$

for instance, from previous page

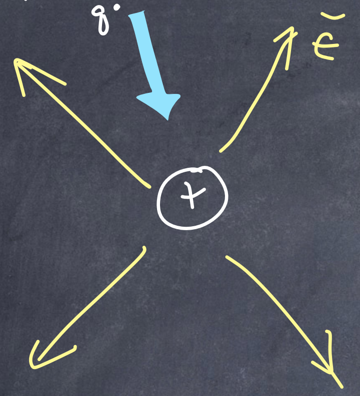
The electric potential is independent of the test charge, q_0.

$$dV = -\bar{E} \cdot d\bar{l}$$

$$\boxed{\Delta V = -\int_a^b \bar{E} \cdot d\bar{l}}$$

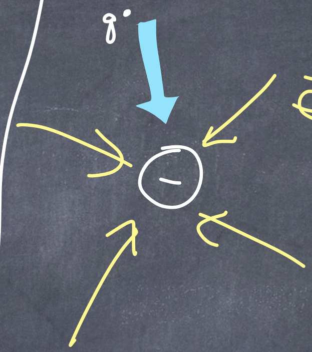
The $(-)$ sign means that ΔV is $(-)$ when movement is in the same direction as the $\bar{E}$-field.

V increases as we move q_0 (+) toward (+) charge



(what would happen to the potential of a (+) charge?)

V decreases as we move q_0 (+) toward (−) charge.



The units for electric potential are Volts

$$1\ V = 1\ \text{Volt} = 1\left[\frac{J}{C}\right] \quad \left[\frac{\text{energy}}{\text{charge}}\right]$$

$$\Delta U = q\,\Delta V$$

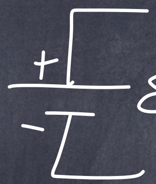
$$[J] = [C]\left[\frac{J}{C}\right]$$

$$\Delta V = \frac{\Delta U}{q} = \frac{-\int \bar{F}\cdot d\bar{\ell}}{q} = \overset{\text{units}}{\left[\frac{N\cdot m}{C}\right]} = V$$

$$\bar{E} : \left[\frac{N}{C}\right] = \left[\frac{V}{m}\right]$$

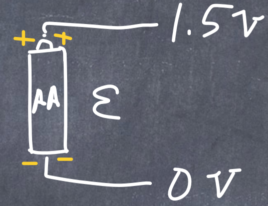
$$1\,V = 1\left[\frac{J}{C}\right] = \left[\frac{N\cdot m}{C}\right]$$

we can make a potential difference with a chemical battery



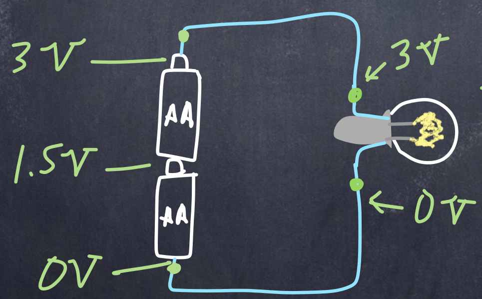
$\mathcal{E}$ or emf is a source of potential difference

we can define 0V to be anywhere, and we often put it at the negative electrode

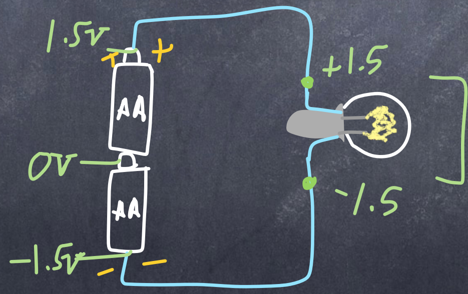


we say $\Delta V = 1.5\,V$

Electric potential is the same everywhere on a conductor. The difference in voltage (across a battery) is what defines the movement of charge + work that can be done.

