Let there be light!

PHY117 HS2024

Today:

Electromagnetic waves

Creating light theoretically + experimentally

Speed of light

Refraction

Week 13, Lecture 1

Dec. 10th, 2024

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We saw: changing magnetic fields produce electric fields.
changing (moving, for instance) electric fields produce magnetic fields.
This positive feedback generates a self-sustaining phenomenon.

If one solves the four inter-related equations of electricity & magnetism, known as Maxwell's equations (Gauss' Law is one), the solution one finds is a wave, moving at constant speed. The wave is referred to as an electromagnetic wave or "light".

This discovery (Maxwell, 1865) generated other discoveries, like the radio, and paved the way for relativity and our modern understanding of forces and interactions.

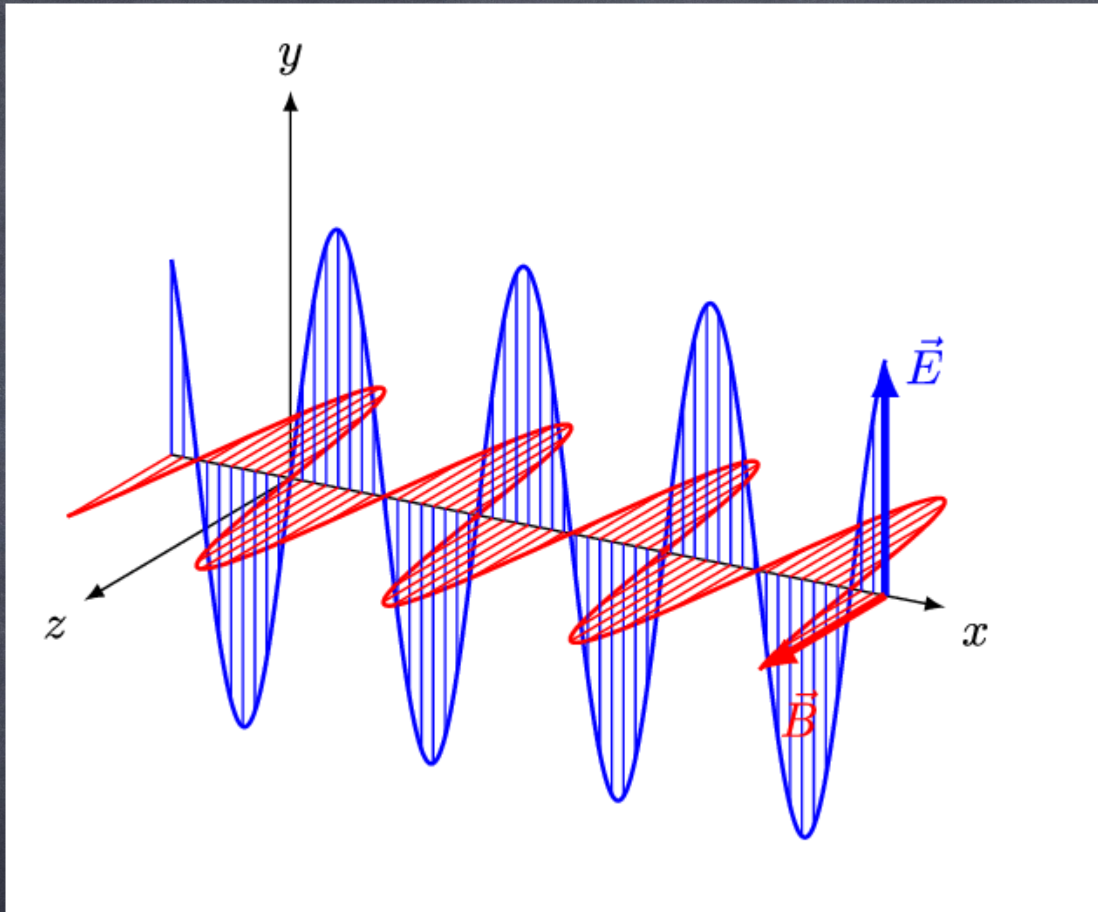
	MAXWELL'S EQUATIONS IN INTEGRAL FORM	
LHS	$X = E$	$X = B$
$\oint X.dA$	Gauss's Law (1813) $\oint E.dA = \frac{Q_{inside}}{\varepsilon_o}$ The Electric field through a closed area is equal to the total charge inside of the area divided by ε_o.	Gauss's Law for Magnetism (1813) $\oint B.dA = 0$ The Magnetic field through a closed surface is zero (as many field lines going out as going in). It means that magnetic monopoles do not exist.
$\oint X.dl$	Faraday's Law (1831) $\oint E.dl = -\int \frac{\partial B}{\partial t}.dA$ The Electric field around a closed loop is just equal to the minus of the rate of change of Magnetic field through the loop.	Maxwell - Ampere's Law (1861) $\oint B.dl = \mu_o I + \mu_o \epsilon_o \int \frac{\partial E}{\partial t}.dA$ The Magnetic field around a closed loop is equal to rate of change of Electric field through the loop times $\mu_o \epsilon_o$ plus the Electric current in the loop times μ_o.

The solution to these 4 equations is a transverse wave with specific directions for $\bar{E}$ & $\bar{B}$ fields, and a fixed wave velocity in a vacuum.

You actually know all these!
Except this one piece.

Electromagnetic (EM waves)

Example of EM wave as a sine wave

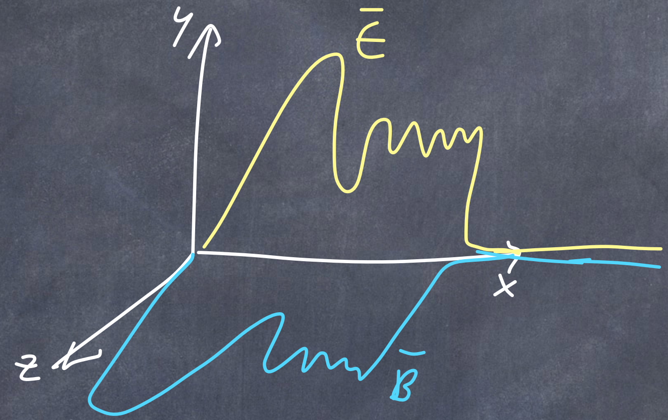


Notice:

1) $\bar{E} \perp \bar{B}$ always
2) $|\bar{E}| \propto |\bar{B}|$ at each moment in time
3) The direction of propagation of our wave ($\bar{V}$) is $\perp$ to $\bar{E}$ & $\bar{B}$

$\longrightarrow \bar{V} \propto \bar{E} \times \bar{B}$ right-hand rule

The shape does not need to be sinusoidal. It can be a weird pulse:



But we can represent any wave or pulse by superpositions of sine waves (using* Fourier transforms) so it is convenient to consider EM waves as sine waves.

