PHY 117 HS2024

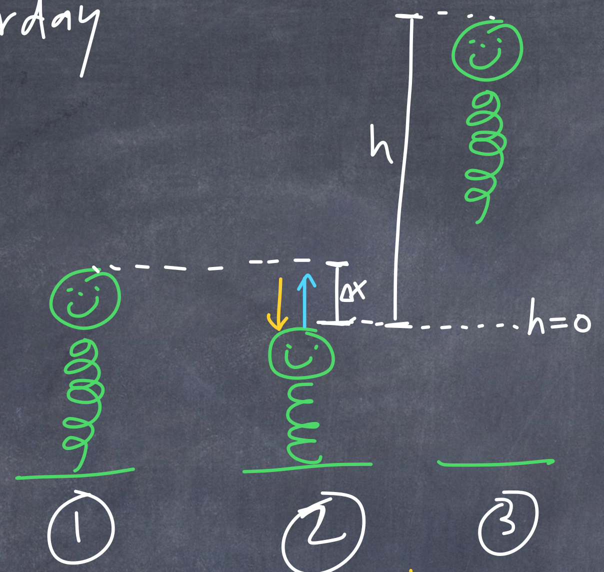
Today:
Energy transformation / conservation
Non-conservative forces
Elastic and inelastic collisions
Momentum conservation
Relationship of momentum and force
Impulse = change in momentum

Quiz #2:
OLAT -> Quizzes -> Quiz 2
100/690 participants.
Average time: 9 minutes.
Please participate !

Note:
K: kinetic energy
units: N.m
k: spring constant
units: $\frac{N}{m}$
Today I write
$\vec{K}$ for kinetic energy
when it is confusing

Week 3, Lecture 2
Oct. 2nd, 2024
Prof. Ben Kilminster

Yesterday



Energies:

1. Spring not compressed

2. I do (+) work
spring does (−) work
$U_s = \frac{1}{2}k(\Delta x)^2$
$U_g = 0$
(because $h=0$)
$K=0$

3. $U_s = 0$
$U_g = mgh$
$K = 0$

we found work for me to compress the spring is $W_{me} = +\frac{1}{2}k(\Delta x)^2$

work done by spring is $W_{spring} = -\frac{1}{2}k(\Delta x)^2$

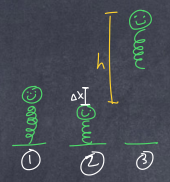
Negative work by a system gives it positive potential energy.

$$\Delta U_{spring} = -W_{spring} = \frac{1}{2}k(\Delta x)^2$$

keep in mind

$$\Delta U = U_f - U_i$$
$$U_f = \Delta U + \cancel{U_i}^{\,0}$$

So $U_s = \Delta U_{spring}$



Applying energy conservation:

Total energy 2 = Total energy 3

$$U_s = U_g$$

$$\frac{1}{2}k(\Delta x)^2 = mgh \Longrightarrow h = \frac{\frac{1}{2}k(\Delta x)^2}{mg}$$

we know:

$k = 612\ \frac{N}{m}$

m = mass of grasshopper
= 0.424 kg

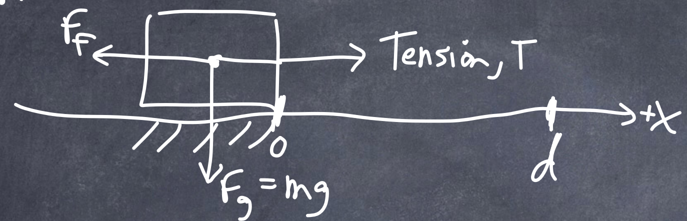
Δx = compression of spring
= 0.14 m

$$h = \frac{\frac{1}{2}\left(612\ \frac{N}{m}\right)(0.14\ m)^2}{(0.424\ kg)(9.8\ m/s^2)} = \boxed{\begin{array}{c}1.44\ m\\ \text{prediction}\end{array}}$$

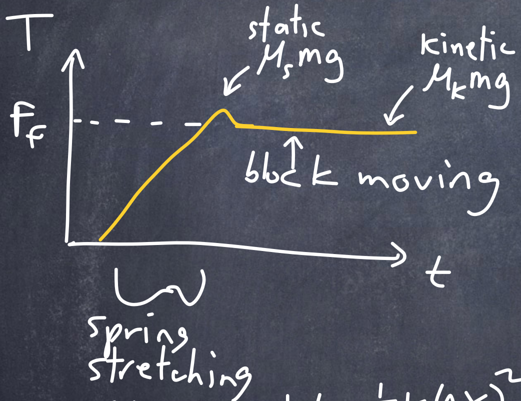
h measured $\stackrel{?}{=}$

Non-conservative force: friction. Friction does work, becomes heat. No longer available for U or $\bar{K}$.