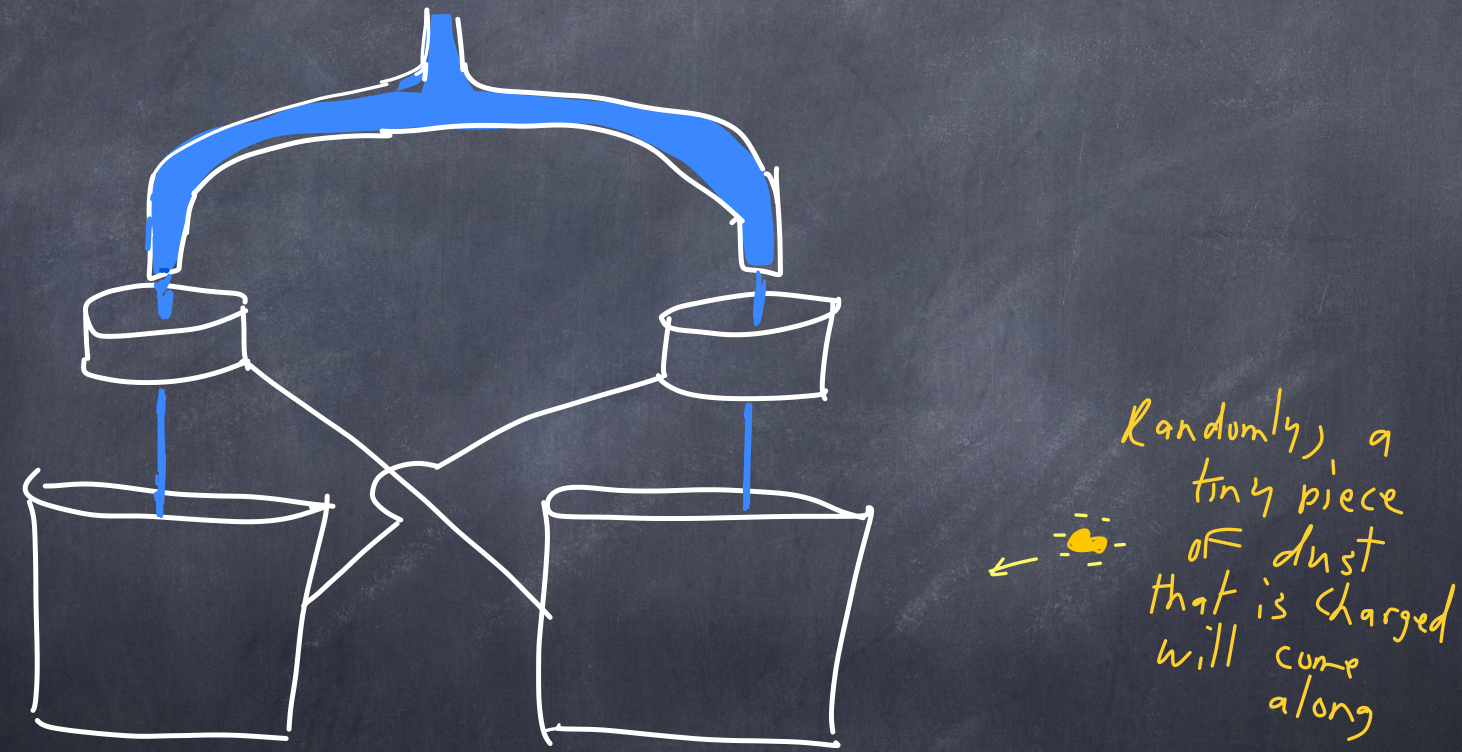


$\Delta V = 3\,V$



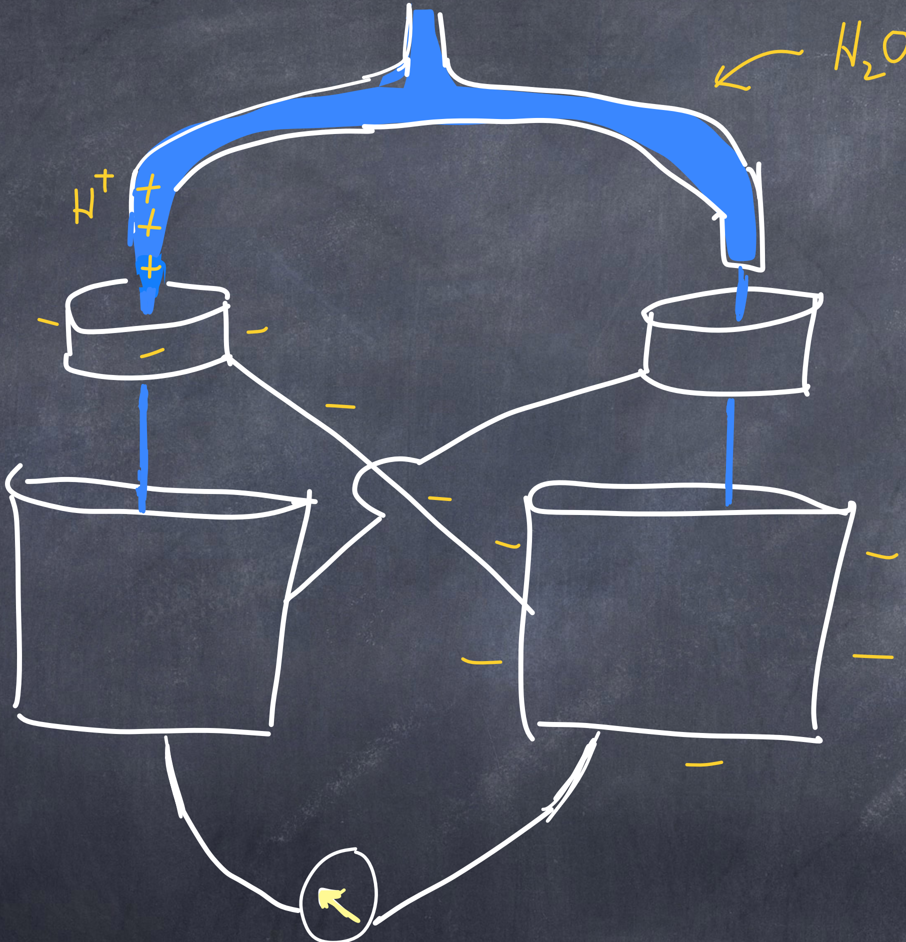
$\Delta V = 1.5 - (-1.5) = 3\,V$

we can charge one conductor with respect to another,
to create a potential difference.