* not covered in this class, but in script.

Solving Maxwell's equations, we find that the velocity of the waves is related to ε_0 & μ_0 (in vacuum)

$$v = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \equiv \text{"}c\text{"} \text{ in a vacuum.} \qquad (4)$$

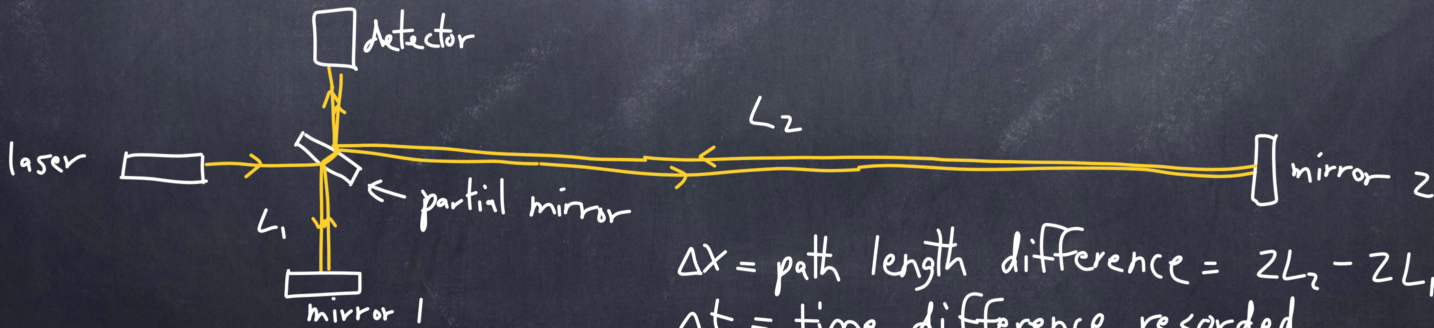
This is pretty cool! If we measure the electric field of a charge, we can determine ε_0. If we measure the magnetic field of an electric current, we can determine μ_0. Together, these "predict" the speed of light.

$$\varepsilon_0 = \frac{Q}{\oint \bar{E} \cdot d\bar{A}} = 8.85 \times 10^{-12} \; C^2/N \cdot m^2$$

$$\mu_0 = \frac{\oint \bar{B} \cdot d\bar{l}}{I} = 4\pi \times 10^{-7} \frac{N}{A^2}$$

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 2.99792458 \; \frac{m}{s} \; \underline{\text{exactly}}$$

speed of light measurement



Δx = path length difference = $2L_2 - 2L_1$

Δt = time difference recorded on oscilloscope

$$c = \frac{\Delta x}{\Delta t} = \sim 3\text{e}8\ \frac{m}{s}$$

Note: wave equations from solving Maxwell's equations

$$\frac{\partial^2 E_y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2} = 0 \quad (3)$$

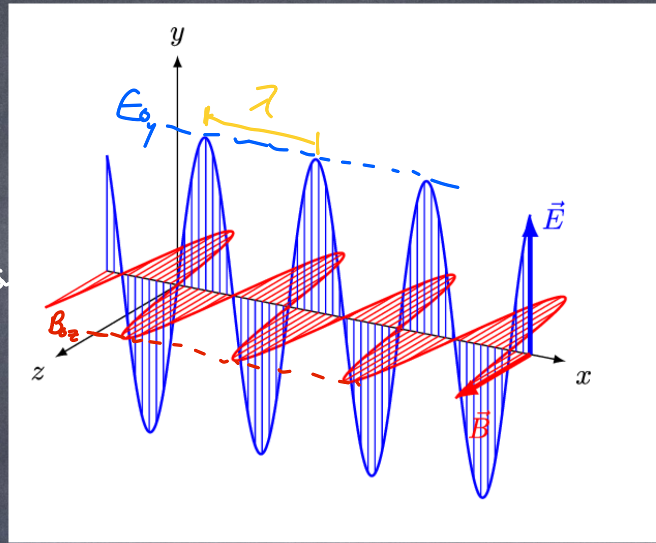
and

$$\frac{\partial^2 B_z}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 B_z}{\partial t^2} = 0$$

The wave functions (e.g., eq (1)) satisfy wave equations for all x and t.

Using what we know about how to define waves, we can write the wave functions for this:

$\bar{E}$ always in $\pm\hat{y}$ direction
$\bar{B}$ always in $\pm\hat{z}$ direction
propagation always in $+\hat{x}$ direction



EM wave function example ⓪

$$\bar{E} = E_{0y} \sin(kx - \omega t)\,\hat{y}$$
$$\bar{B} = B_{0z} \sin(kx - \omega t)\,\hat{z}$$

Since it is a wave, $f\lambda = \frac{\omega}{k}$ = velocity $\equiv c$ = "the speed of light"

we see that $|\vec{E}| \propto |\vec{B}|$. The constant of proportionality is ... you guessed it, c.

For EM waves, $\boxed{E = cB}$ (!) Here $E_y = c B_z$

$$\left[\frac{kg \cdot m}{s^3 \cdot A}\right] \quad \left[\frac{m}{s}\right] \quad \left[\frac{kg}{s^2 \cdot A}\right]$$

In previous example:

$\vec{E} = E_y = E_{0y} \sin(kx - \omega t)\,\hat{y}$

$\vec{B} = B_z = B_{0z} \sin(kx - \omega t)\,\hat{z}$

and $E_y = c B_z$

All EM waves obey $f\lambda = c$

There are different wavelengths of light

visible light: λ: 400 nm $\longleftrightarrow$ 700 nm

400 E-9 m

blue ... red

$$f = \frac{c}{\lambda} = \frac{3E8 \frac{m}{s}}{600E-9\,m} \sim 5E14\ Hz = 500\ THz$$

Table 13.1: Rough classification of electromagnetic spectrum and its applications.

