

PHY117 HS2023

Prof. Ben Kilminster

Sept. 28, 2024

Week 1, Lecture 2

<https://www.physik.uzh.ch/de/lehre/PHY117/HS2023.html>

Template
for
notes
here

Lecture schedule (13 lectures)	Topics	Lecture notes
Week 1 Sept.19-20, 2023	Intro, units, principles of motion	> PHY117_HS2023_Week1_L1.pdf (PDF, 12 MB) ↓
Week 2 (Sept.26-27, 2023)		> PHY117_HS2023_Week1_L2- template.pdf (PDF, 8 MB) ↓ > PHY117_HS2023_Week1_L2- template.key (KEY, 32 MB) ↓

Q: what are these: \hat{x} , \hat{y} , \hat{z}

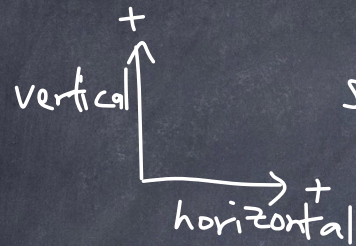
A: They are special vectors with length 1 called unit vectors



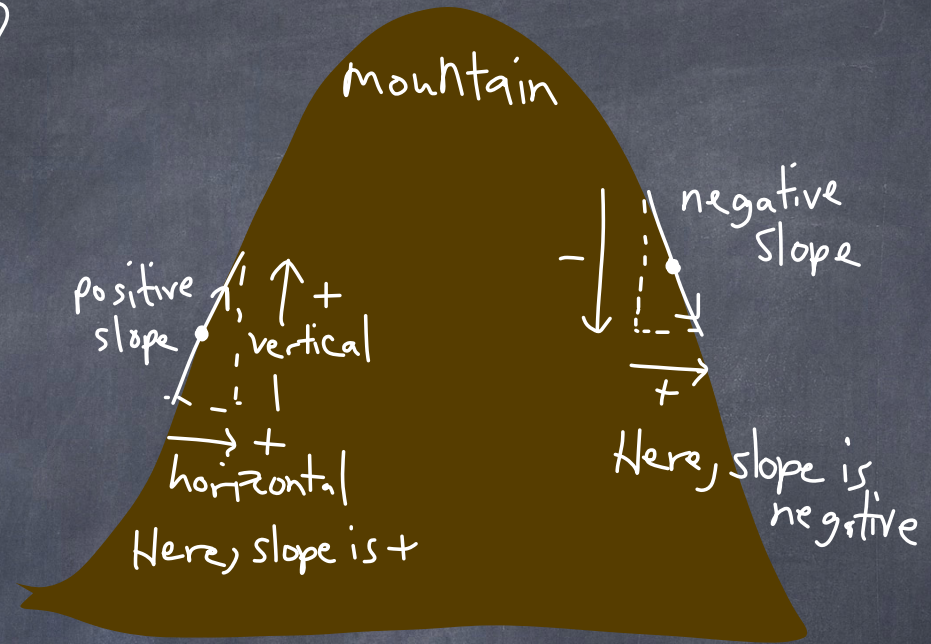
the "A" is called a "hat."

Unit vectors give us a coordinate system to define vectors.

Q: what is a slope?



$$\text{slope} = \frac{\text{vertical}}{\text{horizontal}}$$



Q: what is the difference between velocity & speed?

A: velocity is a vector
speed is a scalar

$$\text{velocity} = \vec{v} \begin{pmatrix} \text{magnitude} \\ \text{direction} \end{pmatrix}$$

$$\text{speed} = v = |\vec{v}| \begin{pmatrix} \text{magnitude,} \\ \text{no direction} \end{pmatrix}$$

Example: $\begin{cases} \vec{v} = 3 \frac{\text{m}}{\text{s}} \hat{i} \\ v = 3 \frac{\text{m}}{\text{s}} \end{cases}$

Q:

Reminder on derivatives:

$$f(x) = \sin(x)$$

$$\frac{df(x)}{dx} = f'(x) = \cos(x)$$

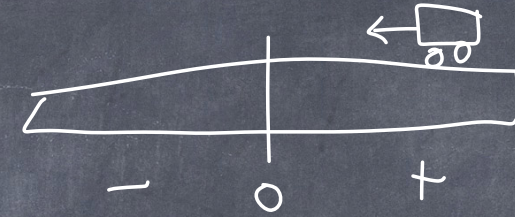
chain rule means we multiply
by $\frac{d(kx)}{dx} = k$