Kelvin generator (Kelvin water dropper)

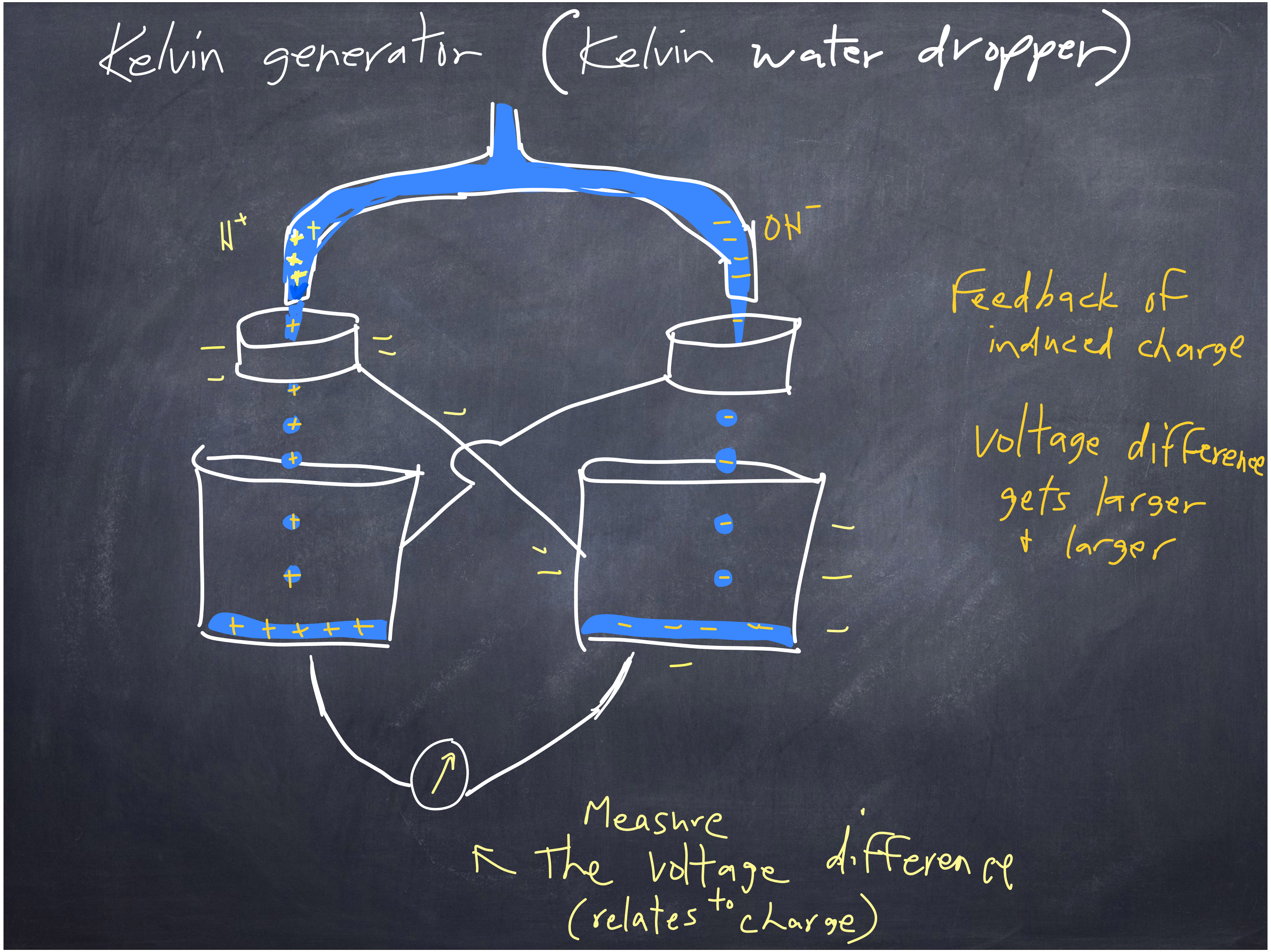
$\leftarrow H_2O, OH^-, H^+$



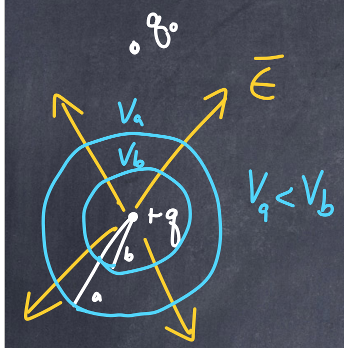
charged dust redistributes along conductor.