Class	Wavelength λ	Frequency f	Application
Radio waves	$> 1\,\text{m}$	$< 300\,\text{MHz}$	Radio & TV broadcast, telecomunications maritime navigation
Microwaves	$1\,\text{mm} - 1\,\text{m}$	$300\,\text{MHz} - 300\,\text{GHz}$	Microwave oven, radar, mobile phones, 4G, Wi-Fi, satellite communications, GPS, cosmic microwave background
Infrared	$750\,\text{nm} - 1\,\text{mm}$	$300\,\text{GHz} - 400\,\text{THz}$	Thermal imaging, TV remote control, night vision, bio imaging, optical fibers
Visible	$400\,\text{nm} - 750\,\text{nm}$	$400\,\text{THz} - 750\,\text{THz}$	Human vision, illumination, photography, microscopes, lasers
Ultraviolet	$10\,\text{nm} - 400\,\text{nm}$	$750\,\text{THz} - 30\,\text{PHz}$	Disinfection, dental curing, black lights, sun tanning, counterfeit detector
X rays	$0.01\,\text{nm} - 10\,\text{nm}$	$30\,\text{PHz} - 30\,\text{EHz}$	Crystallography, radiation therapy, medical imaging, security scans
Gamma rays	$< 0.01\,\text{nm}$	$> 30\,\text{EHz}$	Radioactive sources, cancer treatments, PET scans, cargo container screening

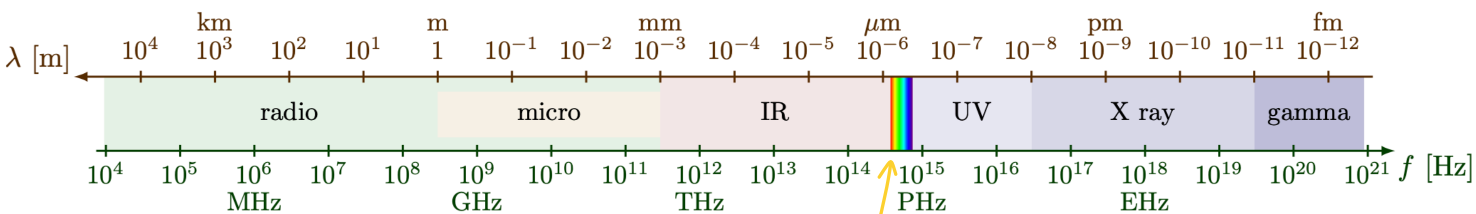
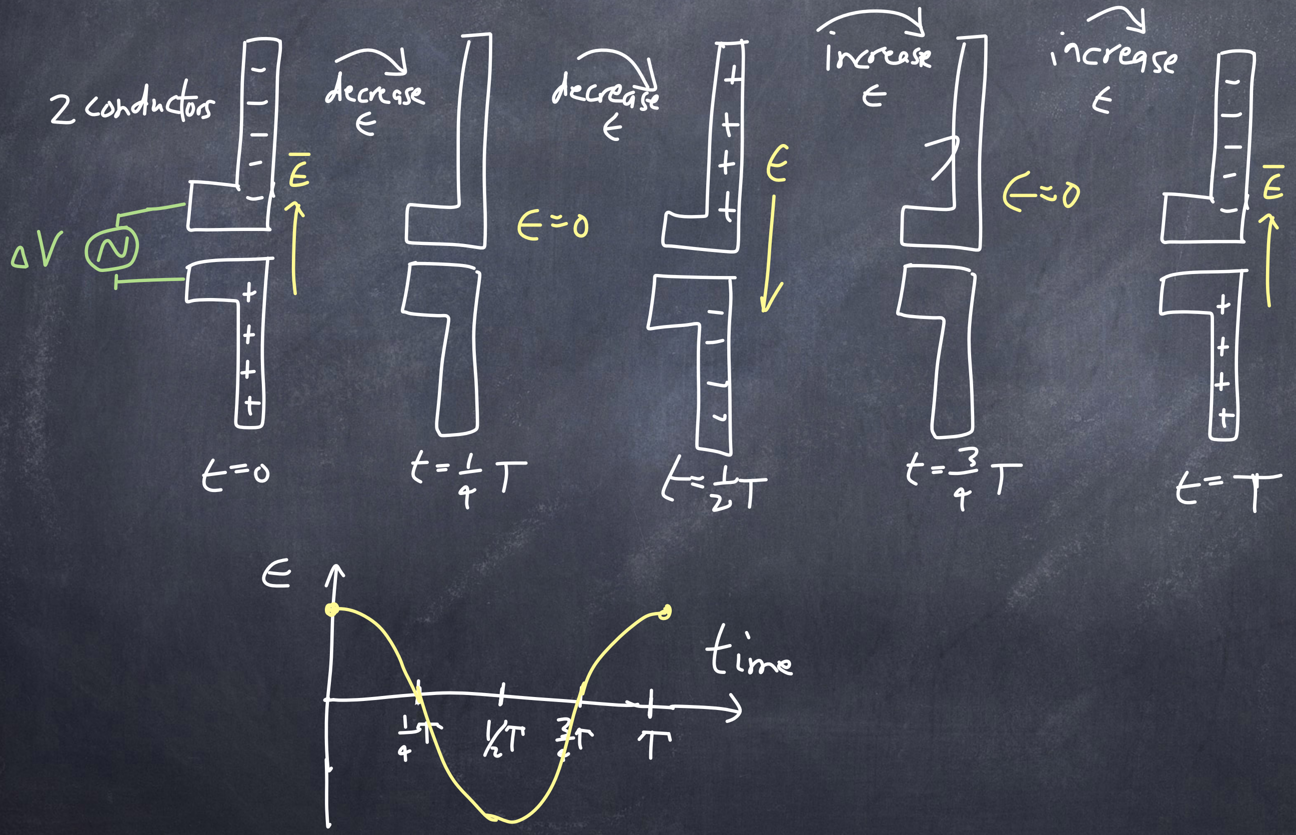


