PHY127 FS2023

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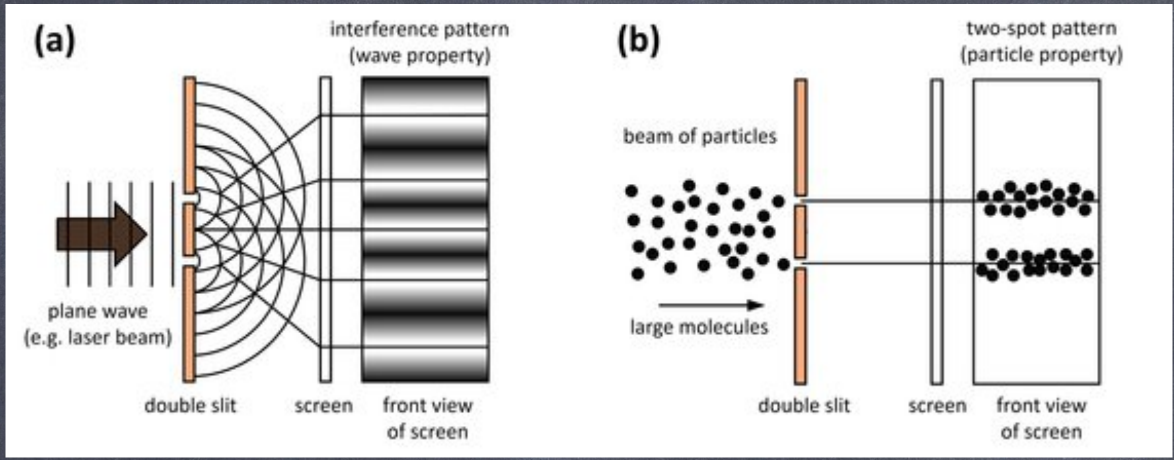
Lecture 4

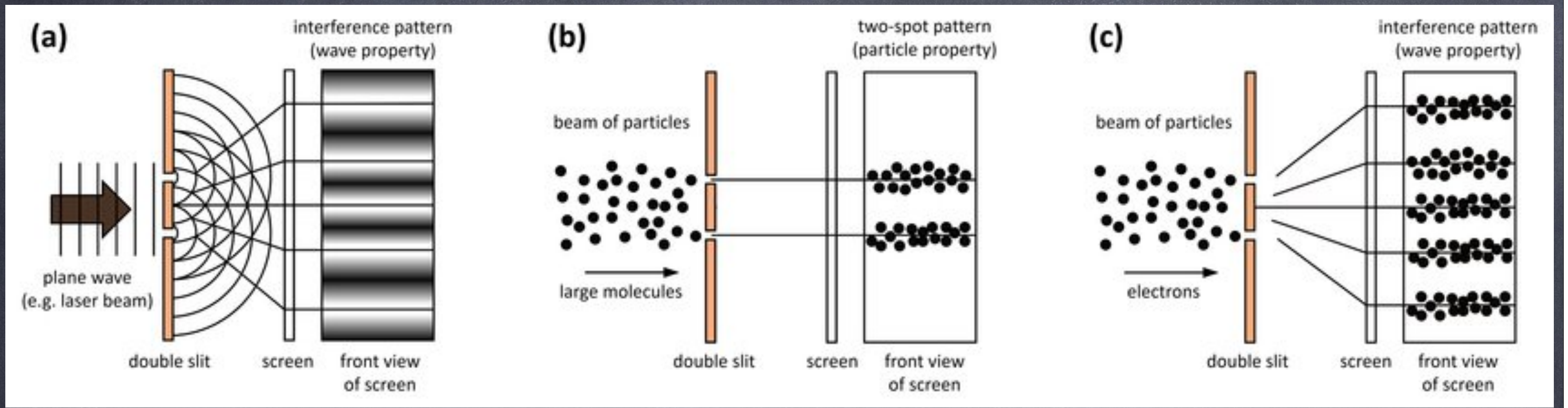
March 15, 2024

Today, we explore the wave functions of particles and how this relates to their quantum probabilities.