F_f is opposite movement
(−)



$U + \bar{K}$ is not conserved.



$F = k\Delta x \rightarrow W = \frac{1}{2}k(\Delta x)^2$

At constant velocity,

$\Sigma F = 0$

$T - F_f = 0$

$T = F_f$ is constant

$d\bar{x} = dx\,\hat{x}$

$$\bar{F}_f = -\mu F_N\,\hat{x} = -\mu mg\,\hat{x}$$

$$W = \int_{x_1}^{x_2} \bar{F} \cdot d\bar{x} = \int_0^d \bar{F}_f \cdot dx\,\hat{x} = \int_0^d -\mu mg\,\hat{x}\cdot\hat{x}\,dx$$

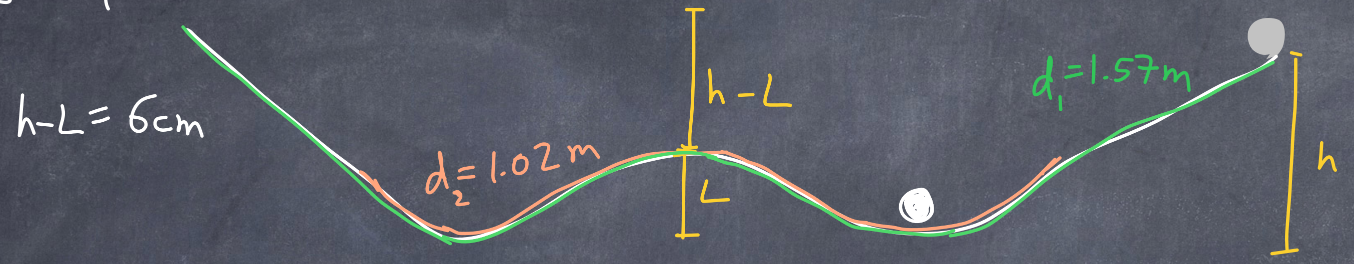
$$W = \int_0^d (-\mu mg)\,dx = -\mu mgx\Big]_0^d$$

$$\boxed{W = -\mu mgd}$$

work done by friction for the case of moving a distance d

we see this is just force times distance because force is constant & ∥ to distance.

How many times will a steel ball roll across this track before getting stuck?



This problem can be solved with energy conservation. We have K, U_g, and the work due to friction, W_f. The ball will roll across the track until it loses gravitational energy through friction, and can't get over the bump. This happens when $U_g = mg(h-L)$ is lost due to the work of friction, $W_f = \mu mgD$

D = total distance traveled

μ_r = coefficient of friction for a steel ball rolling on an aluminum track $= \mu_r = 0.00076$.

d_1 is full length of track (at start)

d_2 is length of track when ball gets stuck (at end)

End is when $W_f = U_g$

$$\mu \cancel{m}\cancel{g} D = \cancel{m}\cancel{g}(h-L)$$

$$D = \text{total distance} = \frac{h-L}{\mu}$$

so $D = \dfrac{0.06\text{ m}}{0.00076} = 79\text{ m}$

How many times across is 79 m?

Length of the track will vary from 1.57 to 1.02 m,

so the average length is $\langle d \rangle = \dfrac{1.57+1.02\text{ m}}{2} = 1.3\text{m}$

so $\dfrac{D}{\langle d \rangle} = \#\text{ times} = 61\text{ times}$

prediction

\# measured = ? =

Collisions

Elastic and inelastic collisions

A collision is elastic if there is no work done by the forces of friction, deformation, or sticking or breaking. Otherwise, it is inelastic.

In elastic collisions, $K + U = \text{constant}$

If U is the same before and after, then K must also be the same. $K_{initial} = K_{final}$

So Kinetic energy is conserved in elastic collisions

Elastic collisions : $K_{initial} = K_{final}$

In elastic collisions, momentum is also conserved.

Momentum is defined as $\bar{p} = m\bar{v}$ (Its a vector)

Conservation of momentum means

If same objects, then $m_i = m_f$

$$\bar{p}_{initial} = \bar{p}_{final} \quad \text{or} \quad \underbrace{\sum m_i \bar{v}_i}_{\text{total initial momentum}} = \underbrace{\sum m_f \bar{v}_f}_{\text{total final momentum}}$$

Experiment:

initial

$\boxed{m_1} \xrightarrow{} v_1$ $\quad$ $\boxed{m_2}$ $v_2 = 0$

$m_1 = m_2$

Final ?

