Today:

torque
angular momentum
rotational inertia
precession

if time:

pressure
atmospheric pressure
fluids

PHY 117 HS2024

Reminder: please ask questions about exercises on the OLAT forum.

Week 4, Lecture 2
Oct. 9th, 2024
Prof. Ben Kilminster

Quiz 2:

	Unanswered	Right	Wrong
The spring constant would be the same on the moon than on the earth. ($k = \frac{mg}{\Delta x}$, and g is different on the moon)		answer 118	~~129~~

K is a constant, independent of the type of force,
If we pull on a spring with any force F, it will extend by Δx, so $K = \frac{F}{\Delta x}$.

More info...

For instance, assume $g_{moon} = g_{earth}/6$

$\Delta x_m = \Delta x_{earth}/6$

To calculate K, we hang a weight.

$$K = \frac{F}{\Delta x} = \frac{m g_{earth}}{\Delta x_{earth}}$$

$g_{earth} = 9.81 \ m/s^2$

$g_{moon} = 1.62 \ \frac{m}{s^2}$

$$K = \frac{m g_{moon}}{\Delta x_{moon}} = \frac{m g_{earth}/6}{\Delta x_{earth}/6}$$

$$K = \frac{m g_{earth}}{\Delta x_{earth}}$$

yesterday

linear motion	rotational motion
$\bar{F} = m\bar{a}$	$\bar{\tau} = I\bar{\alpha}$

I is kind of like mass ($I = mr^2$) for one particle

$$\boxed{\text{Newton's second law of rotation} \quad \Sigma\bar{\tau} = I\bar{\alpha}}$$

I is a measure of how mass is distributed.

for simple objects: $I = \sum_i m_i r_i^2$

For objects with shape: $I = \int r^2\,dm$

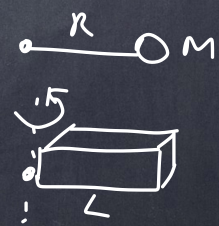
simple case: ring
total mass M, all at a radius R

rotating around center

$I_{ring} = MR^2$

Examples of I, rotational inertia

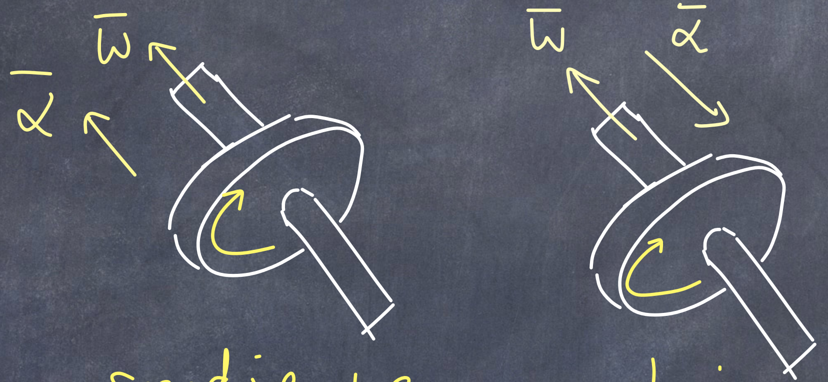
shape	I
* point	$I = MR^2$
* ring	$I = MR^2$
stick	$I = \frac{1}{3}ML^2$
sphere	$I = \frac{2}{5}MR^2$
disk	$I = \frac{1}{2}MR^2$



In the script, I derive some of these.
In this class, you only need to know how to derive these (*)

$\bar{\tau} = I\bar{\alpha}$ what direction is $\bar{\alpha}$?

$\bar{\alpha}$ points toward $\bar{\tau}$



speeding up slowing down

For $\bar{\alpha}$, the angular acceleration, it points in the same direction as $\bar{\omega}$ if $\bar{\omega}$ is getting bigger.

If $\bar{\omega}$ is slowing down, then $\bar{\alpha}$ points opposite to $\bar{\omega}$.

In linear motion, we have $\bar{p} = m\bar{v}$

and $\sum \bar{F} = m\bar{a} = \dfrac{d\bar{p}}{dt}$ (1)

If $\sum \bar{F} = 0$, no net force, then $\dfrac{d\bar{p}}{dt} = 0$,

and momentum is conserved.

$$\underset{\text{initial}}{\bar{p}_i} = \underset{\text{final}}{\bar{p}_f}$$

In a rotating system $\bar{\tau} = \bar{r} \times \bar{F}$. Is there an equivalent of momentum?

start with eq. (1), we take "$\bar{r}\times$" of both sides

$$\underbrace{\sum(\bar{r} \times \bar{F})}_{} = \frac{d(\bar{r} \times \bar{p})}{dt}$$

$$\sum \bar{\tau} = \frac{d\bar{L}}{dt}$$

where we define $\bar{L} \equiv \bar{r} \times \bar{p}$, which is the angular momentum.