$$f(x) = \sin(kx)$$
$$f'(x) = [\cos(kx)](k)$$

If we have $f(g(x))$, then

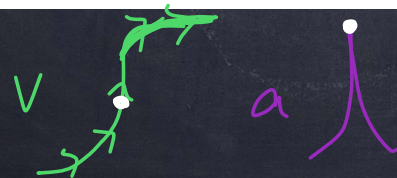
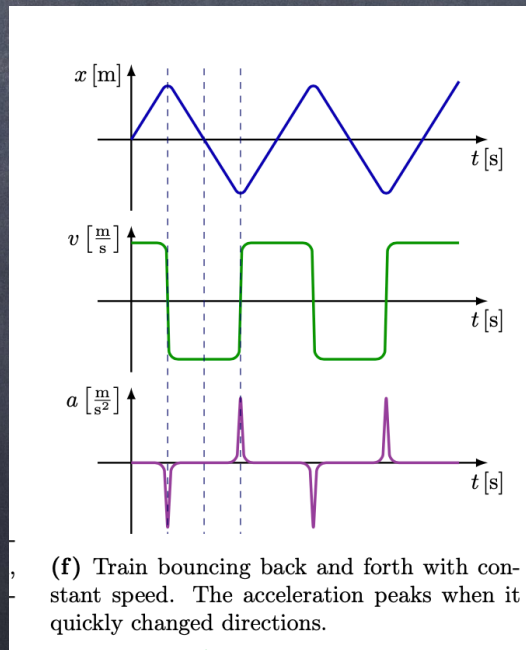
$$\frac{d}{dx} f(g(x)) = f'(g(x))g'(x)$$

Explanation of yesterday's experiment:



velocity is the slope of x vs. t

acceleration is the slope of v vs. t



velocity is + : moving in + direction
 - : moving in - direction

$a = 0$: when velocity is constant
 $a = (-)$: when velocity is decreasing

Here, acceleration is not constant. It has a peak structure. we will discuss this later when we talk about "impulse".

Can we make our air-car accelerate more smoothly?

we can by making use of a "simple harmonic oscillator"

One example is a spring.

The position changes according to the formula

$$x(t) = x_0 \cos(\omega t)$$

↑
starting position

↑
 ω is the angular frequency

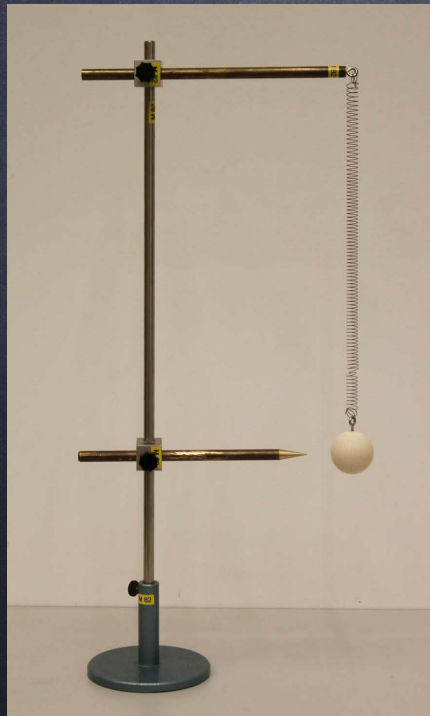
$$\omega = \frac{2\pi}{T}, \text{ where}$$

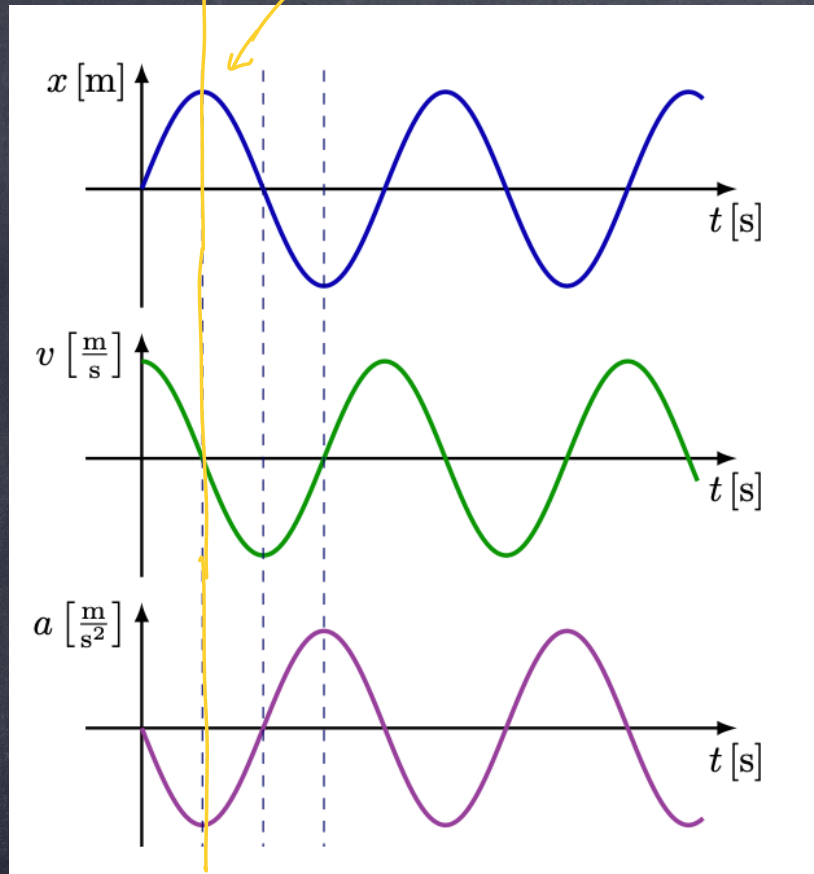
T is the "period", the time it takes to do one cycle.