$\boxed{m_1}$ $v_1^f = ?$ $\quad$ $\boxed{m_2}$ $v_2^f = ?$

conservation of momentum: $m_1 \bar{v}_1 = m_1 \bar{v}_1^f + m_2 \bar{v}_2^f$ (1)

conservation of kinetic energy: $\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 v_1^{f^2} + \frac{1}{2} m_2 v_2^{f^2}$ (2)

Rewrite: (1) : $m_1 (v_1 - v_1^f) = m_2 v_2^f$ (3)

Rewrite (2) : $m_1 (v_1^2 - v_1^{f^2}) = m_2 v_2^{f^2}$

$m_1 (v_1 - v_1^f)(v_1 + v_1^f) = m_2 v_2^{f^2}$ (4)

$m_1 v_1^2 - m_1 v_1^{f^2} = m_2 v_2^{f^2}$

$m_1 (v_1^2 - v_1^{f^2}) = m_2 v_2^{f^2}$

Factorize: $a^2 - b^2 = (a+b)(a-b)$

Divide (4) by (3) : $v_1 + v_1^f = v_2^f$ (5)

Substitute (5) → (3) : $m_1 (v_1 - v_1^f) = m_2 (v_1 + v_1^f)$

$m_1 v_1 - m_1 v_1^f = m_2 v_1 + m_2 v_1^f$

$v_1 (m_1 - m_2) = v_1^f (m_1 + m_2)$

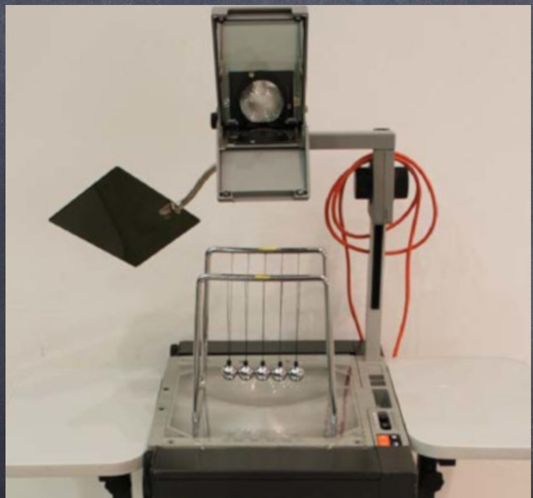
$v_1^f = v_1 \left(\frac{m_1 - m_2}{m_1 + m_2} \right)$ (6)

In our case $m_1 = m_2$, so (6) becomes $v_1^f = 0$

so car 1 will stop, and car 2 will continue with same velocity as car 1 initial

If $v_1^f = 0$, then by (1) → $v_2^f = v_1$ (and $m_1 = m_2$)

momentum and kinetic energy (approximately) conserved

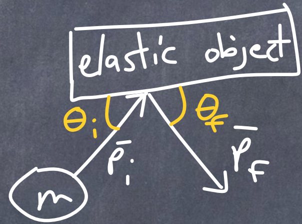


Approximately elastic

What if the collision is inelastic ?

$\bar{K}$ is not conserved.

However, $\bar{p}$ is conserved.



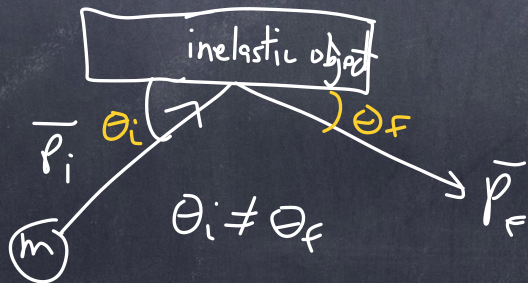
$|\bar{p}_i| = |\bar{p}_f|$

Elastic collision : $\theta_i = \theta_f$

x: $p_{i_x} = p_{i_f}$

y: $p_{i_y} = -p_{f_y}$

Q: why does object change directions, $\Delta p \neq 0$? (see Impulse...)



$\theta_i \neq \theta_f$

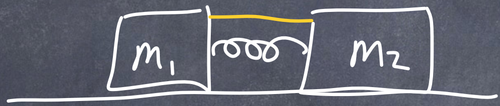
$|\bar{p}_i| \neq |\bar{p}_f|$

what happens?

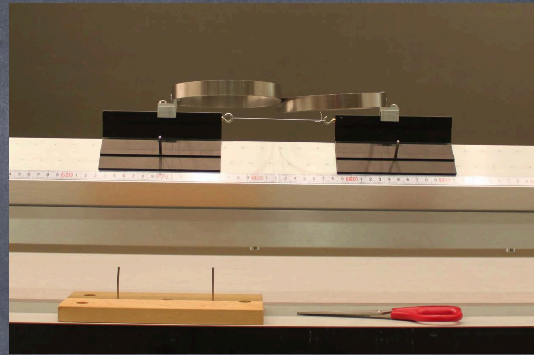
we need to account for the deformation or movement of inelastic object.