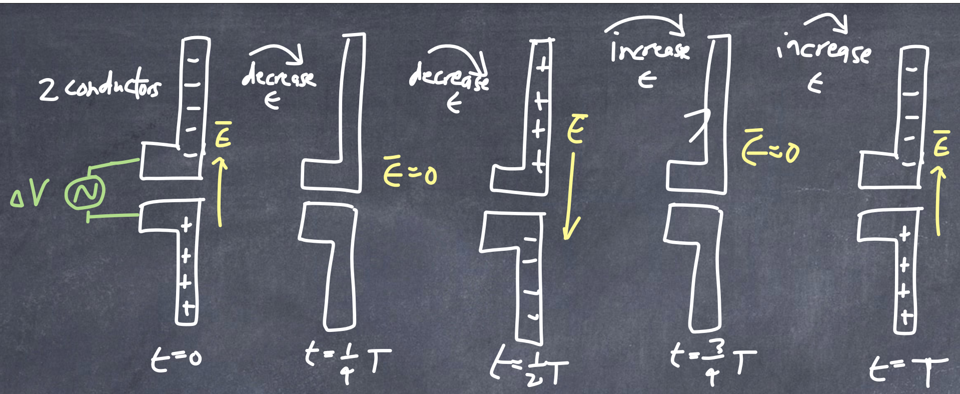
Figure 13.2: Electromagnetic spectrum with rough classification.

human eye sees these

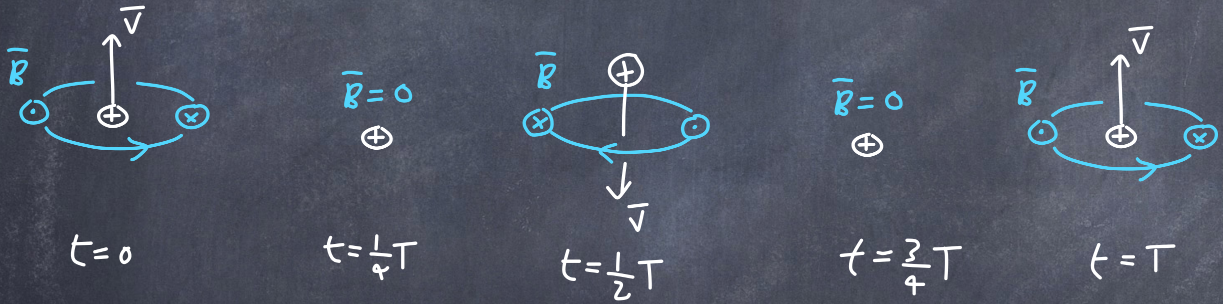
How to create light from moving an electric charge:

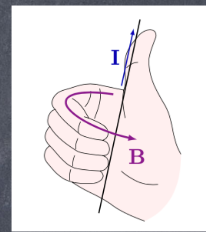
To make an EM wave, we can accelerate an electric charge.



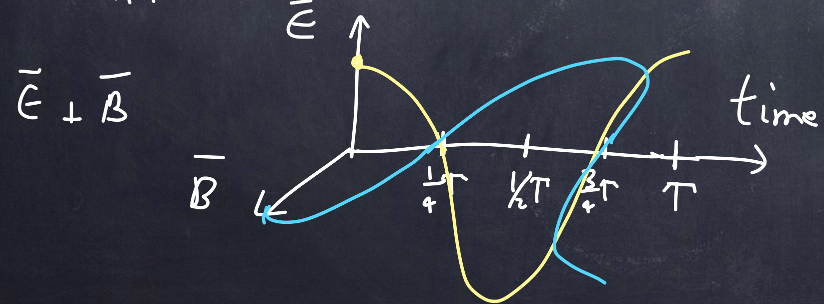


Moving electric charges generate magnetic fields.





Result:

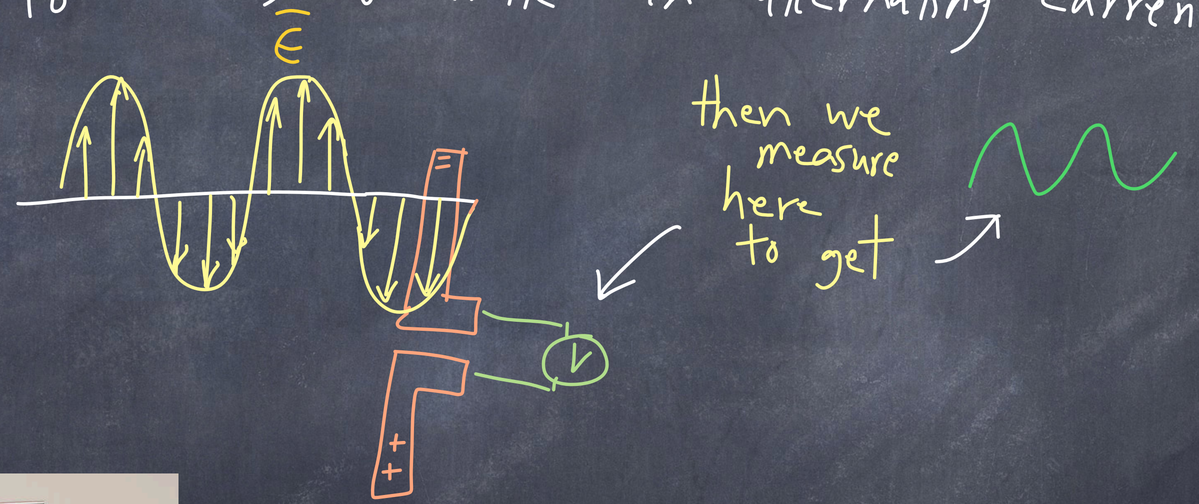


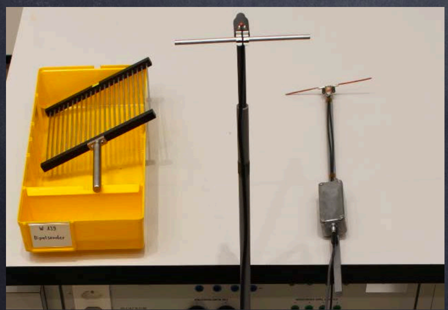
If we make an $\bar{E} \perp \bar{B}$, as above, then light is created!

we have made an EM wave. Detecting an EM wave is the opposite of making one.

