Structural similarity between the magnetic-flux profile in superconductors and the surface of a 2d rice pile

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Abstract. – We analyze the magnetic-flux profile in the critical state of an $YBa_2Cu_3O_{7-x}$ thin film. It is found that the profile forms a self-affine structure in both space and time. Furthermore, the surface of a pile of rice is studied, which also shows a self-affine structure. These experimental results are compared to simulations of a two-dimensional version of the Oslo model for sandpiles. The exponents characterizing the roughness and dynamics of the profiles for both experimental systems agree with each other, the values being $\alpha = 0.39(3)$ for the roughness exponents and $\beta = 0.50(3)$ for the growth exponents. These values are reproduced in the simulations as well. This shows that a simple sandpile model can be used to describe the shape and dynamics of the vortex landscape in the critical state. Conversely, superconductors may be used as a model granular system.

The connection between sandpiles and the critical state in superconductors goes back to de Gennes [1], more than thirty years ago. He invoked the sandpile as a mechanical model for the occurrence of the critical state. In this analogon, the vortices of quantized magnetic flux in the superconductor take the role of sand grains. The friction forces between the grains are mapped onto pinning forces due to impurities and gravity takes the form of a "magnetic pressure" arising from the repulsive inter-vortex interactions [2]. Given these identifications, the building up of the critical state in a superconductor is similar to that of a sandpile, with its self-organizing slope.

In recent years, the image of a sandpile has become a paradigm for the complex behaviour of simple dynamical systems. This is mainly due to the concept of self-organized criticality (SOC) and its sandpile cellular automaton [3]. However, experiments on real sand and its dynamics have been inconclusive for a long time as to whether SOC does in fact apply to them [4]. Recent experiments on the behaviour of one-dimensional (1d) piles of rice have however shown the features of a critical system, such as finite-size scaling [5]. Further investigations into rice piles have also shown that the surface of such a rice pile is a self-affine structure [6, 7] giving further insights into the dynamics of experimentally realized SOC systems. Extending these studies to the structure of a 2d rice pile opens up a way for a proper comparison of the critical state in superconductors with a granular pile. The corresponding flux profile,
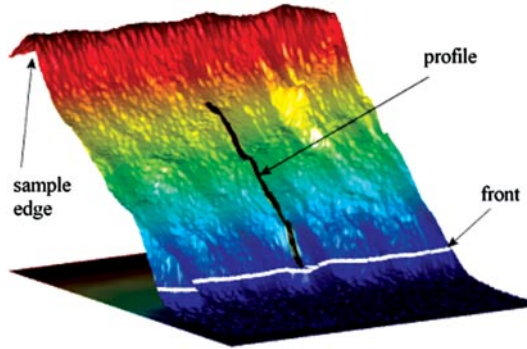
Fig. 1 – The vortex landscape in an YBa$_2$Cu$_3$O$_{7-x}$ film at an applied field of 15 mT. This surface is obtained by plotting the vortex density on the z-axis (corresponding to the height of a granular pile) as a function of position in the sample. Each profile (black line) is defined starting at the front (white line) and going backwards towards the sample edge.

given by the magnetic-flux density $B(x)$ in the penetrating direction, is easily available from magneto-optical experiments. Furthermore, kinetic effects are thought to be suppressed in rice piles, such that simple cellular automata can describe the system [5]. In comparison, kinetic effects are *naturally* unimportant in the study of the critical state in superconductors, as the constituting particles (the vortices) show heavily overdamped dynamics [8]. By determining the self-affinity of the magnetic-flux profile, we also extend work on SOC in superconductors. Up to now, such studies were restricted to the distribution of the size of flux jumps presenting a global measure of the state of the system [9].

In this letter, we show that the experimentally determined structure of these magnetic-field profiles, given by the roughness and growth exponents [10], is in excellent *quantitative* agreement with the exponents characterising the surface of a 2d rice pile. In addition, a cellular automaton developed to describe granular piles [6], is extended to 2d in order to deal with a similar geometry as our experiment. Simulations of such a cellular automaton also are in excellent agreement with the experimental results.