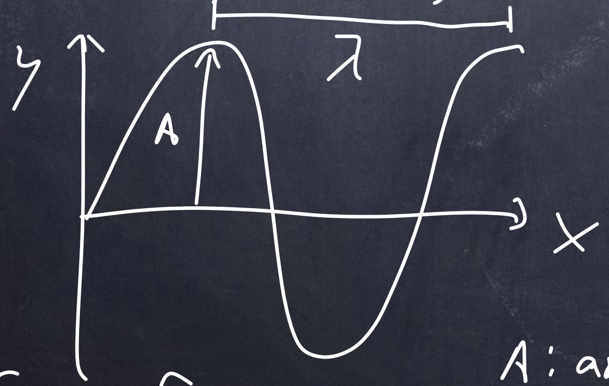
What do individual photon particles do ?

waves particles photons



particles are waves and energy is quantized.
Quantization can be thought of as standing waves. Today we make standing waves. We will see, touch, hear standing waves $\rightarrow$ let's us understand quantized energy levels of an atom.

To describe a sine wave moving with velocity v, where $v = \lambda\nu$



A: amplitude

if we freeze a wave at time $t=0$

if we look at a point $x = 0$

The wave travels a distance λ in a time $T = \frac{1}{\nu}$

the velocity is then $v = \frac{\lambda}{T} = \lambda\nu$

The formula for the wave is called the wave function:

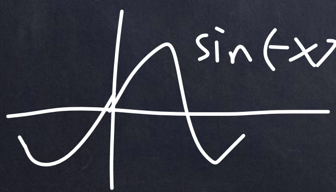
$$y(x,t) = A\sin(kx - \omega t)$$

k: wave number $\quad K = \frac{2\pi}{\lambda}$

ω : angular frequency $\quad \omega = 2\pi\nu = \frac{2\pi}{T}$

If $t=0$, $y(x, t=0) = A\sin kx = A\sin\frac{2\pi x}{\lambda}$

If $x=0$, $y(t=0, t) = A\sin(\omega t) = -A\sin\omega t = -A\sin\frac{2\pi t}{T}$



$\sin(-x) = -\sin(x)$

waves on a string:

depends on tension (F), & the mass density
(Force)
$\mu = \frac{\text{mass}}{\text{length}}$

Using Newton's Laws, $\sum F = ma$, we can derive the wave equation to explain how waves move on a string (see script for physics 1, chapter 3)

wave equation
$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2} \qquad v = \sqrt{\frac{F}{\mu}}$$

The symbol $\frac{\partial}{\partial x}$ means the partial derivative with respect to x.

y: The height of the wave

x: the distance along the string

v: velocity of the wave

t: the time

$y = y(x,t)$! this means the amplitude (y) depends on x & t.

A solution to the wave equation is

$$y = A\sin(kx - \omega t)$$

Standing waves:

A formula for a standing wave is

$$y(x,t) = A\cos\omega t \,\sin kx$$] one solution.

At a time $t=0 \Rightarrow y(x, t=0) = A\sin kx$

string

string is fixed at $x=0$ and $x=L$

The general wave function that is a solution to the wave equation for a standing wave at time $t=0$ is:

$$y(x) = A \sin\left(\frac{2\pi x}{\lambda}\right)$$

To find the solutions, we substitute different λ as a function of L

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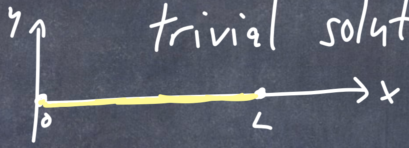
trivial solution (not moving)