Angular momentum: $\bar{L} = \bar{r} \times \bar{p}$

$$= \bar{r} \times (m\bar{v}) = m(\bar{r} \times \bar{v})$$

In a circle, $\bar{v} \perp \bar{r}$

$$\bar{r} \times \bar{v} = rv\sin\theta_{rv} = rv(\sin 90^\circ)$$
$$= rv$$

So here, angular momentum for an object moving in a circle is $L = mvr$

Now we also know that $v = r\omega$

Therefore, $L = m(r\omega)r = \underbrace{mr^2}\omega$
$$= I\omega$$

$$\boxed{\bar{L} = I\bar{\omega}}$$

momentum (angular)

inertia (rotational)

velocity (angular)

analogy

$$\bar{p} = m\bar{v}$$

momentum (linear)

inertia (mass)

velocity (linear)

Direction of angular momentum vector of second hand of SBB clock?

If there are no external forces, then

$$\sum (r \times \bar{F}) = 0 = \sum \bar{\tau} = \frac{d\bar{L}}{dt}$$

If $\frac{d\bar{L}}{dt}$ is $\emptyset$, then $\bar{L}$ is constant.

$$\bar{L}_{before} = \bar{L}_{after}$$

conservation of angular momentum

when there is no external torques

So $\bar{L}$ is conserved like $\bar{p}$

$$\boxed{\bar{L} = I\,\bar{\omega}}$$

this means that if we change I, then ω must also change because L stays constant.

$I = MR^2$ for the weights

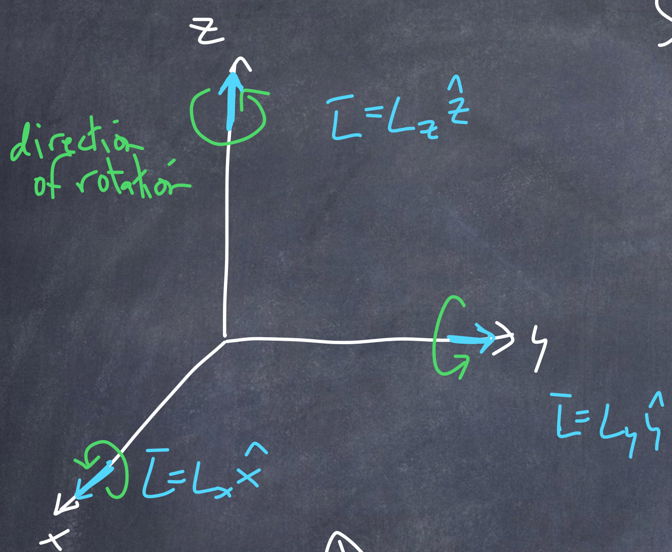
I is big

I is small

$$\underset{\text{same}}{L} = \underset{\text{decrease}}{I}\;\underset{\text{increase}}{\omega}$$

Objects can spin around 3 axes.

So $\bar{L}$ can be



$$\bar{L} = L_x \hat{x} + L_y \hat{y} + L_z \hat{z}$$

Angular momentum must be conserved in all 3 directions, independently.

check your right-hand rule to see that the spin is consistent with the axis direction.

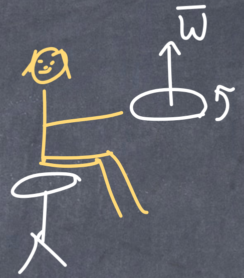
Initial :

$\bar{\omega}$ $\odot$

chair can only rotate around the z axis

$\hat{z}$

Final:



$\bar{\omega}$

wheel now has $\underbrace{\bar{L} = L_z \hat{z}}_{\text{rotating around the z axis.}}$

In the direction, L_z is conserved.