The experiments were carried out on a thin film of YBa$_2$Cu$_3$O$_{7-x}$ (YBCO), with a thickness of 80 nm, grown using pulsed laser ablation on a NgGaO$_3$ substrate [11]. It is the same sample as that used in ref. [12]. The sample was subjected to a slowly increasing magnetic field (in steps of $\mu_0 \Delta H_{\text{ext}} = 50 \, \mu\text{T}$), while held at a constant temperature of 4.2 K. After each field step, the sample was allowed to relax for 10 s after which a series of magneto-optical images was taken. A complete experiment consists of 350 time steps starting at zero applied field. In the following, the results of 10 such experiments are averaged. This corresponds to 128 images for each experiment and a total number of 2.5×10^5 profiles used for the statistical analysis described below. The local magnetic-field profile was determined using a magneto-optical setup [13]. An Yttrium-Iron Garnet (YIG) is placed on top of the thin film, which is located at the object plane of a polarizing microscope. Due to the high Verdet constant of the YIG, the polarization vector of the incoming light is turned locally when a magnetic field is present. This means that in the polarization microscope, regions of high magnetic fields show up as bright spots. In an ideally cross-polarized microscope, only the absolute value of the field can be determined. Moreover, the illumination has to be uniform to a high degree for a reliable determination of this absolute value. Both of these problems can be circumvented by using a recently developed lock-in method [14]. Introducing a modulation on the incom-

ing polarization, the polarization vector can be determined directly for each pixel. Since the rotation of the polarization vector is directly related to the z-component of the magnetic flux (B_z), this method allows a direct determination of B_z, including its sign. An example of such a magneto-optically determined flux-scape can be seen in fig. 1, where the sample is shown in a field of 15 mT. The studied profiles are defined starting at the front of penetrating flux, given by a certain threshold level (~ 2 mT), going backwards to the edge of the sample for a given distance (128 pixels). Over the width of the interesting region we could then study a big number (200) of such profiles from one image. The fact that we take the profiles starting from the front is important in the study of the time behaviour, since this mimics a stationary state, where the height of the pile does not grow in time.

For the study of the rice pile we devised a geometry which is similar to that of the flux penetration in the YBCO thin film. The growth takes place on a line, where the rice is uniformly distributed using a mechanical distributor [15]. The size of the pile is $\sim 1 \times 1$ m^2. The 3d structure of the pile surface is measured using the distortions of a set of projected lines imaged at an angle to the projection direction using a high-resolution camera [15]. This setup allows a resolution of ~ 2 mm over the whole area, which is comparable to the size of the grains ($\sim (2 \times 2 \times 7)$ mm^3). The profiles are defined in the same way as for the flux-scape, where the threshold is taken at a height of 0.1 m and going back 256 pixels. In a typical experimental run, 400 images are taken at intervals of 30 s. The data presented here are from three different experiments, yielding a total of 5×10^5 profiles.

The sandpile model used in the simulations is an extension of the Oslo model to two spatial dimensions. It belongs to the class of sandpile models with randomized updating rules, where the critical slope is dynamically adjusted by the avalanches [16]. In the original version of the model, a line of local slopes $z_i = h_i - h_{i+1}$ of a granular pile is considered. An extension to higher dimensions is easily made. In this case, the z's stand for the gradient of the pile:

$$z_{i,j}^2 = (h_{i,j} - h_{i+1,j})^2 + (h_{i,j} - h_{i,j+1})^2 \tag{1}$$

and the simplest possible extension of the 1d toppling rules [6] to 2d is

$$
\begin{aligned}
z_{i,j} &\longrightarrow z_{i,j} - 4, \\
z_{i\pm1,j} &\longrightarrow z_{i\pm1,j} + 1, \\
z_{i,j\pm1} &\longrightarrow z_{i,j\pm1} + 1, \\
z_{i,j}^{\mathrm{c}} &\in \{3, 4\},
\end{aligned}
\tag{2}
$$

when $z_{i,j} > z_{i,j}^{\mathrm{c}}$, after which the critical value is chosen anew. The updating for this two-dimensional version takes place on a line at $x = 0$, whose values are increased by one after each avalanche (*i.e.* $z_{0,j} \to z_{0,j} + 1$). The height of the pile is obtained from the gradients by integration. In the y-direction, periodic boundary conditions are assumed. The boundaries in the x-direction are open. Together with the updating rules, this geometry leads to a planar surface comparable to the magnetic flux in the critical state of a superconductor, which shows roughening behaviour. The simulations were carried out on a grid of 128×128 pixels. The analyzed profiles are from 128 time steps after saturation of the roughening was reached, which involves waiting for 5000 time steps until profiles are analyzed.