Then: $v(t) = \frac{dx}{dt} = -\underbrace{x_0 \omega}_{v_0} \sin(\omega t)$

$$a(t) = \frac{dv}{dt} = -x_0 \omega^2 \cos(\omega t)$$

Notice $a(t) = x(t)(-\omega^2)$ ← This we will come back to.





(g) Periodic one dimensional motion of a mass on a spring moving back and forth. Velocity is largest when $x = 0 = a$.

$t=0$ Note, if we start $t=0$ here, then
 $x(t) = x_0 \cos(\omega t)$



Here, we start $t=0$
 at $x=0$:

$$x(t) = x_0 \sin(\omega t)$$

where $x_0 =$ initial displacement

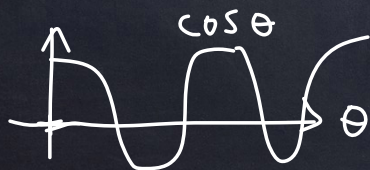
$$v(t) = v_0 \cos(\omega t)$$

where $v_0 = \omega x_0$
 is the maximum speed

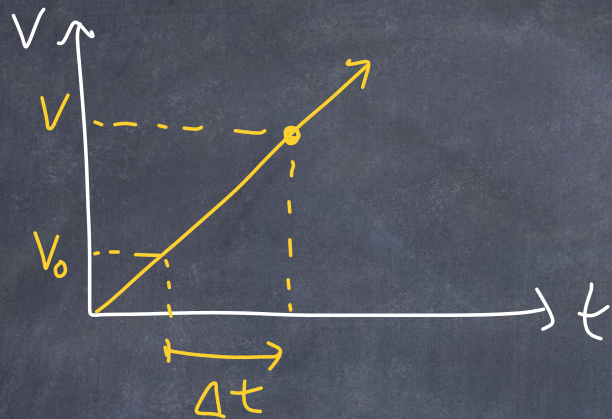
$$a(t) = -a_0 \sin(\omega t) \quad \text{where } a_0 = \omega^2 x_0 \text{ is the maximum acceleration}$$

Notice that depending on when we start $t=0$,

$x(t)$ can either be $x_0 \sin(\omega t)$
 or $x_0 \cos(\omega t)$



Motion with constant acceleration:



$a = \text{slope of } v \text{ vs. } t$
 $= \text{constant}$

$$a = \frac{dv}{dt}$$

We can determine V from a starting V_0 and the slope, a .

$$V = V_0 + at$$

↑ final velocity
↑ starting velocity

$$a \cdot t = \left[\frac{\text{m}}{\text{s}^2} \right] \left[\text{s} \right] = \frac{\text{m}}{\text{s}}$$

Average velocity is half-way between $V_0 + V$:

$$V_{\text{av}} = \frac{1}{2} (V_0 + V)$$

Some other useful formulas for constant acceleration

$$\Delta x = \frac{1}{2}(v_0 + v)t$$

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

↑
final
positia

↑
starting
positia

↑
starting
velocity

↑
acceleration
(constant)

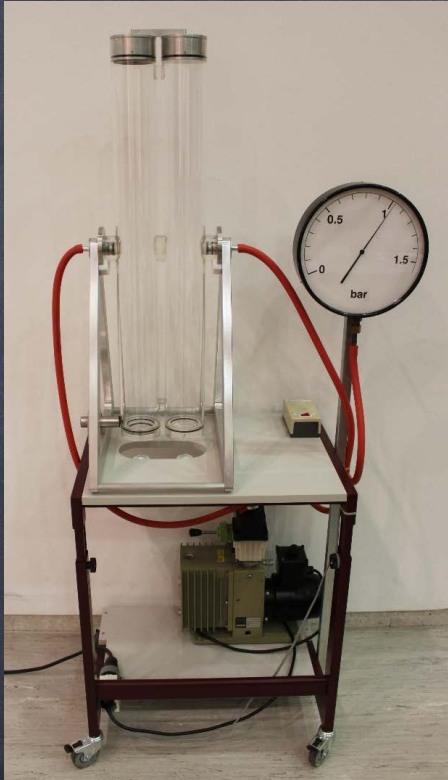
$$v^2 = v_0^2 + 2a \Delta x$$

Most of the time, on earth, the acceleration is due to gravity.