Note: In a conductor, excess charge accumulates on surface.

measure the electricpotential difference between the two pails


Kelvin generator (Kelvin water dropper)
H+
OH−
Feedback of induced charge
voltage difference gets larger + larger
Measure
↖ The voltage difference
(relates to charge)

Potential energy due to point charge.



$$\bar{E} = \frac{kq}{r^2}\hat{r} \qquad d\bar{l} = dr\,\hat{r}$$

$$dV = -\bar{E} \cdot d\bar{l}$$

$$dV = -\frac{kq}{r^2}\hat{r} \cdot dr\,\hat{r} = -\frac{kq}{r^2}dr$$

$$V = \int dV = \int -\frac{kq}{r^2}dr = \frac{kq}{r} + V_0$$

constant of integral

The convention is that the potential is 0 when we are infinitely far away.

$$V(r=\infty) = 0 = \frac{kq}{\infty} + V_0$$

$$V_0 = 0$$

so $V = \dfrac{kq}{r}$ assuming $V=0$ at $r=\infty$

for a point charge

If q_0 is released from b, it will move outward from b to a

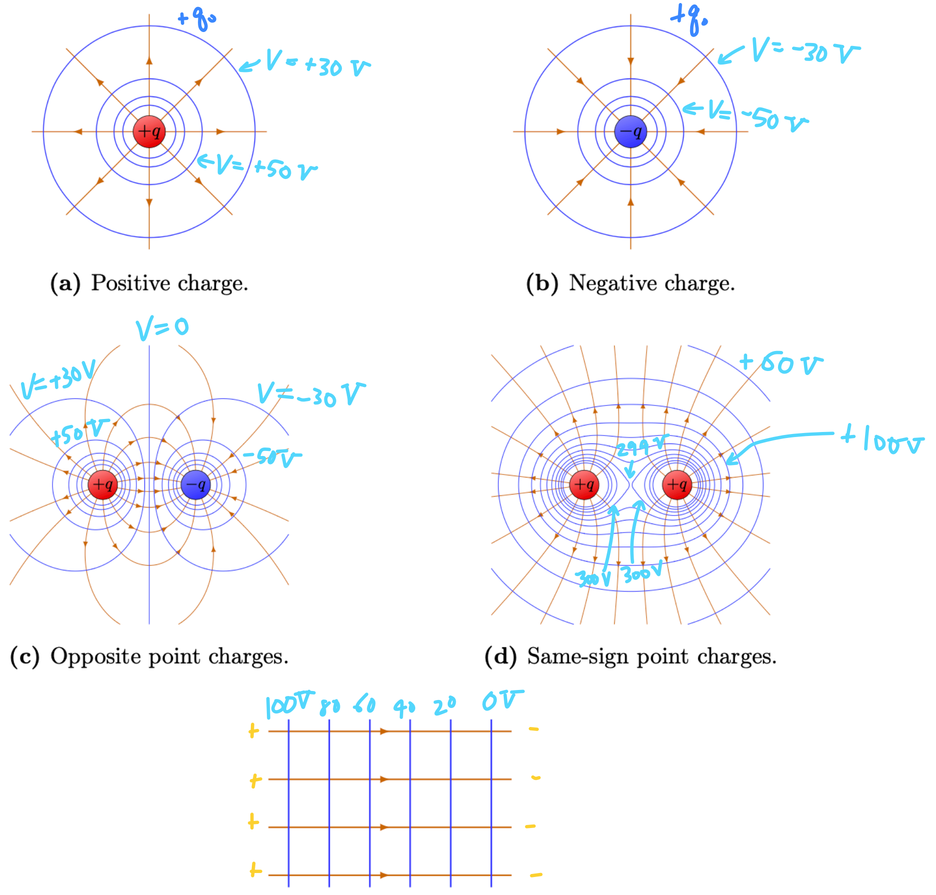
$$V_{final} - V_{initial} = V_a - V_b = \Delta V$$

$$V_a - V_b = \frac{kq}{a} - \frac{kq}{b} = (-)$$

Decrease in potential

Equipotential lines: lines of equal potential

Examples



(a) Positive charge.