In order to determine the self-affine exponents in space and time of the profiles of the magnetic flux, the rice pile and the simulations, we carried out several procedures to minimize systematic uncertainties. The most classical determination of the self-affine exponents is to characterize the roughness of an interface via its width [10]:

$$w(L, t) = \left(\left\langle \left(h(x, t) - \langle h(t) \rangle_L \right)^2 \right\rangle_L \right)^{1/2}. \tag{3}$$

In time, the width will grow as a power law, with a "growth exponent" β from the initial, flat state up to a saturation time, after which it is independent of time. However, when the interface to be studied is the profile of a pile, there is no initial, flat interface and thus the study of the width as a function of *time* is difficult. After a while, when saturation of $w(L,t)$ in time is reached, the width of the interface varies as a power law with the system size L, where the characteristic exponent is usually called the roughness exponent, α.

A more promising way of analysis, which enables the determination of both the roughness and the growth exponents also for times after saturation, is via the two-point correlation function defined as [10]

$$C(x,t) = \left(\left\langle \left(h(\xi,\tau) - h(x+\xi, t+\tau) \right)^2 \right\rangle_{\xi,\tau} \right)^{1/2}. \tag{4}$$

It has been shown that this quantity obeys the same scaling laws as the width, such that it can be used to determine α and β. In the study of the profile dynamics, the correlation function is much more useful, because power law scaling is also obtained for times after saturation occurs. Thus, we can apply this method to the profiles determined at such later times. Furthermore, the determination of the correlation function takes into account more data points than a local width analysis. This leads to better averaging of the data and thus smoother curves for the determination of the exponents. For large length or long time scales, however, the averaging is still only over very few points, such that power law behaviour breaks down before the correlation length is reached.

Another, still more reliable way of finding the self-affine exponents is to calculate the spatial (temporal) power spectrum or structure function, $S(k)$ ($S(\omega)$), of the interface [17]. It is defined as the square of the Fourier transform $\hat{h}(k)$ ($\hat{h}(\omega)$) of the local height $h(x)$ ($h(t)$):

$$S(k) = \left| \hat{h}(k) \right|^2, \qquad S(\omega) = \left| \hat{h}(\omega) \right|^2, \tag{5}$$

where we determine the Fourier transforms via an FFT algorithm after padding the profile with zeros to the next power of two. The distribution function, $\sigma(k)$ ($\sigma(\omega)$), defined via an integral of the structure function

$$\sigma(k) = \left(\frac{1}{2\pi} \int S(k)\,\mathrm{d}k \right)^{1/2}, \qquad \sigma(\omega) = \left(\frac{1}{2\pi} \int S(\omega)\,\mathrm{d}\omega \right)^{1/2}, \tag{6}$$

is equal to the local width w [18]. The advantage of this method with respect to the correlation function is that also at large length or long time scales the averaging will be done over a reasonable sample, such that the results are reliable over a bigger range.

In the following, we use both the correlation function and the power spectrum to determine the roughness and growth exponents of the magnetic-flux profile in our YBCO thin film and the profile of the 2d rice pile. Furthermore, we determine these quantities for simulations of the 2d version of the Oslo model discussed above.

