Experimental study of self organized criticality on a three dimensional pile of rice

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Abstract. The dynamics of a driven, three dimensional rice-pile is studied. When the pile is fully grown, its activity takes place in power-law distributed avalanches. The observation of finite-size scaling in the observed cut-off size indicates that the pile is in a true critical state, as is demanded by self-organized criticality. Before the stationary state is reached, the maximum slope of the pile is increasing towards a critical value, where the critical state is reached asymptotically. The exponent governing this approach to the critical state is related to the exponents determined in the critical state for the avalanche dimension and distribution. This is in good accord with an analytical theory of self-organized criticality. based on extremal dynamics.

INTRODUCTION

The ubiquitous appearance of power-laws in nature has lead Bak *et al.* in 1987 to propose that slowly driven non-equilibrium systems self-organize into a critical (SOC) state, which naturally leads to power-law behavior [1]. While in the past 15 years, much progress has been made in the theoretical foundations of a process that naturally leads a system to its critical state [2, 3], controlled experiments are a rarity in the field. There are less than a handful of experiments actually demonstrating the existence of a true critical state in a slowly driven system, showing finite size scaling of the cut-off size in addition to a power-law distribution. Two such experiments were carried out in two dimensional piles of granular material (confined between two glass plates), with in one case rice [4] and in the other case steel balls [5] as the granular material. An extension to three dimensional piles has only been published this year, where again two independent experiments were carried out, one on a pile of rice [6] and one on a pile of beads [7]. Here we describe the experiment on the three dimensional pile of rice, however going beyond the mere existence of a critical state.

From the theoretical investigations it has been shown that in order to get a proper understanding of SOC, the way to the critical state is of utmost importance, as this allows the self-organization process to be studied [8]. Therefore, we here also study the behavior of the rice pile in the transient regime in the approach to the critical state and concentrate in particular on the maximum slope of the pile, as the critical state is reached. Using the theory that extremal dynamics is what underlies SOC, the approach to the critical state can be described analytically by what is called the Gap-Equation [2]. This results in the fact that the exponent governing the maximum slope to its critical value, δ, is related to the exponents characterizing the critical state, i.e. the avalanche distribution exponent, τ, the avalanche dimension, D and the fractal dimension of the active sites, d_B. Combining the results of the experiments on the critical state with those from the transient behavior, can therefore lead to a confirmation that extremal dynamics indeed lies at the heart of SOC.

EXPERIMENTAL DETAILS

The experiments were carried out on a pile of rice with the surface area of $\sim$1 x 1 m^2 [6]. Long grained rice of dimensions $\sim$2 x 2 x 7 mm^3 is fed continuously at one edge of the pile in a uniformly distributed line and an image of the pile surface is taken every 30s with a high resolution charge-coupled device (CCD) camera (1596x2048 pixels). The driving rate corresponds to the dropping of $\sim$1500 grains between two images. This does however qualify as slow driving, as at each point along the line of growth, this only corresponds to about 2 grains being added each time step. Furthermore, one has to compare the number of added grains with that already in the pile, which is of the order of $10^7 - 10^8$ grains. In order to reconstruct the surface coordinates of the pile, a set of colored lines (red-green-blue) is projected onto the pile perpendicularly. The images are taken at an angle of 45° to the projection direction leading to a distortion of the lines corresponding to the surface properties, with and accuracy and precision of $\sim$ 1-2 *mm* [9]. After identification of the different lines from the image, a simple calculation leads to the surface coordinates as shown in Fig. 1.

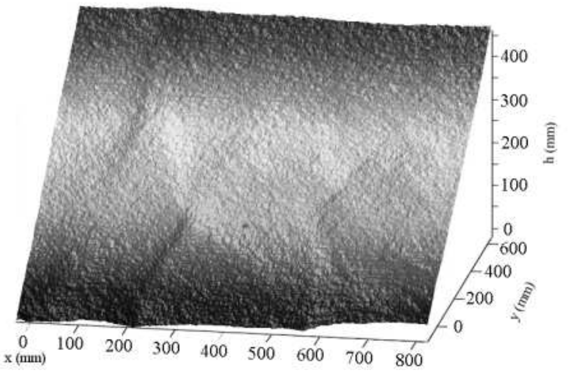

FIGURE 1. A typical image of the reconstruction of the rice-pile surface. Due to the number of lines projected, and the high resolution of the CCD camera, the surface can be reconstructed with an accuracy of $\sim 1-2mm$, which is comparable to the size of a grain.

In the first part we will discuss experiments in a statistically stationary state. In this case, the pile was fully grown when the experiment was started and the average removal of material at the bottom of the pile corresponded to the average material added. Here, each experiment consisted of about 400 images and four separate experiments were analyzed. The volume of rice displaced by an avalanche, ΔV, is then determined from the height-difference between two consecutive time steps

$$\Delta V = 1/2 \int |h(x,y,t) - h(x,y,t+\Delta t)| dxdy, \qquad (1)$$

where $h(x,y,t)$ is the height of the pile at position x,y and at time t. The height difference between two time steps also presents a measure for the distribution of active sites, which shows self-similar behavior. While these results are used below, they are not shown here in detail, see Ref. [6].