$$\vec{a} = -g \hat{z}$$



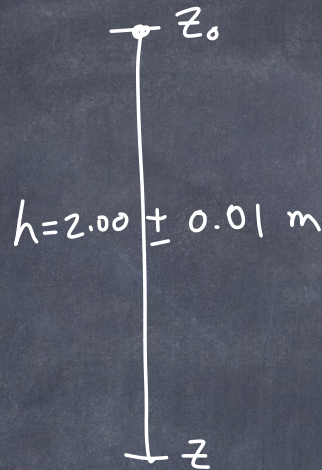
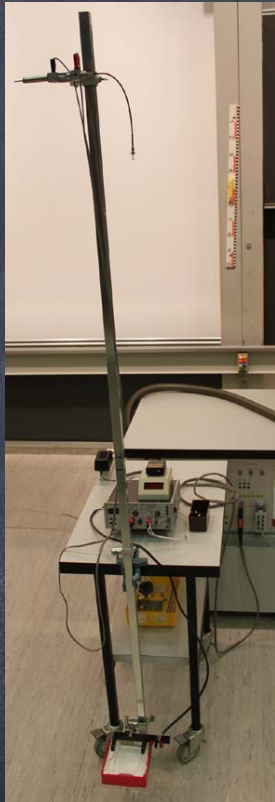
Does gravity accelerate everything the same?



yes (but sometimes other forces like air resistance)

What is g ? $\left[\frac{m}{s^2}\right]$

+z
↑



$$\bar{a} = -g \hat{z}$$

$$z = z_0 + v_{0z} t + \frac{1}{2} a t^2$$

$$\underbrace{(z - z_0)}_{(-)} = v_{0z} t - \frac{1}{2} g t^2$$

$$\underbrace{(z_0 - z)}_{(+)} = \frac{1}{2} g t^2$$

$$h = \frac{1}{2} g t^2 \rightarrow g = \frac{2h}{t^2}$$

measure the time:

0.6396 s

0.6395 s

$$t = 0.6395 \pm 0.0001 \text{ s}$$

$$g = \frac{2(2.00 \text{ m})}{(0.6395 \text{ s})^2} = 9.78 \pm 0.05 \frac{\text{m}}{\text{s}^2}$$

Example 2.4: For something more complicated like

$$f(x, y, a, b) = K \frac{xy^n}{ab^n}, \quad (2.16)$$

with a constant K , we find after some algebra

$$\sigma_f = |f| \sqrt{\left(\frac{\sigma_x}{x}\right)^2 + \left(\frac{\sigma_y}{y}\right)^2 + \left(\frac{n\sigma_a}{a}\right)^2 + \left(\frac{n\sigma_b}{b}\right)^2}, \quad (2.17)$$

$$\sigma_g = g \sqrt{\left(\frac{0.01 \text{ m}}{2.00 \text{ m}}\right)^2 + \left(\frac{2(0.0001 \text{ s})}{0.6395 \text{ s}}\right)^2}$$

$$\sigma_g = 9.78 \frac{\text{m}}{\text{s}^2} \left(\sqrt{2.5 \times 10^{-5} + 9.8 \times 10^{-8}} \right)$$

$\sigma_g = 0.05 \frac{\text{m}}{\text{s}^2}$

Bern:

$$g = 9.8089$$

$$\frac{\text{m}}{\text{s}^2}$$



Assume we throw a ball up at 20 m/s , what is its position in 4 s ?

$$z = z_0 + v_{0z}t + \frac{1}{2}at^2$$

\uparrow \uparrow \uparrow \uparrow \uparrow
 1 m $+20 \frac{\text{m}}{\text{s}}$ 4 s $a = -10 \frac{\text{m}}{\text{s}^2}$ $(4 \text{ s})^2$

$$z = 1 \text{ m} + 80 \frac{\text{m}}{\text{s}} + \frac{1}{2} \left(-10 \frac{\text{m}}{\text{s}^2} \right) (4 \text{ s})^2$$

$$z = 1 \text{ m} \quad (\text{same as initial position})$$

How fast is it moving?

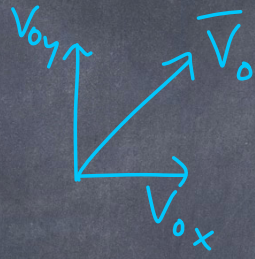
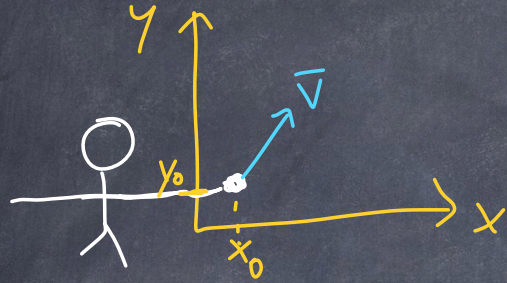
$$v_z = v_{0z} + at = 20 \frac{\text{m}}{\text{s}} + \left(-10 \frac{\text{m}}{\text{s}^2} \right) (4 \text{ s})$$

$$v_z = -20 \frac{\text{m}}{\text{s}}$$

(opposite initial velocity)

Motion in 2-D :

when we throw an object it moves in a parabola.