The roughening of the magnetic-flux profile is shown in fig. 2a, where the distribution function, $\sigma(k)$, as well as the spatial correlation function, $C(x,0)$, are given. The distribution function can be seen to scale over a decade and a half. On small scales, an "intrinsic width" [10], introduced by experimental noise, dominates the behaviour of the correlation function. The resulting growth exponent from both methods, $\alpha^{\text{flux}} = 0.39(3)$, is in excellent agreement with that determined from the rice pile, which can be inferred from fig. 3a, where a value of $\alpha^{\text{rice}} = 0.39(3)$ is found. The results are reproduced in the 2d Oslo model, shown in fig. 4a. Here again, a value of $\alpha^{\text{Oslo}} = 0.38(3)$ is found from both $C(x)$ and $\sigma(k)$. Note the
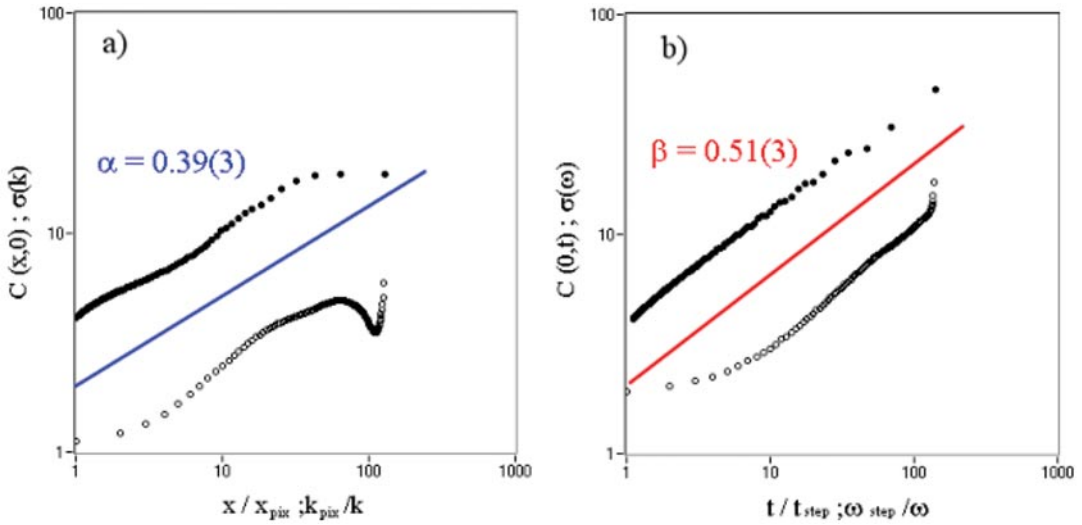
Fig. 2 – a) The spatial behaviour of the magnetic-flux profiles. The slope of the straight line indicates the value of the roughness exponent, $\alpha = 0.39(3)$. b) The temporal behaviour of the magnetic-flux profiles. Here, the growth exponent is indicated by the slope of the straight line, $\beta = 0.51(3)$. Open symbols stand for the two-point correlation function and closed symbols for the distribution function. x_{pix} is the size of a pixel, $k_{\text{pix}} = 2\pi/x_{\text{pix}}$, t_{step} is the duration of a time step and $\omega_{\text{step}} = 2\pi/t_{\text{step}}$. The curves are shifted for clarity.
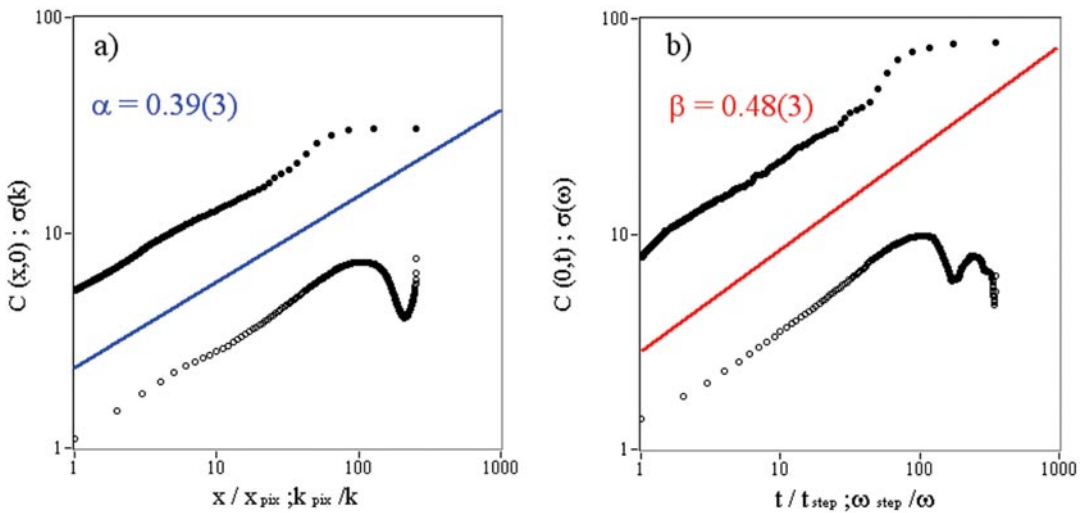

