8 The $\pi^+ \to e^+\nu_e$ / $\pi^+ \to \mu^+\nu_\mu$ branching ratio

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Mesons with positive electric charge (π^+, K^+, D^+, etc.) are described as bound states of an up-type quark and a down-type antiquark. They decay through the electroweak interaction with a (virtual) W propagator which may result in final states with a charged and a neutral lepton (plus γ or e^+e^- pair). As shown in Tab. 8.1 these modes dominate for pions and kaons [2]. The decay rate can be factorized (ignoring numerical constants) in:

- the decay constant F_P accounting for the overlap between the two quark wave functions,
- g^2/m_W^2 for the W propagator, with g the weak coupling constant,
- $V_{CKM}^{q\bar{q}}$ squared describing the reduced strength for the W-quark vertex through quark mixing,
- a correction factor when lepton universality is allowed to be violated so g for the W-lepton vertex depends on generation number (lepton flavor)
- a kinematic factor $m_P(1 - m_l^2/m_P^2)^2$ for the Q-value,
- a helicity factor proportional to m_l^2 originating in the $V-A$ structure of the weak interaction.

The first three factors mostly cancel when normalizing to the total decay rate. An almost perfect cancellation occurs in the ratio $R^P_{e/\mu}$ of the rates for $P \to e^+\nu_e$ and $P \to \mu^+\nu_\mu$:

$$R^P_{e/\mu} \equiv \frac{\Gamma(P \to e^+\nu_e)}{\Gamma(P \to \mu^+\nu_\mu)} \simeq (\frac{m_e}{m_\mu})^2 \times (1 - \frac{m_\mu}{m_P})^2 \times (\frac{g_e}{g_\mu})^2 \, , \tag{8.1}$$

where g_i denotes the coupling strength of the $Wl_i\nu_i$ vertex. One readily identifies the factors associated with the helicity suppression, the Q-value, and the universality violation. Table 8.2 compares the theoretical values including all bells and whistles, which are a few percent below the simple estimate, with the published experimental values for π and K.

Tab. 8.1 –
SM predictions for the $P \to \bar{l}\nu_l$ branching fractions.

initial state			final state		
P	$q\bar{q'}$	$V_{CKM}^{q\bar{q}}$	$e^+\nu$	$\mu^+\nu$	$\tau^+\nu$
π^+	$u\bar{d}$	0.974	1.22×10^{-4}	99.99%	-
K^+	$u\bar{s}$	0.226	1.55×10^{-5}	63.5%	-
D^+	$c\bar{d}$	0.22	7.5×10^{-9}	3.2×10^{-4}	7.2×10^{-4}
D_s^+	$c\bar{s}$	0.98	7.5×10^{-8}	3.2×10^{-3}	2.9×10^{-2}
B^+	$u\bar{b}$	0.003	1.0×10^{-11}	5×10^{-7}	1.0×10^{-4}

Tab. 8.2 – SM predictions and measured values for $R^P_{e/\mu}$ ($P = \pi, K$) defined in Eq. 8.1.

	$\Gamma_{\pi^+\to e^+\nu}/\Gamma_{\pi^+\to\mu^+\nu}$	$\Gamma_{K^+\to e^+\nu}/\Gamma_{K^+\to\mu^+\nu}$
theory	$1.2353(1) \times 10^{-4}$[1]	$2.477(1) \times 10^{-5}$[1]
experiment	$1.2312(37) \times 10^{-4}$[2]	$2.488(12) \times 10^{-5}$[3]

[1] V. Cirigliano and I. Rosell, JHEP **10** (2007) 5.
V. Cirigliano and I. Rosell,
Phys. Rev. Lett. **99** (2007) 231801.

[2] G. Czapek *et al.*, Phys. Rev. Lett. **70** (1993) 17; D. I. Britton *et al.*, Phys. Rev. Lett. **68** (1992) 3000.

[3] C. Lazzeroni *et al.* (NA62 Collaboration),
Phys. Lett. B719 (2013) 326-336

8.1 $\pi^+ \to e^+\nu_e$ measurements

The measured value of the $\pi^+ \to e^+\nu$ / $\pi^+ \to \mu^+\nu$ branching ratio, even if 20 years old by now, still gives the best constraint on deviations from the SM assumption of a flavour independent coupling of W bosons to leptons. Two new experiments [1] are underway which aim at improvements in accuracy by almost one order of magnitude.

For pions at rest the decay $\pi^+ \to e^+\nu$ is characterized by an electron with $E = \frac{1}{2}m_\pi c^2 = 69.8$ MeV emitted with an exponential time distribution with $\tau_{\pi^+} = 26.0$ ns. Since the 4.2 MeV muon from $\pi^+ \to \mu^+\nu$ at rest travels just $\sim$0.1 g/cm^2 and stays in the π^+ stopping target this decay is only observed through the subsequent decay $\mu^+ \to e^+\nu\bar{\nu}$. The decay chain is characterized by an electron with $E < \frac{1}{2}m_\mu c^2 < 52.8$ MeV emitted with a time distribution first rising with τ_{π^+} and then falling with $\tau_\mu = 2.20$ μs. Ideally, the two decay modes can be perfectly distinguished by the

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[2] In π^+ decay the only alternative decay mode is $\pi^+ \to \pi^0 e^+\nu$ with a branching ratio $O(10^{-8})$.

positron energy alone but corrections apply since pions and muons may decay in flight tens of ps before they would have stopped. Radiative corrections and imperfections in the experimental setup result in a low-energy tail in the $\pi^+ \to e^+(\gamma)\nu$ positron energy distribution leaking into the region populated by $\pi^+ \to \mu^+\nu$. The uncertainty in this tail fraction is in fact the major source of systematic error in determinations of $R^{\pi}_{e/\mu}$.