Initial velocity:
 $\vec{v}_0 = v_{0x} \hat{x} + v_{0y} \hat{y}$

x-direction:

$$v_x = v_{0x} + at$$

$$x = x_0 + v_{0x}t + \frac{1}{2}at^2$$

velocity in x-direction is constant

(Neglect
air resistance)

y-direction: $v_y = v_{0y} + at \Rightarrow v_y = v_{0y} - gt$

$$y = y_0 + v_{0y}t + \frac{1}{2}at^2 \Rightarrow$$

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

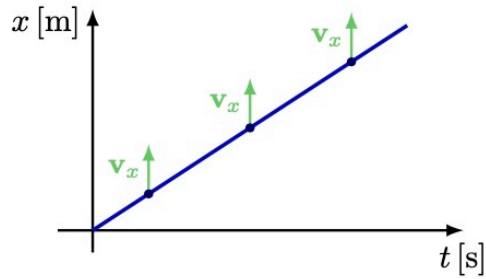
$$V_x = V_{0x}$$

$$x = x_0 + V_{0x} t$$

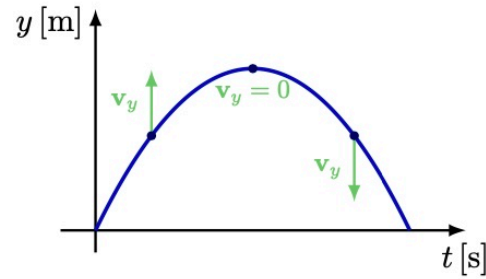
$$V_y = V_{0y} - g t$$

$$y = y_0 + V_{0y} t - \frac{1}{2} g t^2$$

$x = x_0 \rightarrow$

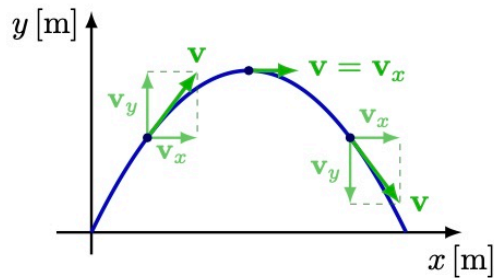


(a) $x-t$ diagram.

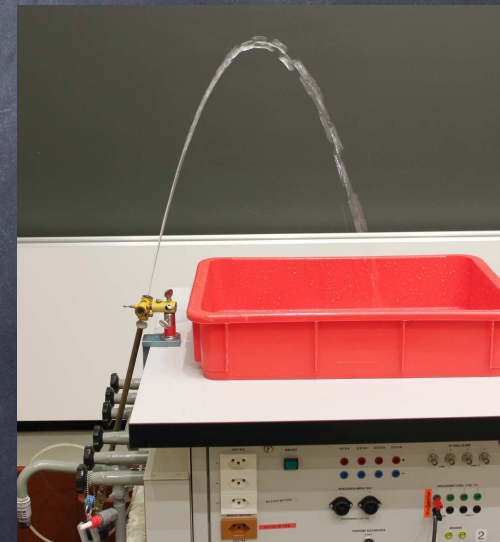


(b) $y-t$ diagram.

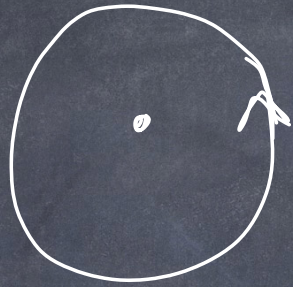
What we see:



(c) $y-x$ diagram.



Motion in a circle:

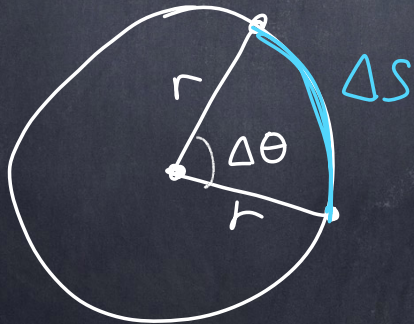


circle has $360^\circ = 2\pi$ radians

$$90^\circ = \frac{\pi}{2} \text{ radians}$$

calculator: "deg" / "rad"