Initial: Angular momentum around z-axis is zero.

$$L_z = 0$$

Finally : in z-direction

$$L_{z_{wheel}} + L_{z_{me}}$$

Because of conservation of $\bar{L}$,

$$\bar{L}_{initial} = \bar{L}_{final}$$

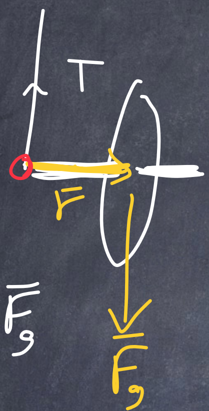
$$0 = L_{z_{wheel}} + L_{z_{me}}$$

$$L_{z_{me}} = -L_{z_{wheel}}$$

precession

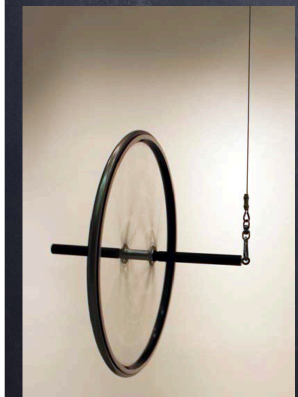
Remember that $\bar{\tau} = \frac{d\bar{L}}{dt}$

when not spinning, $\bar{\tau} = \bar{r} \times \bar{F} = rMg$
causes wheel to fall

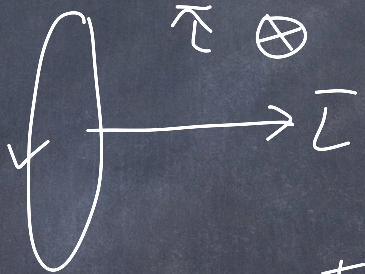


$\bar{\tau}\ \otimes$

$\bar{T} = -\bar{F}_g$



But if we spin the wheel, and apply torque



Since $\bar{\tau} = \frac{d\bar{L}}{dt}$,

then $d\bar{L} = \bar{\tau}\, dt$

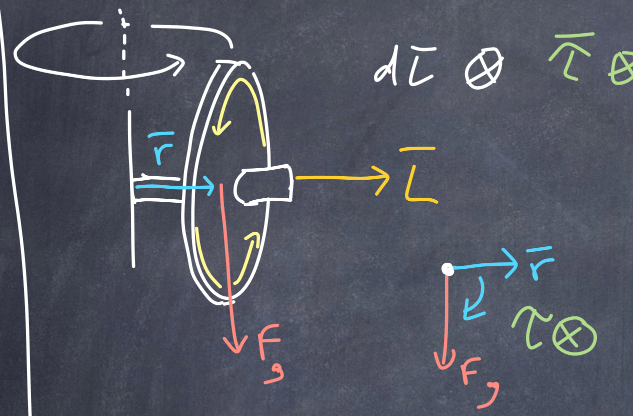
If we have a torque,
then we change the angular momentum.

The direction of $d\bar{L}$ is the direction of $\bar{\tau}$.

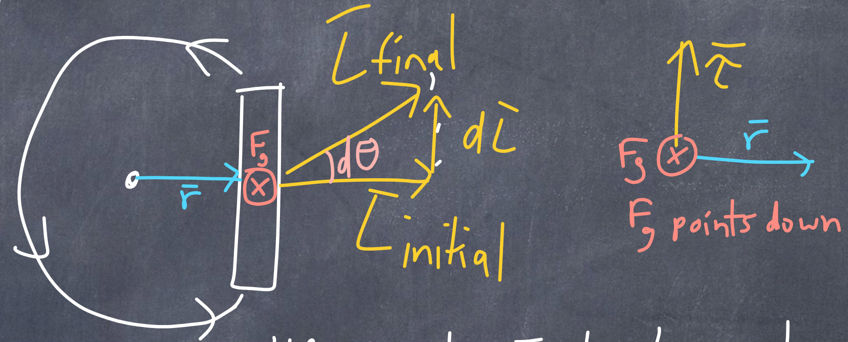
Here

The amount of $d\bar{L} = \bar{\tau}\,dt = rMg\,dt$ ①

view from side:



view from above: ②



we expect $\bar{L}$ to turn, to rotate in the direction of the torque.

from picture ②:

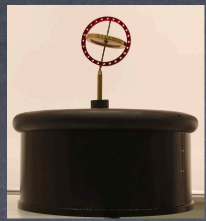
$$dL = L\,d\theta \qquad d\theta = \frac{dL}{L}$$

From ① $$d\theta = \frac{rMg\,dt}{L} \Rightarrow \underbrace{\frac{d\theta}{dt}}_{\omega_p} = \frac{rMg}{L}$$

ω_p : angular velocity of precession

$$\omega_p = \frac{rmg}{L} = \frac{rmg}{I\omega} \leftarrow$$

ω : angular velocity of the spinning wheel




Angular momentum, torque, & precession will come up in NMR & MRI,
(we will discuss in PHY 127)

New topic: <u>Fluids</u>

Fluids – what is pressure?