Other momentum conservation examples:

①



initial state
$\Sigma \bar{p} = 0$



final momentum $= 0$

$$0 = m_1 v_1 + m_2 v_2$$

$$m_1 v_1 = -m_2 v_2$$

If $m_1 = m_2$, then $v_1 = -v_2$

②

$m_2 > m_1$

$$p_i = 0 = p_F$$

Final: $p_F = 0 = m_1 v_1 + m_2 v_2$

$$v_1 = -v_2\left(\frac{m_2}{m_1}\right)$$

If $m_2 > m_1$, then $|v_1| > |v_2|$

Relation of momentum and force:

$$\bar{p} = m\bar{v}$$

$$\frac{d\bar{p}}{dt} = m\frac{d\bar{v}}{dt}$$

(If mass doesn't change, in a rocket, mass cha es also so $\frac{dp}{dt} = m\frac{dv}{dt} + v\frac{dm}{dt}$)

but this is $\bar{a} = \frac{d\bar{v}}{dt}$

so $\frac{d\bar{p}}{dt} = m\bar{a} = \bar{F} \Longrightarrow$ A net force will change an object's momentum.

Momentum conservation:

so $\bar{F} = \frac{d\bar{p}}{dt} \longrightarrow d\bar{p} = \bar{F}\,dt \Rightarrow \Delta\bar{p} = \int_{t_1}^{t_2} F\,dt$

so where $\Delta\bar{p}$ = change in momentum $= \bar{p}_2 - \bar{p}_1$

If $\Delta p = 0 \Rightarrow$ momentum is conserved if no net force

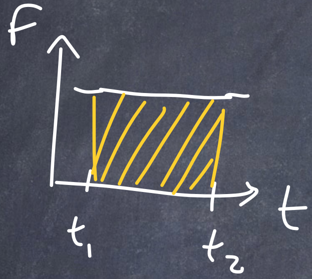
Let's look at forces changing with time.

$$\bar{F} = \frac{d\bar{p}}{dt} \qquad d\bar{p} = \bar{F}\,dt$$

If the force is constant, then $\Delta p = F \Delta t$

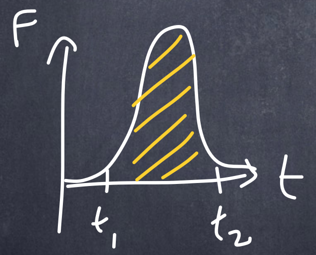
constant force

amount of time of force.

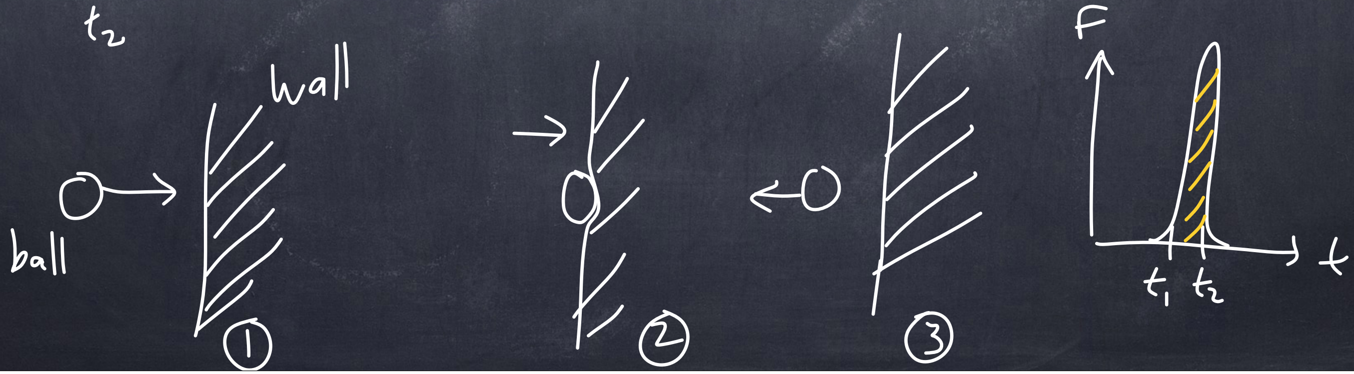


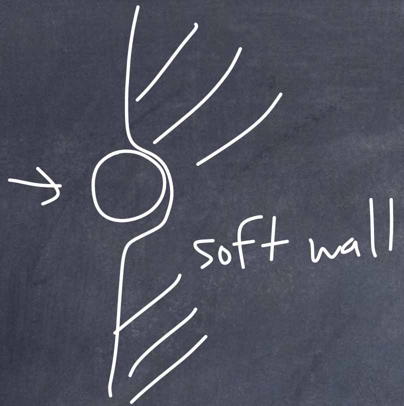
change in momentum is the area.