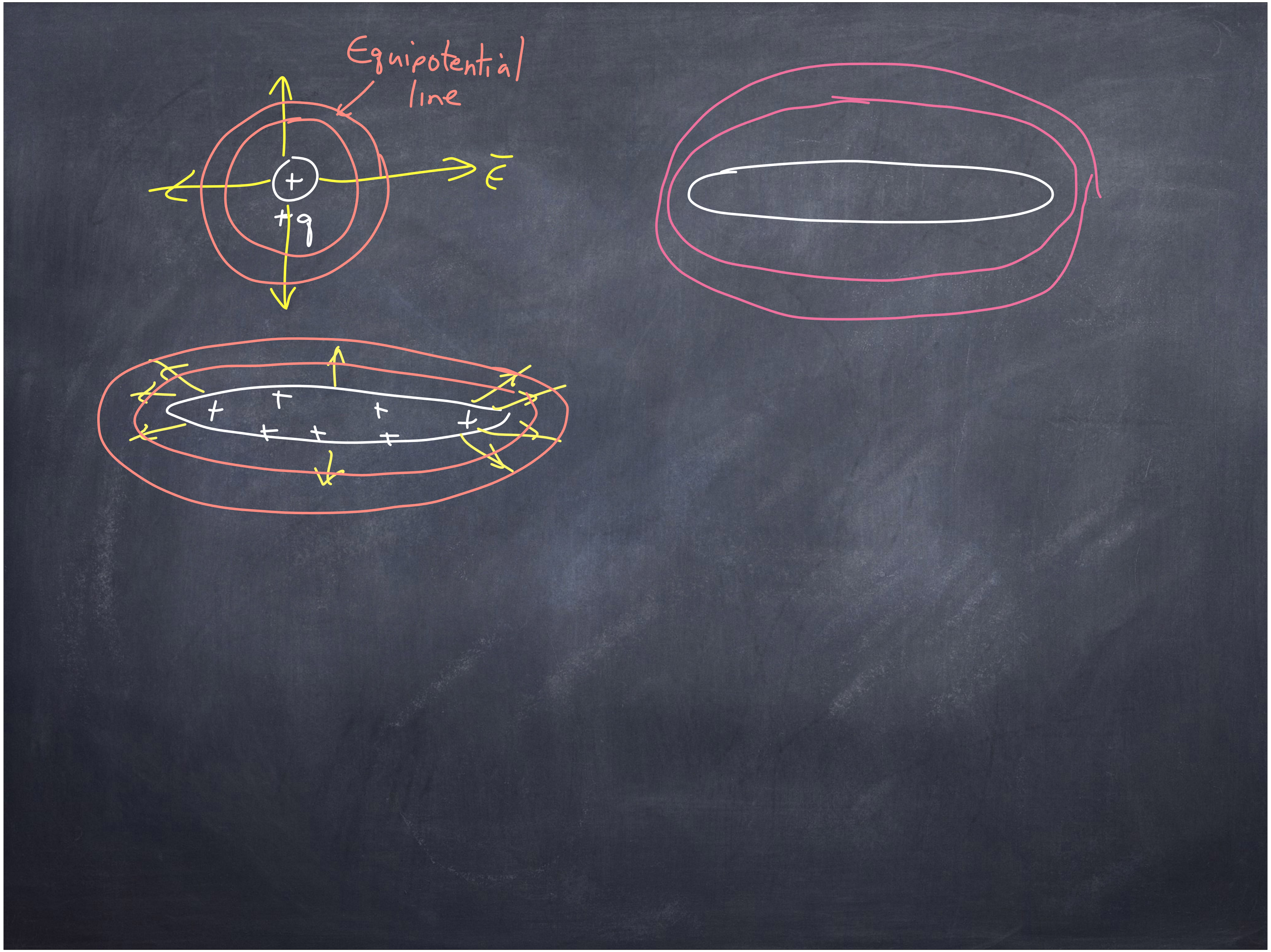
(b) Negative charge.

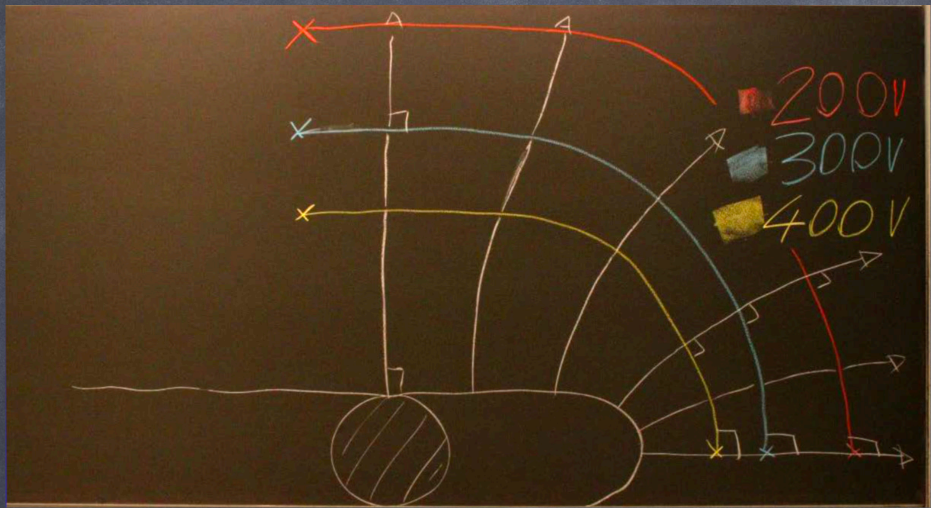
(c) Opposite point charges.

(d) Same-sign point charges.

(e) Uniform field (like for a infinite sheet of charge).