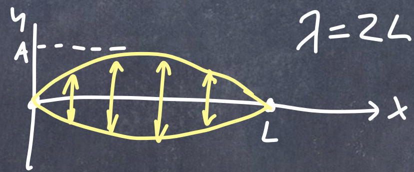
0 bumps



$$y(x) = 0$$

1 bump

$\lambda = 2L$



$$y(x) = A\sin\left(\frac{\pi x}{L}\right)$$

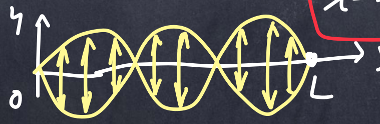
2 bumps

$\lambda = L$

$$y(x) = A\sin\left(\frac{2\pi x}{L}\right)$$

3 bumps

$\boxed{\lambda = \frac{2}{3}L}$

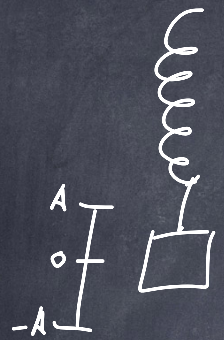


$$y(x) = A\sin\left(\frac{3\pi x}{L}\right)$$

In general, the solutions are $y(x) = A\sin\left(\frac{n\pi x}{L}\right)$ for $n = 0, 1, 2, 3, \cdots$

n = number of bumps

Energy transmitted by a wave



A simple harmonic oscillator has an energy of $E = \frac{1}{2} k A^2$

energy

A: amplitude
K: spring constant

The energy depends on the amplitude squared.

The spring constant is related to the angular frequency by $k = m\omega^2$

m: mass of object
ω: angular frequency

A string oscillating up and down is also a simple harmonic oscillator. Here the energy depends on $m\omega^2$

$$E = \frac{1}{2} m \omega^2 A^2$$

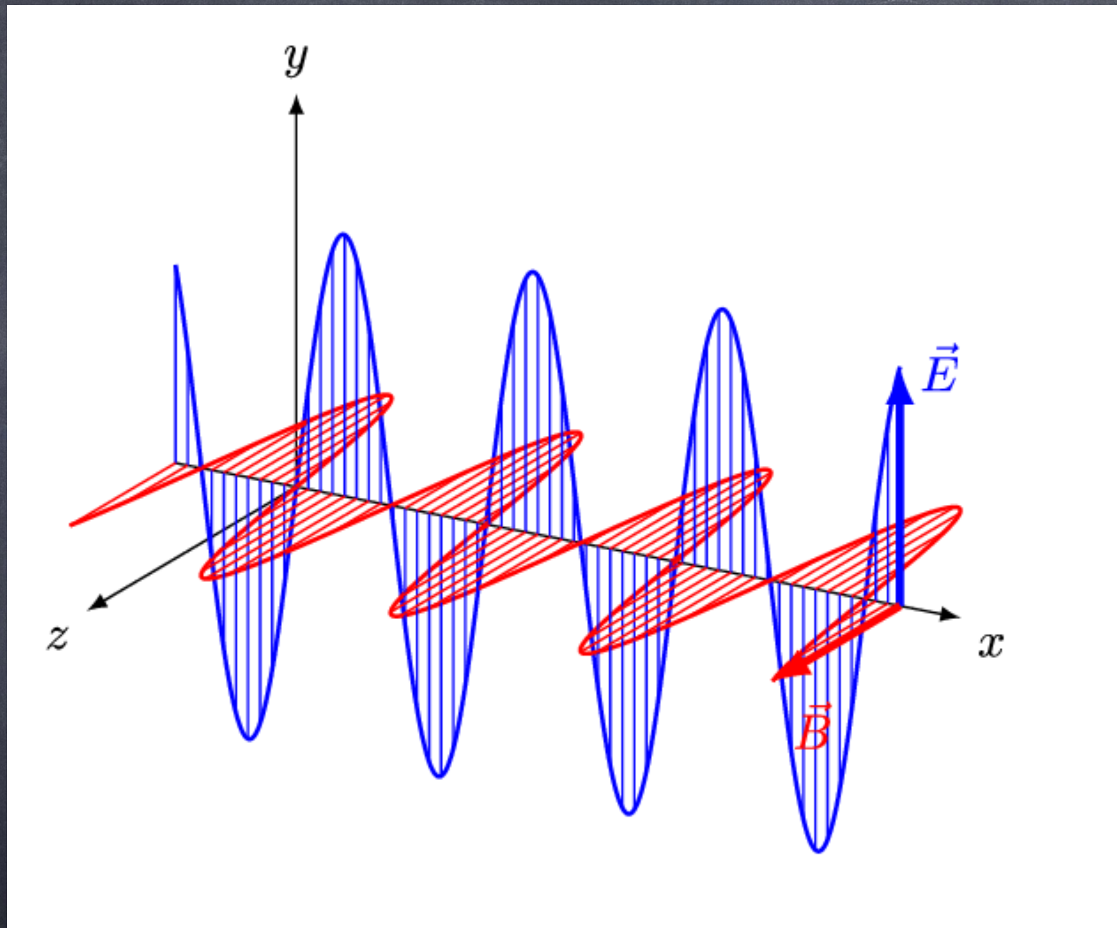
A small part of string carries an energy: $\Delta E = \frac{1}{2} \Delta m \omega^2 A^2 = \frac{1}{2} \mu \omega^2 A^2 \Delta x$