The $\bar{E}$ Field of the wave causes electric charges to move, generate an alternating current

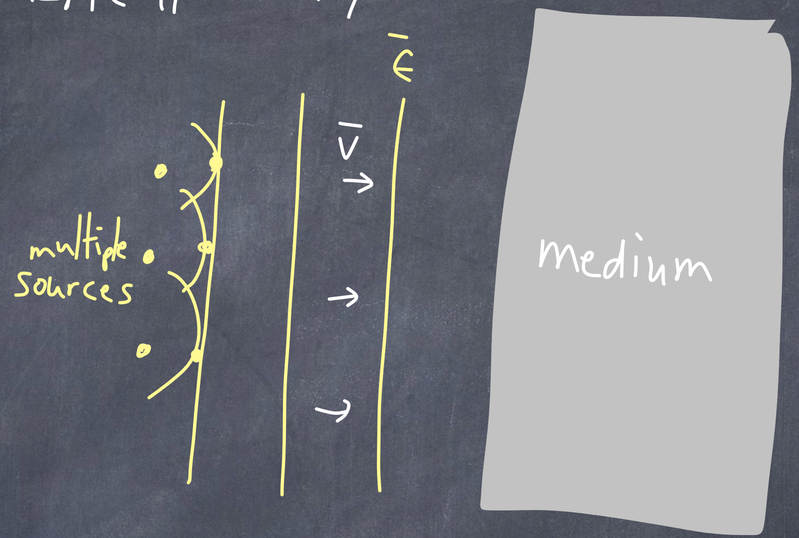
we create:





Light propagates in spherical way according to Huygen's principle

plane waves:



Consider a plane wave entering a medium. What happens?

we know:

$$\boxed{c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}}$$ in a vacuum.

In a medium, with ε, μ $(\varepsilon = k\varepsilon_0, \mu = k_\mu \mu_0)$

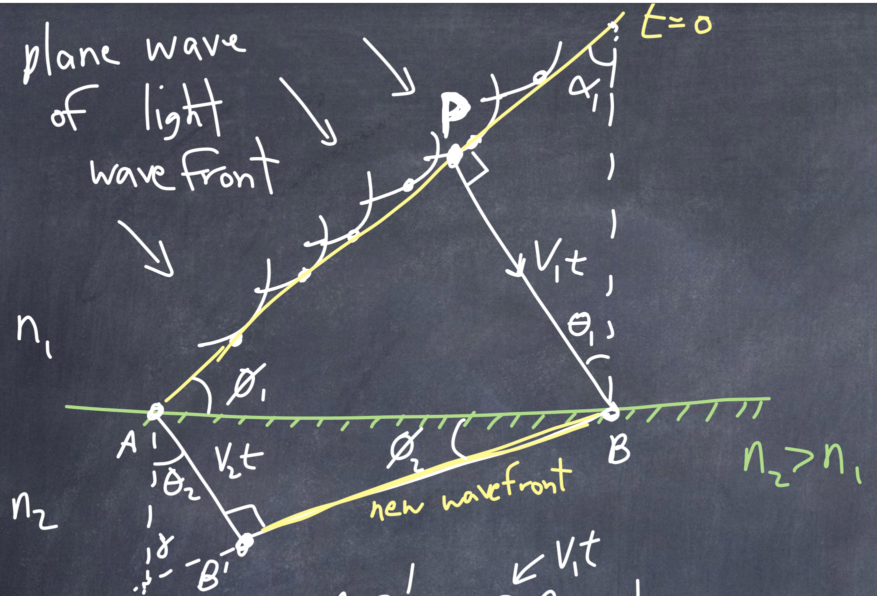
$$c' = \frac{1}{\sqrt{\varepsilon \mu}}$$

The ratio of the speed of light in a medium compared to that in a vacuum is equal to "n"

(5) $$\boxed{\frac{c}{c'} = \frac{\sqrt{\varepsilon \mu}}{\sqrt{\varepsilon_0 \mu_0}} = n}$$

$n > 1$ (for air, $n = 1.00029$)

"n": index of refraction



$V_1 = \frac{c}{n_1}$ $\quad$ $V_2 = \frac{c}{n_2}$

In time time t, the wave propagates a distance.

$$\left.\begin{aligned}\phi_1 + \alpha_1 + 90^\circ &= 180^\circ \\ \theta_1 + \alpha_1 + 90^\circ &= 180^\circ\end{aligned}\right\} \rightarrow \phi_1 = \theta_1$$

$$\left.\begin{aligned}\phi_2 + \gamma + 90^\circ &= 180^\circ \\ \theta_2 + \gamma + 90^\circ &= 180^\circ\end{aligned}\right\} \rightarrow \phi_2 = \theta_2$$

$AB' < PB$ because the velocity is slower in n_2 than n_1 ($AB' = V_2 t$, $PB = V_1 t$)

The new wavefront has changed direction!

Note $\sin\phi_1 = \frac{V_1 t}{AB}$

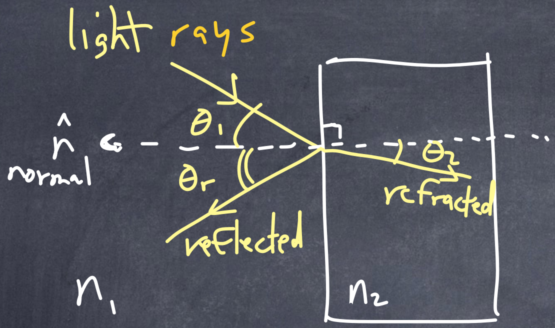
But also $\sin\phi_2 = \frac{V_2 t}{AB}$

$$AB = \frac{V_1 t}{\sin\phi_1} = \frac{V_2 t}{\sin\phi_2} \qquad \phi_1 = \theta_1, \quad \phi_2 = \theta_2$$

$V_1 = \frac{c}{n_1}$ $\quad$ $V_2 = \frac{c}{n_2}$

$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

Snell's Law
The law of refraction



Light reflects and refracts

reflection : $\theta_1 = \theta_r$

refraction: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

where n_1 + n_2 are the indices of refraction such that $n = \frac{c}{v}$

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \qquad v = \frac{1}{\sqrt{\varepsilon \mu}}$$

If $\theta_1 = 0^\circ$, $\perp$ to the surface,

the reflected intensity, $I_r = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 I_0$, then the rest is transmitted (refracted)

As an example for air ($n \sim 1$), glass ($n \sim 1.5$)

then $\frac{I_r}{I_0} = 4\%$ is reflected.