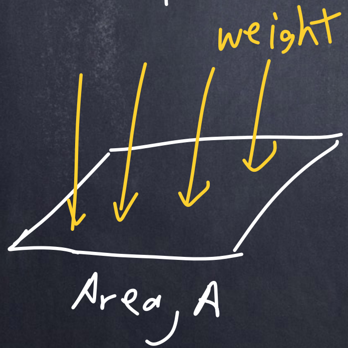
what is atmospheric pressure?

$$P = \text{pressure} = \frac{\text{force}}{\text{area}}$$

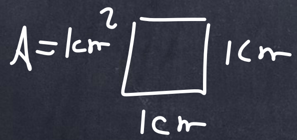
units $\left[\frac{N}{m^2}\right] \equiv [Pa]$
pascal

$$1\ Pa = \frac{1\ N}{m^2}$$

The atmospheric pressure at sea level is 101.325 kPa. This is the weight of all the air above us on some area.
force.



How much weight does the atmosphere feel like on an area of $1\ cm^2$?

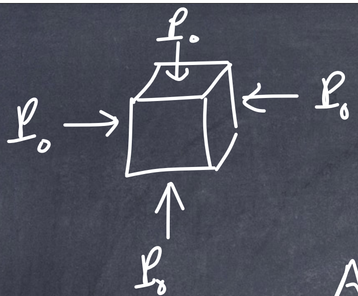


$$P_0 = 101325 \frac{N}{m^2}$$

atm. pressure

$$F = P_0 A = 101325 \frac{N}{m^2} \cdot 1\ cm^2 \cdot \frac{1\ m^2}{(100\ cm)^2}$$

$$\underline{F \cong 10\ N/cm^2} \Rightarrow \underline{m = 1\ kg}$$



Pressure is not a vector.
Pressure pushes in all directions with the same value (at a given height)

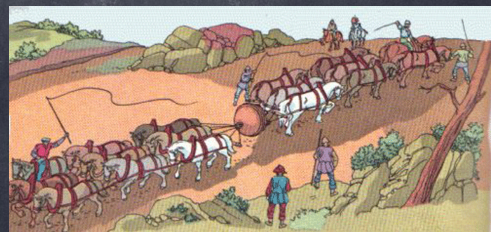
Atmospheric pressure even points up!

$$A = \text{(5 cm)} = \pi r^2 = 75\ cm^2$$

$$F_{ATM} = P_0 \cdot A = 10\frac{N}{cm^2} \cdot 75\ cm^2 = \underline{750\ N}$$

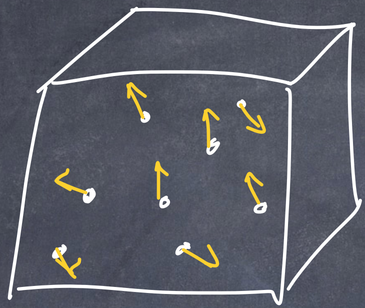
$$F_g = (m_{water})\, g = (0.5\ kg)\left(10\frac{m}{s^2}\right) = \underline{5\ N}$$

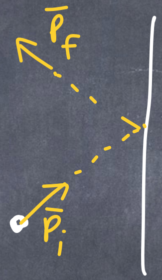
The force from the atmospheric pressure is much more than the weight due to gravity



1650

where does pressure come from?





wall of a box

$\bar{p}_i$: initial momentum of molecule before it hits the wall

$\bar{p}_f$: final momentum

There is a change in momentum $\underbrace{\Delta \bar{p}}_{\text{impulse}} = \bar{p}_f - \bar{p}_i$

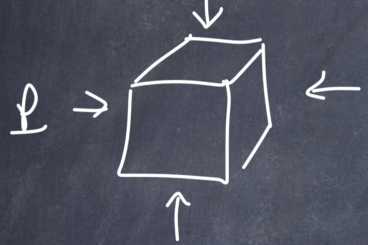
remember
$F = P \cdot A$ $\Rightarrow$ $\bar{F} = \dfrac{\Delta \bar{p}}{\underbrace{\Delta t}_{p:\ \text{momentum}}} = \underbrace{P}_{\text{pressure}} \cdot \text{Area}$

For N molecules,

$$F = N \frac{\Delta p}{\Delta t}$$

depends on velocity.

If we apply a pressure to a substance of volume V, it gets compressed by some amount ΔV. (Note: ΔV is $(-)$)



we can define the Bulk modulus, B, to describe how much a substance resists compression.

$$\boxed{B \equiv \frac{-P}{\frac{\Delta V}{V}}}$$

(B is small if ΔV is large.)

The $(-)$ sign here makes B $(+)$ because ΔV is $(-)$

material	B [GPa $= 10^9$ Pa]
iron	100
lead	8
water	2
air	~ 0.0001

gases: B is very small (depends on temperature)

liquids & solids: B is large because it is hard to compress liquids and solids.