For the experiments on the build-up towards the critical state, we first created a flat pile surface, at an angle far below the critical angle ($\phi_0 \simeq 0.55$ compared to $\phi_c \simeq 0.8$). Each experiment consisted of about 500 images and 9 separate experiments were analyzed [10]. Here, we want to determine a measure for the distance from the critical state (the Gap of Ref. [2]). We therefore determine the maximum local slope of the pile, $f(t)$, as a function of time (see inset of Fig. 5). As the pile gets closer to the critical state, the maximum local angle will approach a critical value, f_c. The Gap is then given by the difference $G(t) = f_c - f(t)$ of the maximum local slope to its critical value.

RESULTS IN THE STATIONARY STATE

Determining the size of an avalanche as described above, the evolution of the rice pile in the stationary state is intermittent, as can be seen in Fig. 2. There the avalanche

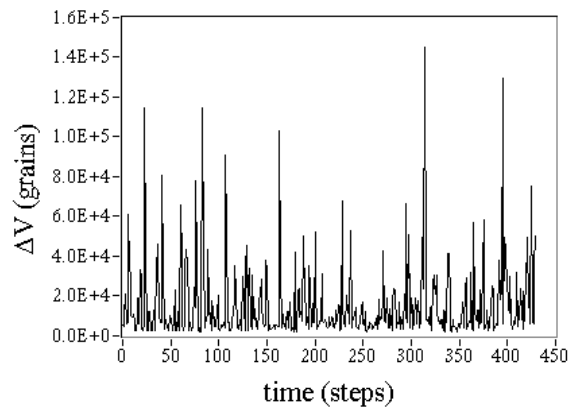

FIGURE 2. The temporal evolution of the avalanche sizes ΔV for one experiment. The activity is clearly intermittent, which already indicates SOC behavior. A proper test however consists of studying the size distribution as well as its finite size scaling. This is done in Figs. 3 and 4.

size as a function of time is given for one experiment. A histogram of this time evolution leads to the avalanche size distribution, which is a central issue in SOC physics. By studying subsets of the whole surface separately, it

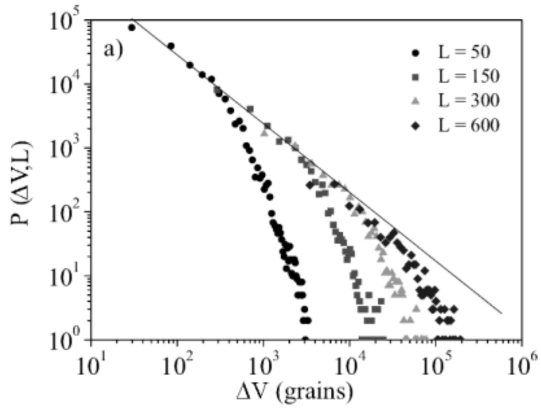

FIGURE 3. The avalanche size distributions for different size subsets of the pile on a double logarithmic plot. The size distributions are obtained from a histogram of the temporal evolution as shown in Fig. 2. As system sizes have a scaling region, where power-law behavior is observed with an exponent of $\tau \simeq 1.2$. The cut-off size, where the power-law breaks down, increases with the linear size of the system. This indicates the presence finite size scaling, which is shown explicitly in Fig. 4.

is also possible to check the data for finite size scaling, which in a true critical state should be observed. The histograms corresponding to different system sizes (with a linear extent of 50, 150, 300, and 600mm respectively) are shown in Fig. 3. As can be seen the avalanche size distribution is a power-law in all data sets, where the exponent can be estimated from a direct fit to be $\tau = 1.20(5)$. The different data sets do however show a cut-off increasing with the system size L. This is a strong indication of finite size scaling, which can be checked by performing a curve collapse. On scaling the avalanche size with L^{-D} and the avalanche probability with ΔV^{τ}, we obtain a curve collapse for these data, as can be seen Fig. 4. Here, $D = 1.99(2)$ is the avalanche dimension and $\tau = 1.21(2)$ is the avalanche size distribution exponent. These values and their errors correspond to the best curve collapse, as shown in Fig. 4. This indicates the presence of finite size scaling and hence the fact that the rice pile is indeed in a critical state. On the other hand, the fractal

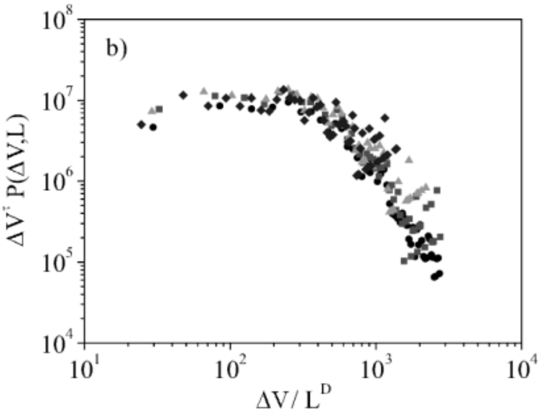

FIGURE 4. The same data as in Fig. 3, where the avalanche sizes are scaled with the system size L^{-D} and the probabilities are scaled with the avalanche size ΔV^{τ}. The data sets for the different subsets collapse onto the same curve, which is a strong indication of finite size scaling and hence the presence of a critical state in the dynamics of the rice-pile.

dimension of the active sites was determined using a box counting method to be $d_B = 1.58(3)$ [6] from the spatial distribution of the height differences.