The smaller the $n_1 - n_2$, the less is reflected. $\Rightarrow$ If $n_1 = n_2$, no reflected light.

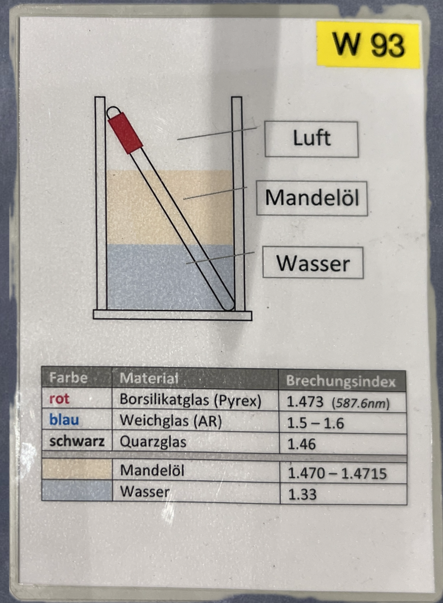
$n_{oil} = n_{glass\ two}$

oil

water

glass one

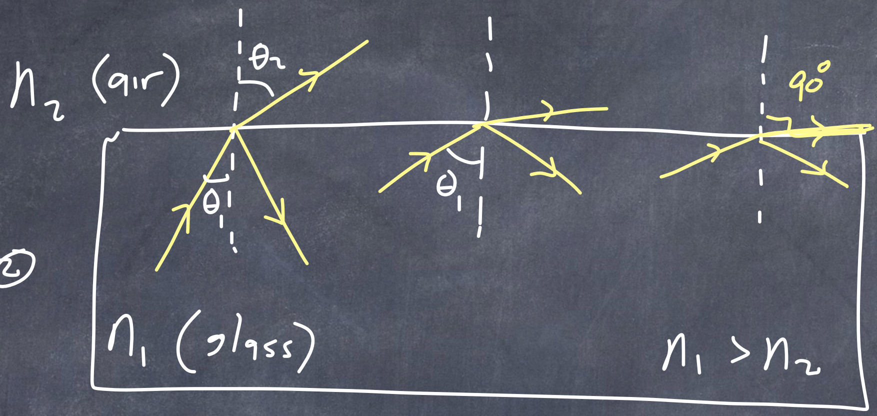
glass two

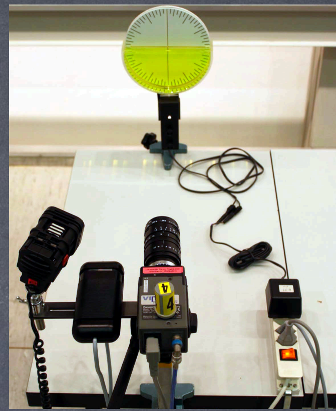


Farbe	Material	Brechungsindex
rot	Borsilikatglas (Pyrex)	1.473 (587.6nm)
blau	Weichglas (AR)	1.5 – 1.6
schwarz	Quarzglas	1.46
	Mandelöl	1.470 – 1.4715
	Wasser	1.33

Reflection and refraction of light

light goes from ① → ② n_1 to n_2





As θ_1 increases for $n_1 > n_2$, θ_2 increases more.
When $\theta_2 = 90^\circ$, no light is transmitted.

$$n_1 \sin\theta_1 = n_2 \underbrace{\sin\theta_2}_{\sin 90^\circ = 1}$$

critical angle $\sin\theta_c = \dfrac{n_2}{n_1}$ for $n_1 > n_2$

θ_c is the critical angle, or θ_1 where above which all light is reflected.

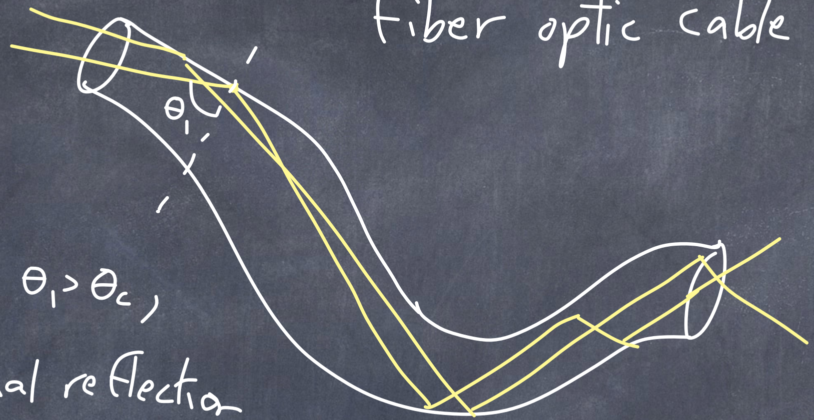
what if $n_2 > n_1$?

then $\sin\theta_c > 1$, which is <u>not</u> possible.

So there is <u>no</u> critical angle if $n_2 > n_1$

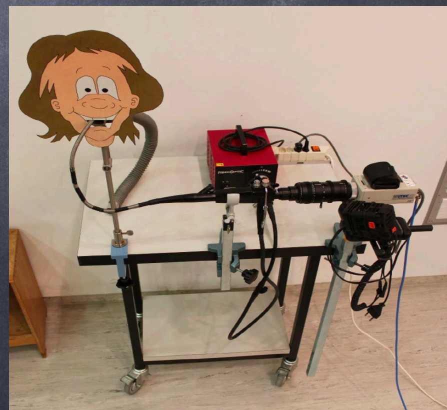
Above θ_c, we have total internal reflection

Fiber optic cable



fiber $n \sim 1.4$
$\rightarrow \theta_c \sim 45°$

Since $\theta_1 > \theta_c$,
total
internal reflection



In refraction, the speed of light changes, $V = \frac{c}{n}$

But the frequency of the light stays the same.
This is because the atoms that absorb the light and re-transmit the light, vibrate and emit light at the same frequency,
to do with atomic energy levels...

we have n_1, n_2

$$f_1 = f_2$$

$$V_1 = \frac{c}{n_1} \qquad V_2 = \frac{c}{n}$$

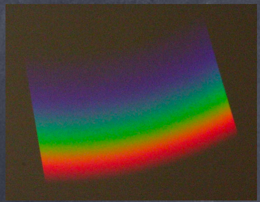
Since $f\lambda = V$, the wavelength must change in a material with different n.