Power transmitted is energy per time:

$$P = \frac{\Delta E}{\Delta t} = \frac{1}{2}\mu \omega^2 A^2 \underbrace{\frac{\Delta x}{\Delta t}}_{v} = \frac{1}{2}\mu \omega^2 A^2 v$$

Both the energy and power are proportional to the amplitude <u>squared</u>.

An electromagnetic wave :



$\bar{E}$: $\hat{y}$

$\bar{B}$: $\hat{z}$

v : $\hat{x}$

For an EM wave: $y(x) \longrightarrow \bar{E}(t)$

height　　amplitude　　electric field

wave functions: $E_y = \underbrace{E_{y_0}}_{\text{maximum amplitude}} \sin(kx-\omega t)$

$$B_z = B_{z_0} \sin(kx-\omega t)$$

simple proportionality: $E = cB$

The energy density of the EM wave is

$$\eta_E = \frac{1}{2}\varepsilon_0 E^2$$

η: energy per unit volume

$$\eta_B = \frac{1}{2}\frac{B^2}{\mu_0} = \frac{1}{2}\varepsilon_0 E^2 \longleftarrow \left\{ c^2 = \frac{1}{\varepsilon_0 \mu_0} \ \& \ E = cB \right.$$

The total energy density is

$$\eta = \eta_E + \eta_B = \varepsilon_0 E^2$$

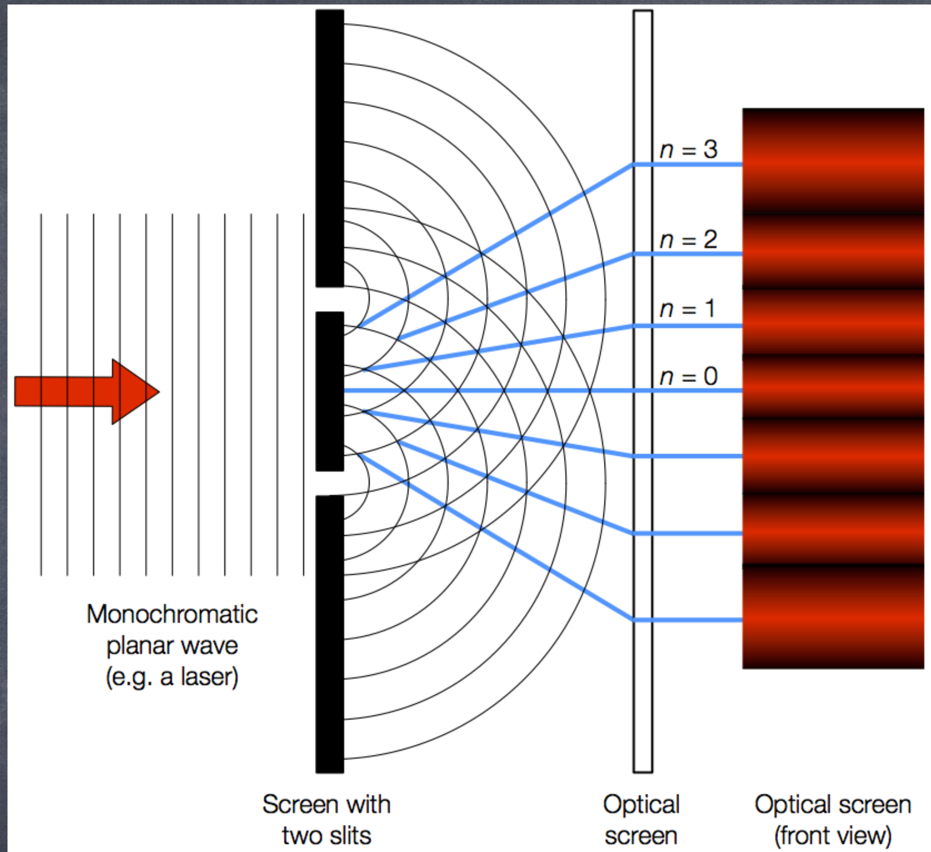
The ~~inten~~ instantaneous intensity I is