If the force is not constant, then

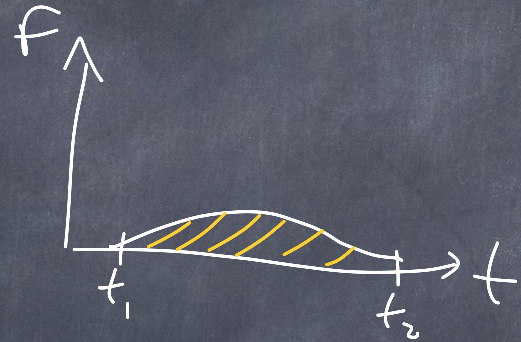


then $dp = \int_{t_1}^{t_2} F(t)\,dt$ = area under the curve of F vs. t from t_1 to t_2





In this case, Δt is longer, so the contact is longer



The force is more constant with time.

$$\Delta p = \int F(t)\,dt = \text{Impulse} = I$$

we can approximate the average force:

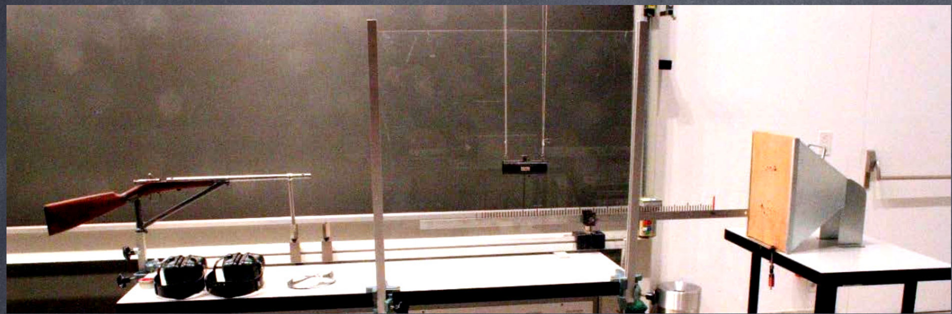
Time average of force $= F_{AV}$ is equivalent to a constant force over time.

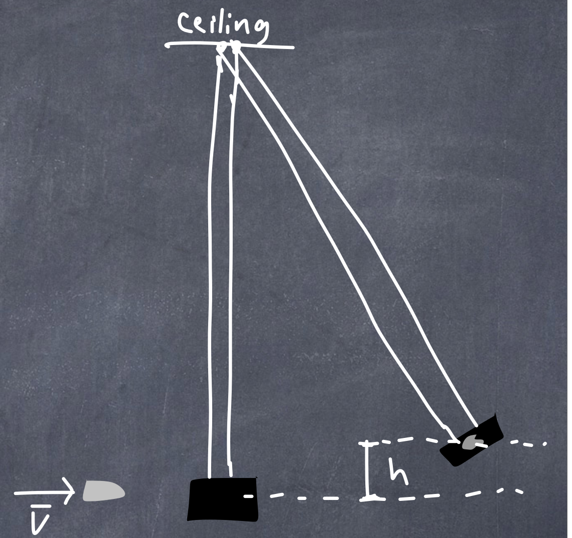
$$(F_{AV})(\Delta t) = \int_{t_1}^{t_2} F(t)\,dt$$

we can minimize the average force in a collision if Δt is long or if the objects deform or break. (Inelastic collision)

will they break?

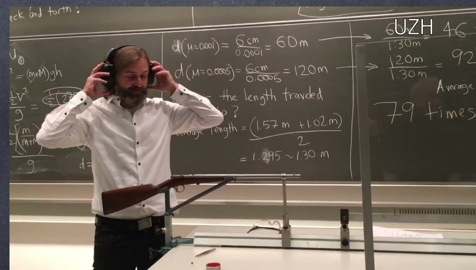
Ballistic pendulum experiment
(can't do in this lecture hall)