Figure 3.6: Equipotential surfaces (blue) of electric field lines (orange) for different configurations of point charges. All the points on the same equipotential have the same electric potential. The equipotential are equidistant to each other: Two neighbouring equipotentials differ by a fixed voltage ΔV.


Equipotential line
+
+q
$\bar{E}$
+ + + + + + +


200V
300V
400V

How do we get $\bar{E}$ from V ?

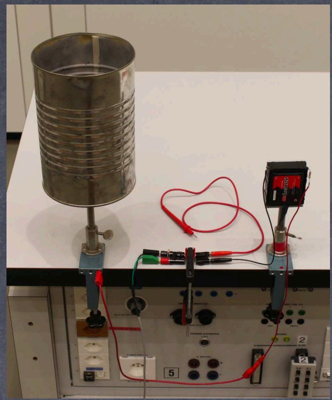
$$V = \int dV = -\int \bar{E} \cdot d\bar{l}$$

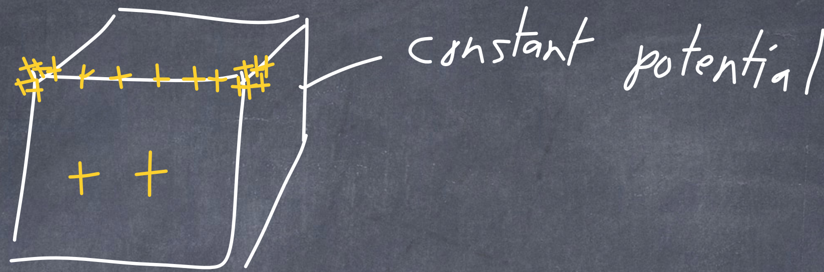
$$\downarrow$$

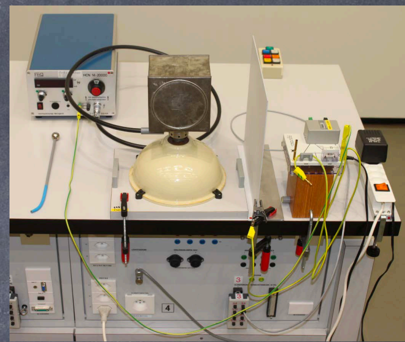
$$\bar{E} = -\frac{dV}{dl}$$ (If $\bar{E}$ changes with l)

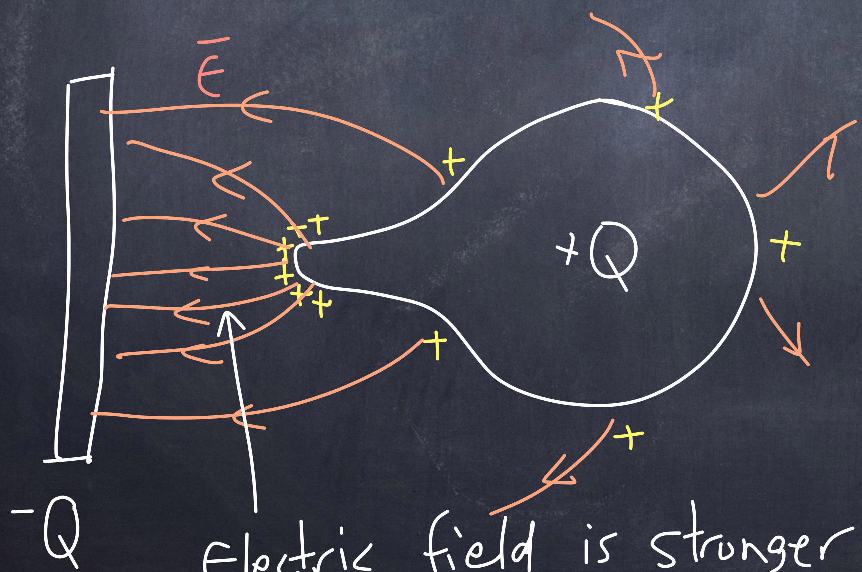
For instance $V = \frac{kq}{r}$ (we have r changing so we use $\frac{d}{dr}$)

$$\bar{E} = -\frac{d}{dr}\left(\frac{kq}{r}\right)\hat{r} = \frac{kq}{r^2}\hat{r}$$