[1] PEN Collaboration, PSI experiment R-05-01 (2005), D. Pocanic and A. van der Schaaf, spokespersons; PIENU Collaboration, TRIUMF proposal 1072 (2006), D. Bryman and T. Numao, spokespersons.

8.2 the PEN experiment

PEN[3] took data at the πE1 beam line at PSI during 2008 – 2010. The beam momentum was typically 75 MeV/c with a spread of ±1% . The main trigger for data readout required a decay positron within a (-30,+220) ns window around the time of an incoming pion. Events with positron energies below ~48 MeV were pre-scaled by 1:64. Whereas ~95% of $\pi \to e\nu$ events in the 3π sr geometric acceptance were recorded the much more abundant $\pi \to \mu\nu$, $\mu \to e\nu\overline{\nu}$ decay chain was suppressed by two orders of magnitude. Still, the recorded number of events in the $\pi \to \mu$ branch is much larger than for $\pi \to e$ and the statistical error in $R^{\pi}_{e/\mu}$ is practically equal to the statistical error in the number of reconstructed $\pi \to e\nu$ events (see Tab. 8.3) so a few 10^{-4}. This is more than ten times smaller than reached before in both π and K decay.

TAB. 8.3 –
PEN vital statistics during the three years of operation.

	2008	2009	2010	total	
calendar days	111	98	68	277	
pion stops	7.5	13.1	16.4	37.0	$\times 10^{10}$
$\pi \to e\nu$ events	4.4	7.8	10.1	22.3	$\times 10^{6}$

A PEN event contains at least a beam pion and a decay positron. The pions crossed various beam counters before stopping in the target scintillation detector. All scintillators were read with 2 GHz waveform digitizers and after 2008 the beam particle trajectories were measured with a TPC (Fig. 8.1). Based on the measured *individual* pion velocity and energy deposits (corrected for light quenching) the pion energy when entering the target is known to a few percent for each event. This knowledge is crucial when testing the target waveform for the occurrence of a decay muon. The decay positrons were tracked in MWPCs and crossed a plastic hodoscope before reaching the pure CsI spherical calorimeter. The active target may be called the heart of the detection system since all particles leave their traces there. At the other hand, the signatures of the two decay branches differ so much that there is little hope to understand a target waveform analysis at the precision required for $R^{\pi}_{e/\mu}$. Still, the target waveforms of well separated π-μ-e sequences (*gold plated* $\pi \to \mu$ *events*) allow a very accurate calibration of the offset of the decay time calculated from the time signals of the dedicated beam and decay particle detectors.

Detector calibrations are almost done now for all years and reconstruction algorithms have been pushed close to perfection. State of the art event simulation all the way down to the data format of the measured events is basically ready for the 2008 data set, and is under development for later years when many readout systems were upgraded.

The data analysis is challenging indeed when the systematic error has to be pushed below the statistical error. An un-binned maximum likelihood program was written, capable of fitting the contributions of 4-5 event types characterized by 3-4 dimensional PDFs in samples of 10^8 events. This achievement is far from trivial.

Crucial in a precision measurement such as PEN is a blind analysis in which adjustments affecting the physics result are only allowed as long as that result is unknown: the result is final by the time it is obtained first. This explains why no preliminary results can be presented.

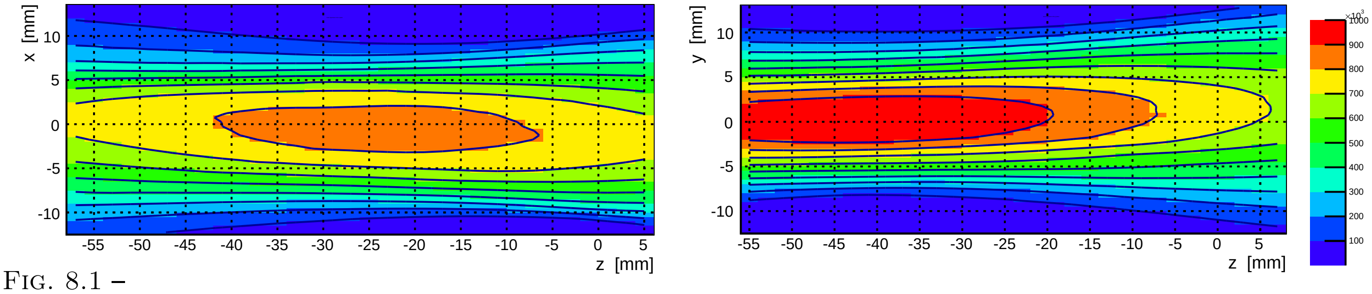

FIG. 8.1 –
xz and yz beam profiles (z is beam axis) during 2010, measured with the mini TPC. The target with 15 mm radius starts at z = -5 mm. Beam tracking not only helps to suppress random background through a constraint on the π-e vertex but also helps to define the 5-dimensional beam phase space used in the simulations.

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[3] see previous annual reports for more details