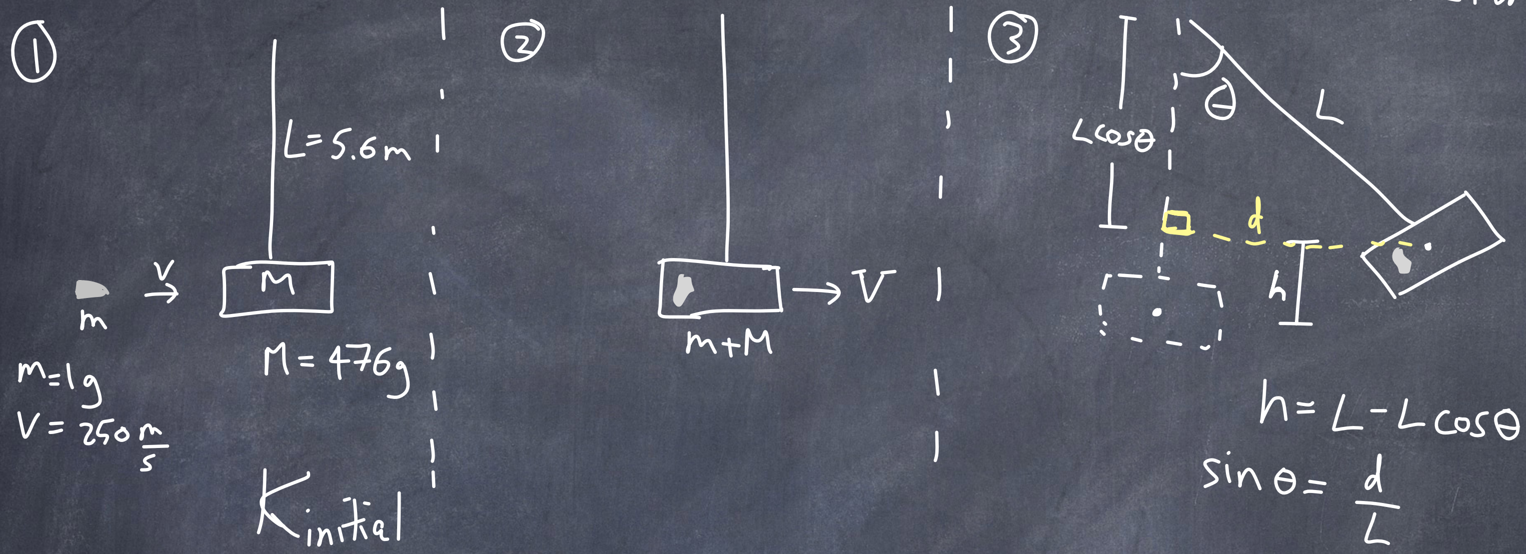
How high will the block go?

see video posted



https://www.youtube.com/watch?v=v2s6Nxc6Dr8

Ballistic Pendulum – Momentum conservation in inelastic collision.



Between ① + ②, momentum is conserved. **Inelastic collision**

Kinetic energy is not conserved.

$$mv = (m+M)V \qquad V: \text{velocity of block + bullet.}$$

Between ② + ③, this is elastic.

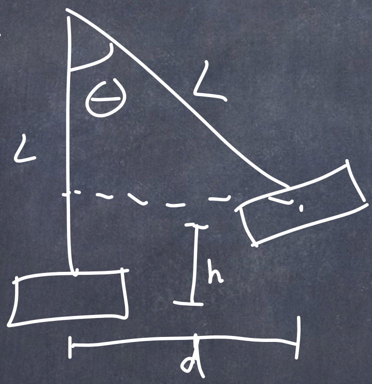
$$K_{②} = U_{③}$$

$$\underbrace{\frac{1}{2}(\cancel{m+M})V^2}_{K_{2}} = \underbrace{(\cancel{m+M})gh}_{U_{3}}$$

solve for $h = \dfrac{\frac{1}{2}V^2}{g} = \dfrac{\frac{1}{2}\left[\frac{m}{m+M}v\right]^2}{g} =$

$\boxed{\begin{array}{l}\text{predicted } h \\ 0.014\,m\end{array}}$

$L = 5.6\,m$



we measure d.

$h = L - L\cos\theta$

$\sin\theta = \dfrac{d}{L} \Rightarrow \theta = \sin^{-1}\left(\dfrac{d}{L}\right)$

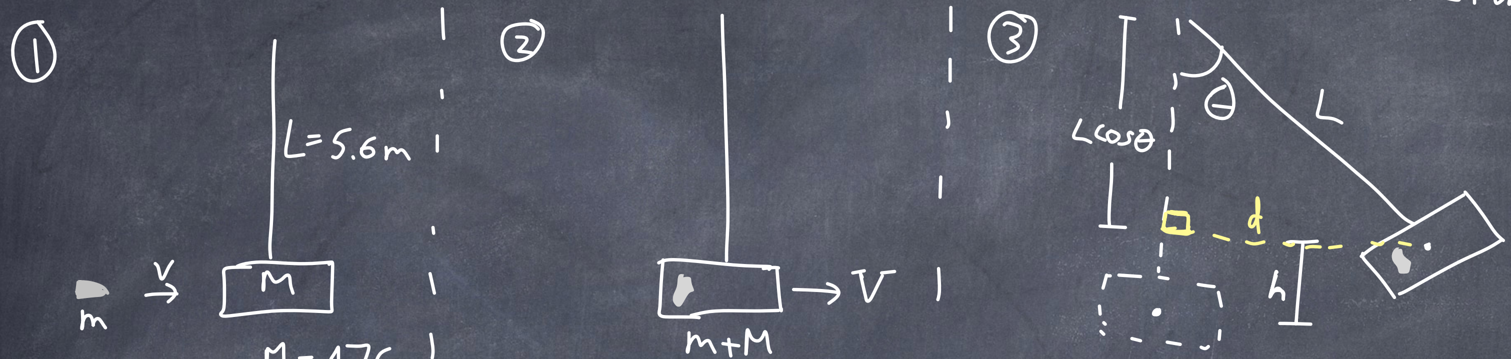
we can measure d, figure out θ, then figure out h.