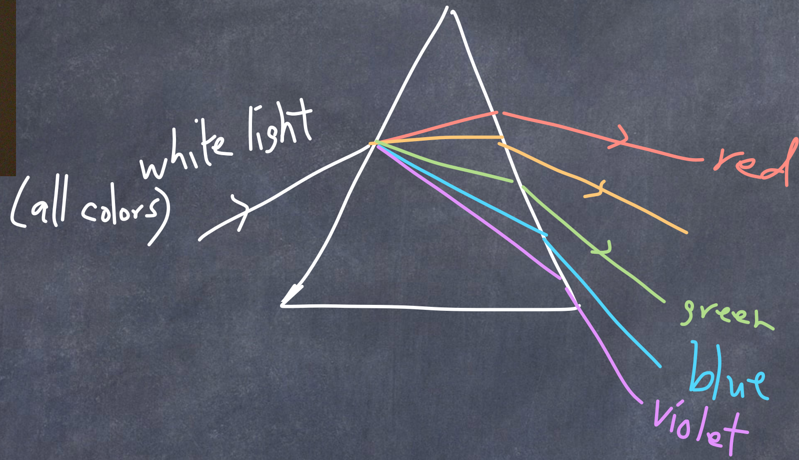
$$f_1 = \frac{V_1}{\lambda_1} = f_2 = \frac{V_2}{\lambda_2} \Rightarrow \frac{c}{n_1\lambda_1} = \frac{c}{n_2\lambda_2} \Rightarrow \left[\lambda_2 = \lambda_1 \frac{n_1}{n_2}\right]$$

change in wavelength inside material 2

For air (n_1) to water (n_2)
wavelengths get smaller.
Light gets "more blue" under water.

In addition, the index of refraction depends slightly on the wavelength of the light.





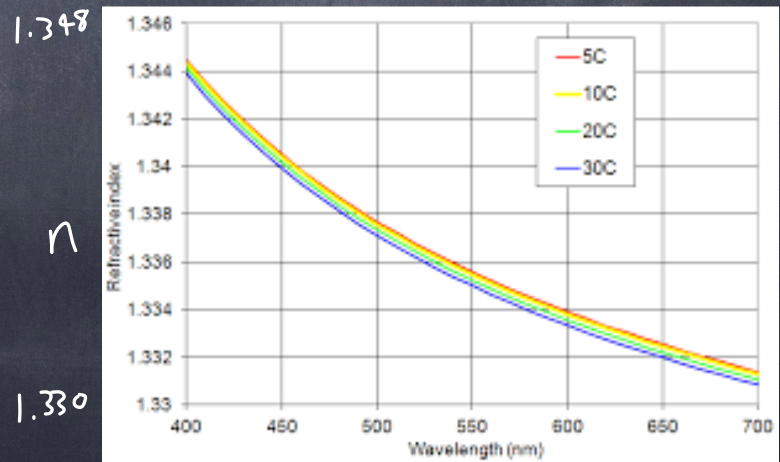
(exaggerated)