RESULTS IN THE TRANSIENT STATE

When the rice pile approaches the critical state, the maximum local slope increases towards its critical value. In the context of extremal dynamics, this process can be described by the Gap-equation, which predicts that $G(t) \propto t^{-\delta}$. Furthermore, the value of the exponent characterizing the approach to the critical state, δ, is directly related to the avalanche size distribution exponent, τ, the avalanche dimension D and the fractal dimension of the active sites d_B in the stationary state via

$$\delta = 1 - \frac{1 - d_B/D}{2 - \tau}. \tag{2}$$

The time dependence of the Gap is shown in Fig. 5, where clearly the approach to the critical state follows a power law over two decades. Experimentally, δ is determined to be 0.8(1), where the biggest contribution of the error comes from the determination of f_c, which was obtained independently from a direct experiment on the maximum possible slope of a small part of the pile to be $f_c = 0.92(2)$. The value for δ obtained in the direct experiment on the transient slopes is in good agreement with the expectation from Eq. 2 of $\delta = 0.75(3)$ using the values of τ, D and d_B determined above.

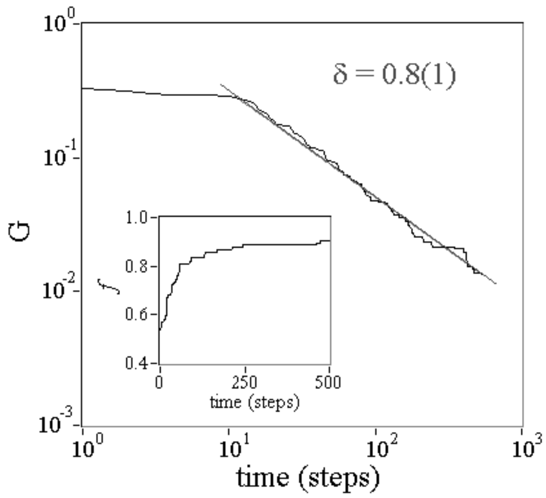

FIGURE 5. The maximum local slope as the critical state is approached is shown directly in the inset. The main figure shows the difference of the maximum slope with its critical value. This Gap asymptotically reaches zero as a power-law, where scaling is observed over two decades. The exponent is found to be $\delta = 0.8(1)$, which is in good agreement with the expectation for an extremal system using the avalanche distribution exponent in the critical state (see above).

CONCLUSIONS

We have shown that a three dimensional pile of rice in the stationary state does show the characteristic properties of a critical state, namely power-law distributed avalanches and more notably finite-size scaling. This implies that indeed the system has been driven to a critical state. Experimentally however, there was no tuning of parameters necessary to reach this critical state, which implies that the rice-pile is in a SOC state. However, in order to understand the nature of the critical state itself, we have further studied the approach of the rice-pile to the critical state. Here, the maximum local slope approaches its critical value according to a power law with an exponent predicted from the theory of systems with extremal dynamics. Thus interpreting the maximum of the local slopes as the Gap leading to critical behavior, it can be seen that the three dimensional rice-pile studied here does self-organize into a critical state according to the law of systems with extremal dynamics. This shows that the critical state does not just fall out of the air also in an experimental system, but can be studied in great detail. Theories of extremal dynamics are intimately connected to those of interface roughening in random media. Indeed there have recently been several mappings [11, 12, 13] of SOC models on interface models similar to the Kardar-Parisi-Zhang equation [14]. This connection between avalanche dynamics and surface roughening can also be seen in our experiments. Here the roughening exponents, which describe the self-affine structure [15] of the rice-pile surface are connected to the avalanche distribution exponents and the avalanche dimension [6]. Furthermore, Paczuski has pointed out that a formal indication for SOC can be obtained directly from the critical state without studying the transient behaviors using the multi-scaling properties of the pile surface in time [8]. Due to the presence of the transient timescale in the dynamics of the pile, there should accordingly be no generic scaling in the temporal behavior. The experimentally determined dependence of the dependence of the growth exponent on the multiscaling moment q is in fact in good agreement with the theoretical prediction [16] as well. This indicates for instance that indeed, Paczuski's criterion can be used to distinguish generic critical behavior from SOC, in systems where the transient stat is unavailable to the experiment.

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