measure $d = 0.4\,m$, $\theta = \sin^{-1}\left(\dfrac{0.4\,m}{5.6\,m}\right) = 0.07$ radians

and calculate $h = L - L\cos(0.07\text{ rad}) = 5.6\,m - 5.585$

$\boxed{\text{measured } h = 0.014\,m}$

Ballistic Pendulum – Momentum conservation in inelastic collision.



$m = 1g$
$V = 250 \frac{m}{s}$

$M = 476g$

$K_{initial}$

$h = L - L\cos\theta$
$\sin\theta = \frac{d}{L}$

Note:

If kinetic energy was conserved, then

$$K_{①} = U_{③}$$

$$\frac{1}{2}mv^2 = mgh \longrightarrow h = \frac{v^2}{2g}$$

$$\underline{\underline{h = 6.7\,m}}$$

wrong!
(collision is inelastic)

Questions from students:

1) what is the difference between elastic & inelastic collisions in terms of F vs. t ?

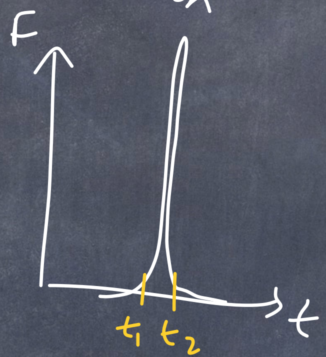
2) If momentum changes directions, wouldn't the average force always be the same ?

3) How can something change directions ? Isn't momentum not conserved if this happens ?

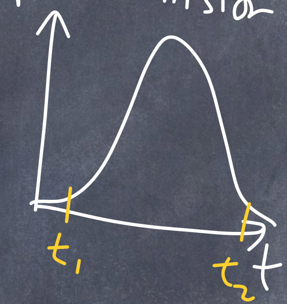
4) what if the problem is 2-dimensional ? How is momentum conserved ?

1) what is the difference between elastic & inelastic collisions in terms of F vs. t ?

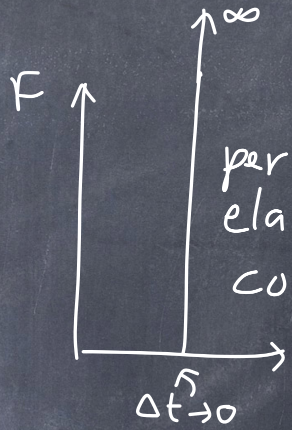
elastic collision



inelastic collision



perfectly elastic collision



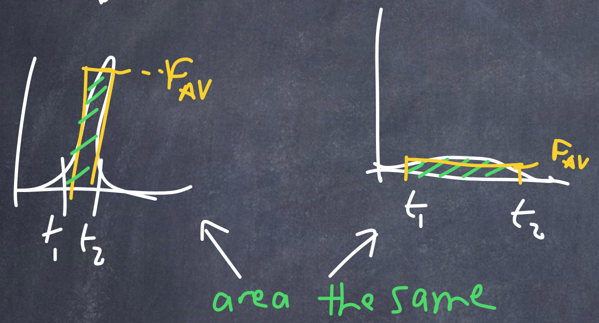
For perfectly elastic collisions,
$\Delta t \to 0$, $F \to \infty$
(these don't happen in the real world.
All interactions have some Δt)

$$\Delta p = I = F_{AV}\,\Delta t$$

$\boxed{\text{2) If momentum changes directions, isn't } F_{AV} \text{ always the same?}}$

The average force depends on Δt.

either

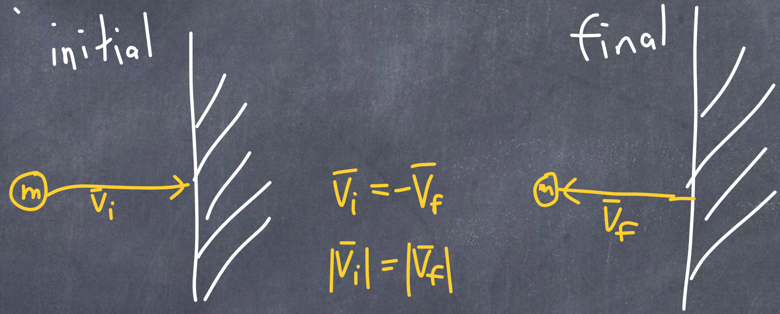


Notice, we can have the same $\Delta p = p_2 - p_1$ in cases where Δt is small and F_{AV} is large, or when Δt is large and F_{AV} is small.