Circumference of a circle is $C = 2\pi r$



$$\Delta s = r \Delta \theta$$

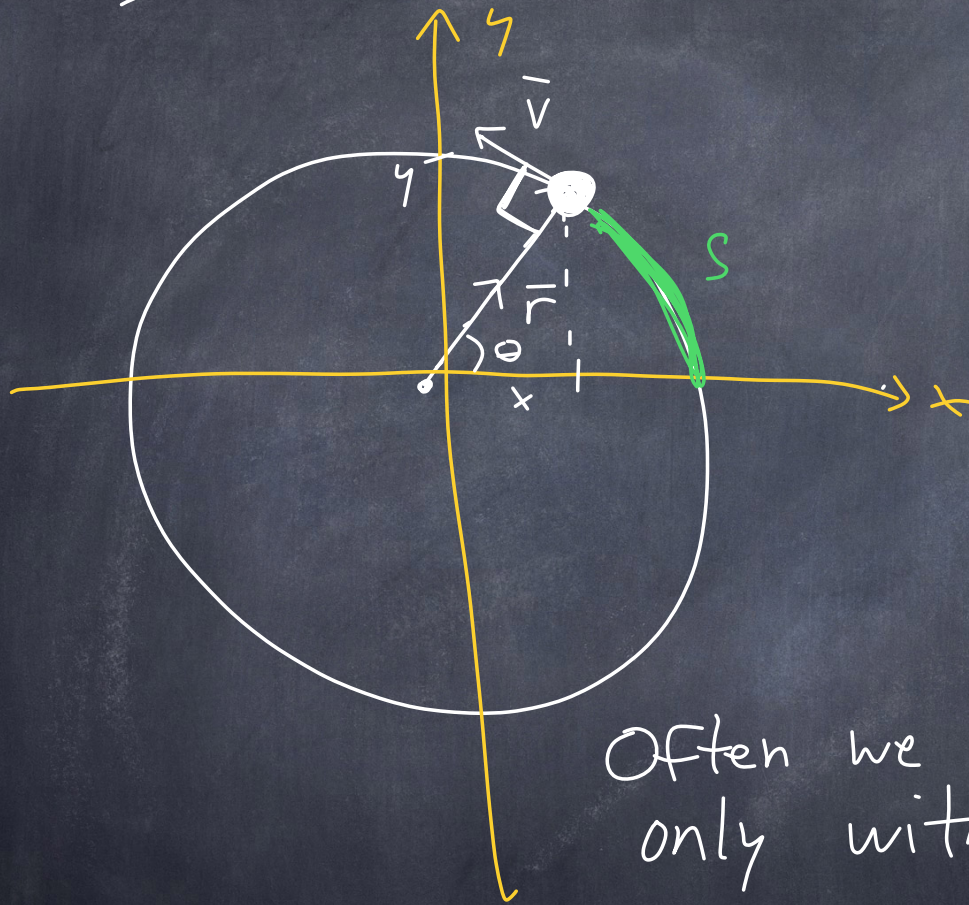
If $\Delta \theta = 2\pi$,
then $\Delta s = 2\pi r$
(circumference)

Half a circle

$$\Delta \theta = 180^\circ = \pi \text{ radians} =$$

$$\Delta s = r\pi$$

Circular motion (in 2D):