$\frac{\text{power}}{\text{area}}$ = energy density $*$ velocity

$$I = \eta c = c\varepsilon_0 E^2 = c\varepsilon_0 E_0^2 \sin^2(kx - \omega t)$$

The intensity of light is proportional to the square of the electric field.

Remember:



measure of intensity

zero intensity

maximum intensity

Intensity has a special meaning $I \propto E^2$

But this is also a measure of the probability of where we find the photons.

$P \propto E^2$

we have $\quad I \propto E(x)^2$ $\quad$ E is the electric field wave function

$$\rho(x) = \psi^2(x)$$

probability

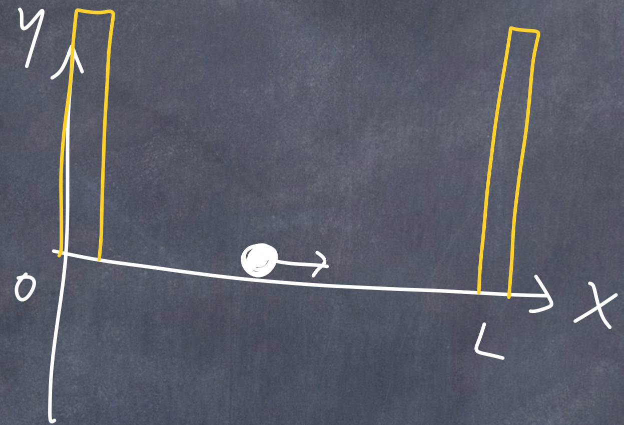
$\rho(x)$ = probability distribution function

$\psi(x)$ = wave function

(a measure of the amplitude of the wave)

Consider the classical world (that we know):

Consider a 1-D box with an electron inside.

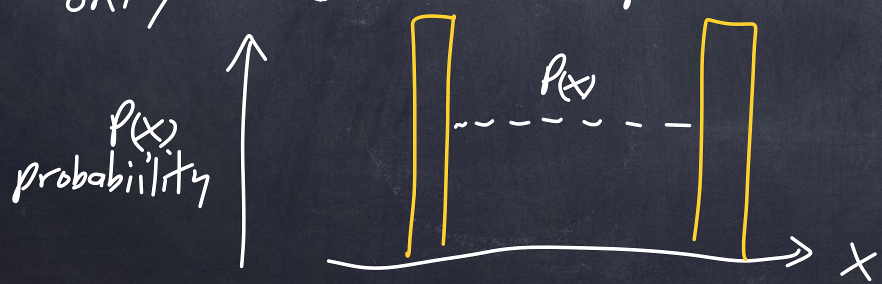


box has length, L

classically, the electron moves back + forth crashing into walls.

If we know the starting position + velocity, we can predict its location at any time.

If we don't know its starting position, then we only know a probability of where it is.



equally likely to be anywhere in the box.