In both cases Δp is the same (area is the same)

How can something change directions? Isn't momentum $\underline{\text{not}}$ conserved if this happens?

Consider this case:



$$\bar{V}_i = -\bar{V}_f$$

$$|\bar{V}_i| = |\bar{V}_f|$$

The momentum changes from mV_i to mV_f

$$\bar{p}_f = -\bar{p}_i \quad (1)$$

substitute in (1)

What happened?

$$\Delta\bar{p} = \bar{p}_2 - \bar{p}_1 = p_f - p_i = -\bar{p}_i - \bar{p}_i = -2\bar{p}_i$$

The momentum changed.

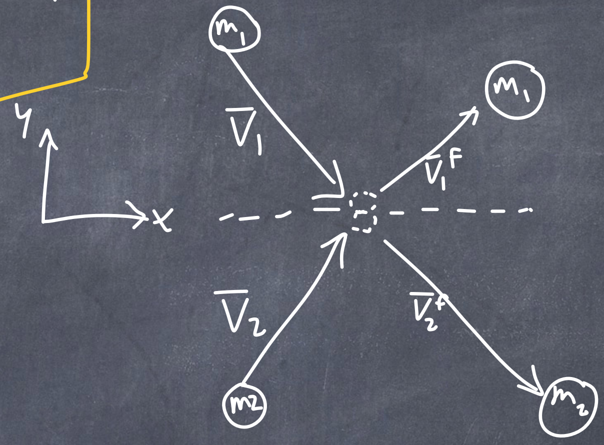
How? The wall provides a force F_{AV} for a time Δt

$$\Delta p = F_{AV}\Delta t \implies \bar{F}_{AV} = \frac{-2\bar{p}_i}{\Delta t}$$

If we know Δt, we can calculate F_{AV}

④ What if the problem is 2-dimensional?
How is momentum conserved?

consider this inelastic collision



Momentum will be conserved in all directions:

Therefore, $\Sigma \bar{p}_i = \Sigma \bar{p}_f$ we have these equations.

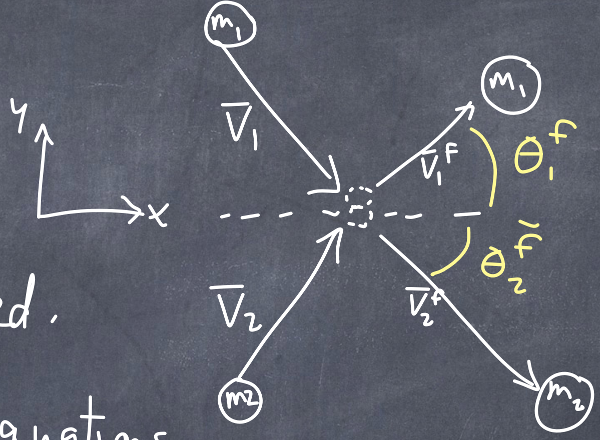
$$\Sigma p_{x_i} = \Sigma p_{x_f} \longrightarrow \begin{cases} m_1 v_{1_x} + m_2 v_{2_x} = m_1 v_{1_x}^f + m_2 v_{2_x}^f \\ m_1 v_{1_y} + m_2 v_{2_y} = m_1 v_{1_y}^f + m_2 v_{2_y}^f \end{cases}$$

$$\Sigma p_{y_i} = \Sigma p_{y_f}$$

If we know the masses and initial velocities, we would have 4 unknowns and 2 equations. So we need more information to solve this.

④ Continued...

If the collision in 2D were also elastic, then K would be conserved.



Then we would have 3 equations and 4 unknowns:

Kinetic energy conservation: $\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}m_1 {v_1^f}^2 + \frac{1}{2}m_2 {v_2^f}^2$

momentum conservation in x: $m_1 v_{1_x} + m_2 v_{2_x} = m_1 v_{1_x}^f + m_2 v_{2_x}^f$

momentum conservation in y: $m_1 v_{1_y} + m_2 v_{2_y} = m_1 v_{1_y}^f + m_2 v_{2_y}^f$

Knowing one of the angles, θ_1^f or θ_2^f, or one final velocity component ($v_{1_x}^f, v_{2_x}^f, v_{1_y}^f, v_{2_y}^f$) would be enough information to solve the problem.