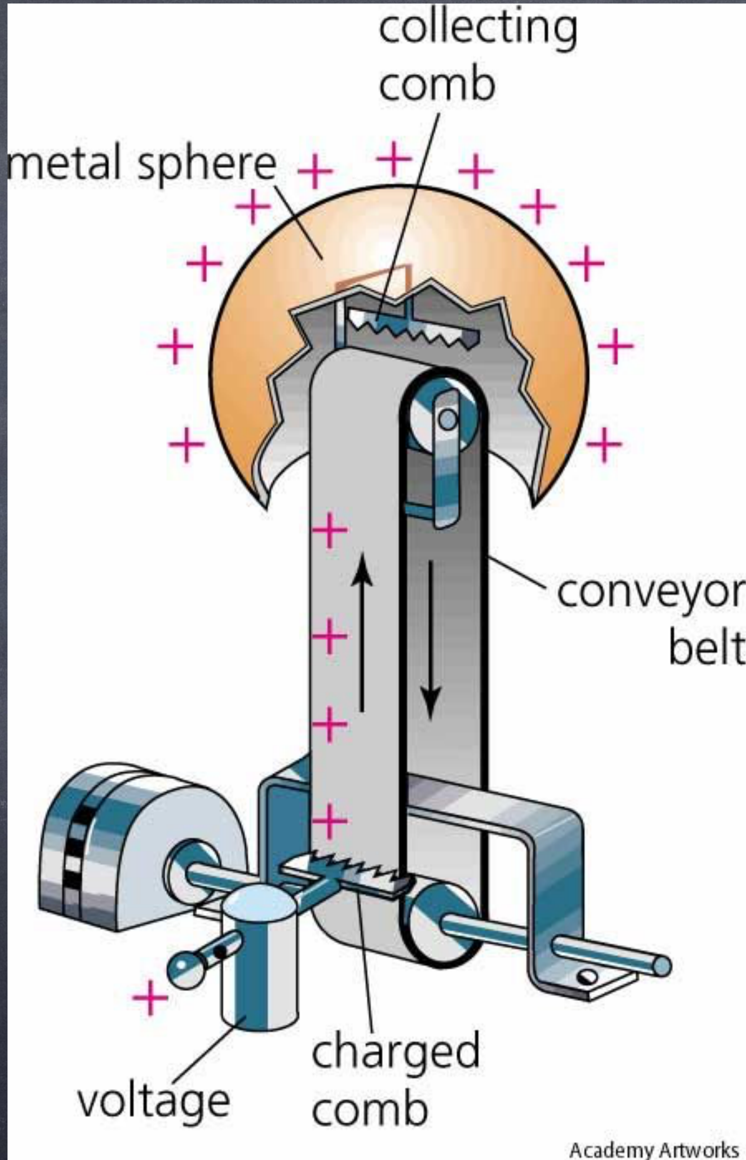




Electric field is stronger at sharper parts of object.

Van de Graaff
voltage generator

Here we add
charge to a
conductor
+
increase its
potentia



Is there a limit to the potential V on a conductor from adding charge Q?

yes. At high electric fields, the air becomes ionized.

$$E_{max} \approx 3E6 \frac{V}{m} \quad (\text{air})$$

This is the dielectric breakdown of air, Air starts conducting electricity above this value. Lightning is electric discharge.

What is the maximum Q + V of a spherical conductor with radius R in air?

$$E = \frac{\sigma}{\varepsilon_0} \Rightarrow \sigma_{max} = E_{max}\varepsilon_0$$

E-field at the surface of a conductor where $\sigma = \frac{Q}{\text{area}}$

For a sphere, $\sigma = \frac{Q}{4\pi R^2}$

$$Q_{max} = \sigma_{max} \cdot 4\pi R^2$$

$$Q_{max} = E_{max} \cdot \varepsilon_0 \cdot 4\pi R^2$$

the bigger the radius, the more charge can be stored before breakdown

Since $E = \frac{Q}{4\pi\varepsilon_0 R^2}$ + $V = \frac{Q}{4\pi\varepsilon_0 R}$ $V = E \cdot R$

Then $V_{max} = E_{MAX} \cdot R$

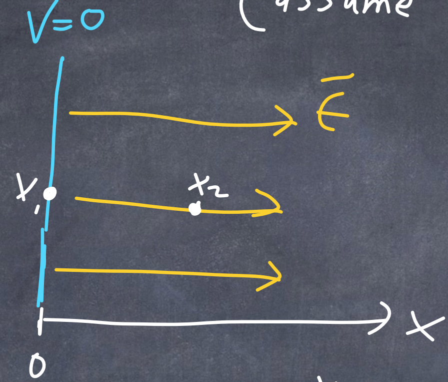
$\Rightarrow$ the maximum potential before discharge increases with R $\Rightarrow$ big R means higher risk of lightning

Skyguide Lägern Radar:
provides airplane navigation
in the hills between
Dielsdorf + Baden

what is $V(x)$ if $\bar{E} = 10\frac{N}{C}\hat{x}$?