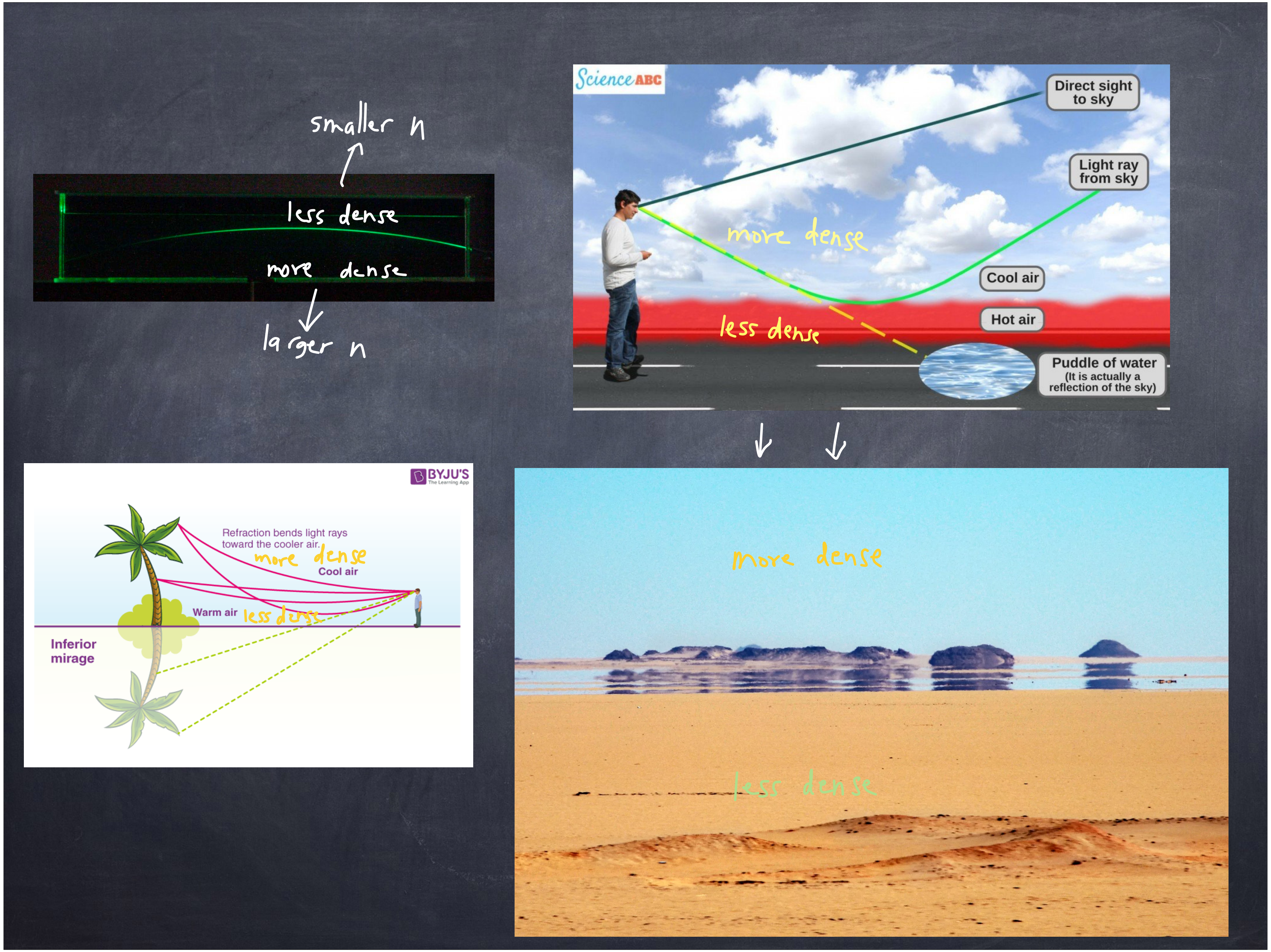
short wavelength is bent more

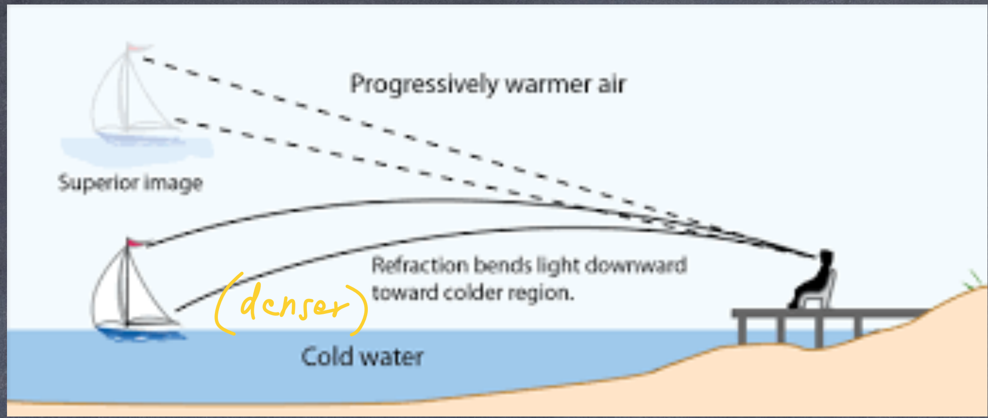
⇒ blue bent more than red in higher n.

Also, n depends on the density.

Hotter air is less dense.




smaller n
less dense
more dense
larger n
Science ABC
Direct sight to sky
Light ray from sky
more dense
Cool air
Hot air
less dense
Puddle of water
(It is actually a reflection of the sky)
BYJU'S
Refraction bends light rays toward the cooler air.
more dense
Cool air
Warm air
less dense
Inferior mirage
more dense
less dense



less dense (warmer) →

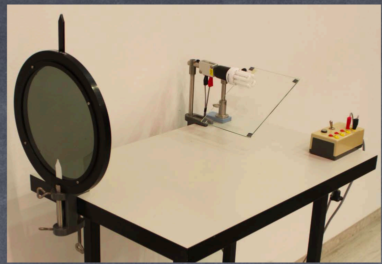
more dense (cooler) →

David Morris took the photographs from the hamlet of Gillan, near Falmouth

Real photo!

Check out "looming effect" on youtube.


W51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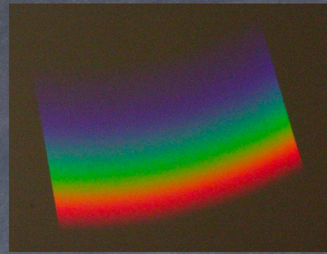
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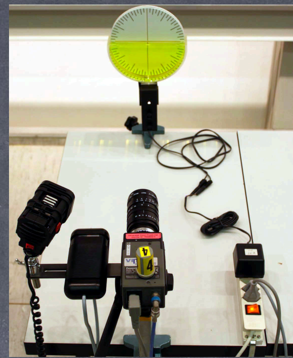
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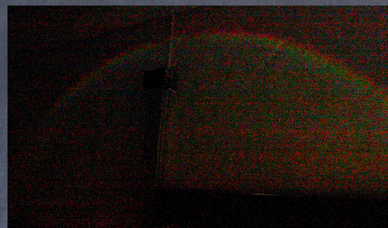
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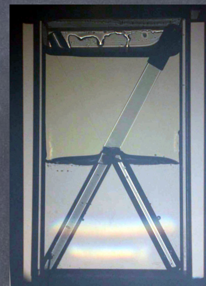
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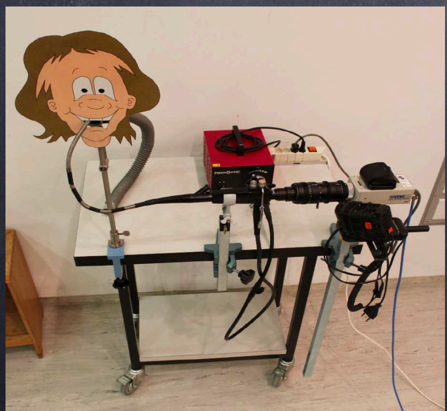
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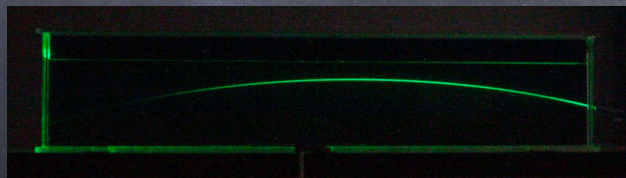
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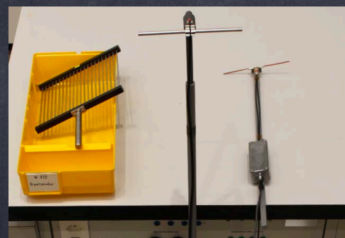
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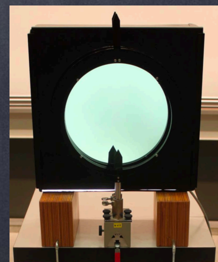
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