position \vec{r} , velocity \vec{v}

$$\vec{r} \perp \vec{v}$$

Distance traveled $= s = r\theta$

$$V = |\vec{v}| = \frac{\text{circumference}}{\text{time to do a circle}}$$

Speed

$$V = \frac{2\pi r}{T}$$

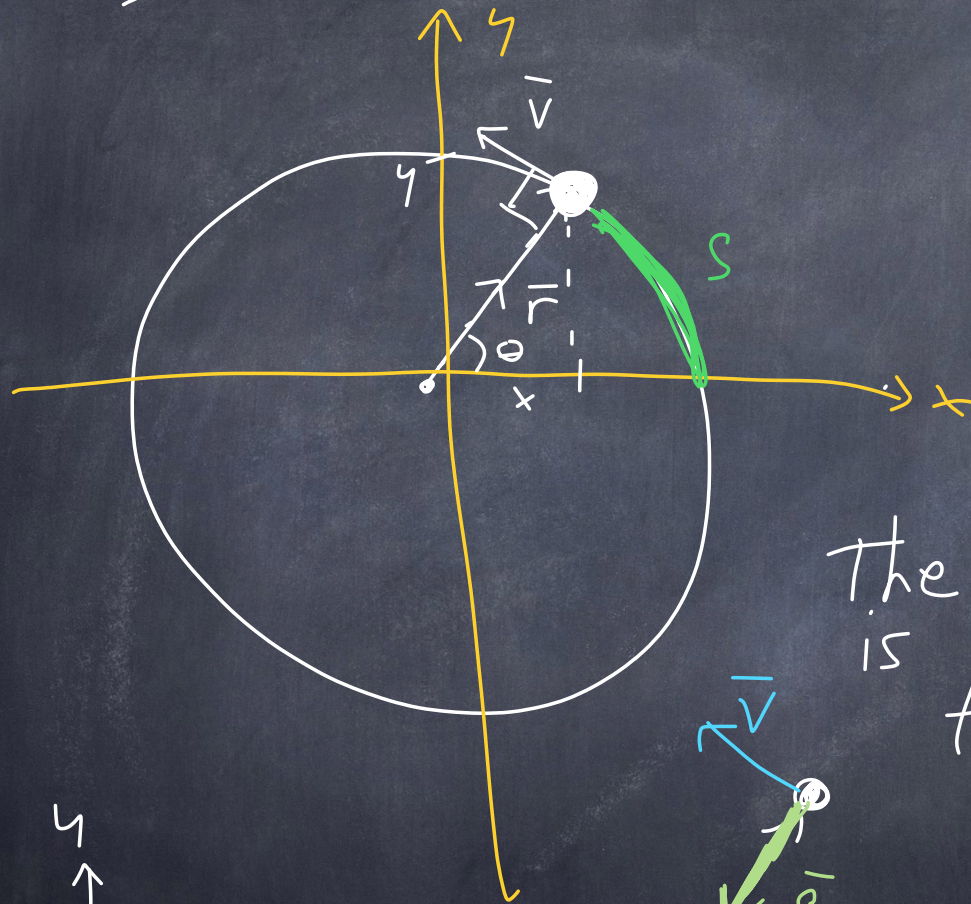
T : period of a cycle

Often we describe the position only with θ , since $|\vec{r}|$ is a constant

For the velocity, we often use $\frac{\Delta\theta}{\Delta t} = \omega$: angular velocity

$$\omega = \frac{V}{r} \left[\frac{\frac{m}{s}}{m} \right] = \left[\frac{1}{s} \right] = \text{radians per second}$$

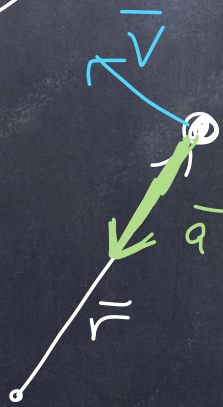
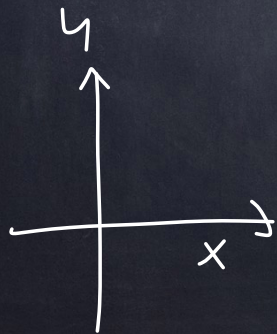
Circular motion (in 2D):



$$a = \frac{v^2}{r} = \text{centripetal acceleration}$$

(Derivation of $a = \frac{v^2}{r}$ on the next page if you are interested)

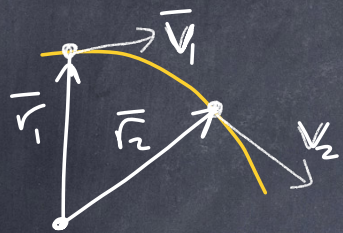
The direction of \vec{a} is toward the center of the circle.



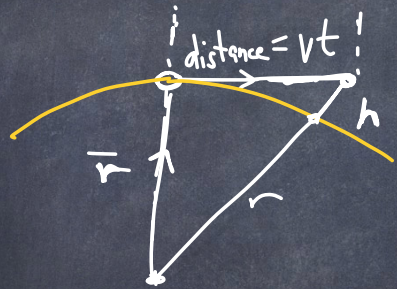
$$\vec{a} = -\frac{v^2}{r} \hat{r}$$

skip Motion in a circle, Ball on a string

assumption: The speed is the same vs. time



The velocity is changing (direction changes)
 If the ball's velocity stayed the same, it would be at a height of $r+h$ after a time t .



Pythagorean theorem:

$$(vt)^2 + r^2 = (r+h)^2$$

$$(vt)^2 + r^2 = r^2 + 2rh + h^2$$

For small time t , h is small

h is very small compared to r ($h \ll r$)

$$\text{so } h^2 \ll rh$$

$$\text{Therefore, } (vt)^2 \approx 2rh + h^2$$

$$\text{so } h \approx \frac{1}{2} \left(\frac{v^2}{r} \right) t^2$$

Compare this to $t = \frac{1}{2} at^2$, we see that

$$a = \frac{v^2}{r}$$

This is the centripetal acceleration. Movement in circle.

A satellite is moving around the earth at a height of 100 km above the ground.
 How long does it take to go around the earth?

$$C_{\text{earth}} = 40,000 \text{ km}$$

$$g = 9.81 \frac{\text{m}}{\text{s}^2}$$



$$C = 2\pi r \Rightarrow r_{\text{earth}} = 6370 \text{ km}$$

$$r_{\text{satellite}} = 6470 \text{ km}$$

we know 2 things:

$$\left\{ \begin{array}{l} a = \frac{v^2}{r} \\ a \approx g \end{array} \right\}$$

therefore, $\frac{v^2}{r} = g \Rightarrow v = \sqrt{rg}$

This is just to get m instead of km

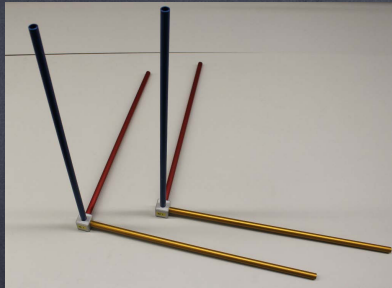
$$v = \sqrt{(6470 \text{ km}) \left(\frac{1000 \text{ m}}{1 \text{ km}} \right) (9.81 \frac{\text{m}}{\text{s}^2})} = 7970 \frac{\text{m}}{\text{s}}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi (6470 \text{ km})}{7.97 \frac{\text{km}}{\text{s}}} \cdot \left(\frac{1 \text{ minute}}{60 \text{ seconds}} \right) = 85 \text{ minutes}$$