(assume that $V=0$ at $x=0$)

$$E = 10\frac{N}{C} = 10\frac{V}{m}$$



$$dV = -\bar{E}\cdot d\bar{l}$$

$$dV = -10\frac{V}{m}\hat{x}\cdot dx\,\hat{x}$$

$$dV = -10\,dx\left[\frac{V}{m}\right]$$

$$V(x_2)-V(x_1) = \int_{x_1}^{x_2} dV = \int_{x_1}^{x_2} -10\,dx\left[\frac{V}{m}\right] = -10x\Big]_{x_1}^{x_2}\left[\frac{V}{m}\right]$$

$$= 10\frac{V}{m}(x_1-x_2)$$

we are told that $V=0$ at $x=0$

so $V(x_2)-V(x_1) = 10\frac{V}{m}(x_1-x_2)$

since $V(x_1=0)=0$

$$V(x_2)-0 = 10\frac{V}{m}(0-x_2) \Rightarrow V(x_2) = -10x_2\left[\frac{V}{m}\right]$$

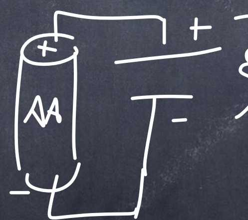
we saw that $$\underset{\substack{\uparrow\\ \text{energy}\\ [J]}}{\Delta U} = \underset{\substack{\uparrow\\ \text{charge}\\ [C]}}{q}\ \underset{\substack{\uparrow\\ \text{potential}\\ [V]}}{\Delta V}$$

A convenient unit of energy is the electron volt $[e \cdot V]$

example

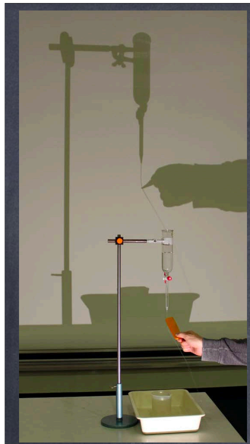
$$\Delta U = 1\,\underset{\substack{\uparrow\\ \text{charge}\\ \text{of electron}}}{e}V = 1\,(1.6\text{E-}19\,C)(1\,V) = 1.6\text{E-}19\,J$$

battery



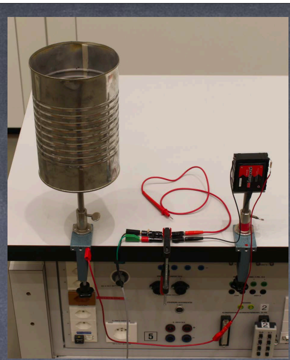
$\Delta V = 1.5\,V$ $\Longrightarrow$ An electron moving through 1.5 V of potential difference will gain 1.5 eV of energy

The Large Hadron Collider (LHC) at CERN has protons with energy 6.5 teV = 6.5E12 eV



ES43

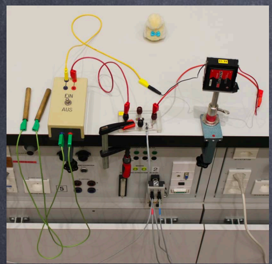
ES62



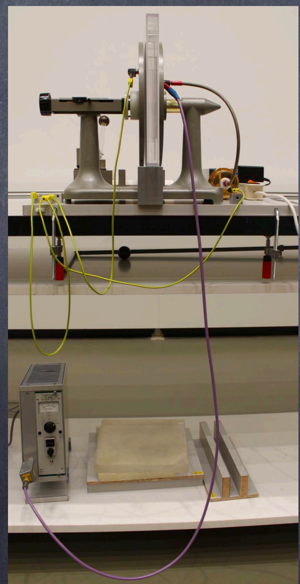
ES12

ES28

ES20



ES70



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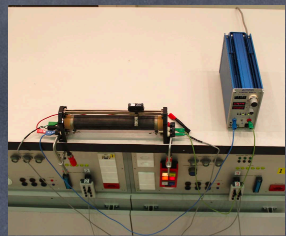

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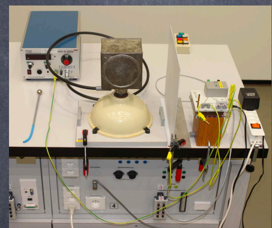

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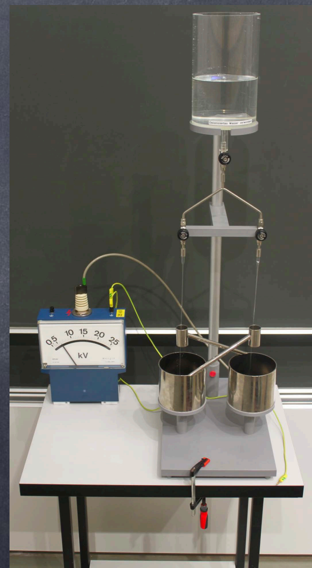

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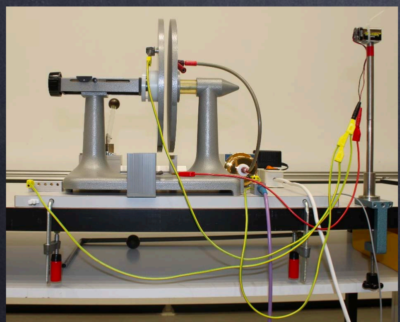

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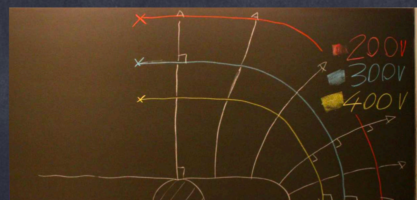

ES10