Fig. 3 – a) The spatial behaviour of the rice pile profile. The roughness exponent is given by the slope of the straight line, $\alpha = 0.39(3)$. b) The dynamics of the 2d rice pile. Here, the slope of the straight line indicates the growth exponent, $\beta = 0.48(3)$. Open symbols stand for the two-point correlation function and closed symbols for the distribution function. The curves are shifted for clarity.
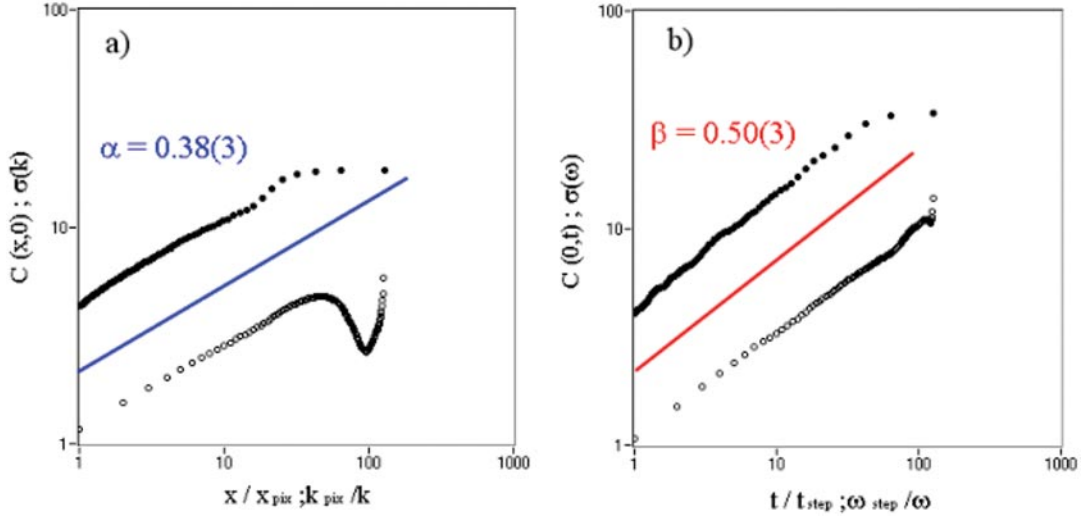
Fig. 4 – a) The spatial behaviour of the 2d Oslo model. The roughness exponent is given by the slope of the straight line, $\alpha = 0.38(3)$. b) The dynamics of the 2d Oslo model. Here, the slope of the straight line indicates the growth exponent, $\beta = 0.50(3)$. Open symbols stand for the two-point correlation function and closed symbols for the distribution function. The curves are shifted for clarity.

larger value of the roughness exponent in the 2d system as compared to that found in [7] for the 1d case, in both simulations and experiments, with $\alpha^{1d} \simeq 0.25$.

The quantification of the *dynamics* of the evolution can be seen in fig. 2b, where the time correlation function, $C(0,t)$, is plotted, as well as the distribution function, $\sigma(\omega)$. We obtain power law behaviour over two decades. This allows a determination of the growth exponent of the flux profile, for which we obtain: $\beta^{\text{flux}} = 0.51(3)$. In fig. 3b, the dynamics of the rice pile surface are characterised in the same way. The value of the growth exponent, $\beta^{\text{rice}} = 0.48(3)$ is in good agreement with that of the flux profile. Again these findings can be compared to simulations of the 2d Oslo model, where the results are shown in fig. 4b. Here we obtain a growth exponent of $\beta^{\text{Oslo}} = 0.50(3)$, in good agreement with the experimental results on both systems.

Thus in the spatial roughness, as well as the dynamics there is quantitative agreement between the 2d flux profiles in YBCO thin films and those of a 2d rice pile. Furthermore, both of these experiments are well described by a 2d version of the Oslo sandpile model.

In conclusion, we presented measurements on the magnetic-field profile in the critical state of superconductors. This profile is a self-affine surface, which can be described by a growth and a roughness exponent, which we have determined in two different ways. These exponents numerically agree with those obtained from an analysis of the surface structure of a 2d rice pile, as well as simulations of the 2d Oslo sandpile model which we developed. This is a quantitative confirmation of the classical analogue of the critical state as a granular pile [1], however extending it further to include the more detailed structure and dynamics of the piles as well. Therefore, magnetic vortices in superconductors may be used as a model system for granular matter in the future. In contrast to other experimental systems for studying granular phenomena, there are clear advantages in the study of superconductors: i) The inter-particle interactions between vortices are well known [8]. ii) The size and shape of the particles (vortices) are well known and also easily controllable by changing the temperature. iii) The density of particles is easily and continuously changed via the applied magnetic field.

iv) Different types of driving are easily implemented, since the driving is being done by either the magnetic field or an electrical current [19]. v) The quenched disorder can be controlled and changed in similar samples [20]. This makes possible to study the influence of static disorder on the structure of granular matter.

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