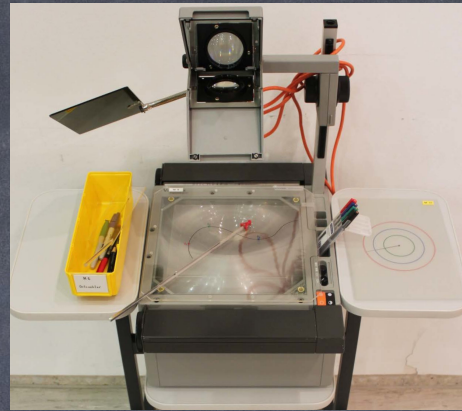
\Rightarrow Space station takes 92 minutes

we are very close!

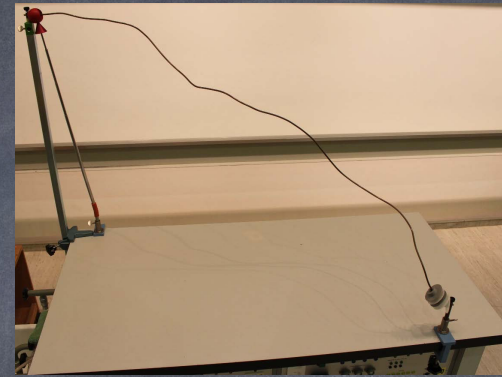
Experiments



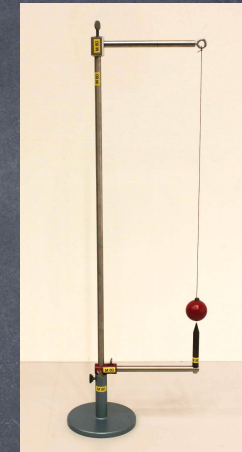
M1



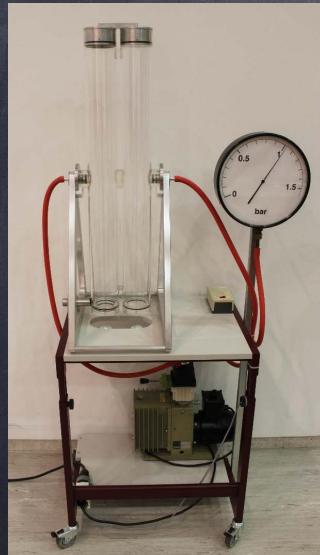
M6



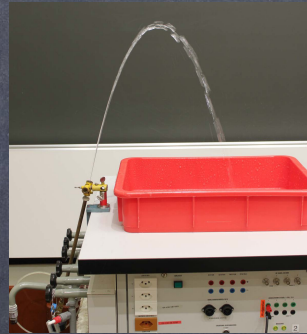
M10



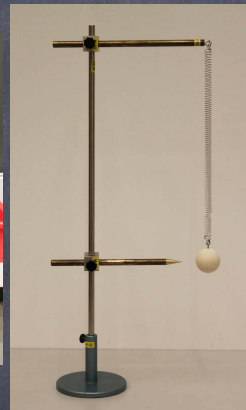
M80



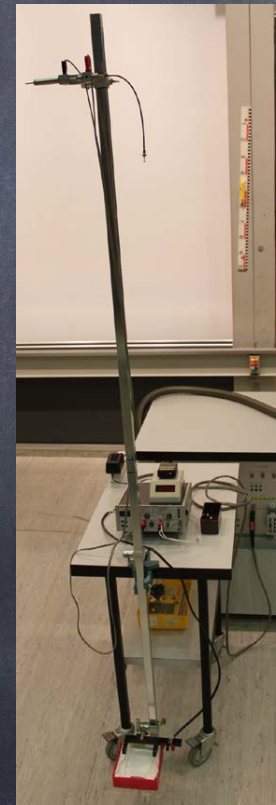
M46



M48



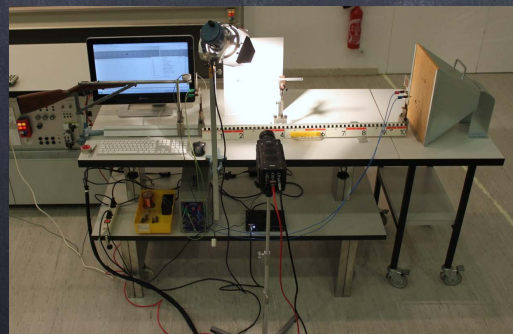
M82



M40



M198



M12



M27