But it must be in the box.

$$\text{So} \quad \int_0^L P(x)\,dx = 1 \qquad \int_0^L P\,dx = 1$$

because is constant with x.

so we can solve this: $\left. Px \right]_0^L = 1$

$$P(L) - P(0) = PL = 1$$

$$P = \frac{1}{L}$$

So actually we want a probability for it to be in a finite space.

$$P_{x_1 \to x_2} = \int_{x_1}^{x_2} P(x)\,dx = \int_{x_1}^{x_2} \frac{1}{L}\,dx$$

$$= \left. \frac{x}{L} \right]_{x_1}^{x_2} = \frac{x_2 - x_1}{L}$$

For $t_1 = 0$, $t_2 = \frac{1}{2}L \Rightarrow \dfrac{\frac{1}{2}L - 0}{L} = \dfrac{1}{2}$

A wave is a particle,
and a particle is a wave.
This is true for any particle.

The wavelength of a particle is $\boxed{\lambda = \dfrac{h}{p}}$

p: momentum

This is the de Broglie wavelength.

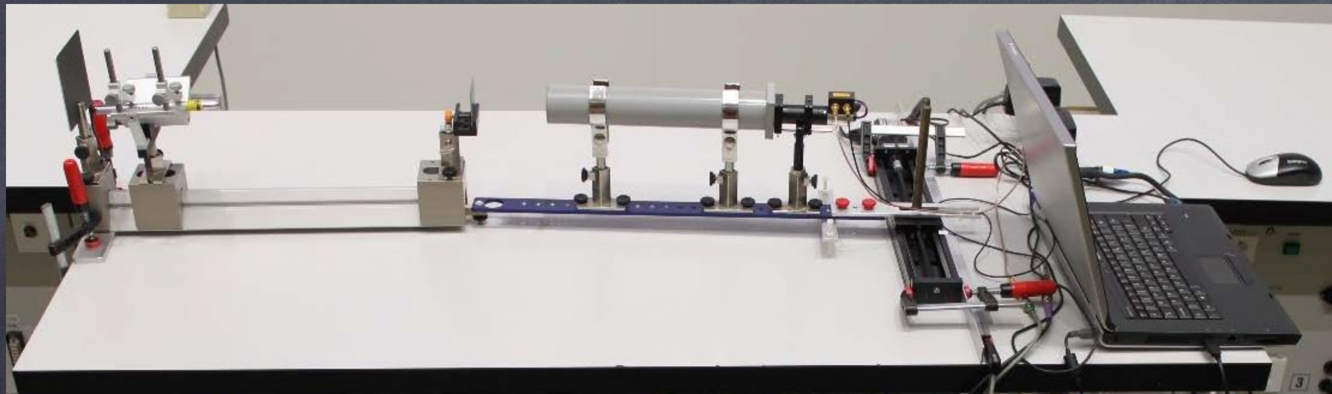
An electron also behaves like a wave.

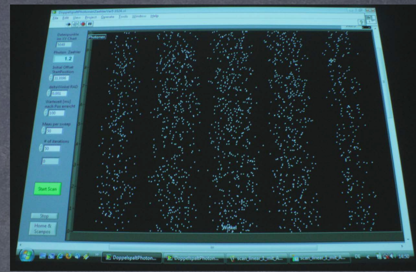
The energy of a particle : $E = \underbrace{K}_{\text{kinetic energy}} + \underbrace{U}_{\text{potential energy}}$

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$p = mv$$

$$E = \frac{p^2}{2m} + U = \frac{h^2}{\lambda^2 2m} + U$$





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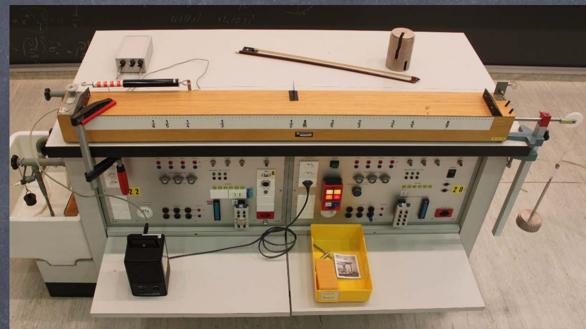

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